Compliant Mechanisms: Principles and Design
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Lecture – 07
Empirical formula for flexure joints

Welcome to the second week of lectures on Compliant Mechanism: Principles and Design. This week we are going to cover two things one is Elastic Pairs as we are calling them in this course and how to model them and how to use them, that would be the first one part of this weeks lectures, this is the second week. The other part of this weeks lectures are going to focus on using Finite Element Analysis for both small displacements and large displacements to model Compliant Mechanism; not just with the elastic pairs, but with the elastic pairs as well as elastic segments. There are two parts the first part would be focusing on the first three lectures and second part will be taken up in the next 3 lectures. In this lecture and the one that follows this, we are going to talk about elastic flexures and elastic pairs.

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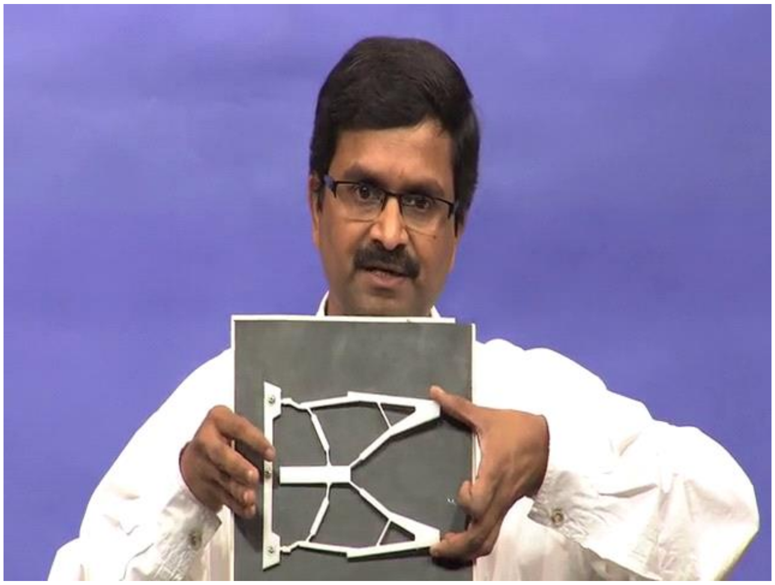

Let me first start with by showing a device that as elastic flexures you see, we have over here and here here here and there there, we have elastic pairs at joints which connect relatively rigid portions meaning that if I look at this segment. It has an elastic pair here this is rigid and if I look at this one so, there is an elastic pair here as well as down here. There are elastic pair meanings joint that connect it is still a monolithic mechanism or what we call a fully compliant mechanism, but it has this elastic flexures which give it a discrete compliance and you can see how it works. This particular one if I hold it at these two points and do this its going to go like that, such if I put a load here like I put my finger, I can see the mechanical advantage a little force I am applying small force or here and a large force over here. This is the elastic flexure Based compliant mechanism.

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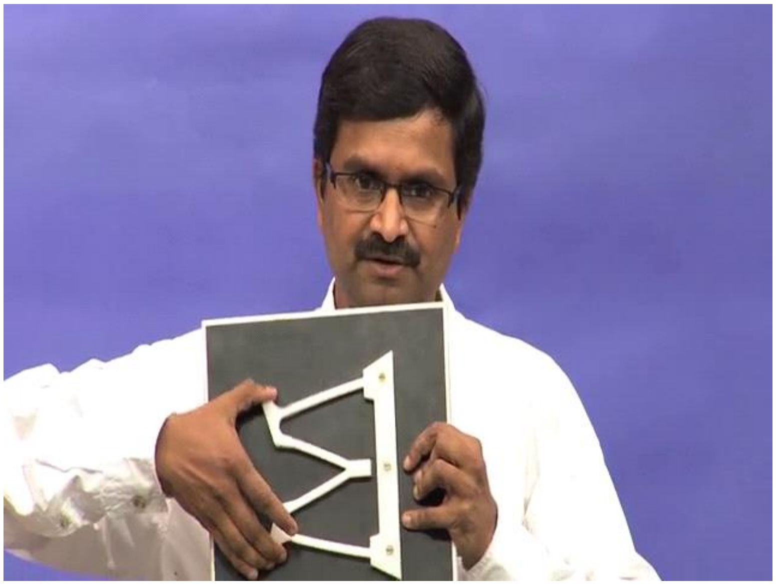

Lets contrast that with one that as elastic segments. Here apply force and it goes like this or if I keep it like this, the way I have I apply force here, it goes if I put my finger here I can feel the amplified force over here. This elastic segment Based on that is not we are going to talk about today. We are going to talk about this one; before I start to say how do we compare the two, the one that is discrete compliance like this, the other one that I showed which is this, which has distributed compliance. Which is better? If you were to play with these things, you will see the difference. Let me show it to you by first taking the discrete compliance one, if apply force it is going like this but what would happen is that if I do not take enough care I can flex these elastic pairs a lot. In fact, if you were to look closely here, they already undergo some plastic deformation. In fact, the colour of this polypropylene changes becomes more whitish indicating that it has undergone plastic deformation. Some time there are called Plastic hinges as well not because they are made of plastic materials, but they have undergone plastic deformation and in fact, their stiffness also changes a little bit. If I do not take enough care this thing can actually go out of planes. If when I am applying force like this, it can actually go out of plane because out of plane stiffness sometimes can be lower than influence stiffness, but here we made it thick enough that it is not doing, but if you do not take care it can go. That does not happen in the distributed Compliant Mechanism. Other thing is that the range of motion here will be quiet limited because if I deform it by large motion so, that you know these things undergo relative rotation a lot, then these things will undergo a lot of stress and they might fail like I said they already have undergone plastic deformation, but even then they are used quiet a bit in various applications.

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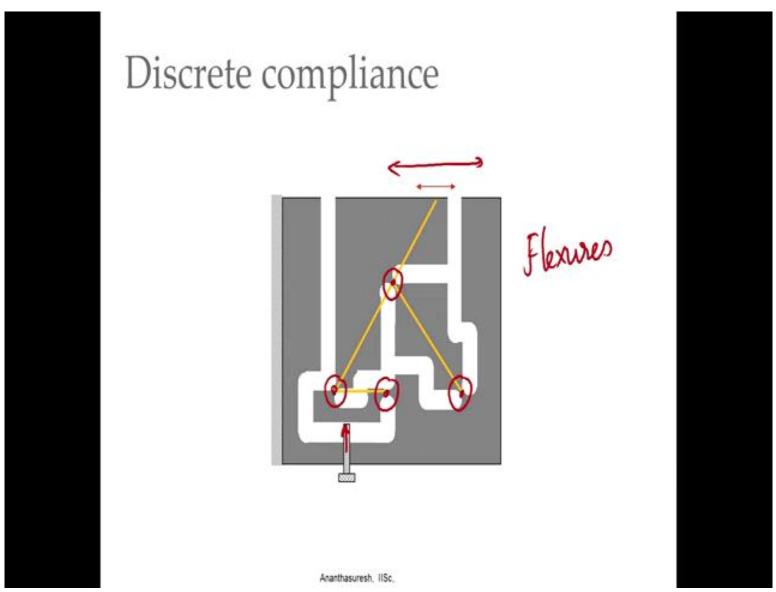

Lets look at this discrete compliance today made of elastic pairs, this we had already seen in the first lecture. We take a sheet of material and you can create something like this very easily. All these are elastic pairs we have joints there, there all are elastic pairs. In fact, we had said that these actually comes from a four bar linkage, if apply force over here by turning this screw this is going to move like this. If I move it back and forth the screw of touches I can make this move it is suitable for precision motions which are small motions because these elastic pairs or flexures as they are called; there are flexible joints their are used for very small motions so, that you can get precise motion where resolution or the smallest motion that you can repeatedly apply. That is the definition of (Refer Time: 06:03) mechanism were repeatability is important; that mean that should not be any effects from the environment noise, there could be temperature or fiction backlash those are not there in compliant in any way they are certainly not there in flexures that is why people use them in precision mechanisms.

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This is the Plastic hinge or living hinge as we call it. Again plastic not because its made of plastic material such as this. This is the Plastic hinge, but it has undergone plastic deformation most often, injection moulding people do this specially to make this little bit more flexible than the rest of the material. We would see when you take plastic boxes we would see d colouring or slide softening of the colour in this portions there are number of these things that we see around.

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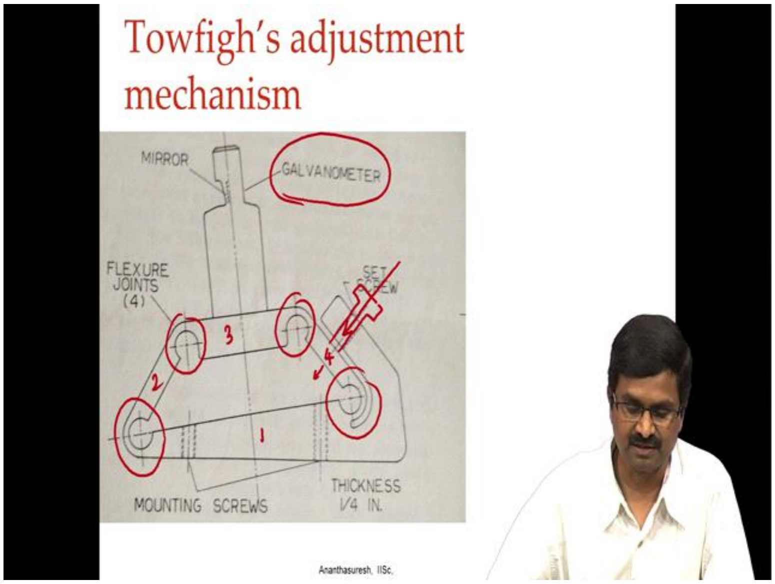

It is not at all new a person named Keivan Towfigh had a used this mechanism back in 1960s, where he had a used that in a galvanometer adjustment mechanism. Here there is a set screw so; that means, that we can apply force on this if I were to number the segments 1, 2, 3, 4. It is like a four bar linkage essentially on the four segment apply force like this by having a screw over here, according the screw will apply a force like this it would move ever so, little what is needed in optical instruments or these some of these expression instruments like galvanometer. There are mirror here mirror has to be turned little bit. This is in 1960s what people have used. Focus on this geometry of the flexure here there is a circular segment which is very shorts we can called it an elastic pair rather calling elastic segment. This is the crucial thing for this. this can be analysed and we can see how much it moves in what direction and whether it is effected by thermal fluctuations and so, forth. That is why people worry about in these flexure Based mechanism or elastic pair mechanism.

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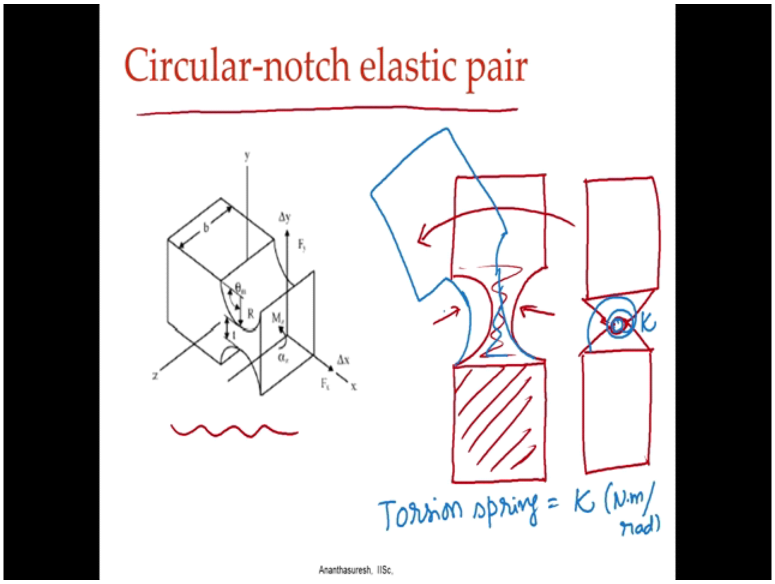

The circular notch elastic pair is very common so, what you need to do is if you have a large plate if you want create circular of flexure there you have something like this you remove this materials.

Let me use that by erasing the whole things goes if I try to erase. I will just redraw it then I thought I can erase selectively, but if I have a plate it does work in a way I can actually do this. I can erase this line and then redraw it. Basically I have (Refer Time: 09:11) still in the eraser mode. We have this circular flexure it is like this. That is what is shown over here in 3 dimensions, we are removing basically cylinder amount of material, but in two half's right this is one part that is removed and the other part of that is remove. If I were to fix one of this parts and apply a moment or a torque on it then it will flex. This portion elastically deform because this is narrow all right. This portion elastically deform and that forms the elastic pair so, we can actually think of this as there is a rigid body over there, there is another rigid body here. These two I can say are connected with effectively pin joint. That is an elastic pair of course, this pin joint is not without its stiffness so, the effect of stiffness can be modelled by putting a torsion spring, that blue one that we have put there is a torsion spring; which would imitate the elastic deformation of this portion there so, will denote by Kappa which will have units Newton meter per radian, meaning that if you multiply this Kappa with the angle of rotation of this, because this is going to deform in some manner. Something like this it will deform it wont extend like I have drawn, but it will bend. That bent bending is denoted with a pin joint here with torsion spring Kappa there, that give you the same resistance this circular notch elastic pair would give and it has been used extensively in precision machines.

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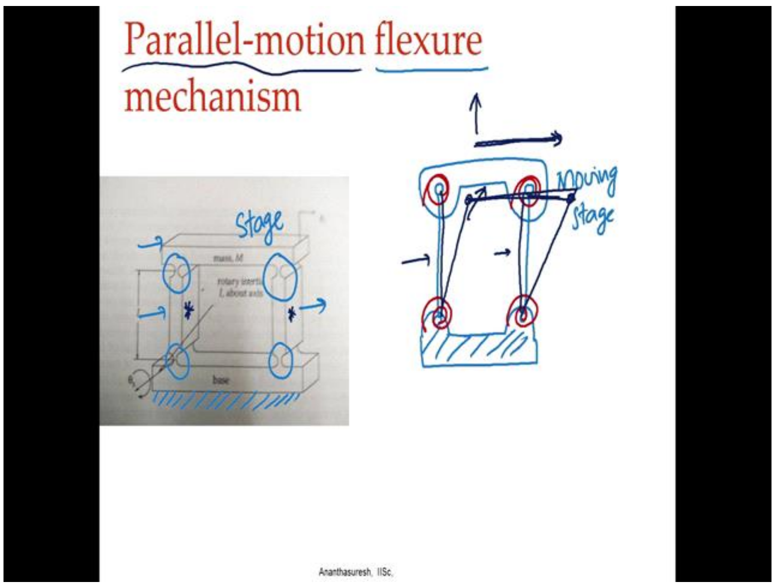

Here is one of those what is called a Parallel-motion flexure mechanism and you would notice these circular notch elastic pairs are circular notch hinges sometimes people call it also a flexure. what is this do it is actually called a Base.

This is like a fixed frame and there are these things it like a parallelogram or rectangle four bar mechanism; this is the one that is we can call it as Stage. This Stage here when you apply force anywhere we can apply force on this body or this body or this body wherever you apply this Stage is not going to rotate it is going to basically translate in two directions, it will definitely move because if I want to draw the equivalent of that here, each flexure of joint, the Base is here. This is the Base that is fixed and then here we have the moving Stage. Because we have this elastic pairs we must show the torsion springs here. There is torsion springs torsion spring torsion spring torsion spring. When we model this we assume that these things are actually rigid. These things are not flexible you see, this portion and this portion are relatively rigid compare to the flexure joints.

Now, if there were to be a force applied anywhere, anywhere for this mechanism it can be like this whichever way you do because of its geometry which is like a rectangle like a parallelogram, it is going to only move in that direction and in that direction. In fact, if you move it in this direction its actually going to come down because it is going to go like that and this joint is going to come there, this joint is going to come there, it is going to move horizontally that is why it called Parallel-motion. The motion of this Stage it is going to be parallel to the Base and that is what people want you do not want unnecessary rotations of the Stage and one can improve this further.

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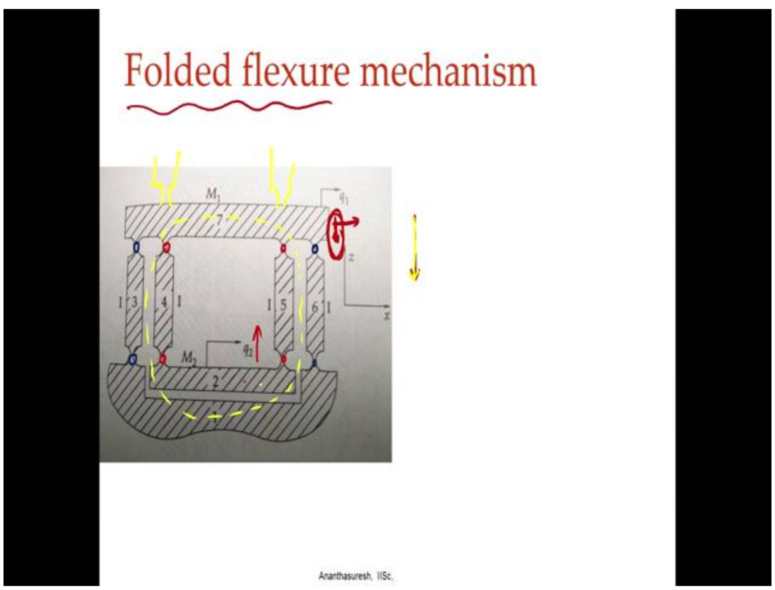

Now, what you see here original earlier one we had this much of four joints. This was the Stage now it has been moved by putting 4 more, let me show them in a different colours we have 4 more flexure here. It is folded back. This was the Stage that was capable of moving like this and like this in two dimensions, but no rotation because of the constrains that are put in with elastic pairs. Now another one is added on top of that. This one will still have this motion and this motion or what you can do by putting it down this component, that is a vertical or this component that moves in this direction can be compensated over here so, this Stage now does not move much in this.

I will not say it will not move at all, but compared to the previous Stage that we have here the motion in this direction will be reduced over here by having this compensative v one. You might actually think that instead of this portion let me encircle what I mean, let us say this portion it could have been put on top of this also. One can put this like that whatever I am showing here would have been flipped and put the other side; when you do that it does not compensate this one. It has to be put downwards and the way and the rotation happens and what it moves it is going to compensate it will have much less motion in the vertical direction is the folded flexure mechanism feature is that. People came up with very clever configurations. Here is another one where the Stage does not move because of the symmetry.

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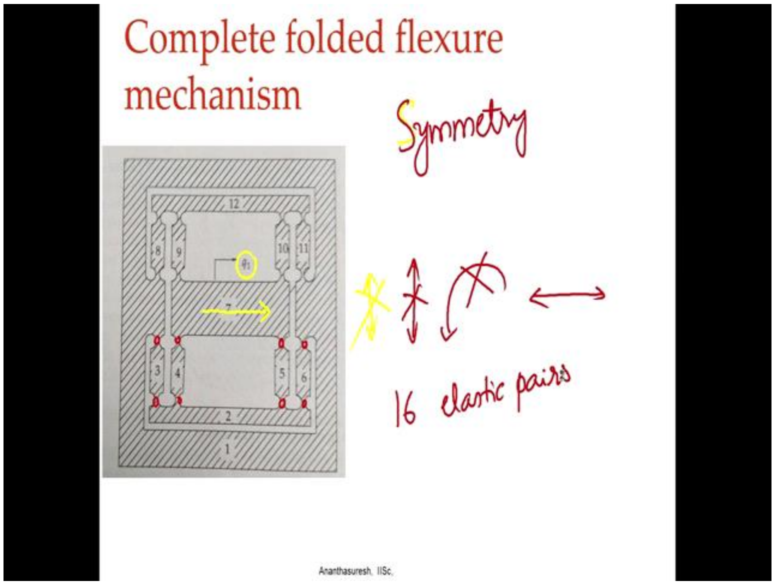

Now if you look at this central one because of symmetry it will only move in the if I call that x direction or q1 direction it will not move in this direction at all.

It say symmetric no here what is used is symmetry by using symmetry here we are able to get a Stage that has only one degree of freedom, one translation the other translation is the vertical direction is removed and there is rotation also that is absent here. Only this motion is possible for this mechanism. That is how we can use this 8 actually there are more 16 now. They are all the elastic pairs 8 and either side total 16 elastic pairs are used to let at precision Stage.

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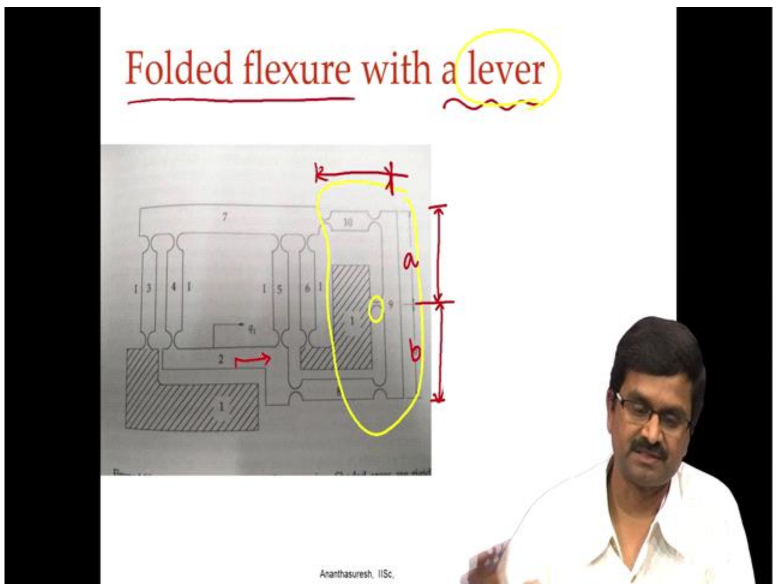

Here is another one which is also the Folded Flexure, but there is also a lever again look at the portion, let me use the colour that we can see. This is the extra portion now, the 1, 2, 7 are already there in the previous one; now we have 8, 9 and 10 this segments are there. Now you can see the lower Stage and upper Stage are coupled through a lever. This is the lever there is a pivot here, this is there is a here and b you can adjust this a and b; let me change the colour of the ink there is a here and b here; this is length b is this length a is this by adjusting this a and b and of course, there is some distance that you you can have here also by adjusting these lengths you can couple them so, that this Stage and that is Stage you can do further compensation previously I said that by having this folded flexure you can reduce the vertical motion of that things, so, that it mostly that very little this way by coupling it further you can reduce it even further. People use this to get Stages that have the motion the way that you want mostly in 1 axis or 2 axis.

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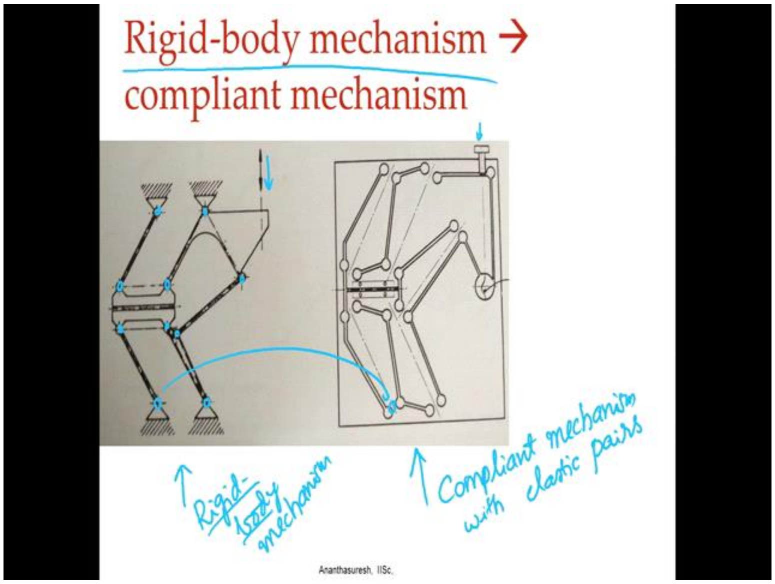

Sometimes you can do even more complicated motion with this just flexure. One of the ways that you can do is to start with a rigid body mechanism which is over here. Everything you can see the joints, there are joint here there is one more joint, joint, joint joint, joint and joint. In fact, these are good examples for trying your mobility formulas Grubler's formula and then Maxwell's rule which are equivalent when there only bars and joints not other pairs. These are all good examples see what is the mobility of this this particular rigid body mechanism, which has rigid bodies and kinematic joints this is rigid body mechanism whereas, this one here which has exact equivalent of that if you notice wherever these joints are we have these flexure.

Let me put a flexure, corresponding to that and likewise if I want to see where this flexure is we can see that that is also over here. Everywhere that you know we have this flexure coming and the actuation here you know that is the force over there. This is a Compliant Mechanism. Compliant Mechanism with elastic pairs, if you look at this is one would think that it is very easy to design Compliant Mechanism which it is, we take a rigid body mechanism and replace all the joints with the elastic pairs and you get a Compliant Mechanism.

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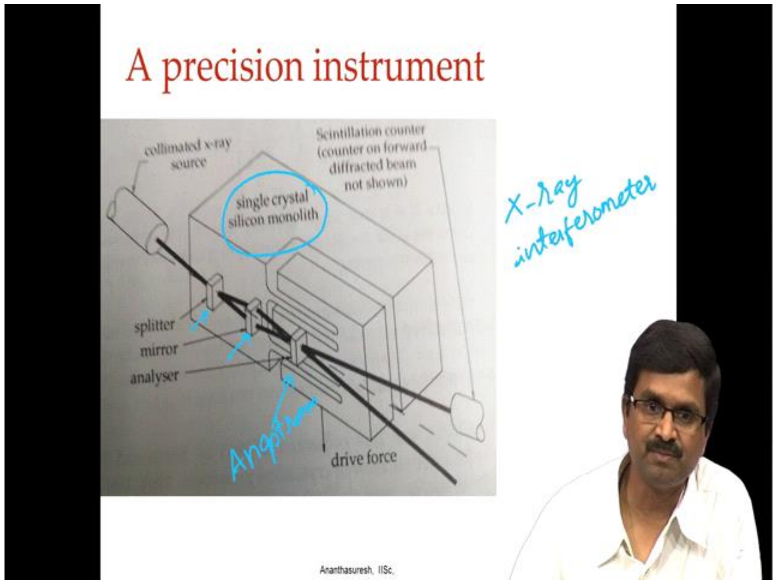

These are although they look very simple, they are used in very sophisticated instruments and precision instruments applications, these actually an X-ray interferometer, where there is precise motion that is required for something called this analyser. Where X-ray is split the beam splitter and there is a mirror. They have to be moved ever so, little not all of them this analyser is what is moved, when you move that this X-rays actually the movement of this has to be of the order of Angstroms, that can be done with these mechanism. This is made of as you can see single crystal silicon monolith very small and the motion of this are going to very very small by achieving such small motion you can create very sophisticated instruments, even though mechanism when the mechanism view point they look very simple, but they have very important applications.

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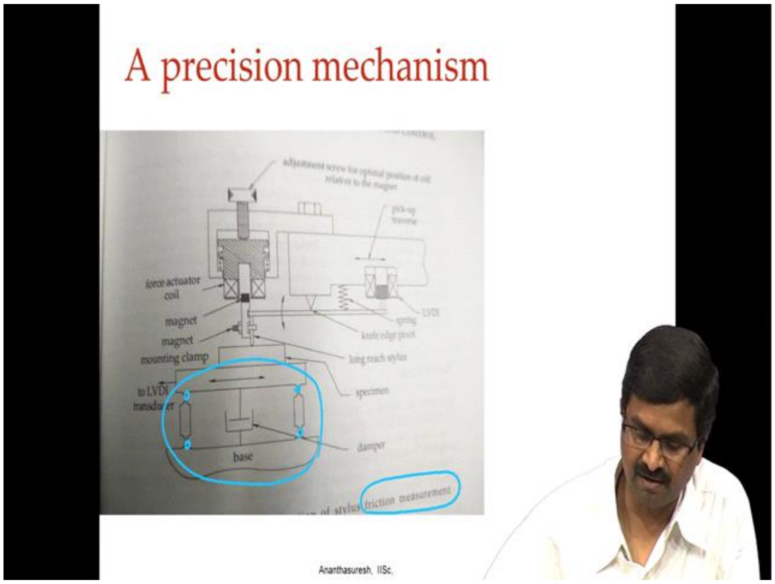

Here is another one which is for fiction measurement. Again you can see our familiar parallel motion flexure there with 4 elastic pairs. There are lot of applications of these in precision machines.

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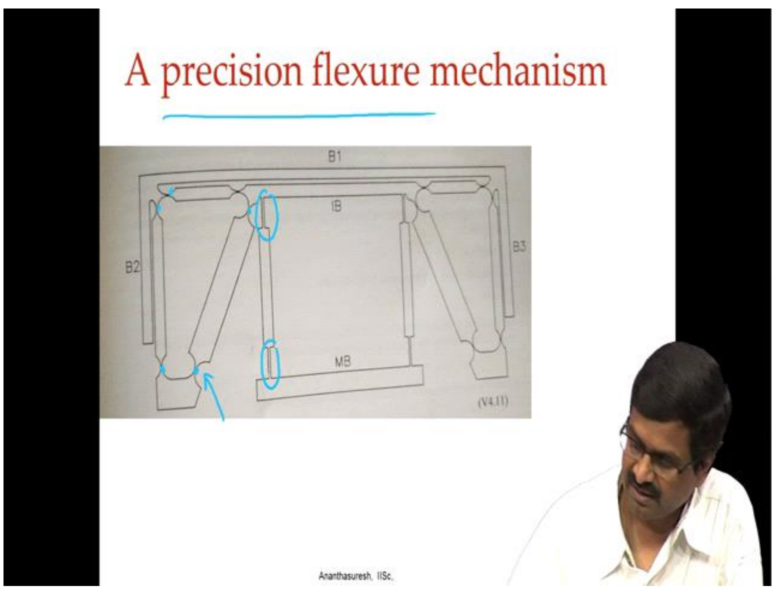

Here is another Precision Flexure Mechanism again you can see the joints. You also see slightly different type of flexure now. These are circular corner flexure as they are called a circular notch flexure were has these are little longer they are called small-length rotational elastic pair or small length flexure. What is important for them is basically this segment that can bend as it shown here. It was straight to began with like this let say this is the Base.

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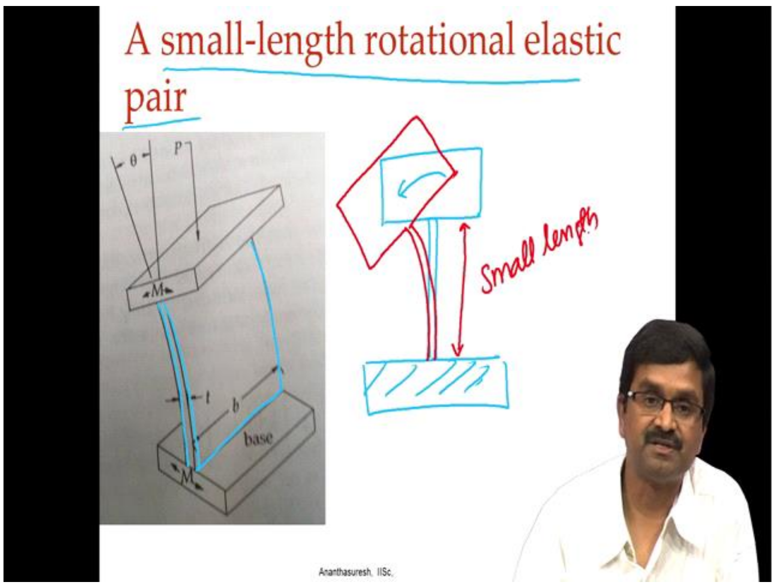

There is a relatively rigid body over here, if there were to be a movement acting on it these going to deflect like this accordingly this rigid body portion is going to rotate. They are small length because this length is small. Small compared to the size of the rigid bodies.

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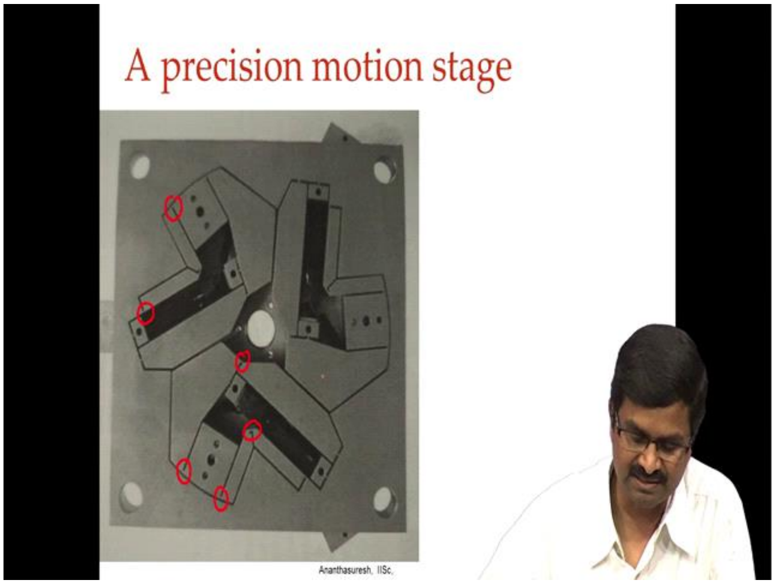

There also used widely in precision motion Stages, you can see if you look closely here you can see this small length flexure over there over here, over there over there and over here and there. This symmetric arrangement you take a parallel manipulator and replace the all the joints with flexures we get precision motion Stages and these are used widely as I already said and they are very expensive also all microscopes and well all microscope (Refer Time: 23:48) and everything that needs scanning need Stages that move very precisely people use this flexures. This technology has been around for a very long time.

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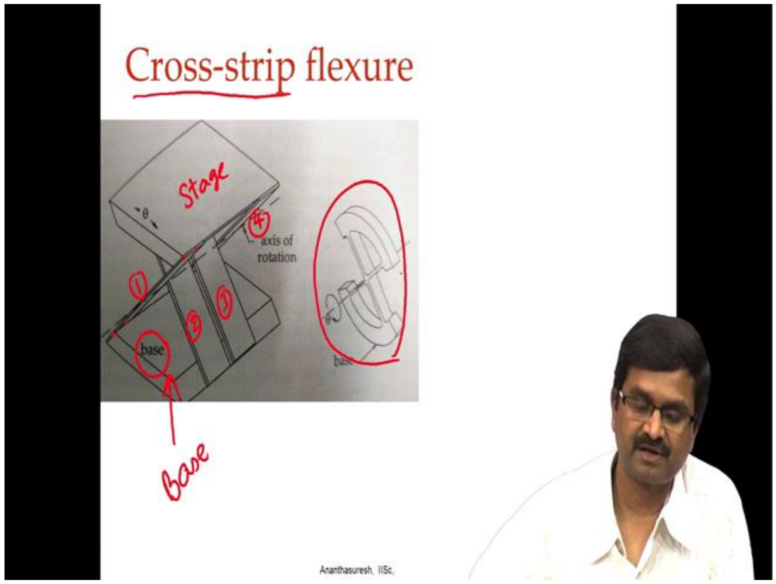

There are different forms of flexures we saw circular notch one and small-length flexures they are also what are called Cross-strip flexures. Now here you see there are 4 strips. This 1, 2, 3 and 4; which are arranged in a cross fashion and here also there are 4 flexures, but they are attached in the centre. Here may be some assembly is required to you know bolt these things between Base on the Stage, this is the Base and this is the Stage. This is the Base. You can connect them and its monolithic.

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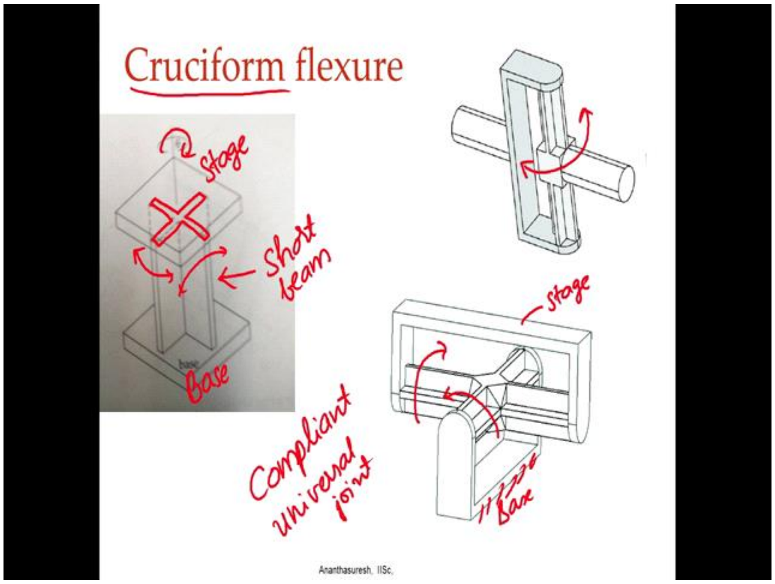

There are a lot of different types of flexures possible here another one called Cruciform flexure. What you have here is the cross section of a beam or a small flexure is plus it could be a cross also or this type plus or Cruciform are as it called. This is very interesting because it actually twists, it is a torque is applied over here by if you think of this as a short beam.

You have to when applied this is the again the Base and this is the Stage, here the Stage rotates but cannot bend as easily. Because the plus cross section will have very high second movement of area either direction. The bending about let say you know this way or that way is going to be very difficult, the stiffness is very high whereas, rotational or torque stiffness here is low. Here is a manifestation of that so, that you know it is able to rotate about this axis more easily but cannot bend and you can put two of those. That it rotates about that axis, rotates about this axis you have a universal joints Compliant universal joint. What we have here is a Compliant universal joint because it allows two rotations between the Base which is the Base here and this I can call it Stage or whatever rigid body that we have. I will have two rotation equipped to the Base.

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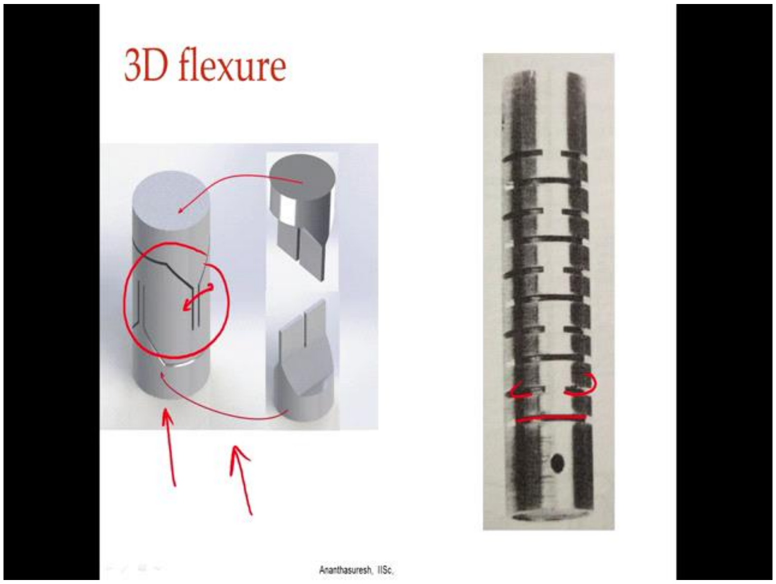

You can also have them in 3 Dimensional fashion. Here there is something shown you can take a solid cylinder cut out which let us say y d m or what is it cutting where you can make sections, the way it is shown there I have a little piece here, that you can see it may be difficult to see, but may be will have little focus in happening. What I shown there, this is a very little piece cylinder that can tilt like this, that can also tilt like this, that is what I maintain universal joint. If I attach a body here look to the Base this little cylinder with those couple of cuts can tilt like this and tilt like this. This has an interesting features that. You cannot tilt more than certain angle because we come back to the slide, we can see that when it tilts this going to just touch.

Over here when it tilts it is going to touch the other surface it cannot move much there is limited motion, but it is going to be strong, because it is not going to excessively deform with such a think if you take a long cylindrical rod by putting these cuts this is different design what you saw here. The easiest things to cut half a through like this and then turn around half a through there if you keep on cutting like this a steel rod can be bent like an elephants trunk or for that matter anything granite pillar if you cut like this, that can also be bent, but limited motion at limit motion like that is shown in this particular design.

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You might also have seen these toys. So, what we can do with elastic flexures. Here is little toy with the wooden where this tail like what I meant that if you take a cylindrical rod and put this thing we can do that. It has actually that so, this crocodile toy that you find in the markets you know that is all with flexures so it with flexure, you can get any amount of displacement per at each elastic pair the motion is limited and 3D flexures.

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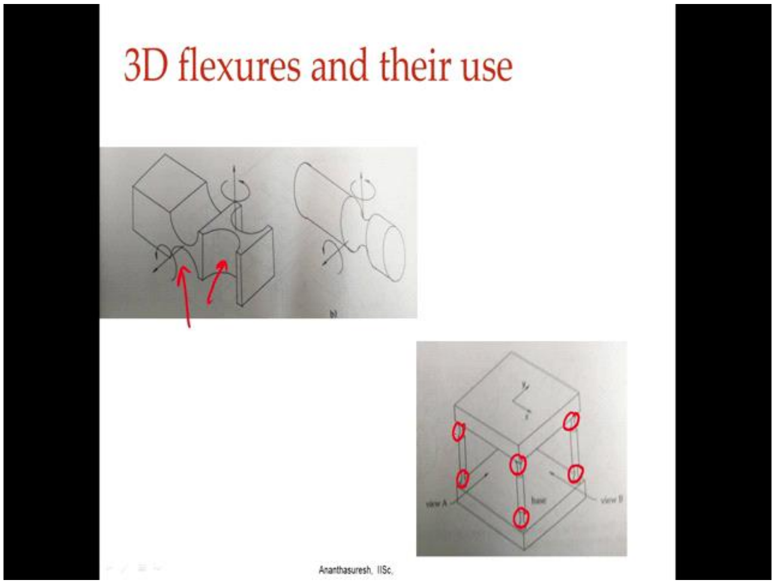

If you look at the slides again you can have 2 circular notch flexures with access rotated or you can have a 3 Dimensional equivalent of this circular notch hinges with which you can make a Stage like this in 3 Dimensions. You can see now these things are three dimensional versions of circular notch hinges.

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Now, you are removing hobs of sphere from the cylinder to create this. Lots of thing they can do with flexures. In the next part of the lecture we will look at the details what the stiffness is and how we can model this flexures.