Heat Transfer And Combustion in Multiphase Systems
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Lecture 11
Governing equations: Averaging techniques-I

In the last lecture what we did is that we say that how you can actually invoke some kind of an averaging technique. So, that you do not have to explicitly track all these interfaces rather take a very macroscopic view of a multiphase systems which may be enough to do some design some analysis builds newer systems. It may not be always the same type of fidelity that may be needed.

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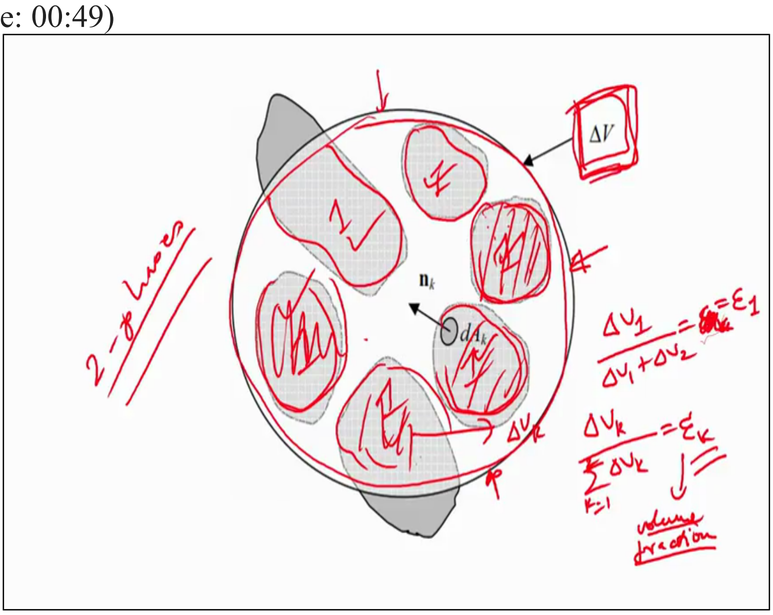

So, you introduce the Eulerion volume averaging technique which if you recall or something like this okay where there is a volume element and then there are within that volume element there are some volume elements like of each phase okay.

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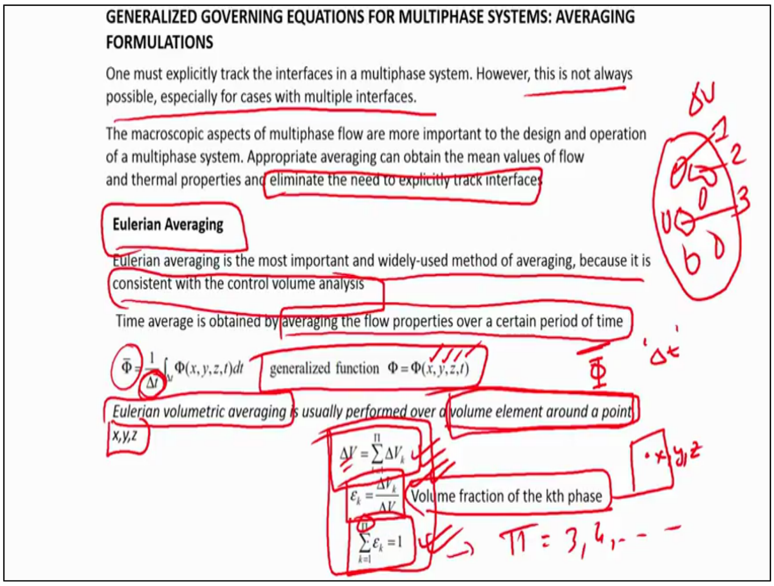

GENERALIZED GOVERNING EQUATIONS FOR MULTIPHASE SYSTEMS: AVERAGING FORMULATIONS

One must explicitly track the interfaces in a multiphase system. However, this is not always possible, especially for cases with multiple interfaces.

The macroscopic aspects of multiphase flow are more important to the design and operation of a multiphase system. Appropriate averaging can obtain the mean values of flow and thermal properties and eliminate the need to explicitly track interfaces

Eulerian Averaging

Eulerian averaging is the most important and widely-used method of averaging, because it is consistent with the control volume analysis

Time average is obtained by averaging the flow properties over a certain period of time

$\bar{\Phi} = \frac{1}{\Delta t}\int_{\Delta t} \Phi(x,y,z,t)dt$ generalized function $\Phi = \Phi(x,y,z,t)$

Eulerian volumetric averaging is usually performed over a volume element around a point x,y,z

$\Delta V = \sum_{k=1}^{\Pi} \Delta V_k$

$\varepsilon_k = \frac{\Delta V_k}{\Delta V}$ Volume fraction of the kth phase

$\sum_{k=1}^{\Pi} \varepsilon_k = 1$

And we also said defined a few rules we said the net volume is a sum total of the individual volumes we define something called a volume fraction and then we also did something called this volume fraction summation of this volume fraction has to be equal to 1 which is logical.

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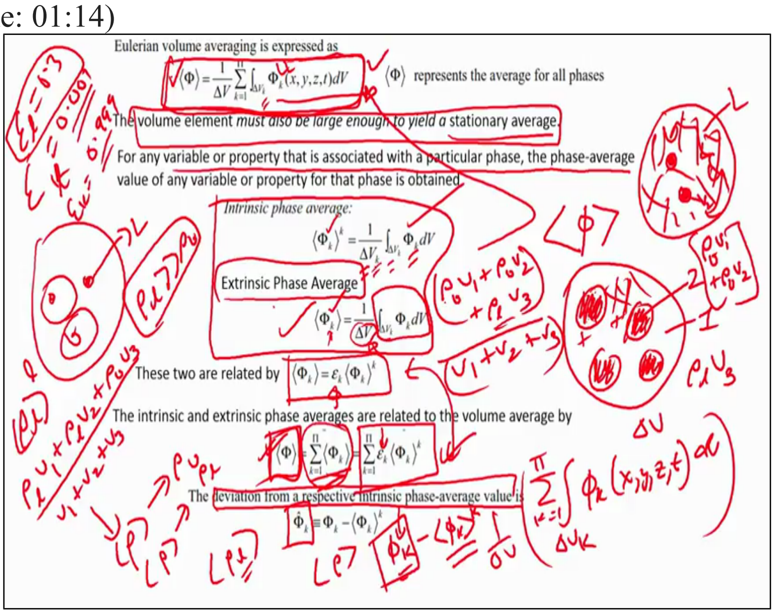

Eulerian volume averaging is expressed as

$\langle \Phi \rangle = \frac{1}{\Delta V}\sum_{k=1}^{\Pi}\int_{\Delta V_k} \Phi_k(x,y,z,t)dV$ $\langle \Phi \rangle$ represents the average for all phases

The volume element *must also be large enough to yield a* stationary average.

For any variable or property that is associated with a particular phase, the phase-average value of any variable or property for that phase is obtained

Intrinsic phase average:

$\langle \Phi_k \rangle^k = \frac{1}{\Delta V_k}\int_{\Delta V_k} \Phi_k dV$

Extrinsic Phase Average

$\langle \Phi_k \rangle = \frac{1}{\Delta V}\int_{\Delta V_k} \Phi_k dV$

These two are related by $\langle \Phi_k \rangle = \varepsilon_k \langle \Phi_k \rangle^k$

The intrinsic and extrinsic phase averages are related to the volume average by

$\langle \Phi \rangle = \sum_{k=1}^{\Pi}\langle \Phi_k \rangle = \sum_{k=1}^{\Pi}\varepsilon_k \langle \Phi_k \rangle^k$

The deviation from a respective intrinsic phase-average value is

$\hat{\Phi}_k \equiv \Phi_k - \langle \Phi_k \rangle^k$

So, the Eularion volume averaging is expressed as this, where this is basically an averaging notation okay, this is an averaging notation. So, this actually means an average okay. So, the volume average means if you look at this expression what we have done is that for each of the phases k1 2Pi, we have taken that quantity whatever that property is Phi k across those each elemental volume that is occupied by these individual phases, right.

And we have integrated it over that particular phase and then we have summed it across all the phases that is what we have done, right. And then we have divided it by the volume, right. So, imagine a situation, so, these are say; let us keep it simple. So, this is like phase one, this is like phase two and this is the elemental volume V.

So, what we have done is that we have taken whatever property that is there we need this control volume will be within this elemental volume there is property variation, we have averaged them, over each of those elemental volume right. So, that the total gives you a total average okay.

So, this Phi k x, y, z comma t dV integral over this delta Vk basically means what you are basically averaging out the quantities in each of those elemental phases, right, is over each of those elemental phases you are basically averaging them out, got it. So, that is the most important part of this. So, no information that how it varies in each of these elemental volume is actually retained, right.

You are basically averaging the whatever variation of Phi that you have you are averaging it out across that phase, right, okay. So, it might vary in some way but you are averaging that out and not only that now you are averaging it over all the phases also, this is for one phase okay. So, now for this phase also you are doing the same so you add this plus this okay and you get this okay.

And then you are dividing this entire thing by that elemental volume. So, this will give you the phase averaging or the volume averaged quantity okay. So, this Phi that you see over here it does not belong to any one phase. It is the sum total, so, if that is density say for example it is the density of phase one and the density of phase two and it gives you some kind of an average density of the system, right.

So, if you do it something like this it is a water, liquid water, water vapour kind of a system. If you average the things out, right, it will give you an average quantity for the density okay. So, say we take this example a little forward let us say for example there are only two bubbles in a predominantly liquid system, right.

So, what we are exactly doing over here, over each of this volume we are averaging the density say for example. And for the liquid we are averaging that density over this whole thing right, okay. So, what will happen you are basically doing this averaging of the vapour over that small volume V1.

Let us assume okay then you are doing the same thing for this volume V2 which is this volume these are all very small elemental volume say for example, right. Okay. So, you add these two quantities together okay that is what you have done okay by averaging and then for the larger volume where it is all liquid water say for example you do liquid water into some volume V3 right.

So, in your essential thing comes out to be Rho 1 V1 + Rho2 V2 rather not Rho1 this is Rho V plus let me do this, let me show you this in a more logical way. So, it is a Rho V V1 there are only two bubbles say Rho V V2 + Rho l V3 right and this is divided by V1 + V2 + V3, all these are elemental volumes, right.

That is exactly what this expression means got it, clear, okay. So, if for example in this particular case obviously the liquid density is a lot higher than the vapour density all right. So, you will get an average which is more closer to the liquid density right because liquid density is multiplied with a very large volume.

And the vapour density is multiplied by a very small volume. So, it is very natural that you are going to get a density right which will be more closer to the liquid density, correct. However if I take the reverse scenario where there are say for example liquid droplet exactly like this in a predominantly vapour volume.

These are liquid droplets is the reverse problem right, instead of bubbles now you have droplets okay. Now the same thing same methodology will be valid here now instead of Rho V what I will do is that Rho l into V1 plus Rho l into V2, right, plus Rho V into V3 divided by V1 + V2 + V3, right.

Now here of course there is a little bit of an issue that is because it Rho l is much, much greater than Rho V, right. So, you cannot say that the density will be very close to the vapour phase okay. But however if the liquid droplets are very, very small say for example very small that is a very sparsely populated system.

Remember that Epsilon k that we defined let us say for liquid water this is about 0.01, right. So, it is .1 % of for the rest it is like 0.999 right, for the vapour. Now if that is the case in that case you can safely say that this average to Rho, that means this Rho average will come very close to the average to the web vapour phase in that particular phase because it is very sparse, right.

On the other hand if they these bubble, these droplets are very giant droplets, right. That means this epsilon l say becomes equal to say .3 okay 30%of the volume of this vapour space. Then this Rho okay will approach closer to Rho l because of the simple reason that Rho l is much, much greater than Rho V okay.

Understand the averaging procedure okay is dependent on volume, obviously that the extent not just volume is basically the volume fraction, it is dependent on. It is also dependent on the actual nature of this quantity for example if it is Rho how does Rho change across the two phases okay. In some other kind of a substance the density difference may not be this much.

Now this is not just for densities a valid for other types of things as well. So, the key feature for choosing this kind of an averaging is that the volume element must be large enough to yield a stationary average, that the first thing okay. And for any variable or property that is associated with a particular phase okay.

We can also do something which is called a phase averaging okay. So, what is phase averaging? First let us look at these two quantities here; one is called an intrinsic phase average. What does intrinsic phase average means? You are integrating the property over the elemental volume that is occupied by that particular phase only, right.

And then you are dividing it by the total volume of that particular phase, got it. So, it is the average that you are getting okay, for a particular property longing to a particular phase over that phase only. It is not overall, it is over that phase only you are getting a variation, right. On other hand the extrinsic phase averaging you are averaging a particular quantity, a particular property belonging to a particular phase, right.

But you are averaging it over the; you are dividing it by the net volume. So, instead of the volume of that particular phase you are averaging it over the total volume. But you are still doing the average; doing the integration over the phase volume only okay. So, that is the two differences and give two are related just by applying the volume fraction, right.

You can apply the volume fraction to this you will find that this will boil down to that, okay. So, extrinsic and intrinsic phase averages are related to each other, okay. So, you know the volume fraction you can convert one from the other okay. They are also related now to the volume average. Volume average is this quantity, volume averages all phases combined right this is for a particular phase only.

Remember this is still valid for a particular phase this is the property that is belonging to a particular phase okay. So, it is like the way if you from the density a problem that we just showed for example is in that case it is Rho l it is averaged over the liquid phase that is intrinsic, when Rho l is averaged over the entire thing okay but still it is Rho l, right.

That is actually called the extrinsic averaging okay whereas this volume average is nothing but this Rho okay. So, that is the information; that is how we are actually defining it okay. So, the Rho average or this average, the volume average is related to the individual averages for example the extrinsic averages, if you just some the extrinsic averages over all the phases we get the corresponding volume average.

If you do the same for the intrinsic phase multiplied by this Epsilon k, you recover the volume average okay. These are also related, so, phase average and volume averages are related intrinsic and extrinsic phase averages are also limited to each other, clear, okay. Now there are some deviations from a respective intrinsic phase average value.

Say you take any quantity Phi k, we remember here Phi k is a field quantity, so that means it varies. Even within that in elemental volume of that particular phase also it will vary, right. So, when you take this Phi k averaged over the individual phase out, there will be some kind of a dispersion right. It is like a fluctuation just like always happens.

So, for example there is the very air in this room right I take the average concentration of air, right, okay. If you subtract that out from that individual, from the local concentration of air, right. You will get a variation, right, so this is exactly like a dispersion, a deviation okay. So, this is like a first moment; you can consider this like a first moment, right.

So, this is the intrinsic phase average it has been taken out from the local fluctuation but that means the actual quantity Phi k that varies within that control volume or that elemental area space, got it, okay. So, this is; so, there are three key concepts that we have found one is what is the volume average? What is an intrinsic phase average? What is an extrinsic phase average? How these intrinsic, extrinsic and the volume averages are related to each other.

And lastly we have found out what is called the deviation from our respective intrinsic phase averaged value, got it, okay. So, these are several important things that we learnt in this particular slide okay.
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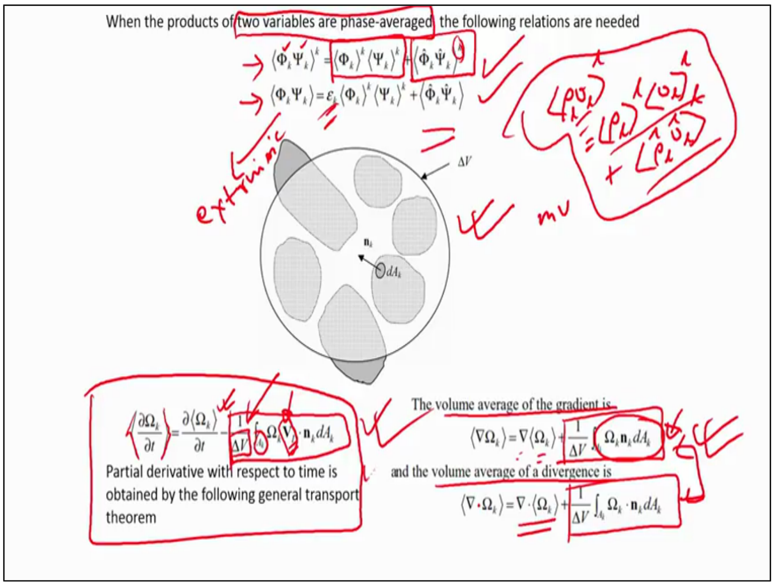

When the products of two variables are phase-averaged the following relations are needed

$$\langle \Phi_k \Psi_k \rangle^k = \langle \Phi_k \rangle^k \langle \Psi_k \rangle^k + \langle \hat{\Phi}_k \hat{\Psi}_k \rangle^k$$

$$\langle \Phi_k \Psi_k \rangle = \varepsilon_k \langle \Phi_k \rangle^k \langle \Psi_k \rangle^k + \langle \hat{\Phi}_k \hat{\Psi}_k \rangle$$

$$\left\langle \frac{\partial \Omega_k}{\partial t} \right\rangle = \frac{\partial \langle \Omega_k \rangle}{\partial t} - \frac{1}{\Delta V} \int_{A_k} \Omega_k \mathbf{V}_I \cdot \mathbf{n}_k \, dA_k$$

Partial derivative with respect to time is obtained by the following general transport theorem

The volume average of the gradient is

$$\langle \nabla \Omega_k \rangle = \nabla \langle \Omega_k \rangle + \frac{1}{\Delta V} \int_{A_k} \Omega_k \mathbf{n}_k \, dA_k$$

and the volume average of a divergence is

$$\langle \nabla \cdot \Omega_k \rangle = \nabla \cdot \langle \Omega_k \rangle + \frac{1}{\Delta V} \int_{A_k} \Omega_k \cdot \mathbf{n}_k \, dA_k$$

There are some two or three important rules that I am going to narrate over here and so listen to them very carefully. This picture I am putting over here so that there is no confusion you can still correlate this was the same picture that we saw earlier. Now here what you can see is that if this is the products of two variables or phase averaged okay.

So, one variable say is mass, other variable maybe say velocity okay. In it can be any other combinations like that when two quantities are basically phase averaged this is intrinsic phase averaging as you can see over there, right. Intrinsic phase averaging it is given by the individual intrinsic phase averages, product of the individual intrinsic says averages plus whatever is the dispersion or the deviation average of that, got it.

It is very similar to those kind of methodologies that if you take Rho and V say for example if you average them Rho l dl, if you average them right that is averaged over they say the liquid okay. This is what it is this is Rho l averaged over l, right; Vl averaged over l correct plus Rho l Vl averaged over l, right.

So, this is nothing but is Rho V okay, it can be any other things obviously and just giving you an example this is written in that particular fashion okay. Similarly this is intrinsic this is extrinsic phase average. It is nothing but you write basically the same thing except there is an Epsilon k now, okay.

Because Epsilon k comes from the volume fraction and here of course it is not average to it is not with respect to k anymore, right. This is the complete phase averaging, which this is the extrinsic part, got it. So, this is the rule number one that we establish, got it. In the product of the two things rule number two is this particular thing stuff that we have here, right.

When you phase average, when you average basically this is also a extrinsic phase average okay of a partial derivative, of a particular quantity belonging to a particular phase. This is given by the quantity itself, right. This is the phase average of the temporal derivative, right. So, that is equivalent to the variation of the temporal variation of the average quantity, got it.

The temporal of the averaged quantity minus this flux term okay. This flux term, this is I basically the inter phase velocity. So, it is the interface velocity and this Ak is once again the area of that individual component of that individual phase okay. And this is the total volume this is the extrinsic phase average, so, the total volume comes into the picture, got it.

So, this is the phase extrinsic phase average of the temporal variation of a quantity is nothing but the temporal variation of the phase average, extrinsic phase averaged quantity minus this additional term which is given by the interface which is to be given by the; so, it is basically like an through the interface something is going out.

It is like a flux term it is essentially looks like a flux term, right. So, that is what it is okay. Similarly okay, so, that is rule number one okay. Similarly the volume average of our gradient, so, you have a gradient of a particular property right it can be anything more than a it is a grad of a particular property.

It is basically the grad of the average property right, plus once again you have this particular kind of a thing coming into the picture okay. So, this is once again another additional term that you incur similarly the value average of a divergence of a quantity this is divergence operator right.

That is given as the divergence of the volume averaged of the phase average, extrinsic phase averaged quantity plus once again the same term, got it. So, three rules this, this and this all of this you can actually work out and why they are so, got it, okay. So, once we established these rules things becomes a little bit more simple, right.

So, these are the rules which will come in very handy so just to do a recap we did define how the averaging should actually work. And then we define that how we can actually average when there is a product of two variables. And then we establish certain ground rules that when you actually have a temporal derivative.

If we have a grad and if you have a divergence how to deal with them, right, because this would all be needed when you actually do the momentum balance and other things, right, okay. So, this basically ends, this particular thing for the rest have to; we have to go start looking at that.

When we now actually do such a thing how we actually can get into the; we can write the generalized equation now. How we can write the generalized equation like the volume average multi fluid models, if we talk about that. How quick we can use this rule okay basically to formulate those equations.

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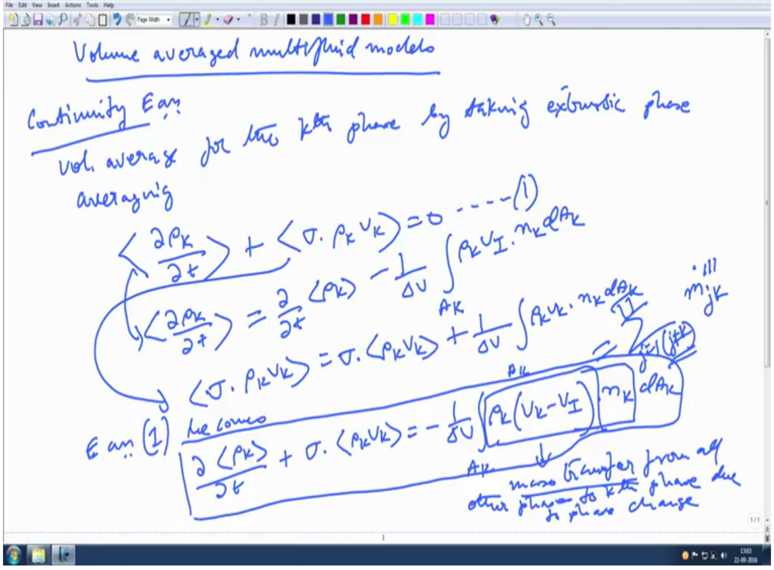

Volume averaged multifluid models

Continuity Eqn:

vol. average for the kth phase by taking extrinsic phase averaging

$$\left\langle \frac{\partial \rho_k}{\partial t} \right\rangle + \langle \nabla \cdot \rho_k v_k \rangle = 0 \quad ----(1)$$

$$\left\langle \frac{\partial \rho_k}{\partial t} \right\rangle = \frac{\partial}{\partial t}\langle \rho_k \rangle - \frac{1}{\Delta V}\int_{A_k} \rho_k v_I \cdot n_k \, dA_k$$

$$\langle \nabla \cdot \rho_k v_k \rangle = \nabla \cdot \langle \rho_k v_k \rangle + \frac{1}{\Delta V}\int_{A_k} \rho_k v_k \cdot n_k \, dA_k$$

Eqn (1) becomes

$$\boxed{\frac{\partial \langle \rho_k \rangle}{\partial t} + \nabla \cdot \langle \rho_k v_k \rangle = -\frac{1}{\delta V}\int_{A_k} \rho_k (v_k - v_I)\, n_k \, dA_k}$$

$$\sum_{j=1\,(j\neq k)} \dot{m}'''_{jk}$$

mass transfer from all other phases to kth phase due to phase change

So, let us go to the windows journal and volume averaged so if we look at now volume averaged multi fluid models, this is the important thing that we are going to do over here, okay with all the experience that we had gathered okay. So, for the continuity equation we will just introduce it and then we will take it up in the next lecture okay.

So, the country; so, there are; so, the volume average multi fluid equation means you are doing the volume averaging for each of the phases and each of them fluid components basically the phase is over here okay. So, the continuity equation is the volume average for the kth phase by taking extrinsic phase averaging okay.

So, basically this equation you already knew right from the differential formulation without any assumption of incompressibility that is what you have, right. Now if you take the first term, you already know what it is going to be, it is going to give you right -1 over the volume Ak, Rho k, VI, VI is the interface nk dAk right.

That is first term, similarly the second term over here so this was the first term led to this. The second term plus this is the rules that we established right using the same Rho k Vk. nk dAk got it okay. So, basically nothing but we have applied the rules. So, using these two okay we can write if this equation is say one okay so equation one now become okay.

So, that is what you get if you sum total two things okay. So, what does this term actually means we have already covered that this is the mass transfer right, okay, from all other phases we already knew about this right all other phases to the kth phase is due to phase change, is that right, okay.

So, this is this we already did it earlier right, this entire thing we did earlier right. So, this is the all the mass transfer that is happening from all other phases coming over to this kth phase because this is this equation we are writing it for the kth phase, okay. All other all of our mass transfer means it can be mass transfer from this phase to those phases also right, it does not mean that it is always a mass coming in okay.

And so therefore this particular quantity is nothing but summation of j = 1 to Pi where j is not equal to k. If it is the kth phase m dot triple prime jk right, transfer from all other phases to the kth phase, that is why j = 1 to Pi but not j cannot be equal to k, got it, okay. So, this is the transport equation when you actually do the phase averaging thing right.

So, this represents actually the phase average. So, in the next class we will extend this for the next lecture we will extend this and see in what ways this can be used okay for developing the other equations. And let us see that those equations are basically, the as I said the volume averaged multi fluid models.

We look at the energy there are lots the standard equation okay. And we will also look at that if we have next we can also look at what is the; what about the homogeneous equations also, okay. So, this is for each individual phase, so, we have introduced it and in the next class in the next lecture we will pick it up and see how we can take it forward. Thank you;