Heat Transfer And Combustion in Multiphase Systems
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science-Bangalore

Lecture 10
Governing equations and interfacial conditions

Okay! Welcome to this lecture, last lecture we stopped after doing if you recall we did the jump condition of the interface and we did the continuity for all the mass conservation and the interface.

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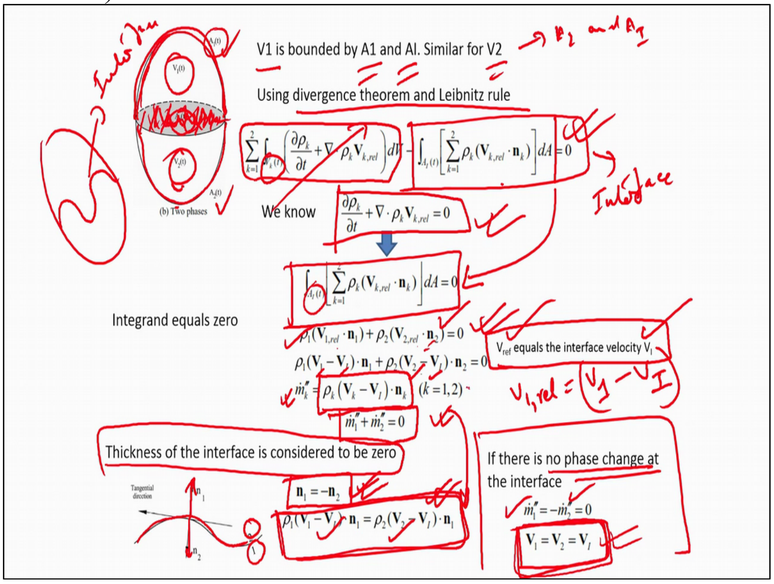

V1 is bounded by A1 and AI. Similar for V2 → V_2 and A_I

Using divergence theorem and Leibnitz rule

$$\sum_{k=1}^{2}\int_{V_k(t)}\left(\frac{\partial \rho_k}{\partial t}+\nabla\cdot\rho_k\mathbf{V}_{k,rel}\right)dV-\int_{A_I(t)}\left[\sum_{k=1}^{2}\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\right]dA=0$$

Interface

We know $\frac{\partial \rho_k}{\partial t}+\nabla\cdot\rho_k\mathbf{V}_{k,rel}=0$

$$\int_{A_I(t)}\left[\sum_{k=1}^{2}\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\right]dA=0$$

Integrand equals zero

$$\rho_1(\mathbf{V}_{1,rel}\cdot\mathbf{n}_1)+\rho_2(\mathbf{V}_{2,rel}\cdot\mathbf{n}_2)=0$$

$$\rho_1(\mathbf{V}_1-\mathbf{V}_I)\cdot\mathbf{n}_1+\rho_2(\mathbf{V}_2-\mathbf{V}_I)\cdot\mathbf{n}_2=0$$

V_{ref} equals the interface velocity V_I

$V_{1,rel} = V_1 - V_I$

$$\dot{m}_k''=\rho_k(\mathbf{V}_k-\mathbf{V}_I)\cdot\mathbf{n}_k \quad (k=1,2)$$

$$\dot{m}_1''+\dot{m}_2''=0$$

Thickness of the interface is considered to be zero

$$\mathbf{n}_1=-\mathbf{n}_2$$

$$\rho_1(\mathbf{V}_1-\mathbf{V}_I)\cdot\mathbf{n}_1=\rho_2(\mathbf{V}_2-\mathbf{V}_I)\cdot\mathbf{n}_1$$

If there is no phase change at the interface

$$\dot{m}_1''=-\dot{m}_2''=0$$

$$\mathbf{V}_1=\mathbf{V}_2=\mathbf{V}_I$$

And we concluded that for no phase change the velocity in phase 1 and the velocity in phase 2 and the velocity of the interface all have to be the same. If there is of course phase change then you have an alternative relationship which is basically this okay.

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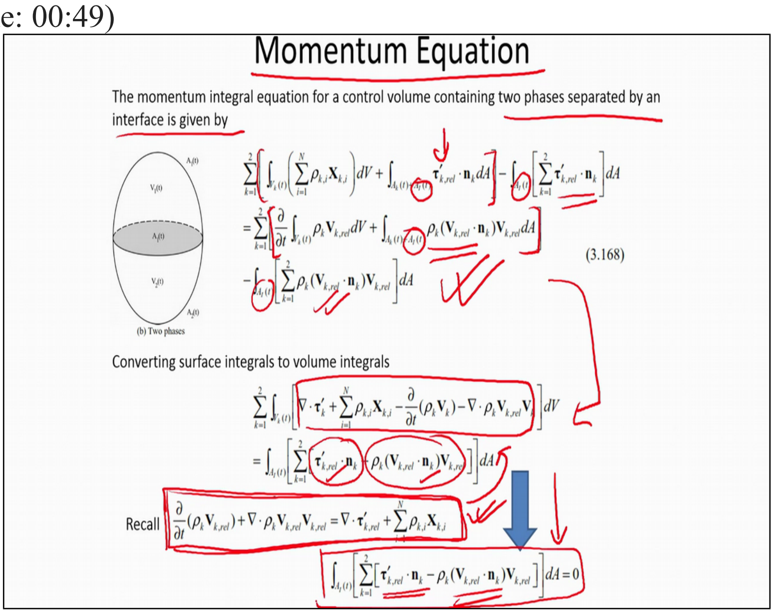

Momentum Equation

The momentum integral equation for a control volume containing two phases separated by an interface is given by

$$\sum_{k=1}^{2}\left[\int_{V_k(t)}\left(\sum_{i=1}^{N}\rho_{k,i}\mathbf{X}_{k,i}\right)dV+\int_{A_k(t)+A_I(t)}\boldsymbol{\tau}'_{k,rel}\cdot\mathbf{n}_k dA\right]-\int_{A_I(t)}\left[\sum_{k=1}^{2}\boldsymbol{\tau}'_{k,rel}\cdot\mathbf{n}_k\right]dA$$
$$=\sum_{k=1}^{2}\left[\frac{\partial}{\partial t}\int_{V_k(t)}\rho_k\mathbf{V}_{k,rel}dV+\int_{A_k(t)+A_I(t)}\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\mathbf{V}_{k,rel}dA\right]$$
$$-\int_{A_I(t)}\left[\sum_{k=1}^{2}\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\mathbf{V}_{k,rel}\right]dA \qquad (3.168)$$

Converting surface integrals to volume integrals

$$\sum_{k=1}^{2}\int_{V_k(t)}\left[\nabla\cdot\boldsymbol{\tau}'_k+\sum_{i=1}^{N}\rho_{k,i}\mathbf{X}_{k,i}-\frac{\partial}{\partial t}(\rho_k\mathbf{V}_k)-\nabla\cdot\rho_k\mathbf{V}_{k,rel}\mathbf{V}\right]dV$$
$$=\int_{A_I(t)}\left[\sum_{k=1}^{2}\left(\boldsymbol{\tau}'_{k,rel}\cdot\mathbf{n}_k+\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\mathbf{V}_{k,rel}\right)\right]dA$$

Recall $\frac{\partial}{\partial t}(\rho_k\mathbf{V}_{k,rel})+\nabla\cdot\rho_k\mathbf{V}_{k,rel}\mathbf{V}_{k,rel}=\nabla\cdot\boldsymbol{\tau}'_{k,rel}+\sum_{i=1}^{N}\rho_{k,i}\mathbf{X}_{k,i}$

$$\int_{A_I(t)}\left[\sum_{k=1}^{2}\left[\boldsymbol{\tau}'_{k,rel}\cdot\mathbf{n}_k-\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\mathbf{V}_{k,rel}\right]\right]dA=0$$

Now Let us go to the momentum equation. So, the momentum integral equation for a control volume which has got two phases separated by an interface is given by this equation form. Let us look at each of the terms over here once again. What we have done like last time is that we have added that AI and we have subtracted the same okay.

We have added it and we subtracted it, it is essentially the same thing that we did in our continuity equation okay. So what we have done is that we have added the stress at the interface and we have taken and we have subtracted the same out. Similarly for the flux term that is the convective term okay what we have done is that we have added the momentum flux are the interface and we have taken the same out, okay.

So, these are the only two things that we have done. Now the next procedure is basically to convert this to a volume integral now. In the volume integral once you do the transformation okay as we know the volume, these; all these things can be transferred to the volume integral. This portions in the in these brackets which I am highlighting they can be all transferred to the volume integral.

So, this is what you get after you do the volume integral of the same okay and the right hand side term that you see over here it is basically the interface terms. So, the stress are the interface and the corresponding flux of the interface okay the momentum flux and the stress okay. Now recall the original momentum equation written in the differential notation.

This is the original momentum equation, right, okay. Now in the original momentum equation this was the term, this is applicable for any phase, right. So, therefore if we substitute this, back into this particular equation we get this as a result of that, okay. So, here what it says is that the shear stress.

The total stress and the interface and the momentum flux at the interface has to be equal to zero that means this is the in this integral equation has to be satisfied. Now as we know earlier if the integral portion is equal to 0, right. This entire integral is equal to 0, the integrand which is basically nothing but this also has to be 0.

Because it has to be valid for any sized and shaped a control volume okay. So, the size shape does not really matter, it can be as small as you want.

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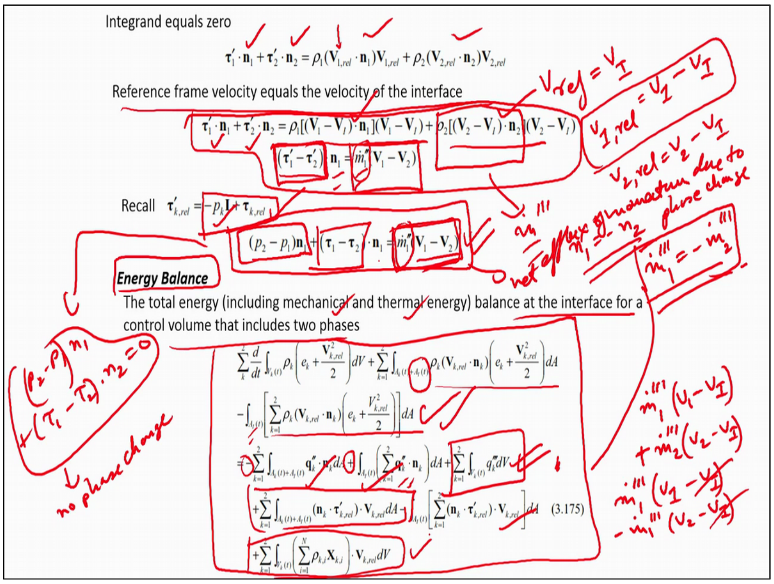

Integrand equals zero

$$\boldsymbol{\tau}_1' \cdot \mathbf{n}_1 + \boldsymbol{\tau}_2' \cdot \mathbf{n}_2 = \rho_1(\mathbf{V}_{1,rel} \cdot \mathbf{n}_1)\mathbf{V}_{1,rel} + \rho_2(\mathbf{V}_{2,rel} \cdot \mathbf{n}_2)\mathbf{V}_{2,rel}$$

Reference frame velocity equals the velocity of the interface

$$\boldsymbol{\tau}_1' \cdot \mathbf{n}_1 + \boldsymbol{\tau}_2' \cdot \mathbf{n}_2 = \rho_1[(\mathbf{V}_1 - \mathbf{V}_I) \cdot \mathbf{n}_1](\mathbf{V}_1 - \mathbf{V}_I) + \rho_2[(\mathbf{V}_2 - \mathbf{V}_I) \cdot \mathbf{n}_2](\mathbf{V}_2 - \mathbf{V}_I)$$

$$(\boldsymbol{\tau}_1' - \boldsymbol{\tau}_2') \cdot \mathbf{n}_1 = \dot{m}_1''(\mathbf{V}_1 - \mathbf{V}_2)$$

Recall $\boldsymbol{\tau}_{k,rel}' = -p_k\mathbf{I} + \boldsymbol{\tau}_{k,rel}$

$$(p_2 - p_1)\mathbf{n}_1 + (\boldsymbol{\tau}_1 - \boldsymbol{\tau}_2) \cdot \mathbf{n}_1 = \dot{m}_1''(\mathbf{V}_1 - \mathbf{V}_2)$$

Energy Balance

The total energy (including mechanical and thermal energy) balance at the interface for a control volume that includes two phases

$$\sum_k^2 \frac{d}{dt}\int_{V_k(t)} \rho_k\left(e_k + \frac{\mathbf{V}_{k,rel}^2}{2}\right)dV + \sum_{k=1}^2 \int_{A_k(t)+A_I(t)} \rho_k(\mathbf{V}_{k,rel} \cdot \mathbf{n}_k)\left(e_k + \frac{\mathbf{V}_{k,rel}^2}{2}\right)dA$$
$$-\int_{A_I(t)}\left[\sum_{k=1}^2 \rho_k(\mathbf{V}_{k,rel} \cdot \mathbf{n}_k)\left(e_k + \frac{V_{k,rel}^2}{2}\right)\right]dA$$
$$= -\sum_{k=1}^2 \int_{A_k(t)+A_I(t)} \mathbf{q}_k'' \cdot \mathbf{n}_k dA + \int_{A_I(t)}\left(\sum_{k=1}^2 \mathbf{q}_k'' \cdot \mathbf{n}_k\right)dA + \sum_{k=1}^2 \int_{V_k(t)} q_k''' dV$$
$$+\sum_{k=1}^2 \int_{A_k(t)+A_I(t)} (\mathbf{n}_k \cdot \boldsymbol{\tau}_{k,rel}') \cdot \mathbf{V}_{k,rel} dA - \int_{A_I(t)}\left[\sum_{k=1}^2 (\mathbf{n}_k \cdot \boldsymbol{\tau}_{k,rel}') \cdot \mathbf{V}_{k,rel}\right]dA \quad (3.175)$$
$$+\sum_{k=1}^2 \int_{V_k(t)}\left(\sum_{i=1}^N \rho_{k,i}\mathbf{X}_{k,i}\right) \cdot \mathbf{V}_{k,rel} dV$$

So, the integrand becomes equal to 0 that means that there are two shear stress; two stress terms and two flux terms corresponding to each phase. Now if the reference frame velocity equals to the velocity of the interface, like in the case of your momentum expression it is equal to the velocity of the interface which is VI, right, okay.

Moment that particular thing happens what we get is; you can substitute this V1 relative as equal to okay V1 minus the corresponding VI like we did in the last time in the case of a momentum expression. So, this is the total expression that you get after substitution of this particular expression this and a correspondingly relative also V2 relative, right.

So, what it tells you is that the shear the stresses are the interface okay acting on the interface is basically nothing but the flux term okay. So, the stresses is equal to the flux term that is essentially what it means? Now recall that this part of the expression is nothing but that m dot triple prime that we defined earlier in the case of continuity, right.

So, this is what? This is the mass transfer due to phase change across the interface, right. So, what and we also noted that n1 is basically negative of n2 that also we noted, right. So, what it tells you is that tau 1 into dot n1 + tau 2 dot n2 becomes tau 1 prime - tau 2 prime dot n1 we are substituting n1 is the same as - n2, right.

Similarly on the right hand side we can see this is the corresponding mass flux, right, okay. Which we know if you go to this particular expression you will see that this is the corresponding mass flux expression let me highlight it here alright. So, that is the mass flux expression okay.

So and noting also that m1 triple prime is minus m2 triple prime right. So, basically where do we actually expand these two quantities, if you look at these two quantities what these two quantities will look like is that m1 triple prime into V1 - VI that is the first expression, right. Minus or rather plus here m2 triple prime V2 - VI right.

But using m1 triple prime is the same as -m2 triple prime okay what we can do is that this expression will become V1 - VI - m1 triple prime V2 – VI, right. So, if you add these two quantities this VI's will drop out and you will get this expression okay. And on the right hand side; on the left hand side we have the difference in the shear stress okay.

Now as we know the total shear stress is given by P and the corresponding shear stress for each phase, right. So, just if we substitute this, what it becomes it becomes P2 - P1 into n1 tau2 - tau1 into n1 or dot product of n1 is nothing but equal to m1 dot double prime multiplied by V1 - V2.

So, that is the interface condition that you will get if you deal with a interface, from the momentum part of the interface, right. So, from the continuity part we got m1 is the negative of -m2, right. Here we have seen that the difference in pressure across the interface plus the stress that is acting at cross the interface.

The difference in stress factor of the interface is basically what is responsible for the mass exchange or the momentum exchange due to phase change. This is nothing but the relative momentum exchange due to phase change okay. Relative momentum exchange due to phase change this is the net you can say the net flux of momentum due to phase change okay.

Now if for example m1 double; if there is no phase change say for example is happening right. So, what we will have is P2 - P1 into n1 + tau1 - tau2 dot n2 should be equal to 0 in the event of no phase change, right. So, the difference in pressure across the interface okay is that equal to the plus the difference in shear stress of stress factor of the interface is given by, is equal to zero if there is no phase change.

This is the net efflux right off momentum okay, due to phase change, right. So, this concept should be very clear okay that this is what happens, this is what the equation means. Similarly let us look at the energy balance. Now the total energy mechanical and thermal energy balance at the interface if you recall the original equation which contains two phases.

This is a slightly bigger palette with a lot of terms okay. But let us not be scared about it we have done essentially the same thing they have added the interface, we have taken out the interface, right. Across the interface this is basically the flux across the interface, right. And we have taken the added and subtracted it.

Similarly on the right hand side of the expression okay what we have done is that we have taken the; this is the diffusion term we have added the interface; we have taken out the interface okay, over here, sorry. So, here if you look at it, so this is the expression that has been added and the total heat release of course the internal heat generation of course remains the same.

On the other hand this is a work done by the stress if you recall that also has been added and subtracted, right. And there is a body force term because it is on the volume basis only okay it does not have any interface component to that, right. So there are three terms that we that have been brought in. One is basically the flux term okay across the control surface that means across the interface.

There has been a diffusion term which has been brought in okay. This should be negative actually okay and correspondingly or actually it will be positive and sorry because this is negative that transfers it to positive sign okay so, let me just, this is correct okay. So, that part is correct okay. So, this plus sign is correct because there is a negative sign in front of the heat diffusion term.

So when you actually subtract it out it becomes a positive quantity and similarly the stress term that is the stress that is act work done by the stress on the control surface has been also added and subtracted the interface term, right. So, we have added basically three terms we have subtracted three terms that is all that we have done okay.

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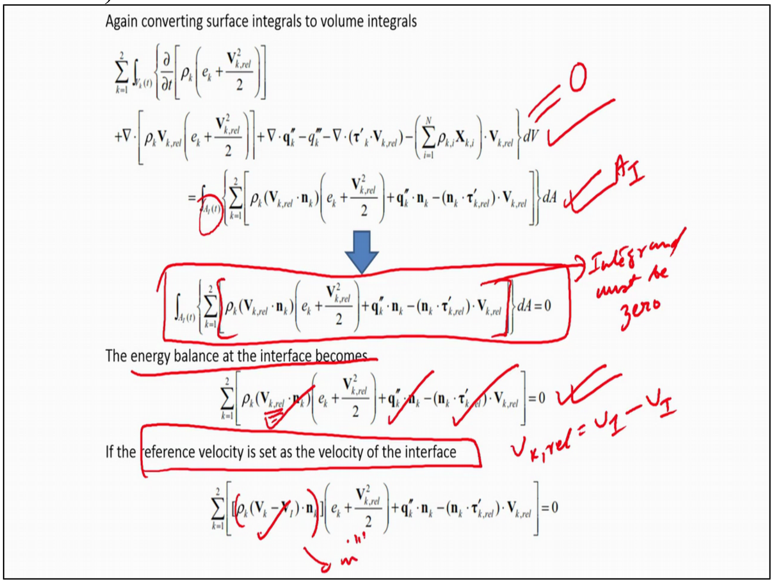

Again converting surface integrals to volume integrals

$$\sum_{k=1}^{2}\int_{V_k(t)}\left\{\frac{\partial}{\partial t}\left[\rho_k\left(e_k+\frac{\mathbf{V}_{k,rel}^2}{2}\right)\right]\right.$$

$$\left.+\nabla\cdot\left[\rho_k\mathbf{V}_{k,rel}\left(e_k+\frac{\mathbf{V}_{k,rel}^2}{2}\right)\right]+\nabla\cdot\mathbf{q}_k''-q_k'''-\nabla\cdot(\boldsymbol{\tau}'_k\cdot\mathbf{V}_{k,rel})-\left(\sum_{i=1}^{N}\rho_{k,i}\mathbf{X}_{k,i}\right)\cdot\mathbf{V}_{k,rel}\right\}dV = 0$$

$$=\int_{A_I(t)}\left\{\sum_{k=1}^{2}\left[\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\left(e_k+\frac{\mathbf{V}_{k,rel}^2}{2}\right)+\mathbf{q}_k''\cdot\mathbf{n}_k-(\mathbf{n}_k\cdot\boldsymbol{\tau}'_{k,rel})\cdot\mathbf{V}_{k,rel}\right]\right\}dA \quad A_I$$

$$\boxed{\int_{A_I(t)}\left\{\sum_{k=1}^{2}\left[\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\left(e_k+\frac{\mathbf{V}_{k,rel}^2}{2}\right)+\mathbf{q}_k''\cdot\mathbf{n}_k-(\mathbf{n}_k\cdot\boldsymbol{\tau}'_{k,rel})\cdot\mathbf{V}_{k,rel}\right]\right\}dA=0} \rightarrow \text{Integrand must be zero}$$

The energy balance at the interface becomes

$$\sum_{k=1}^{2}\left[\rho_k(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\left(e_k+\frac{\mathbf{V}_{k,rel}^2}{2}\right)+\mathbf{q}_k''\cdot\mathbf{n}_k-(\mathbf{n}_k\cdot\boldsymbol{\tau}'_{k,rel})\cdot\mathbf{V}_{k,rel}\right]=0$$

If the reference velocity is set as the velocity of the interface

$V_{k,rel} = V_I - V_I$

$$\sum_{k=1}^{2}\left[[\rho_k(\mathbf{V}_k-\mathbf{V}_I)\cdot\mathbf{n}_k]\left(e_k+\frac{\mathbf{V}_{k,rel}^2}{2}\right)+\mathbf{q}_k''\cdot\mathbf{n}_k-(\mathbf{n}_k\cdot\boldsymbol{\tau}'_{k,rel})\cdot\mathbf{V}_{k,rel}\right]=0$$

$\rightarrow \dot{m}''$

Again what we do follow the same logic converts the surface integrals to the volume integrals right. So, this is the volume integral okay which looks suspiciously like the energy equation and this is the corresponding additional terms that we have incorporated because of the interface that is AI, right. So, because of AI we have added all these terms right.

So, similarly this term goes to 0 that is like an energy equation, right. That is the energy equation that we did, so, this is; so, we are left with just the interface term, right. So, the energy balance in that; for again the, because this is an integral form, the integrand has to be equal to 0, right. The integrand must be equal to 0, so, the energy balance at the interface becomes this.

So, that is nothing but this term in the square bracket that is all that is equal to 0, what does this terms contain one is the flux term across the surface okay and the other one is the diffusion term that is also across the interface and then there is a work done term because of the stresses also at the interface right. These are the three quantities that we have okay.

Again we set the reference velocity as the same as a velocity of the interface. So, once again in this Vk relative becomes equal to V1 - VI okay depending on the phase that you are belonging to. So, this becomes the first term okay. So, the first term wherever there is VK a relative that becomes like this right.

So, if you look at this particular term now this suspiciously looks like that m dot triple is not that so. It looks like that m dot triple prime exactly what we did earlier.

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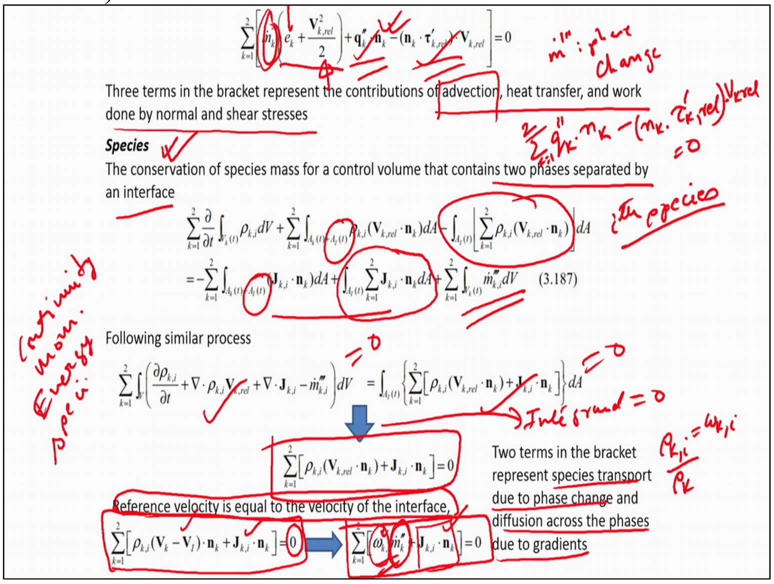

$$\sum_{k=1}^{2}\left[\dot{m}_k''\left(e_k+\frac{\mathbf{V}_{k,rel}^2}{2}\right)+\mathbf{q}_k''\cdot\mathbf{n}_k-(\mathbf{n}_k\cdot\boldsymbol{\tau}_{k,rel}')\cdot\mathbf{V}_{k,rel}\right]=0$$

$\dot{m}''$: phase change

Three terms in the bracket represent the contributions of advection, heat transfer, and work done by normal and shear stresses

$\sum_{k=1}^{2}\mathbf{q}_k''\cdot\mathbf{n}_k-(\mathbf{n}_k\cdot\tau_{k,rel}')\mathbf{V}_{k,rel}=0$

Species

The conservation of species mass for a control volume that contains two phases separated by an interface

$$\sum_{k=1}^{2}\frac{\partial}{\partial t}\int_{V_k(t)}\rho_{k,i}dV+\sum_{k=1}^{2}\int_{A_k(t)+A_I(t)}\rho_{k,i}(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)dA-\int_{A_I(t)}\left[\sum_{k=1}^{2}\rho_{k,i}(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)\right]dA$$
$$=-\sum_{k=1}^{2}\int_{A_k(t)+A_I(t)}(\mathbf{J}_{k,i}\cdot\mathbf{n}_k)dA+\int_{A_I(t)}\sum_{k=1}^{2}\mathbf{J}_{k,i}\cdot\mathbf{n}_k dA+\sum_{k=1}^{2}\int_{V_k(t)}\dot{m}_{k,i}'''dV \quad (3.187)$$

ith species

continuity
mom.
Energy
speci.

Following similar process

$$\sum_{k=1}^{2}\int_V\left(\frac{\partial\rho_{k,i}}{\partial t}+\nabla\cdot\rho_{k,i}\mathbf{V}_{k,rel}+\nabla\cdot\mathbf{J}_{k,i}-\dot{m}_{k,i}'''\right)dV=\int_{A_I(t)}\left\{\sum_{k=1}^{2}\left[\rho_{k,i}(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)+\mathbf{J}_{k,i}\cdot\mathbf{n}_k\right]\right\}dA$$

$= 0$ $= 0$

Integrand $= 0$

$$\boxed{\sum_{k=1}^{2}\left[\rho_{k,i}(\mathbf{V}_{k,rel}\cdot\mathbf{n}_k)+\mathbf{J}_{k,i}\cdot\mathbf{n}_k\right]=0}$$

Two terms in the bracket represent species transport due to phase change and diffusion across the phases due to gradients

$\frac{\rho_{k,i}}{\rho_k}=\omega_{k,i}$

Reference velocity is equal to the velocity of the interface,

$$\sum_{k=1}^{2}\left[\rho_{k,i}(\mathbf{V}_k-\mathbf{V}_I)\cdot\mathbf{n}_k+\mathbf{J}_{k,i}\cdot\mathbf{n}_k\right]=0 \Rightarrow \sum_{k=1}^{2}\left[\omega_{k,i}\dot{m}_k''+\mathbf{J}_{k,i}\cdot\mathbf{n}_k\right]=0$$

So, that is what we have done okay. So, m dot triple prime k means the mass flux across each phase because of phase change only this is due to when is it coming due to phase change okay that we already knew earlier, right. So, and this equation involves the internal energy component as well as the kinetic energy component right.

This particular term over here includes the corresponding mass heat transfer component and this is once again the work done by the stress by the normal and shear stresses. That is what we have said over here advection, heat transfer and work done by normal and shear stresses. So, that is the balance for the interface, right.

So, across the interface there is a term which is advective because of this change in and the phase, because of change in phase okay. There is also a diffusive transport and then there is a work done transport. Now for example if in some ways there is no phase change okay what you will have over here is that your qk double prime dot nk - nk dot k relative.

And this will be like Vk relative is equal to 0 that is what we are going to get, right, okay. If there is no mass component of course this is summed over the k = 1 to k = 2, right. So, if there is once again the dissipation terms are not that important you can further, reduce it further. So, it will essentially boil down that the diffusion of heat across the interface has to be equal to 0 from, both the phases, right, okay.

So, that takes care of the interface conditions for the energy. Now similarly if we look at the species balance equation now. Now the conservation of species mass for example for a control volume that contains two phases separated by an interface once again because the species balance is up next, right, that is what if there are multi-component systems.

So, we are doing it for the ith species essentially okay. Once again what you do standard thing we add this is take this out. This is basically nothing but the flux component okay and we do the same thing for the diffusion component like we did for the heat, right. We had this component for the heat where did we get it; we got it in a method similar to this, right.

So, here also we add the diffusion across the interface of course here you do not need to do anything this is because simply this is the species that is created or the production of production or the sink term okay. So, following a very similar process we divide it into two parts, right. This part is equal to 0, so this part automatically becomes equal to 0.

So, that means this particular portion becomes equal to 0, right. Once again the integrand we once again set the reference velocity equal to the interface velocity and we make that the integrand is actually equal to 0. So, this is this takes into account that the integrand has to be equal to 0 okay. So, the reference and the reference velocity is equal to the velocity of the interface okay.

So, we ultimately get to this particular expression okay which is nothing but Vk - VI plus whatever is the diffusion component of the same that is equal to 0 okay. And if you look at this expression what is Rho ki? Rho ki is basically Rho ki by Rho k by Rho, k is basically Omega ki, right something like that.

So, if we incorporate that definition over here this is the expression that you get. This is basically the mass fraction into the corresponding mass transport of the ith species. This is the total transport of mass from the kth phase to the corresponding adjoining phase, right, okay. And this is basically the mass fraction for that particular ith species.

And summed over all the phases and the corresponding flux of ith species in the kth phase okay across the interface this has to be equal to 0, okay. So, the two terms in the bracket represents species transport due to phase change this is the first term and diffusion across the phases due to the gradient.

So, there is a phase change due to the species due to the phase change there is a species transport and there is also a species transport due to diffusion across the phases due to such gradients okay. So, this part should be clear now that this is what we have for the species balance okay. So, these are the three conditions or the four conditions for continuity, momentum, energy and species.

And then can be various special forms of each but essentially you should always remember there is a component always that is associated with phase change. And then there is a component that is associated with diffusion that is what the two things are right in the case of species okay.

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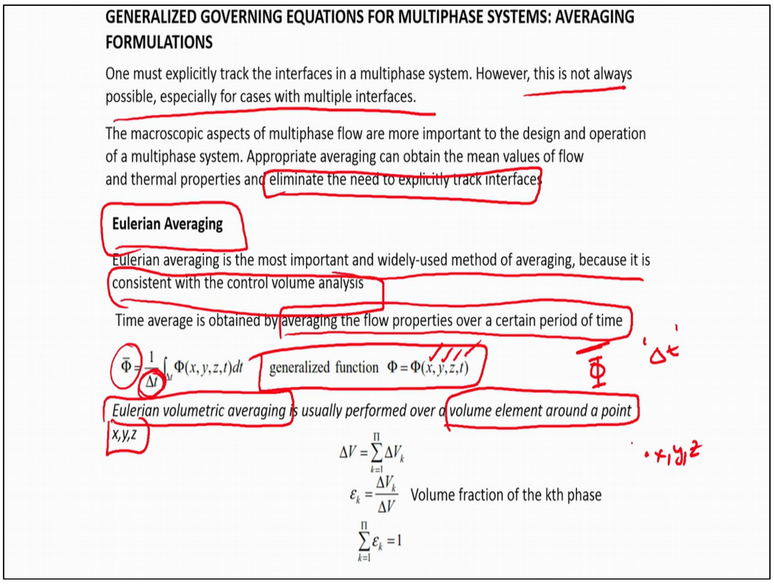

GENERALIZED GOVERNING EQUATIONS FOR MULTIPHASE SYSTEMS: AVERAGING FORMULATIONS

One must explicitly track the interfaces in a multiphase system. However, this is not always possible, especially for cases with multiple interfaces.

The macroscopic aspects of multiphase flow are more important to the design and operation of a multiphase system. Appropriate averaging can obtain the mean values of flow and thermal properties and eliminate the need to explicitly track interfaces

Eulerian Averaging

Eulerian averaging is the most important and widely-used method of averaging, because it is consistent with the control volume analysis

Time average is obtained by averaging the flow properties over a certain period of time

$\bar{\Phi} = \frac{1}{\Delta t}\int_{\Delta t} \Phi(x,y,z,t)dt$ generalized function $\Phi = \Phi(x,y,z,t)$

Eulerian volumetric averaging is usually performed over a volume element around a point x,y,z

$$\Delta V = \sum_{k=1}^{\Pi} \Delta V_k$$

$$\varepsilon_k = \frac{\Delta V_k}{\Delta V}$$ Volume fraction of the kth phase

$$\sum_{k=1}^{\Pi} \varepsilon_k = 1$$

Now, that we have done up to this particular portion okay. Let us look at a couple of more things in a slightly more details okay. Imagine a system which is very complicated right, that there may be like the first day's lecture that we gave that there were a lot of you know bubbles they distributed. They may be complicated shapes okay.

They there will be multiple inter tangles interfaces okay. So, those kind of things could be there okay. There may not be a clear cut separation between the two phases or between the phases not just two, any number of phases in a multiphase system okay. So, there so in order to the other formulation that we so far did always consider to the interest is kind of easily tractable that means he can track the interface, correct.

In many of the real life cases that may not be the scene at all the interfaces can become corrugated they can become very complicated there can be lots of things okay. So, this is not always possible for cases with multiple interfaces. Interfaces to track the interfaces itself can be a little nightmare. So, the macroscopic aspects of the;

In those cases however we still want to analyze a multi-phase system right. So, what can we do we can either track each and every point is we can take a flow and we can track the interface by using different types of methods like level said there are other types of methods also. You track the interface then you let the interfaces actually evolve.

So, that is a very complicated and onerous task but however macroscopic aspects of the multi-phase flow may be more important for the design and operation of a multiphase system. So, you may not need all those information, so, if you can on the other hand develop some kind of averaging technique which gives you the mean values of the flow, the thermal properties and eliminate the need to explicitly track the interfaces.

Then that would be like a great thing to have that would be a great tool to have okay. So, what are the ways by which because we can write governing equations not a problem. But if the problem involves such complicated interfaces it may not be always feasible you know to track the interfaces.

And to analyze the problem okay in a very simple way okay, so, one such averaging will be the Eulerion averaging. What does it do? Eulerion averaging is the most important and widely used averaging because it is consistent with the control volume analysis, right. So, whatever we did that was like an Eulerion kind of a framework, a field concept okay.

So, if we take any generalized function Phi say for example where Phi is a function of x, y, z and t the four dimensions, it is a function of space, it is the function of time. So, the time average is obtained by average the flow properties over a certain period of time right. So, when we talk about average that Phi bar okay.

It is basically averaged over a particular time, this time slot this is delta t okay, over a particular time it is averaged okay. If we talk about Eulerion volumetric averaging on the other hand it is usually performed over of volume element allow no point x, y, z okay. So, it is performed over a volume element this there is a point say x, y, z in space right, and so, it is performed across one such volume.

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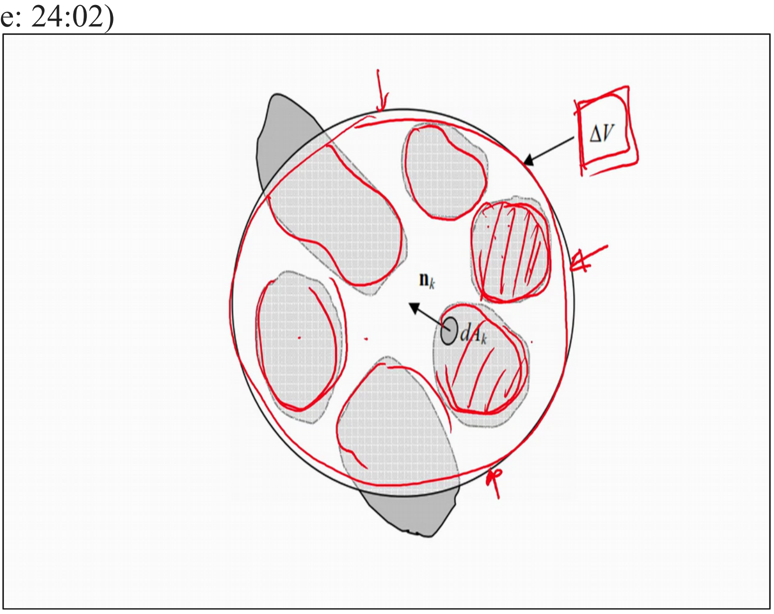

If I put this particular figure up you will see so, this is the volume element delta V if you look at it it has got those individuals patches which are basically the different phases. This is one case this is another field where any other phases also, is another phase, this is another phase, right. All these are different phase's right and this is the volume element that we are talking about okay.

So, inside the volume element there are this small elemental areas of the different phases and they can be distributed in whatever way that we want. So, basically we you can see that we are losing the resolution over this window. When we actually do the averaging out with an average whatever properties that you want to average.

That may be mass energy whatever it is right. If we want to develop this concept then if we do it over this elemental control volume whatever is inside this okay. Whatever is the individual property variation within this right that is kind of averaged out got it. So, we are not looking at probably gradients inside okay it is like completely averaged out these things right.

So, if you look at this particular expression. So, it is performed across a volume element around the point x, y, z. So, I hope that makes sense now that what I am talking about okay. So, the total volume element is basically the individual volume elements that are occupied by each of the phase's right. So, if you look at this these are like those individual volume elements right.
So, this is like this dVk or delta Vk right that is like delta Vk right. So, these are the individual volumes okay of each of the phases. And this is the volume fraction of the kth phase. So, that means if you look at this. So, this is phase one, phase one, phase one, phase one, phase one, phase one, phase one all the various right. So, you calculate the volumes of all these phases, right, okay.

So, say you get something like some delta V1 if you divide it by delta V1 plus delta V2 right. So, that is what actually gives you epsilon k or rather here. So, actually sorry it will be like epsilon 1 but in a normal case this can be written as Vk, summation of delta Vk k equal to 1 to whatever.

Here we are taking two phases that would be like epsilon k got it. So, this is like what we call the volume fraction of each of the phases got it. So, the volume fraction of each of the phases is basically what we are defining over here right. Now if we sum total of all the volume fraction should be equal to 1 by definition it has to be less than 1.

So, there can be any number of phases present for example here there are 5 number of phases. So, if there is a volume element delta V which has got phases of 1, in phases of 2 and phases like 3 okay. You can put that Pi into whatever is the magnitude 3, 4, 5 whatever it is that you think it is going to be, got it okay.

So, if you look at this, this is clear it is only done for two, two phases is equally applicable for any number of phases that you can think of. I am giving you an idea of what this volume element should be right and what are the individual volume fraction should be. And how each of the elemental volumes are actually defined okay.

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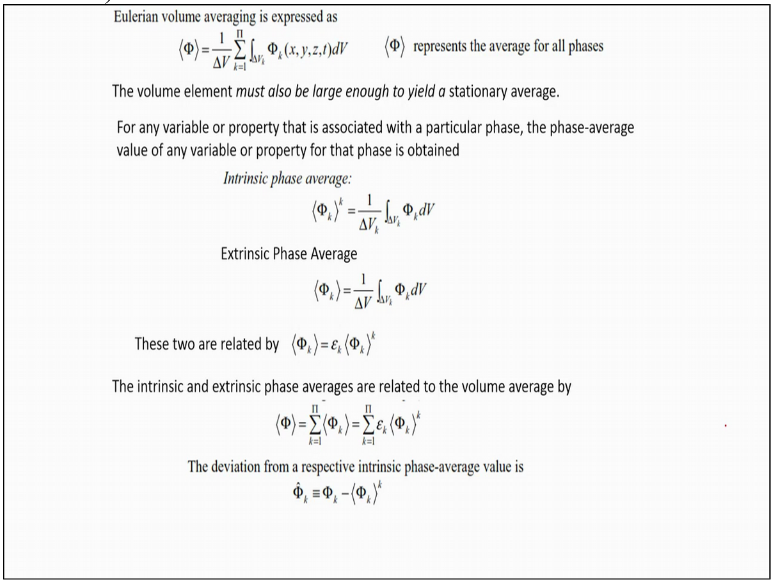

Eulerian volume averaging is expressed as

$$\langle \Phi \rangle = \frac{1}{\Delta V} \sum_{k=1}^{\Pi} \int_{\Delta V_k} \Phi_k(x,y,z,t)\, dV$$

$\langle \Phi \rangle$ represents the average for all phases

The volume element *must also be large enough to yield a* stationary average.

For any variable or property that is associated with a particular phase, the phase-average value of any variable or property for that phase is obtained

Intrinsic phase average:

$$\langle \Phi_k \rangle^k = \frac{1}{\Delta V_k} \int_{\Delta V_k} \Phi_k\, dV$$

Extrinsic Phase Average

$$\langle \Phi_k \rangle = \frac{1}{\Delta V} \int_{\Delta V_k} \Phi_k\, dV$$

These two are related by $\langle \Phi_k \rangle = \varepsilon_k \langle \Phi_k \rangle^k$

The intrinsic and extrinsic phase averages are related to the volume average by

$$\langle \Phi \rangle = \sum_{k=1}^{\Pi} \langle \Phi_k \rangle = \sum_{k=1}^{\Pi} \varepsilon_k \langle \Phi_k \rangle^k$$

The deviation from a respective intrinsic phase-average value is

$$\hat{\Phi}_k \equiv \Phi_k - \langle \Phi_k \rangle^k$$

So, in the next lecture what we are going to do is that we are going to basically look at how this Eularien averaging is performed okay. So, that we are going to do in the next lecture.