

Compliant Mechanisms: Principles and Design
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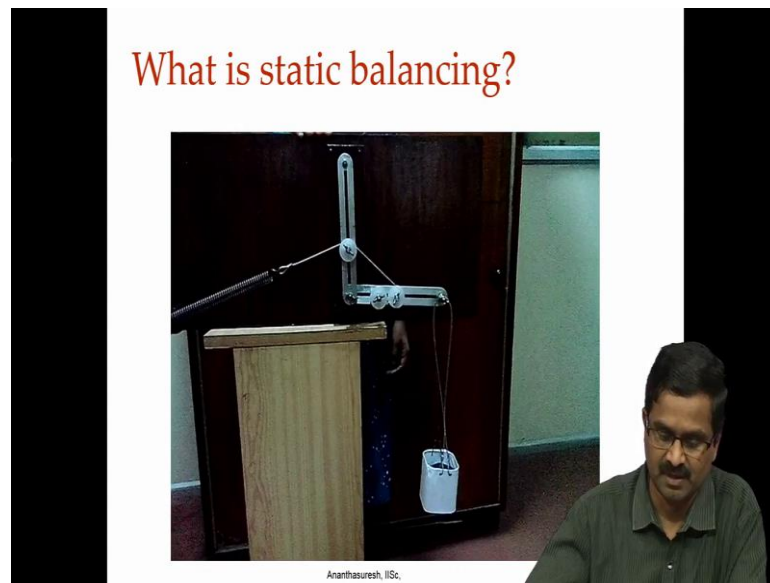
Lecture - 59
Static balance of a compliant mechanism using a linkage

Hello, this is second lecture on static balancing again this is one of the final points of Compliant Mechanisms, third final point the first one being mechanical advantage, second one by stability, now third we are discussing static balancing, all of this have the theme of making compliant mechanisms better, because in compliant mechanisms part of the input energy is used in deforming the mechanism. So, that is not available as output work and that is a disadvantage. So, now, as we discussed in the last lecture if we consider static balancing for compliant mechanisms, that will involved usually a preload then we would have illuminated that disadvantage that part of the energy that is used in the complaint mechanism is actually not loss to do the output.

So, we look at this static balancing now in last lecture we discussed, the general way of statically balancing a rigid body linkage with a spring load or a constant force load using only springs without adding any auxiliary bodies that was the previous method, where as the new method that we discussed involved only springs and that is a good thing for a compliant mechanism, because compliant mechanism is itself a spring in todays lecture we would see how by treating a complaint mechanism as a spring how that can be balanced with a linkage, when you say linkage there will be a linkage and a spring load the spring load comes from the compliant mechanism itself because complaint mechanism is nothing, but a spring.

So, let us look at this that we have static balancing of a complaint mechanism using a linkage with a spring load and that spring load is related to the compliant mechanism itself. So, our spring load is related to be complaint mechanism itself. So, that is our discussion today just to recap what is static balancing.

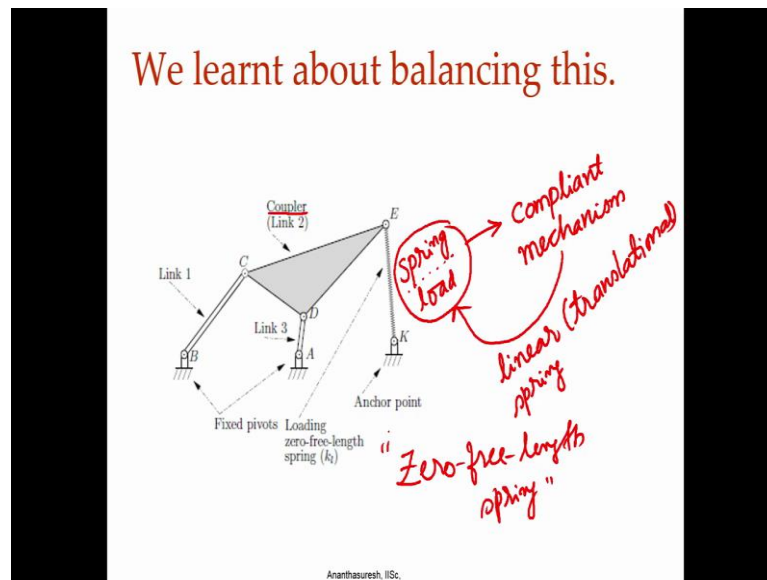
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So, here is where we have a particular device it is basically, a linkage that has constant force here that is like a gravity its actually, gravity balancing here and we can position it wherever we want as you will see and it is not because, of the friction at the joints there is a spring here that is a positive free length spring, where as that is converted with the help of strings that spring and string here convert and have a 0 free length spring between that point and this point.

This is the method of Lucien Lacoste as we had discussed in the last lecture. So, let us play this video where now it is statically balanced you move it somewhere else its stays, there if you move it somewhere else it stays also. So, it is perfect static balancing as we have seen in the last lecture. So, we can put wherever and it will stay put even though gravity load is acting it is like angle poise lamp, where a table lamp concept.

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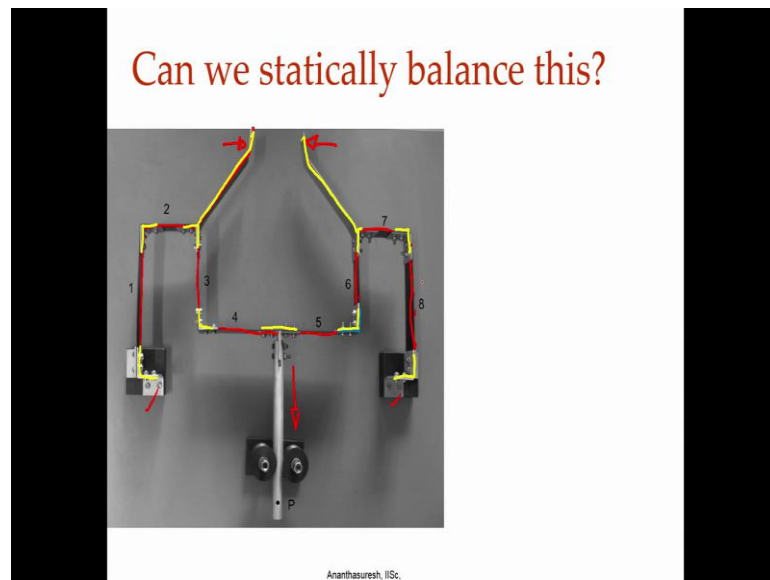


We had also discussed in the last lecture, if I have a 4 bar linkage such as the one that is shown here, if we have a 4 bar linkage with a spring load we have a spring here between points e and k otherwise a 4 bar linkage with a coupler body and so forth.

Now, can we statically balance and that is what we had discussed in the last lecture by adding only springs are not additional bodies. So, couple of springs added here and there, will compensate the spring load that is in this mechanism we already discussed how to do that in this lecture what we do is how about we treat this spring load as a compliant mechanism itself. So, a compliant mechanism is essentially a spring if we consider a law small range of motion in which the stiffness is the complaint mechanism can be thought of as a spring whose spring constant is constant where as short range.

Once we have that spring we can perfectly shorted balance, but approximation is treating a complaint mechanism as a linear spring and is a linear in two senses, one is that it is translational and the other is that its force versus displacement characteristic is linear. So, this is what our discussion today, that we know how to balance a 4 bar linkage with a spring load and that is spring load should be a zero-free length spring load that is something, that we have zero-free length spring by using another such zero-free length spring we can statically balance this linkage with that linear spring load.

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So, here is the spring that we are taking by when it is a spring actually we have compliant mechanism it is a big complaint mechanism. So, it has this sender beams which are made of spring steel here, and these additional things are made of aluminum with this fixed connections here then symmetric. So, this is a compliant mechanism that when we apply force in this direction the output is going to be in these two directions, that is more like a gripper you pull it down and these two things come together, now can we statically balance this, because when apply a force you are deforming these beams here. So, will remove the forces is going to go back, but we want it to stay wherever it is that if you do then it will be statically balanced.

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Key reference

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Application of Rigid-Body-Linkage Static Balancing Techniques to Reduce Actuation Effort in Compliant Mechanisms

There are analytical methods in the literature where a zero-free-length spring-loaded linkage is perfectly statically balanced by addition of more zero-free-length springs. This paper provides a general framework to extend these methods to flexure-based compliant mechanisms through (i) the well known small-length flexure model and (ii) approximation between torsional springs and zero-free-length springs. We use first-order truncated Taylor's series for the approximation between the torsional springs and zero-free-length springs so that the entire framework remains analytical, albeit approximate. Three examples are presented and the effectiveness of the framework is studied by means of finite-element analysis and a prototype. As much as 70% reduction in actuation effort is demonstrated. We also present another application of static balancing of a rigid-body linkage by treating a compliant mechanism as the spring load to a rigid-body linkage. [DOI: 10.1115/1.4031192]

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This is a key reference for this work by Sangamesh Deepak and Amrith N Hansoge, my students at the time and it is actually published recently, it has to parts and will be discussing one in this lecture and another one in the next lecture. So, if you see the idea here is actually reduce the actuation effort; that means that when you have a compliant mechanism we have to move it first for it to transfer the energy. So, for that we have to put in some input effort in force and some energies used for that, but how about reducing it the reduction is what we do with this static balancing. So, we are going to use static balancing reduce the effort needed to move or deform a compliant mechanism.

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Details of the compliant mechanism

3.1 Design. Spring-steel strips of 1 mm thickness and lengths of 180 mm (strips 1 and 8), 140 mm (strips 3–6), and 100 mm (strips 2 and 7) were cut and assembled as in Fig. 22. The corner bracket elements as shown in the figure are of aluminum and have a uniform thickness of 3 mm and arm length of 40 mm on each side.

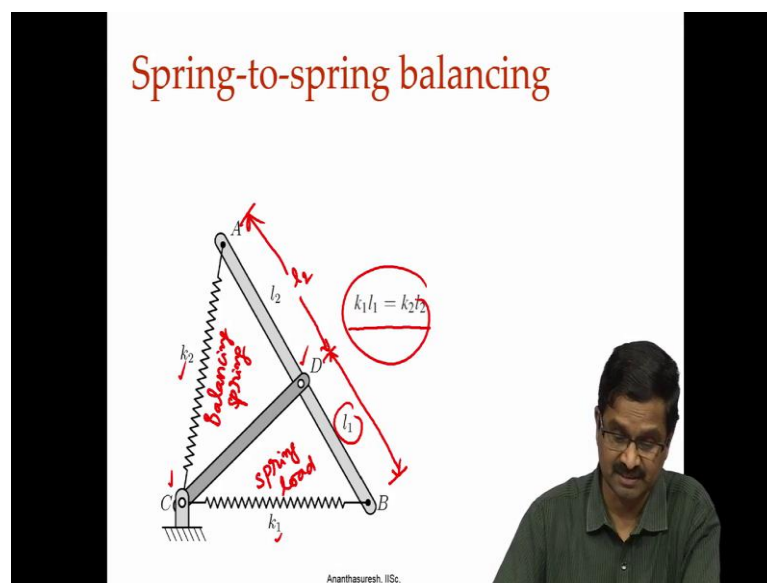
The length of rigid link AB was assumed to be 80 mm with the center at D . According to Eq. (38), the length of link CD was fixed at 40 mm. The stiffness constant of spring CB was fixed at 550 N/m. The deductions and explanation for these assumptions are given in Sec. 3.2. The following data are used in the further calculations:

- Young's modulus of spring steel = 200 GPa
- Young's modulus of aluminum = 70 GPa
- Yield stress of spring steel = 550 MPa
- Yield stress of aluminum = 120 MPa
- Factor of safety for aluminum and spring-steel parts = 1

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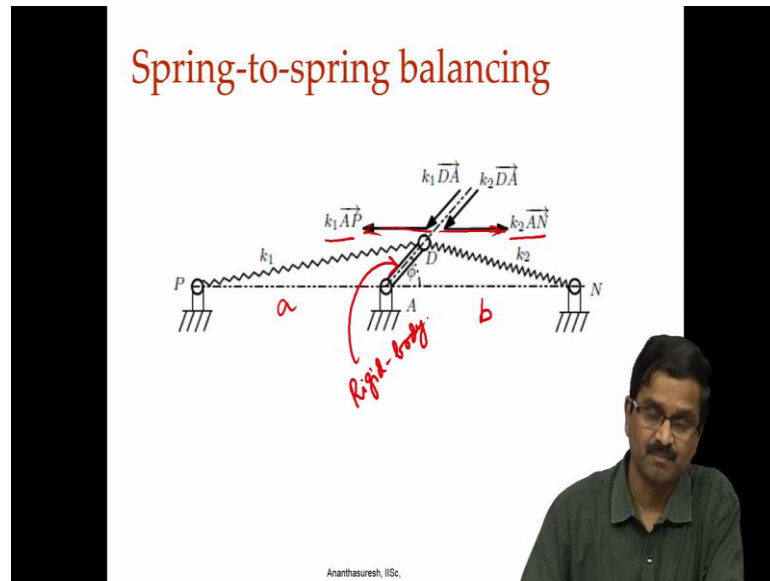
So, the mechanism details are taken from that paper where we have used spring steel strips to make the beams, and the strips are the lengths and width and everything is given spring steel young's modulus and aluminum connections to or brackets to connect them to make up to mechanism aluminum comes in this here. So, there are these aluminum brackets lets use a different color. So, this is aluminum bracket and this aluminum bracket and. So, is this so whatever I am doing in yellow now these are all aluminum? So, it is are all aluminum and so is here and here. So, red once are spring steel so red one there is another one here. So, there is red, red, red and red and is clamped over here and here. So, all the details are in the paper and summer them are given here as well and while we design we should also pay attention to the yields strength of both spring steel as well as aluminum.

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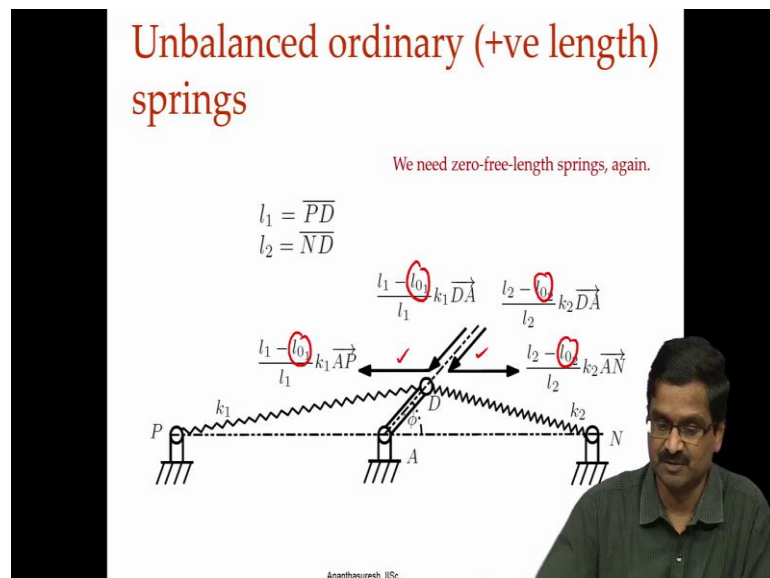
Now, our approach here to do this since we are treating a compliant mechanism as a spring we can go to a 4 bar linkage that is one way or we can go a simple 1 or a 2 bar linkage. So, in this case it is more like a 2 bar meaning that there is one revolved joint their another revolved joint here, there are two springs k_1 and k_2 , one of them we can think of that as the spring load other one as the balancing spring, we had discussed this in the last lecture where we said this is like spring to spring balancing one spring balances the other. When does it happen if k_1 times l_1 , that is l_1 here from there to here and l_2 which is here if k_1, l_1 is equal to k_2, l_2 then this will be perfectly balanced in to all positions.

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We had seen this in the last lecture by arguing in terms of the forces that is $k_2 a_n$ should be equal to $k_1 a_p$, $k_1 a_p$ is this, if I call that a and a_n as b , $k_2 b$ should be equal to $k_1 a$ then the components that are showed here the horizontal component get compensated with each other and the other parts of this spring force $k_1 d_a$ and $k_2 d_a$ will go to the ground prevent and there is no torque it will stay put wherever it is what will stay put this rigid body, here will stay where it is that similar statically balanced.

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We use this approach and we had said that zero-free length is critical here, if we do not have that all this terms will not go to be balanced that is this and this portion this portion now we balanced, because l_0^1, l_0^2 at different and we have l_1, l_2 and they depend on the configuration of this crank. So, it would not be statically balanced we need zero-free length space.

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Let us follow the energy method.

$$SE = \frac{1}{2} k_1 (l_1 - l_{10})^2 + \frac{1}{2} k_2 (l_2 - l_{20})^2$$

$$l_1^2 = PD^2 = a^2 + l^2 - 2al \cos(\pi - \phi)$$

$$l_2^2 = ND^2 = b^2 + l^2 - 2bl \cos(\phi)$$

$$SE = \frac{k_1}{2} (l_1^2 + l_{10}^2 - 2l_1 l_{10}) + \frac{k_2}{2} (l_2^2 + l_{20}^2 - 2l_2 l_{20})$$

$$SE = \frac{k_1}{2} l_1^2 + \frac{k_2}{2} l_2^2 - \frac{k_1}{2} (2al \cos(\pi - \phi)) - \frac{k_2}{2} (2bl \cos(\phi))$$

$ak_1 = bk_2$

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We had used energy method in the last lecture. So, we can do that you know if we say this free length of this is l_0^1 free length is l_0^2 , there is k_1 and k_2 the current length is l which we depend on this angle ϕ . So, we had done this last time. So, we have one pivot, another pivot a third pivot, and is where we have the rigid body l and we have springs here, here bring constant k_1, k_2 and then a and then b if you look at a because a force balance $k_1 a$ is equal to $k_2 b$, it will be balanced if you want to do energy method then we have to write this strain energy for it there is no force rather only springs one spring compensates the other.

So, strain energy is half k_1 into l_1^2 that is the current length from here to here is l_1 and this will be l_2 , we say this is A and this is P this is N and this is D . So, l_1^2 minus l_{10}^2 square half k_2 l_2^2 minus l_{20}^2 square and l_1 that is a length PD , we can write it using cosine rule applied to triangle $p d$ then l_1 is going to be or rather I can write l_1^2 square itself cosine rule that will be, a^2 square let us call this length l a square plus l^2 square minus $2 a l \cos$, if this is $\pi - \phi$ other angle may be $\pi - \phi$ that is l_1^2 square

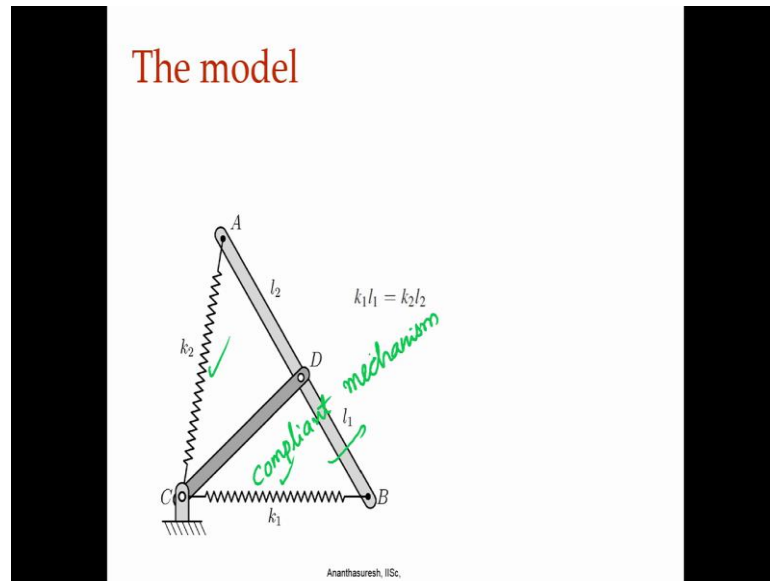
likewise l^2 square, which is $N D$ that will be again using cosine rule this will be $b^2 + l^2 - 2 b l \cos \phi$, now let us expand this let me change color expand strain energy is k_1 by 2 into $l^2 + l^2 - 2 l l \cos \phi$, and then k_2 by $2 l^2$ square plus l^2 square minus $2 l^2 \cos \phi$.

What we want is that we would like to have the strain energy not depend on the angle ϕ it is in two places of course, it is also there in well that is all actually that is the only two places, where you have ϕ . So, you should make show that coefficient of that cosine ϕ should be 0. So, now, $l^2 + l^2 - 2 l l \cos \phi$ and then we have l^2 and $l^2 - 2 l l \cos \phi$ that again depends on ϕ , but then if you make it zero-free length spring this term will disappear and likewise this l^2 square, we have the expression when if you think about that where as these terms $l^2 - 2 l l \cos \phi$ anyway it is a constant we are going to make it 0 and this also we going to make it 0, what will remain if you make $l^2 - 2 l l \cos \phi$ and $l^2 - 2 l l \cos \phi$ then what I will be left with there will be strain energy k_1 by $2 l^2$ square and then k_2 by $2 l^2$ square.

Now, I have $l^2 + l^2 - 2 l l \cos \phi$ right. So, now, we had to see when we add them up k_1 times this, k_2 times this a square and $b^2 + l^2 - 2 b l \cos \phi$ they are all constant. So, not a problem what will be left with I will just write instead of going to be the next slide. So, that is write this it will become minus $2 a l \cos \phi - 2 b l \cos \phi$, now also there will be this k_1 that we need to multiply that is k_1 by 2 k_1 by 2 square as square as substitute with this and then plus I will I have this one minus $2 b l \cos \phi - 2 b l \cos \phi$ in to k_2 by 2 . So, this 2 and this is k_2 . So, when you have this. So, 2 and this 2 gets canceled this 2 this 2 and then l^2 will be gone. So, here we have k_1 into a into cosine ϕ minus ϕ that is minus cosine ϕ . So, that will become this minus it will become plus other side is cosine ϕ and b and k_2 .

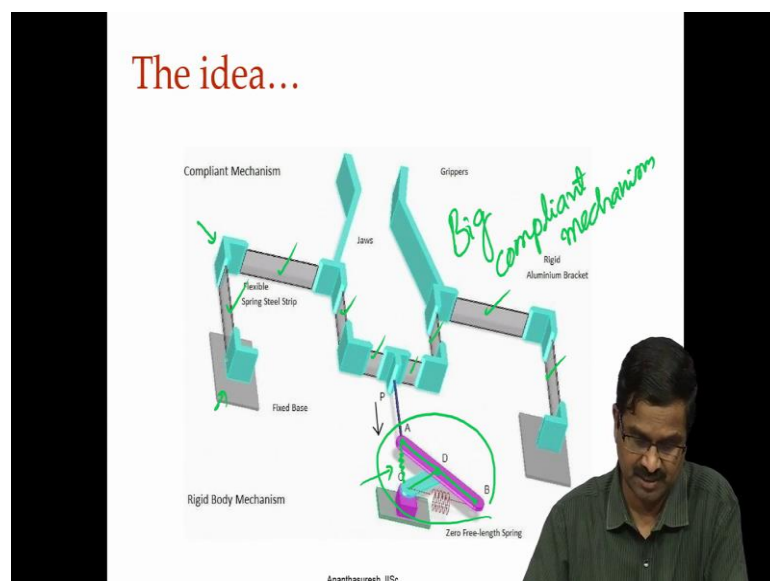
So, cosine ϕ cosine ϕ get cancels. So, what we left with it will be $a k_1$ equal to $b k_2$ that is our condition for this. So, if you have 2 springs that I have to be balance each other we need to have if this is a and this is b that distance and this distance. So, times k_1 should be equal to b times k_2 you can position it wherever you want that is the approach will take.

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So, this energy method we you just use rather than force method. So, if $k_1 l_1$ is equal to $k_2 l_2$ you can position it whichever way you want, whether you what we considered was we had put the pivots here. So, we had put the pivots there and this was also a pivot and as I turned this way that way it was balanced, but you can actually consider inversion, because static balancing is when you do not have external forces should be staying in that configuration is now fix in them you can fix this here and as you move and as you rotate this, whichever way you want relative to this will says statically balanced.

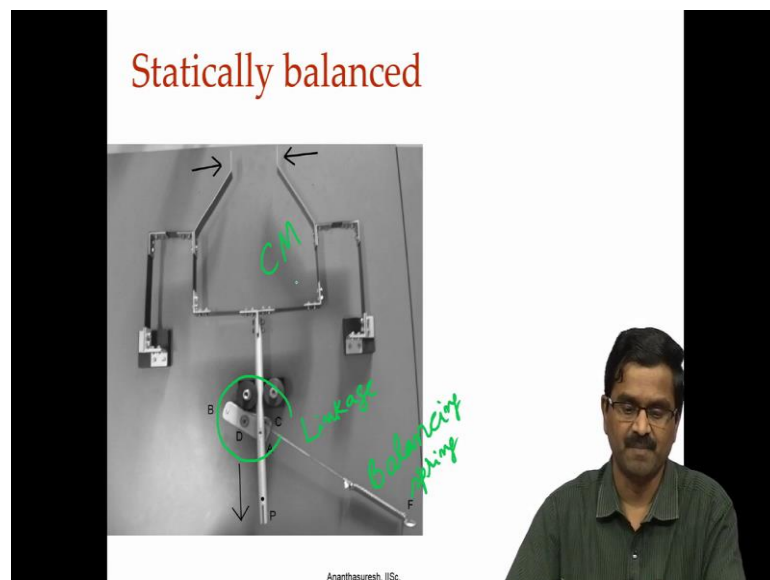
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This approach we take here. So, what we do is that spring there are 2 springs there is a spring 1 and the spring 2 1 of them we make it a compliant mechanism, again when the range of motion is small a compliant mechanism can be treated like a translational linear spring liner in a sense of linear non-linear and the other sense of linear is translation as appose to rotational. So, now, this is like as if there is a spring here that is exactly what we had. So, we have one body with a joint another body here with one spring another spring attached is exactly what we had statically balanced just now. So, now, we can extend that by treating this entire compliant mechanism as a translation spring.

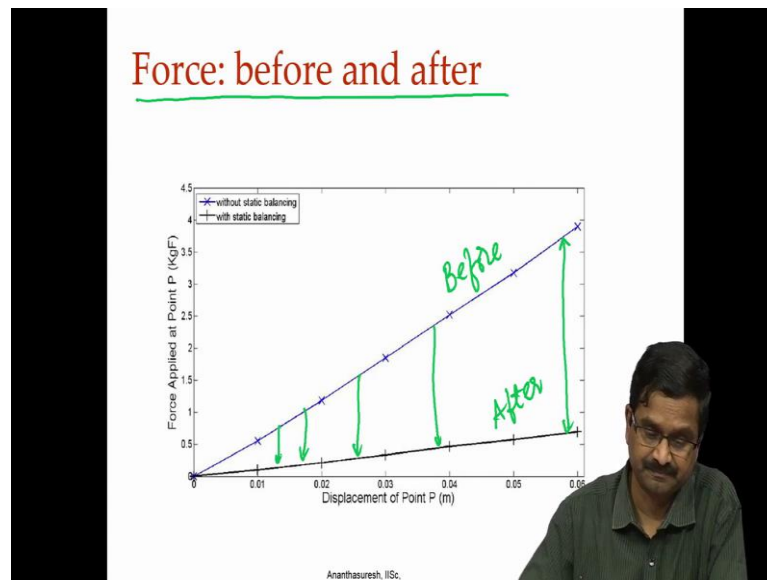
So, that is the approach here that is the idea here. So, in order to make this realistic we need to have a compliant mechanism that under goes large absolute displacement, but that is still linear for the compliant mechanism one way to do that is to take a big compliant mechanism, if I have big compliant mechanism even though it still operates in the linear regime it is going to give us substantial displacement, that is how this big one was done with spring steel strips everywhere and this aluminum brackets lot of them that is how this is done.

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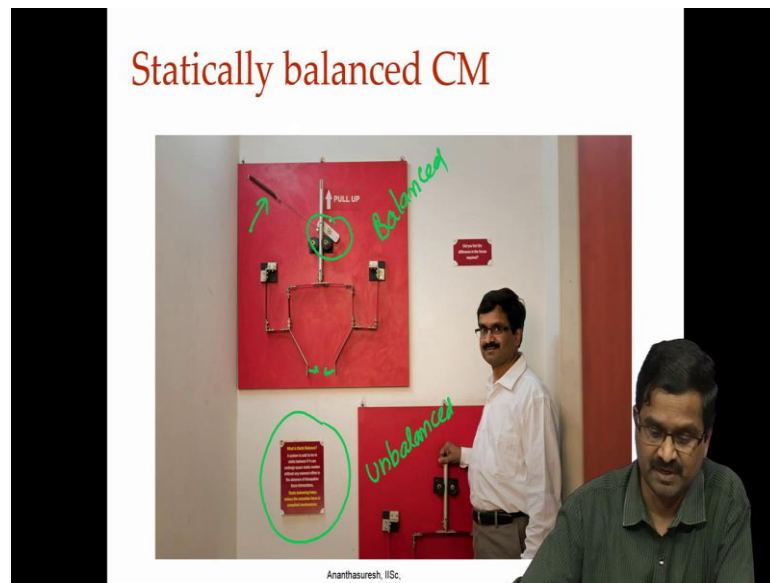
And this is the result if we see we have big compliant mechanism here and the static balance linkage that little thing that we thought is the one that balances, it is small and this is the balancing spring with the string ropes and all that we do we can statically balance this.

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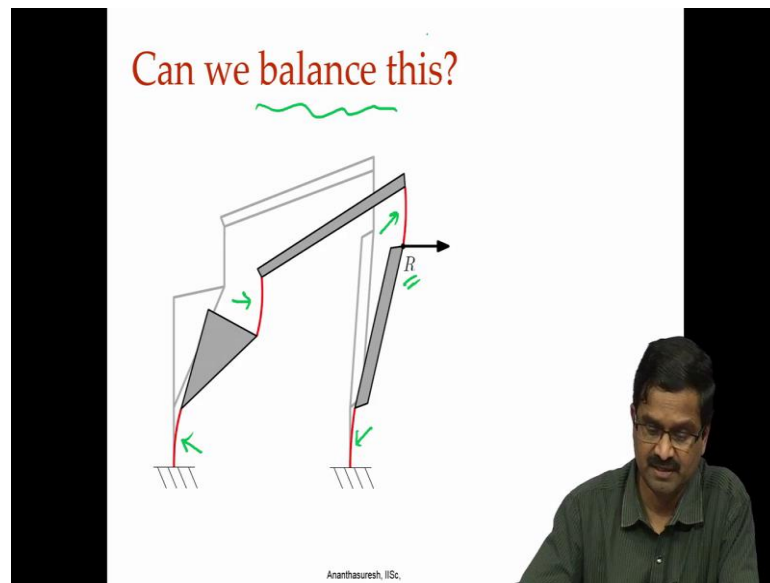
So, if you do that if you look at actuation force to pull that. So, that this joint come together before and after, so this was before that is without static balancing this is after effort is reduced a lot, you might ask why is it not 0 that is what should be static balancing means that as you move the point it should be actually 0, but here it is not because our thing is this that complaint mechanism is linear approximation does not hold good when we go further, and also there could be friction at the joints and other things which have to be avoided if you want to look at really perfect static balancing. At least there is substantially improvement that is our goal, our goal is to bring this down or effort is being drowned.

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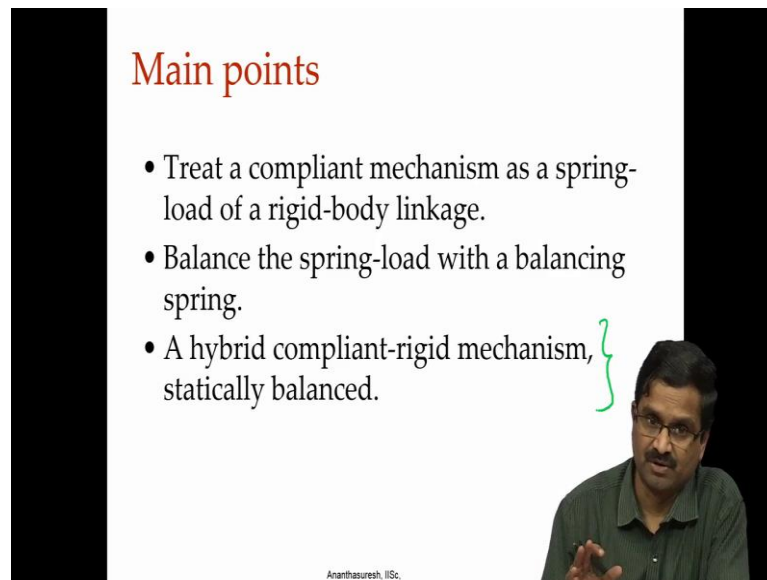
And this is the one that is there in our lab, where anybody can lift this one this is unbalanced and then seen how much effort is needed to lift it and this is balanced. So, the idea is static balancing when you do that, when you lift it up it will be much less effort over here. In fact, if it is kept horizontally which is more difficult to put it. So, we mounted on a wall there you can see the difference, you can feel the difference that when something is statically balanced you require much less effort to do the same thing that is deform it. So, that these two things come together you can see it, but you are hardly applying any force in pulling this up and again the credit goes to the static balancing spring and there is a string and there is this linkage that we talked about over there in order to give us this behavior.

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So, next thing is can we balance this mechanism, right now what we have done is indirect method that is we have treated a compliant mechanism as a spring and put that as a load to a rigid body linkage and statically balance that rigid body linkage, that is somewhat against to a philosophy of a compliant mechanism, because in a compliant mechanism we did not want to have joints, we do not want to go near the rigid body linkages, but here we just used it and we made a compliant. So, big that the role of a rigid body linkage is relative to just statically balance it, but now what you have compliant mechanism such as the one that is shown here with the flexure hinges. So, with some force applied can we statically balance this?

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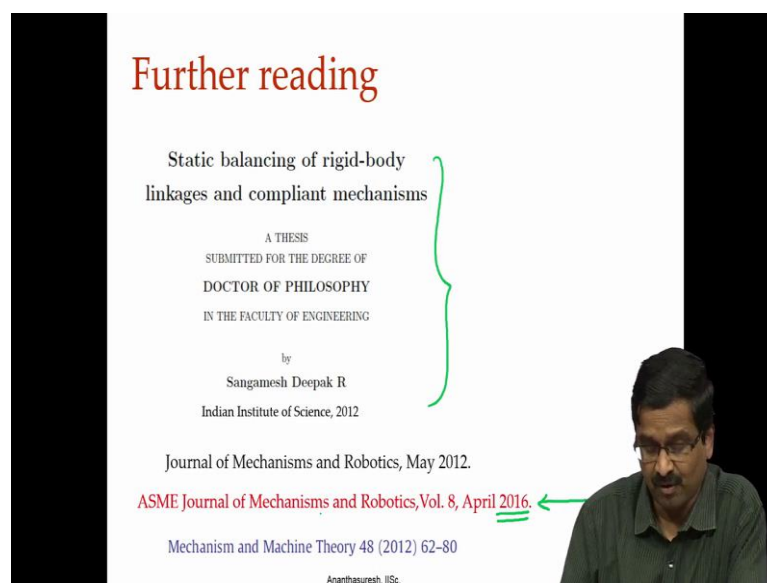
Main points

- Treat a compliant mechanism as a spring-load of a rigid-body linkage.
- Balance the spring-load with a balancing spring.
- A hybrid compliant-rigid mechanism, }
statically balanced.

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These what will discuss in the next lecture. So, to summarize for this lecture the main points are we can treat a complaint mechanism as a spring load of a rigid body linkage. And then balance that rigid body linkage with this spring load the spring load is again due to complaint mechanism, now the method which I discussed is a hybrid compliant and then rigid mechanism and how we can statically balance it that is the basically, the theme here it is a hybrid a complaint mechanism and a rigid body linkage both of them are balanced together by adding a single static balancing spring that is all nothing else.

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Further reading

Static balancing of rigid-body linkages and compliant mechanisms

A THESIS
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For further reading there is special thesis where there are lot of static balancing techniques discussed and in particular thus the one in the red the recent paper are 9 months ago. So, we have a method that we have just discussed and also what we discuss in the next lecture.

Thank you.