Compliant Mechanisms: Principles and Design
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Lecture – 06
Using compatibility and force equilibrium matrices to identify degrees of freedom and state of self-stress in trusses

Hello; so, far we have talked about compliant mechanisms and their mobility analysis because the Grubler's formula that has been developed for rigid body mechanisms can also be used for compliant mechanisms which is what we saw in the last lecture. Today we will contrast and compare this Grubler's formula that is used in rigid body mechanisms literature with what is used by structural people that is let us say civil engineers, who do not want the structures to move at all or move very little they use something called Maxwell's rule and with that also one can find the mobility or degrees of freedom and there is another concept that we learn today called states of self-stress, this comes from the stiff structure literature and as we emphasized in this course compliant mechanisms lie in between rigid body linkages and stiff structures. So, we have the luxury of using concepts from both fields and enrich compliant magnetic field. So, let us look at the Grubler's formula and what its deficiencies are and then consider Maxwell's rules and what its deficiencies are turns out both are actually equivalent.

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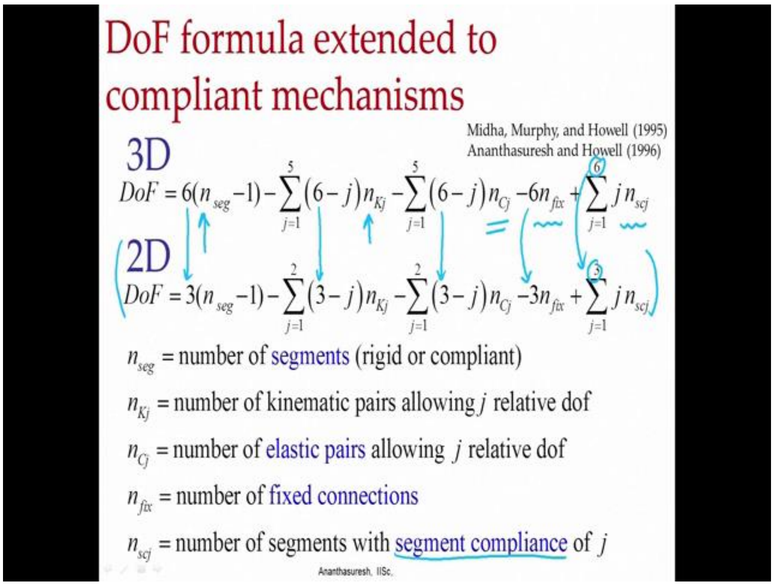

So, let us start with what we have already discussed which is this degrees of freedom formula for compliant mechanisms where we have modified it; the modifications once again I will underline, first of all this number of segments instead of just number of bodies and inclusion of elastic pairs while this corresponds to kinematic pairs, this corresponds to kinematic pairs now, we have elastic pairs and then we have this fixed connections which are needed whenever an elastic segment is connected another elastic segment or elastic segment is connected to a rigid segment; we have to have this fixed connections and then we had this concept of segment compliance which we have dealt with in the last lecture and at this point we all are familiar with how to use this formula.

We had done it for 2 D also which is shown here it is just that freedom 6 becomes 3 that 6 again becomes 3 here in all of these places and number of fixed connections, when we have against all 6 we have three because in planar case there are only 3 degrees of freedom and this goes only from 1 to 3 as opposed to 1 to 6 over there that is all the difference there is between 3 D and 2 D formulae.

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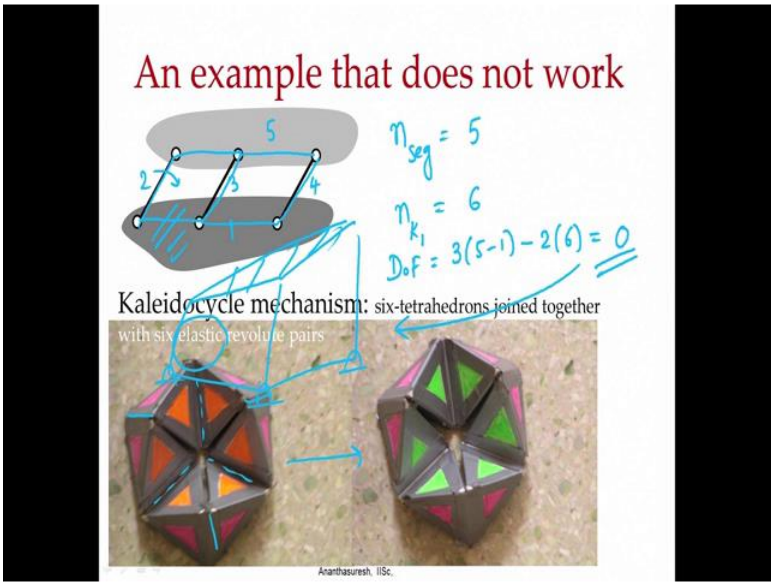

Now, let us consider this example which again should be familiar to all of you that if I say this is fixed frame and there are 2, 3, 4 and 5. So, n here or n seg whichever way we look at, we can call it n seg here there are no elastic segments here that is 5 and then n k 1 joints there are 6 of them here. So, degrees of freedom for this there are no n k 2 joints or elastic segments elastic pairs here. So, we have 3 into 5 minus 1 minus 2 multiplied by

6 that gives us 12 here 12 here 0. So, according to this formula is not supposed to move, but it does because it has parallelograms there is 1 parallelogram here, there is another parallelogram over there, because of that we can easily see that when I turn this it happily moves into 1 degree of freedom.

So, this formula here is not working and that is not surprising because we are only counting in Grubler's formula we are counting number of bodies or in modified form number of segments and the number of pairs whether they are kinematic pairs are elastic pairs and there are segments and assigning the segment compliance the number that we will learn to interpret with practice.

So, we are only counting we are not actually taking into account the exact geometry here, these 5 bodies are arranged to form 2 parallelograms and that is not reflected in the formula if I were to take arbitrary quadrilaterals here like this, where I have this ternary link and then here is where we have the fixed joint then it would not be able to move then this formula is actually correct with a special geometry that comes is what is making this formula not work.

There are a number of other examples here what you see are 2 configurations of what is called a Kaleidocycle mechanism and this one has a degree of freedom you have to make with 6 tetrahedrons join together with 6 revolute pairs. In fact, we have called it elastic pair here because we have joined with this flexible tape; there are 3 joints there and the 3 joints are like this they have particular configuration here, because of that even though it is (Refer Time: 05:53) with 1 degree of freedom. In fact, what we are shown from here to here is actually turning it around right just it just rotates about these axis in a very interesting way it makes an interesting toy.

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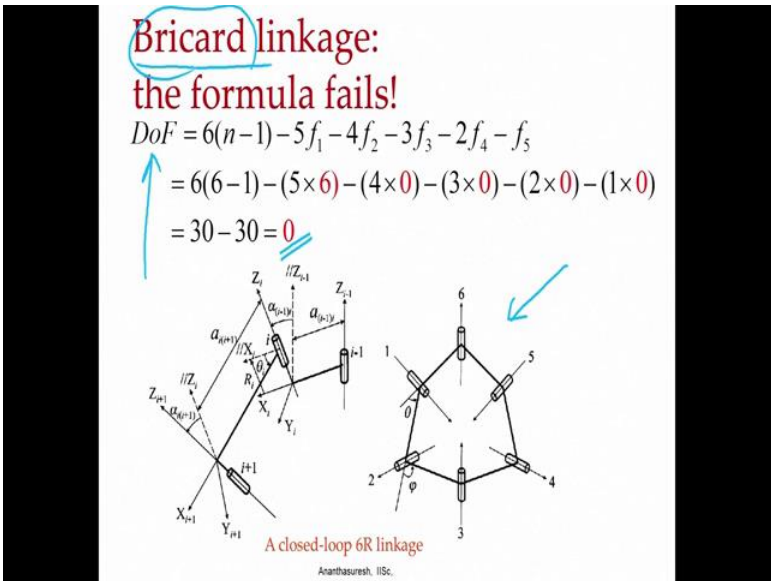

But actually has a name it is called a Bricard linkage because of the special geometry it has if you see over here in this figure all the revolute joints they are all hinges, they all meet at a point and they have a certain angle and circle symmetry to it, again that symmetry is not accounted for in our formula for degrees of freedom and hence, it predicts zero degrees of freedom whereas, this has a single degree of freedom, there are many more examples especially the special linkages where the formula fails with great glory, it is not the formulas mistake it is just that method that we consider are special geometries. So, special that we actually give the name of a person whenever we give name of a person to something it is very profound here Bricard realize that if you configure them in this particular way you get a degree of freedom that normally does not happen if you were to take 6 links 6 bodies arbitrarily with the 6 revolute joints. So, the formula fails.

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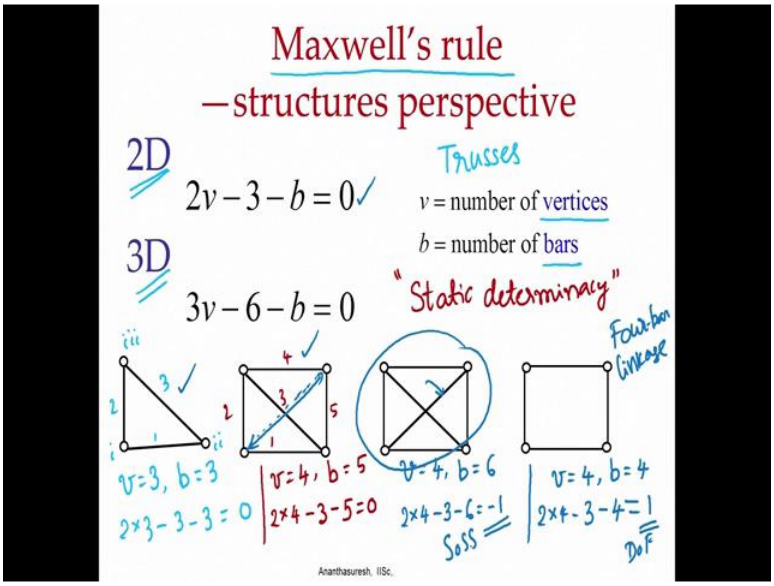

So, we look for alternatives; one of the alternatives because compliant mechanisms can get enriched by taking concerts from rigid body linkages as well as stiff structures, we go to this what is known as Maxwell's rule the same James Clerk Maxwell who gave us the equations of electromagnetism, he also had worked on structures a lot and optimization structure optimization mainly to build his instruments, he was such a great person he was able to contribute fundamentally to these fields as well, here is what we see a simple and elegant rule which named after Maxwell which again has a 2D version, planar version and a 3D version it is a very simple formula 2 v minus 3 minus b equal to 0 where v is the number of vertices and b is the number of bars.

So, here he is looking at only Trusses are very important elements of structures you know we find lot of trusses around us he was analyzing trusses and he found this interesting relationship which is called Maxwell's rule now, 2 v minus 3 minus b equal to 0 in 2D, 3 b minus 6 minus b equal to 0 in 3D.

What he says is that if a truss satisfies this relationship in 2D or 3D depending on as a planar truss are a spatial truss, that such a truss will be just stiff meaning that it has enough constraint so that it does not deform it can deform by last deformation oh sorry different thing it would not be able to move meaning that it is stiff. So, if we take let us say our first example of a triangle 3 bars. So, here equal to v is equal to 3 and b is also equal to 3 there are 3 bars 1, 2, 3 and then 3 trusses I can call it 1, 2 and 3, 3 vertices and

3 bars. So, if you see this formula 2 times v that gives 2 times 3 minus 3 minus number of bars 3 that is equal to 0 so; that means, that a triangle is just stiff we can feel that now we take triangles made with 3 bars it will not have any play it is actually stiff.

Now, let us take this example where the 2 triangles are connected together for this one let us write the Maxwell's rules let me choose a different colour. So, we do not mix it up. So, here vertices are 4 and number of parts is 5, 1, 2, 3 we have added 2 extra bass let us call this 4 and 5 while this is 1, 2 and 3, if I write this formula this is a planar truss. So, 2 times 4 minus 3 minus 3 minus 5 again it is equal to 0, meaning that this is also just stiff as Maxwell said when you say just stiff it also has another connotation that these trusses are statically determinate. Meaning that the internal forces in the members of the truss can be determined purely from equations of static equilibrium that is what we mean by static determinacy, that I am sure you would have heard in your mechanics of materials class when you discussed beams and trusses and so forth. Static determinacy what it means is that from equations of static equilibrium alone we can determine the internal forces in the truss.

So, now let us take this one. So, let me erase this in a little bit and choose a different colour. So, there is no confusion again now how many vertices are there 4 only 4 like the previous 1 and how many bars are there, there are 6 now let us write the formula Maxwell's rule 2 times 4 minus 3 minus 6. So, this one has minus 1 because this is 8 minus 3 5 minus 6 minus 1. So, this shows that we have a case where it does not satisfy the so called Maxwell's rule, in that case this particular truss is not statically determinate which we know and it is not just stiff; it is more than being just stiff we can see that from here to here when I am adding this extra line I am not actually stiffing in the structure I am over constraining it. In fact, if 6 bars are given which have arbitrary lengths when you assemble if they follow triangle inequality you can always assemble and same thing happens with 2 triangles if the bars are a given satisfy the triangle with equality, but here after this if I give the sixth bar to be put in here if it is shorter than or longer than this distance in order to assemble this one either I have to stretch it or contract it if it is shorter I have to stretch it if it is longer I have to contract it and then assemble.

When I do that this particular structure will be in a state of stress it will have some residual stress even before you apply any external forces, such a thing is called a state of self-stress it comes from the way it is assembled and not because of external load acting on it and that is what Maxwell was getting at. So, he said that whenever it is 0 then it is just stiff if it is negative then it has this state of self-stress. So, in this case it is minus 1 it has 1 state of self-stress. So, this all interesting, but if you take an example such as this 1 what I have shown here is a basically a rectangle. So, here if I were to write it number of vertices 4 number of bars only 4 now if I write Maxwell rules 2 times 4 minus 3 minus 4 equal to this is 8 minus 7 that is 1.

So, this particular 1 actually gives degree of freedom if you notice this is nothing, but our 4 bar linkage. So, it actually gives a degree of freedom that this one has the Maxwell's rule can be applied instead of Grubler's formula because it gives you degrees of freedom, but also gives you what we call state of self-stress. So, let us look at that in a bit more detail.

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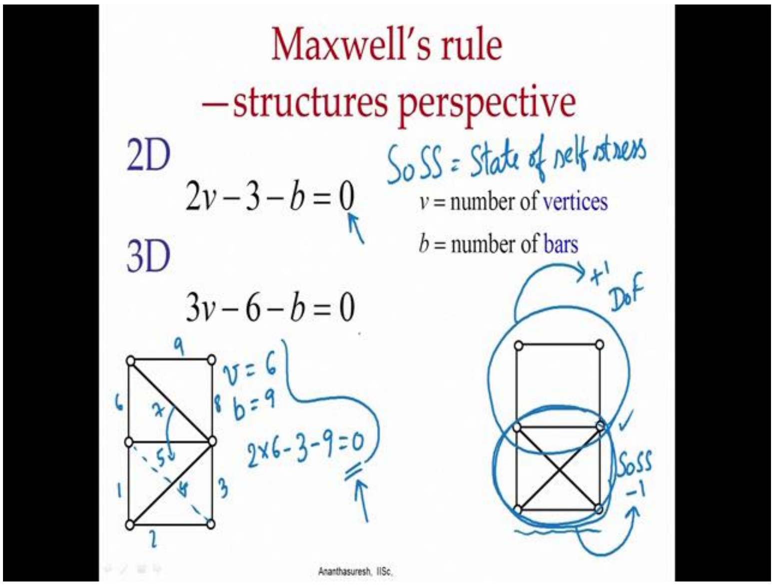

Let us take this example now. So, here again if I say number of vertices is 6 number of bars 1, 2, 3, 4, 5, 6, 7, 8, 9, 9 bars and if I write this formula 2 into 6 minus 3 minus 9 equal to 0, it is just a stiff that is obvious when we look at it because you can only triangles, triangles are just stiff; they do not have any over constraint, but now if I go for this where I have taken this bar and put it like that that is all that is a change that we see over here. So, in terms of number of vertices, number of bars whatever we have written holds for this one also, but then here it says 0, but we just discussed that this part of the structure has 1 state of self-stress.

So, this SoSS what we mean is that it is state of self-stress or we can understand this has over constraint just like a degree of freedom require certain mobility some movement to a truss or a linkage both are actually equivalent if you view from the perspective of Maxwell's rule. Something that has over constrained such as the one that we have encircle here it has a state of self-stress that is you have to stress it in order to assemble if you are given 6 arbitrary links, but then the top portion here is a 4 bar linkage. So, that has a degree of freedom this has a set of self-stress. So, according to the formula this will have positive 1 degree of freedom this has accounted instead of 0 will have negative 1 as we saw. So, this particular portion from the formula x minus 1 this other portion gets plus 1 both of them added give 0 which is what we see here.

So, there is confusion when you apply Maxwell's rule that sometimes you want to get 0 you do not know if some part has a degree of freedom some part has is over constraint or state of self-stress.

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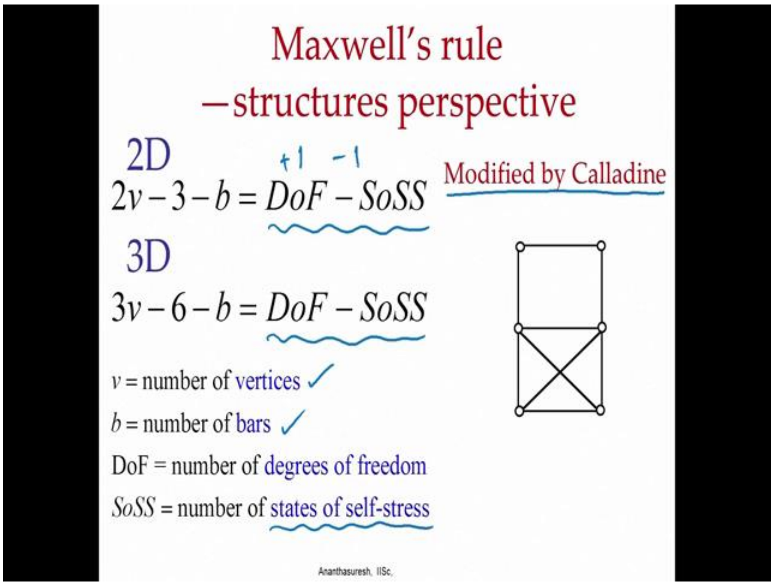

Hence the Maxwell's rule was modified by a professor at University of Cambridge called Christopher Calladine, what he did was instead of 0 he put this he said degrees of freedom minus state of self-stress or SoSS states of self-stress in plural what it means is that. So, much over constraint is there which is the opposite of degree of freedom that is how we should understand. So, again number of vertices v number of bars b now you get DoF minus SoSS the same thing applies to 3 dimensional trusses as well when you have something like this when it is 0 it actually means there is a plus 1 and then a minus 1, 1 state of self-stress 1 degree of freedom.

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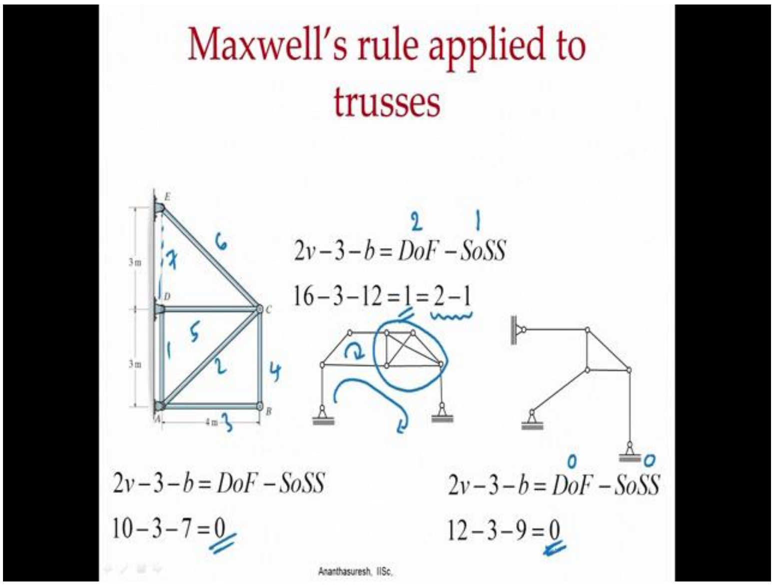

We have some examples this is taken from an engineering mechanics book which is a plain old truss we have there are 5 vertices here which you can count and there are spin bars. So, we see 1, 2, 3, 4, 5, 6 you may wonder where is seventh one the seventh one is here because it is connected of the fix frame that we have that bar has to be connected to make this stress by the way Maxwell's rule does not consider the fact that 1 body is fixed it is just that the truss as if you are thrown into space. So, if you see that it comes to be 0. So, it is just stiff it statically determinate that you would have learnt when you consider trusses let us take 2 other examples.

If I look at the middle one if I again count the number of vertices here there are 8 and the number of bars there are 12 then you get this Maxwell's rule is 1 here as we will discuss a little later in this lecture, it actually turns to be 2 minus 1 meaning that there are 2 degrees of freedom 1 state of self-stress 1 over constraint and 2 degrees of freedom, maybe you can see over constraint already in this portion because we consider those examples and you would also see degree of freedom in this loop that is a four bar linkage and then in this loop once it is stiff there is another 4 bar linkage here. So, there are 2 degrees of freedom and another hand if I go to the third example turns out it be 0 it has 0 degrees of freedom 0 states of self-stress. So, given a truss we do not know how to find even if formula presents a number here the Maxwell's rule we do not know how many degrees of freedom are there how many state of self-stress out there for which we have to have a method.

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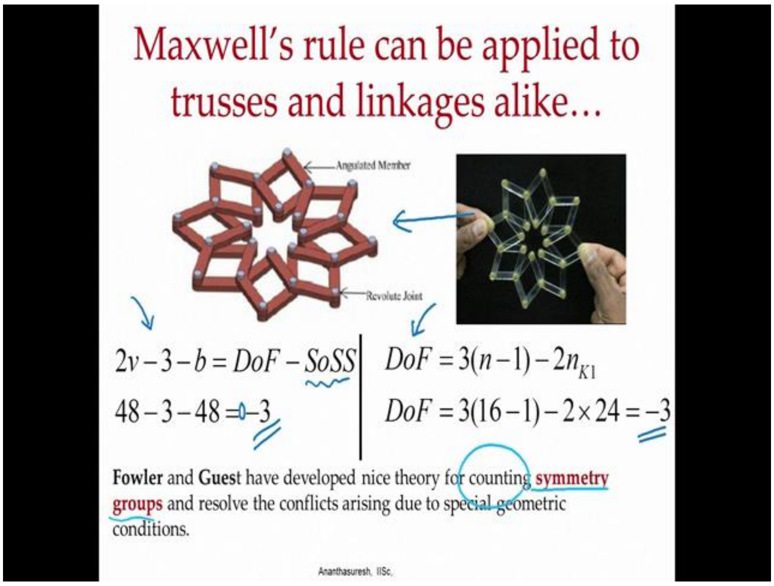

Before that we have this Maxwell's rule applied to a linkage normally we would use the degrees of freedom formula of Grubler, but here if I do with the Maxwell's rule we get the same answer meaning that there is minus 3 that is minus 3. So, minus 3 whenever degrees of freedom formula given by the Grubler's formula is negative that actually means that over constraint, that is negative degrees of freedom which according to this is like 0 minus 3; that means, there are 3 states of self-stress this particular one again is a is a linkage that you have seen it moving in the first lecture itself it has clearly a degree of freedom the formula both formulae the Maxwell's rule as well as Grubler's formula predict minus 3 degrees of freedom or 3 over constraints account state of self-stress.

Clearly there is no stress here we are able to move it with 1 degree of freedom the reason again is that very special symmetry in this is highly symmetric mean there are special geometric conditions that exist in many special linkages or our parallelogram linkage and so forth. And when those are there the formulae that we have seemed not. So, useful that is true, but when there is arbitrary geometry the formulae are very useful when you have special geometry formulas are not able to predict the degrees of freedom or states of self-stress for that we need to use group theory using some special Symmetry Groups, I think I have to get that (Refer Time: 21:44).

So, it does not fly we have to use this symmetry groups which chemists actually use a lot the group theory to look at the molecules which have similar constraints of course, in a different context altogether those configured arise due to the symmetries of special geometric conditions can be done by still going for counting if you notice both Grubler's formula and Maxwell's rule rely on counting there is no computation there is only counting when you count, if you want to stick to that counting method then you have to go to this group theory and then count the symmetric groups that exist in a particular geometric entity such as the truss or a linkage or a compliant mechanism or whatever, but that requires entirely different tools of mathematics, but if we stick to engineering mathematics there is a way out of that which is what we will discuss now.

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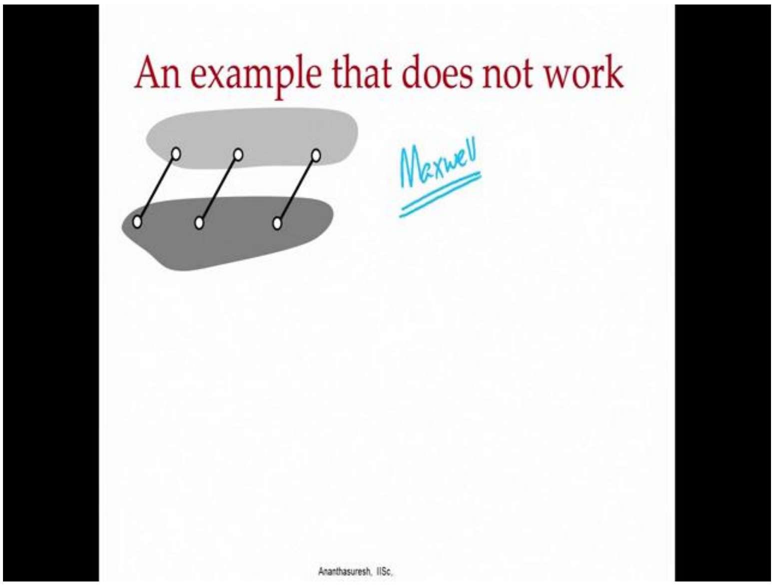

Again if you were to do this Maxwell's rule I would recommend that you try and you would see that Maxwell's rule also does not work.

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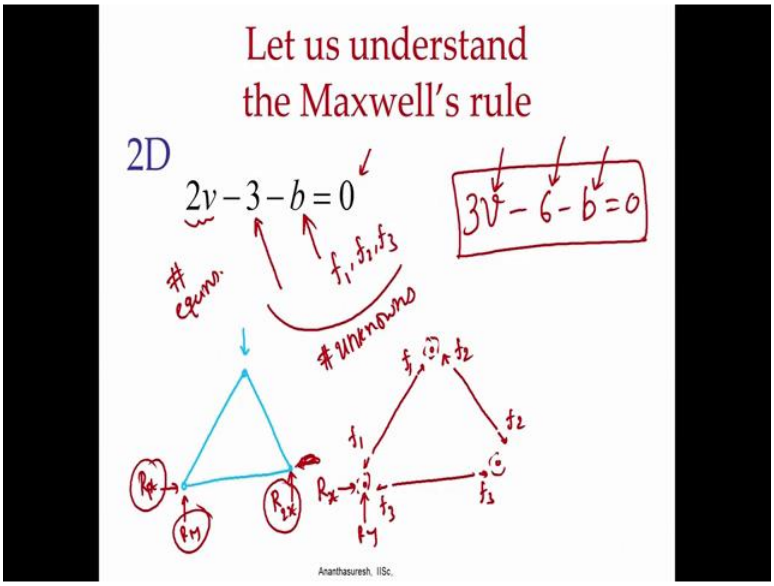

Now, let us go to just understand this Maxwell's rule before we move on to a technique with which we can identify degrees of freedom and states of self-stress even when there are the special geometric conditions of symmetries a computational techniques that gives you rather than merely counting. So, before the lenssor and Maxwell how does it come about? It comes about because this whenever you have let us say I take a truss such as this and there are some forces acting and there could also be the ground reactions as we call let us say this one has a pin point to the ground that the third link is fixed. So, we have that and if it is sliding there will be only vertical reactions we can freely slide in this direction I have to call simply support condition.

If you put that and draw the free body diagram for this truss, for each vertex at each vertex there will be the bar forces let us say we assume that all bars are in tension let us call this is f 1 hence it is f 1, f 2, f 2, f 3, f 3 and those will come on to this vertices also in opposite directions, at this vertex and I have this ground reactions I can write 2 equations in equilibrium at each of the vertices that is why we have 2 v which is the number of equations.

What are the unknowns? Unknowns are each bar has 1 internal force that is in this example it is f 1 f 2 and f 3, if it is static determinate which case this becomes 0 this minus 3 should be here because that is also the unknown that is reaction forces we have I can call this for x or y and then there is another one for not this one over here. So, if I call this R x, R y let us say R 2 x or something if I call R 1 x and r 1 y. So, that 3 reactions for static determinacy that is where we have this 3 coming.

So, basically we are counting the number of equations and number of unknowns, when that is equal the bar is just stiff satisfies Maxwell rule the same thing applies in 3 dimensions where we have 3 v minus 6 minus b equal to 0 that is a three d version of Maxwell's rule, at each vertex you can write 3 equations f x f y f z equilibrium and the number of bars remains the same and then 6 unknowns because in order to make it 3 dimensional truss just stiff we have to have 6 reactions and that is how the Maxwell's rules comes.

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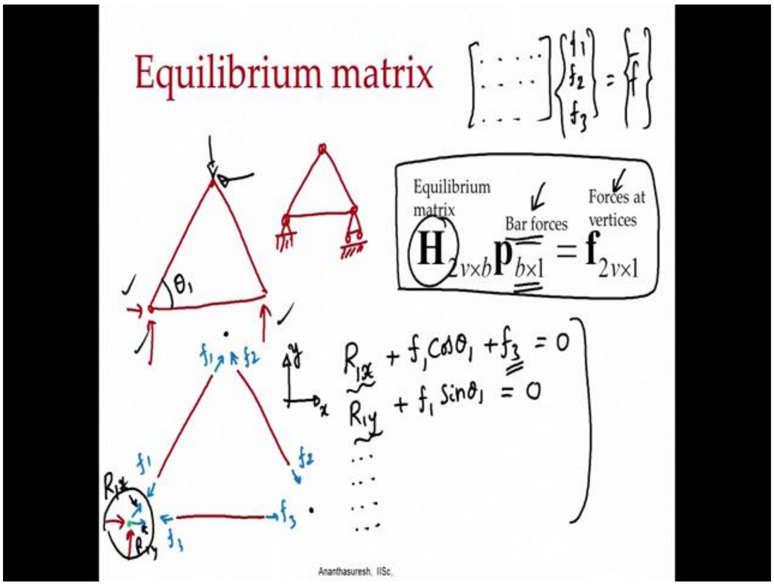

Now, let us understand a computational method for identifying special geometries as well and for that whatever I just wrote there if I have a truss I want to write the equilibrium equation teach each vertex, let us I take this vertex let say there are ground reactions like that meaning that our truss is fixed here whereas, here it is spinned and can slide whereas, this just a spin if I has such a thing these are 3 reaction if I take each bar let us I take a separate out each bar like that let us show the reaction the bar what I just called f 1, f 2, f 3 assuming that all are in tension. So, this is f 1, f 2, f 2, f 3, f 3 and let us show on the vertex map this vertex we already have this forces. So, let me show them in that colour. So, here we have that force and this force at this vertex and then the forces due to the bars also act on that in the opposite direction if it is like that it will be like this will be like this.

Now, let us equally break this vertex the vertex that we have let us study equilibrium equation for that let us let me give some symbols for these calling this R 1 x this is R 1 y and then f 1 f 2 we have I can write an equation in the x direction r one x in the positive if I take my x axis like this and y axis like that that will be R 1 x and then I will have f 1 if I take this angle to be let us say theta 1 I can write this as f 1 I am looking at this force now, f 1 cosine theta 1 and then I have also f 3 plus f 3 I am looking at that 1 now that should be equal to 0. Similarly I can write in the y direction I have R 1 y and I will have f 1 sine theta 1 the vertical component of our f 1 at that vertex and then I do not have anything else equal to 0 those are the 2 things in vertical direction.

Similarly, I can write for this vertex and that vertex I get one more equation one more equation one more 4 more equations, they can be put into the form of a matrix which is what is shown here, what it means is that what I called this f 1 f 2 are the bar forces. So, let us I have f 1, f 2, f 3 there only 3 bar that why size of this is d by 1 is equal to I can take this f 3 and external forces not f 3 this R 1 x, R1 y all of those are external forces I can put them here. These are the external forces which is this f vector here, whatever entry is that we get that is these equations put into matrix form is this Equilibrium Matrix all we are doing is we are getting an equilibrium equations put in the matrix form. So, that there are bar forces which are unknowns forces vertices are known to us of course, the reactions we do not know, but if we apply the static equilibrium for the whole truss you can actually calculate this reaction forces for given applied forces on the truss you know how to do that; that is what this equilibrium matrix means.

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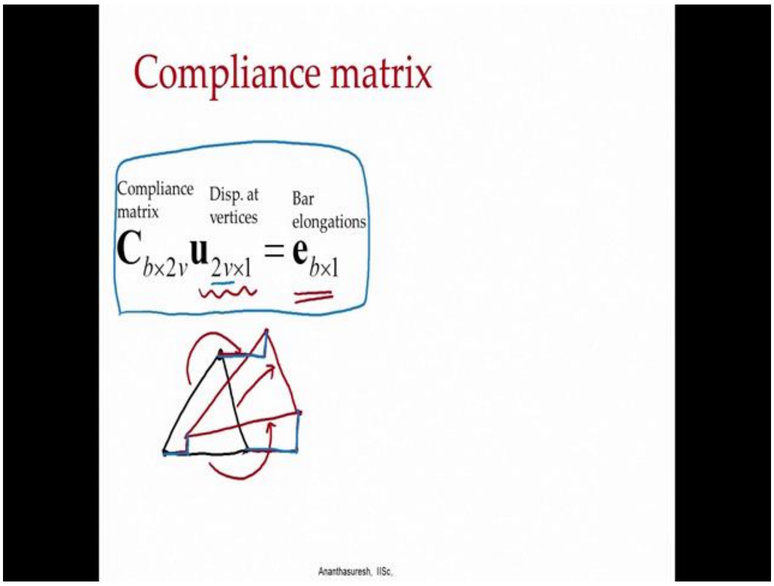

This equal matrix has significance in the sense that when it is not a full rank it gives you certain thing as we will discuss later before we do that let go to something that takes care of displacements and elongation, again if I have this three bar thing let us say I move them arbitrarily. So, this goes there this goes here this vertex goes there which it become something like that. So, this bar has elongated this bar has elongated this whereas, this bar has contracted. So, there are these elongation; this bar elongations can be compared to or related to vertices displacement for example, this one has x displacement, y displacement this has x displacement, y displacement these are x displacement, y displacement let me draw them with a different colour. So, you can see x displacement y displacement x displacement y displacement x displacement y displays vertices that is why it size is-2 v by 1. So, these can be related using (Refer Time: 31:38) trigonometry linearize so that you can put in the form of a matrix like this that is called a compliance matrix this also has interesting properties that we will understand.

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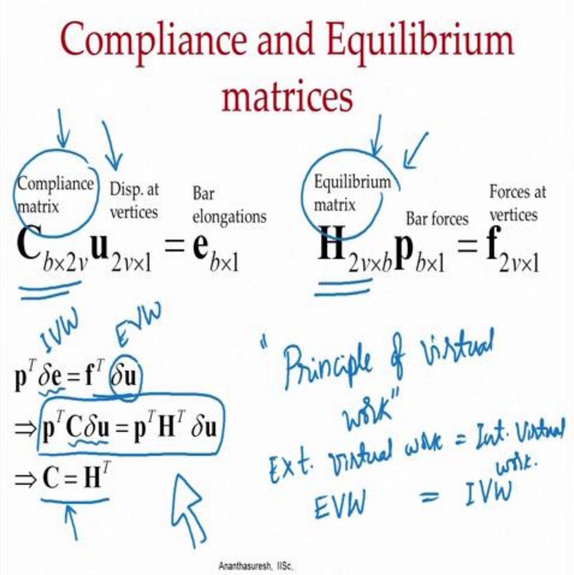

And interestingly these two matrices the C matrix and H matrix the compliance equation matrix equal equilibrium matrix have a relationship that is shown here which follows from the principle of virtual work.

If you recall principle of virtual work states that external virtual work is equal to internal virtual work and here we have that external virtual work is equal to internal virtual work, this is external virtual work this is internal virtual work, what is it mean we imagine virtual displacement delta u the truss is that we just imagine they are not real displacement. They are virtual displacements then what is the work done by external forces f transpose delta u is external virtual work internal forces bar forces are pleased they multiplied they are multiplied with elongations which are again virtual elongations because of virtual displacement that gives internal virtual work, they being equal to each other is the principle of virtual work and here if you substitute taking sort of principle of virtual work for delta e you substitute C delta u that comes from this relationship and likewise for f we get H times p again f transpose p transpose, if I look at this equation now you conclude that C is equal to H transpose. So, you can either start a equlibrium matrix or compliance matrix they both are transpose of each other, you should try this we have not written this we just wrote partly this one for a simple truss if you do that you will see that its true and the proof of course is right here.

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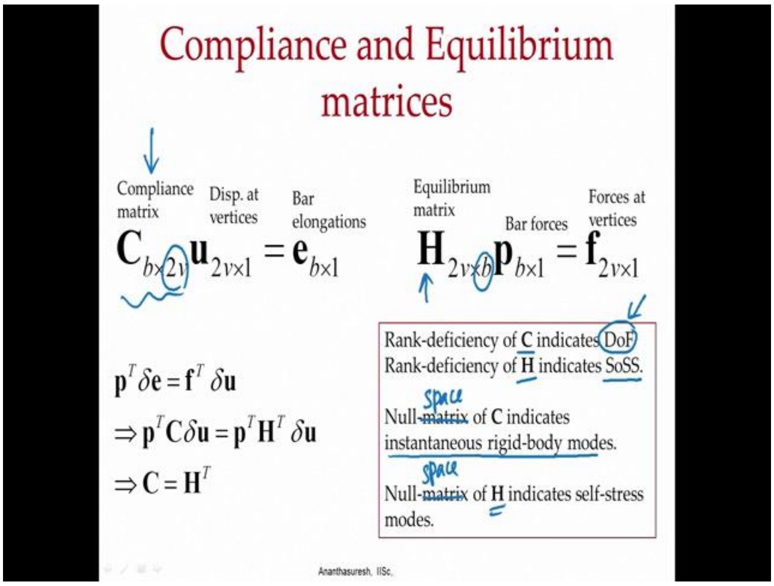

Now, why do we care about this, it turns out that the rank deficiency of the C matrix indicates degree of freedom of a truss or a compliant mechanism if it is made of bars which most complaint mechanisms are, likewise ranked deficiency of the H matrix indicate the state of self-stresses states of self-stress and this null space of the C matrix indicates instantaneous rigid body modes and similarly the null space of this H matrix indicates states of self-stress and their modes meaning. If there is a truss and which part of that truss has a degree of freedom and which part of the truss has set of self-stress will be indicated by looking at the null space of this rank deficient, if it is not rank deficient it is a full rank; that means, that there are no degrees of freedom and this are full rank there are no states of self-stress, whenever there is a rank deficiency here we should look at the column rank or this in this case 2 v in this case it is b or what you type in math lab rank of a matrix, it usually gives this column rank and we look at that if it is not a full value; that means, that that particular truss has a degree of freedom looking at compliance matrix and the corresponding null space vector or null space vectors if rank deficiency is more than one gives you the rigid body modes and likewise for equilibrium matrix we get states of self-stress.

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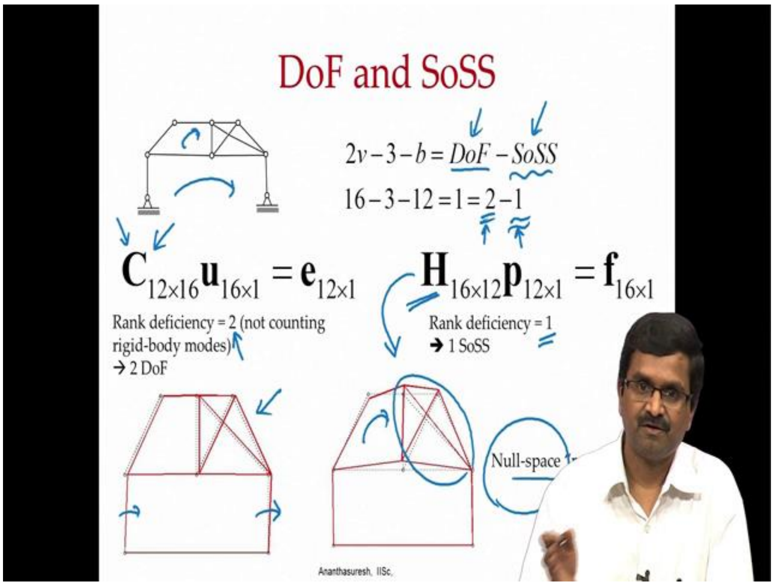

So, it is best to see that with an example that we already considered that this one has 2 degrees of freedom and one state of self-stress.

So, if I construct the C matrix you will find that it is ranked deficient by 2 and that is exactly what we have here 2 degrees of freedom, we look at H matrix for this truss you will find that it has ranked deficiency of one which exactly what we have here. So, if I look at the null space modes of this matrix I get this. So, here in one case this part is simply moving that is 1 degree of freedom that we already said there is one degree of freedom for this portion there is another degree of freedom here which we see over there. So, this is moving like a 4 bar and this part is movingly more like a rigid body because that is what it is and if you were to look at the modes null space modes of this H matrix now, you will also find how the stress is distributed among these elements just like this gives motion null space vectors of H matrix give you the state of self-stress. Looking at one way this physical example of trusses using this compliance matrix and this equilibrium matrix we actually understand the null space another linear algebra concepts from physical perspective and you should review some of the linear algebra you will not understand what null space means for these matrices, but important thing is that by doing little bit of computation we are able to actually get the degrees of freedom and state of self-stress split from the Maxwell's rule.

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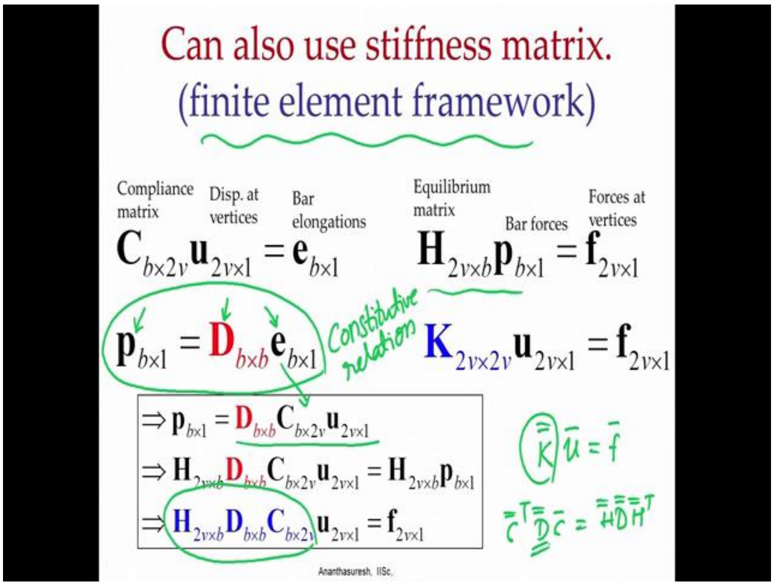

And we can also use stiffness matrix which is from the finite element framework here we have the compliance matrix relating displacements and bar elongations equal to the matrix relating bar forces and forces acting on the vertical external force, but what we want is actually the stiffness matrix.

So, let us let me erase this little thing what we want is a stiffness matrix which is going to relate stiffness matrix K relates vertex displacement to forces on the vertices K u equal to f like K x equal to f like a spring equation, and that comes out here because for that we need additional relationship which can be thought of as constitutive relationship constitutive relation, what this is saying is that how are bar forces internal bar forces are related to elongations that will be a function of cross section properties as well as material properties which are contained in D for a bar you know this a e by l which the stiffness of spring constrain of the bar which is what this is the bar force versus its elongation. If you define that then we can get the stiffness matrix we start P equal to D e and then substitute for e C u and then we have the H into p, p we have now that h into p comes from this equation H into p that we have here, now if you look at this again you try to substitute for this H p that we have now which is from this equation is that force itself force vector.

So, what we have here in this thing what we have there is our stiffness matrix. So, by having this constitutive relationship that D we get H D C or C transpose D C or H D C a or equal to H D H transpose because we already discussed that C and H are related by transpose, this is what we have in the finite element framework where we normally use a letter b to relate strains and displacements instead of C we use a b it will be b transpose D b, D is basically stress strain relationship there that is what we have in this framework for the trusses.

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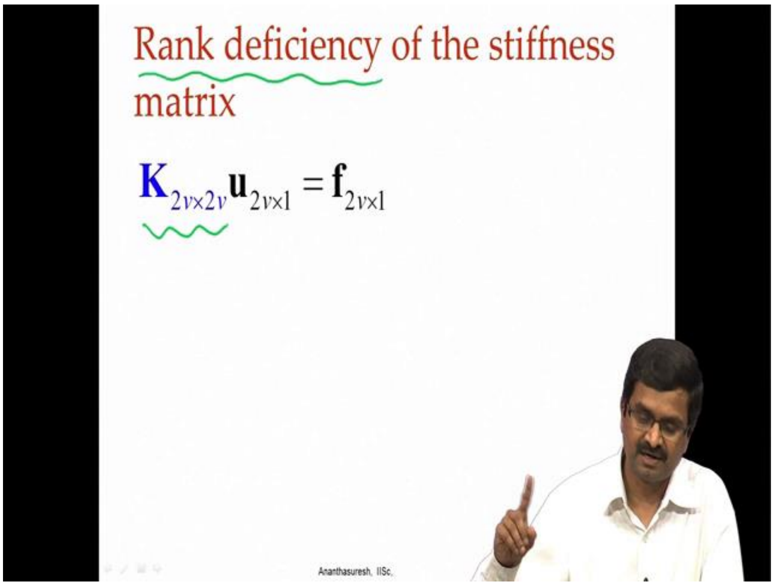

If you look at the stiffness matrix it is ranked efficiency indicates rigid body modes again; that means, that the structure is not stiff there may be over constraint which it will not be able to tell you, but if there is a ranked deficiency then it tells you that it has a degree of freedom; that means, the thing is not properly constrained which has like Grubler’s formula like degree of freedom, that is the stiffness matrix can also be used if we computed using finite element framework for a truss that also will give you the degrees of freedom, but not state of self-stress for that it construct the equilibrium matrix H and look at the rank deficiency of that matrix just to conclude this lecture.

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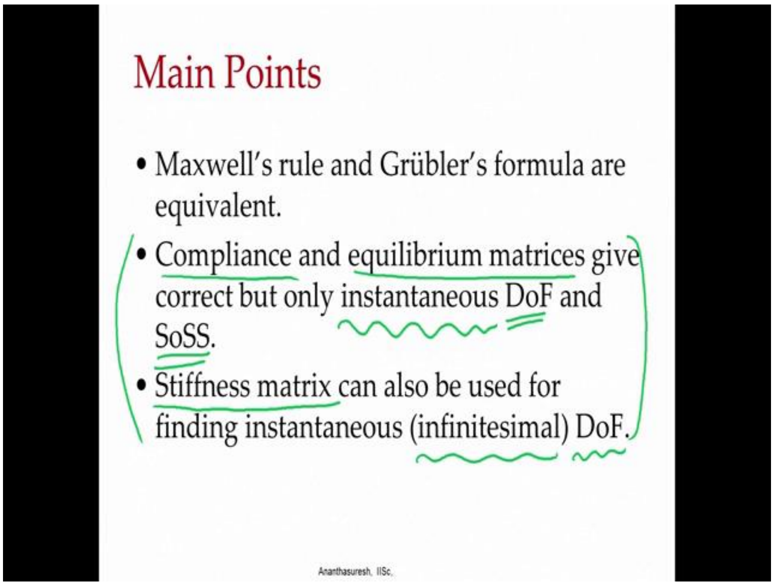

We have a few points to note that Maxwell's rule and Grubler's formula are equivalent when one fails the other one also fails, both work by counting in the case of Maxwell's rules number of rule number of vertices the number of bars Grubler's formula number of bodies which is number of bars, but also the joints and the type of joints and so forth.

That will not tell you whether there are any self-stresses or degrees of freedom it only tells you the total number or DOF minus SoSS, but if you look at the compliance matrix and equilibrium matrix their rank deficiency not only tells you the degrees of freedom state of self-stress, but one thing to note is that that this computational method also tells you only instantaneous meaning in that particular configuration what are the degrees of freedom which sometimes are called infinitesimal modes that is all it will tell you, but not or a finite range because we have linearized equations over there. Degrees of freedom and state of self-stress come from rank deficiency of respectively compliance matrix equilibrium matrix, but they are only instantaneous, but if you want in the finite element framework stiffness matrix that can be used or rather it is ranked efficiency gives you instant instantaneous or infinitesimal degrees of freedom in that configuration if you change the configuration it will re do the rank of the stiffness matrix and find if that is ranked efficient or full rank, there are some linkages or trusses whose rank will change from configuration to configuration they are called metamorphic mechanisms or metamorphic trusses. So, they are infinitesimal modes can be found in any configuration when doing this rank analysis. So, we have discussed the counting method which fails if there are special conditions and a computational method these two, where we can find instantaneous or infinitesimal degrees of freedom are states of self-stress and this mobility analysis is crucial to design a compliant mechanisms, it would not guide you to design something, but after you design you can analyze it to see if it is properly constrained or not we will see some examples later on in the course.

Thank you.