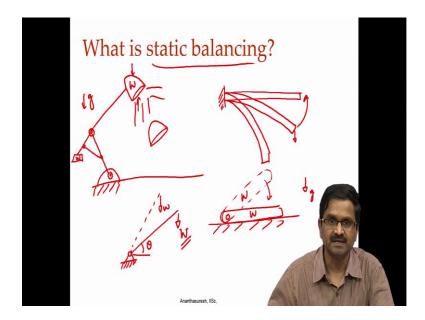
## Compliant Mechanisms: Principles and Design Prof. G. K. Ananthasuresh Department of Mechanical Engineering Indian Institution of Science, Bangalore

### Lecture – 58 Static balancing and zero-free-length springs

Hello we have been talking about final points in compliant mechanisms after spending a lot of time discussing six different design methods for compliant mechanisms. One of the final points we discussed is mechanical advantage in the last few lectures and now and then we discussed bistable compliant mechanisms the third final point that we will discuss on a related to compliant mechanisms is static balancing. This is a very important aspect for compliant mechanisms because whenever a compliant mechanism works it has to work by elastically deforming elastic deformation is like a cut in the input energy as we discussed in the case of mechanical advantage of compliant mechanisms is hampered by this necessary evil, if you call it that is necessary evil being that part of the input energy is used in the deforming the compliant mechanism itself; after that cut is done that is the input energy that is there and then you use some of it to deform the mechanism to enable it to serve its function and the remaining energy is what will be available at the output.

So, this is the necessary evil that is elastic deformation which is essential for compliant mechanisms to work, but that also is a little sink in terms of energy. So, its mechanical advantage effected, mechanical efficiency is effected and one way to overcome that is to use preload as we discussed earlier in the context of mechanical advantage, but now we will discuss another way which also in a way uses preload, but it is a concept of static balancing. So, we will look at static balancing in this lecture and then elaborate on how it could be used in the context of compliant mechanisms in the next two lectures. So, let us look at static balancing as a way to fix this necessary evil of compliant mechanisms where part of the input energy is to be used to deform the elastic body of the compliant mechanisms, it is a solution or a remedy for that and that is static balancing.



Let us look at static balancing of compliant mechanisms and the first thing we have to understand is what is static balancing. Static balancing is when there is a load that can be a force or it can be a spring load as you take the mechanism a linkage mechanism whatever, if you take it from position to position will it stay in those positions if it stays then such a mechanisms called statically balanced.

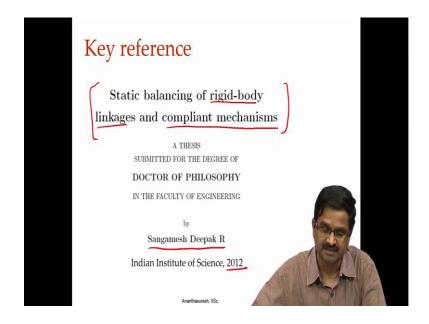
So, let us take an example to understand let say that if there is a table lamp let us just draw a schematic of a table lamp, there is let say a base here and we have the lamp gives the light, this has some weight and there is always gravity we cannot escape that. So, I want to hold it at different heights now when I hold it at different height will it stay there? It would not. In fact, if there is a table if there are no static balancing features for this linkage it will simply fall down and it will hit the table, but it is stays if you think of a table lamp that we have to used wherever, we put at it will stay there sometimes people think that it is the friction in this joints that make its stay where you want it could, but we do not rely on that then usually either a counter weight is added some weight is added. So, that it will stay there, but then that weight has to added for this if it is only one lever it could have been fine, but otherwise something else has to be connected something has to be done to make it statically balanced meaning that wherever I position this lamp that is a weight it should stay there.

So, to contrast that if I draw a cantilever beam for that matter any beam we take a simple cantilever beam now I take this and deform it like this, in order to deform I have to apply a force we remove the force it going to comeback. So, I will apply the force it would be like this the moment I remove it going to go back; it would not stay. So, what will be statically balanced if a take a cantilever beam when I bend it will bend and when I leave it will stay there and I bend it more let us say let us say it bends, like this its stays there when we remove the force is that possible if that is possible then we would say this cantilever beam is statically balanced that is the idea.

So, if I take a rigid body linkage let us say I have just a link like that I mean plane if I rotate it let say something like this it will stay if there is gravity it will stay, but the moment there is gravity and if this is the table if I put it like this is going to fall back on to the table, how do I make it stay where it is where it has some weight or a mass; how I do that if I am able to do that then I would say that system is statically balanced. So, if I have a abstraction like this that is I have something like this and there is a weight constant weight, irrespective of this angle theta if I put it here the weight will still act around like this it should stay; right now where you put theta and there is a weight it is just going to fall down, but we need to constrain it who do I do that if I am able to do that then I would say such a thing is statically balanced basically, in all configurations of a mechanism if it stays where it is without any external force then it is statically balanced that is important that is for example, cantilever as long as upraise the force is stay there remove the force is going to come back it should not be like that without external forces the system should be in equilibrium by itself, when I say external force w can be thought by the external force also that is there that is gravity balancing gravity load will be there, but no other force to hold it.

If there is a lamp as if I apply a force here the weight force here both will be (Refer Time: 07:48) stay there that is not the intend the gravity loads can be there if it is gravity balancing, but no other forces there should be something inherent in the system that will make it hold its configuration in any which way that you move it without having to put additional loads whatever, gravity load is there that will be there and that should be compensated if you do that such a thing called static balancing.

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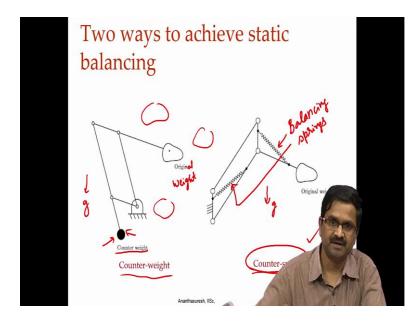


So, this is the key reference for this work its PhD Thesis of doctor Sangamesh Deepak in 2012 in Indian Institute of Science now he is a professor at IT Guwahati and his thesis is very comprehensive it talks about a lot of techniques for static balancing both rigid body linkages as well as compliant mechanisms.

So, when it was doing PhD thesis it was realized by us that if you want to statically balance compliant mechanisms, first you have to understand rigid body linkage static balancing not everything was done that is why he ended up doing a lot on rigid body static balancing and then extended some of it to compliant mechanisms. What do you want to statically balance compliant mechanisms as I said in introduction to this lecture just now, that if a compliant mechanism is statically balanced then there is no need for any external force to hold it in different configurations, that is in any configuration whatever input energy you give to it all of it will be available at the output, because you do not need extra energy to move the mechanism from one configuration to another configuration because it is statically balanced. If you have a compliant mechanism like a cantilever beam if it is a spring or a beam or a mechanism even if there is a elastic deformation we do not need energy to move from one position to another position, one configuration to another configuration. So, that is the advantage of compliant mechanisms if there are statically balanced, then all the energy that is given to the compliant mechanism will be all over the output just like rigid body linkages that do not have any friction in their joints, then they are also perfect in terms of providing all the

input energy at the output; compliant mechanisms always there is a cut a part of the energy should be stored as strain energy inside the mechanism now, we want to avoid that that is why I want to do static balancing that is what this PhD thesis considered.

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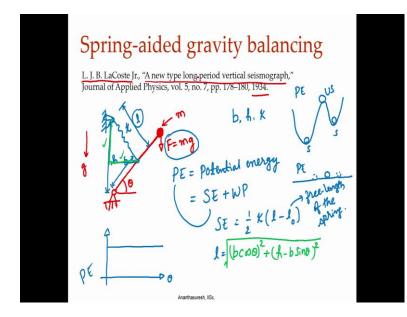


So, there are two ways to achieve static balancing of linkages in general in this lecture we will talk about linkages in the next two lectures will talk about how to balance compliant mechanisms using what we discuss today. There are 2 ways to statically balance; one is called the Counter-weight other is called the Counter-spring. So, counterweight if I have an object such as this is the original weight and as I turned this I take this over here, over here wherever if there is gravity this is gravity balancing and there is gravity in this also.

So, can I balance it if I have a counter weight that is this one with a parallelogram linkage like this it will actually be balanced that is traditional way of doing; now, in this particular case instead of a counter-weight we have springs. These are the Balancing springs, there are two balancing springs. So, this is the other method and this is good for compliant mechanisms because compliant mechanisms are springs by themselves. So, when you want to balance a compliant mechanism statically; then using springs make sense because we can add a segment of a beam up to in order to imitate what these balancing springs do so it will become easy.

So, both of these as you know this original weights wherever you want either the counter-weight or counter-springs will give you static balancing, we will focus on counter-spring method because that is more amenable for compliant mechanisms. A lot is done in counter-weights that is a well known concept now, spring will be more compact and very innovative designs can be obtained with counter-spring static balancing methods.

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The fundamental work in this is done by Lucien LaCoste way back in 1934 a long time ago; this LaCoste had some very important work, but if you look up the title of his paper it says A new type long-period vertical seismograph; completely away from mechanisms, but, in fact, he relied on a mechanism in order to get very long time period vertical seismograph more like a pendulum that moves very accurately basically a very low stiffness pendulum, if you work any minute seismographic activity will be recorded by that. In the context of that he came up with a very innovative idea that is what we will discuss now, let us say we have a rigid body just like a length that is not about its mass there is gravity, but we think this line does not have any mass, but there is a force acting here constant force that could be due to a weight.

So, there is gravity and I put a weight like a big mass here then this F is going to be equal to m g, where this mass is m let us say this angle is theta and this is the masses rod. Will it be statically balanced meaning for any value of theta, if I put it there it should stay there, is that possible? What he came up with this that counter-weight is always 1, but he wanted an alternative. So, what he did was he actually added a spring let me use the different color for it. So, here it is spring between 2 points one point is anchored other point is attach to this, there is a spring constant you took K that is a linear spring translational linear spring and the height here from here to here this height let us call that h let us call this length where it is attached let call this b or something now let us see what happens what should be the values of b, h, k so that this will be statically balanced meaning wherever you position it whatever the value of theta is it should stay there.

He did it in different way than what we will be doing let follow a simple method by writing the Potential energy of this system. In the last few lectures when you talked about bistable phenomena we said that minimum potential energy principle gives you stable equilibrium. So, when I take potential energy, the potential energy if it has a minimum it is stable, maximum; unstable and bistability we had taken; this is minimum, minimum stable, stable; maximum unstable.

So, instead potential energy if it is flat somewhere that is neutrally stable that is if it is flat like this energy landscape is flat you can position this ball wherever it will stay there that is exactly what static balancing is, bistability was having minima and maxima going up and down where as static balancing means that potential energy is constant. In fact, strain energy is constant because potential energy is strain energy plus work potential. According to the definition of static balancing we said that there should not be any external forces; work potential is the negative of the work done by external forces.

So, in static balancing there should not be any external force. So, that is not there; it is equal to 0. So, potential energy is just strain energy that is all there is and that strain energy now has to be flat meaning that whatever is the value of theta if I were to plot this here theta versus strain energy potential energy, whichever we look potential energy here in this case w this force is there. So, we can actually call it potential energy also when there is gravity loading later on we will be taking spring loads then there is no potential everything is strain energy in this particular case if you treat this as an external force then this will be there.

So, potential energy which is negative of the external force that will be there work potential will be there and we can take that let us write this. So, this potential energy in this case for this example it should be flat meaning potential energy expression, if you write that should not have a term that depends on theta only then it will be flat like this. So, let us write potential energy for this; first let us write strain energy for it. Strain energy we have a spring there. So, let us say half k into the length of this from there to their if I call it 1, 1 minus let us say it is free length that is where there is no load on the spring like your ball point pen spring we will have a length when there is no load that is when there is no load on it, that even you are not stretching or compressing it will have a length that is the free length; 1 0 is the free length of the spring. What is 1 here; 1 that we have indicated that one let us use some (Refer Time: 19:07) (Refer Time: 19:08).

So, I need to know this length this let me is a different color. So, I need to know this k different color I said I need to know this length and this length then I will get that l. So, what is that length if I call this b that is going to be b cosine theta; so I will be if I use Pythagoras theorem this square plus this square under root; so, under root b cosine theta square plus this length. So, we took care of this what about that there will be the total was h minus b sine theta. So, h minus b sine theta square that is our l. Let us remember this go to the next slide.

So, we can see what happens to the strain energy. So, 1 is b cosine theta square plus h minus b sine theta square. So, here 1 let us write it again that is the b cosine theta square plus h minus b sine theta square that is our 1. So, our strain energy here is half k 1 minus 1 0 square.

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Energy method  $\frac{Jz}{SE} = \frac{1}{2} \frac{k(l-J_0)^2}{k(l-J_0)^2} = \frac{k}{2} \left( \frac{J^2}{J^2} + \frac{J^2}{J^2} - 2JJ_0 \right)$   $= \frac{k}{2} \left( \frac{b^2 co^2 \theta}{b^2} + \frac{h^2 + b^2 J_0 f^2 \theta}{b^2} - 2bf \sin \theta + \frac{J^2}{J^2} - 2JJ_0 \right)$   $= \frac{k}{2} \left( \frac{b^2 co^2 \theta}{b^2} + \frac{h^2 + b^2 J_0 f^2 \theta}{b^2} - 2bf \sin \theta + \frac{J^2}{J^2} - 2JJ_0 \right)$   $= \frac{k}{2} \left( \frac{b^2 + h^2}{J^2} + \frac{J^2}{J^2} - 2JJ_0 - 2bf \sin \theta \right) + mgl fina$   $= \frac{k}{2} \left( \frac{b^2 + h^2}{J^2} + \frac{J^2}{J^2} - 2JJ_0 - 2bf \sin \theta \right) + mgl fina$ 

So, I can write it as k by 2 into 1 square plus 1 0 square minus 2 11 0. Let us substitute for 1 square then I get k by 2 1 square is b square cosine square theta plus this square we have written that and this one will be h square plus b square sine square theta minus 2 b h sine theta, that is 1 square and then there will be 1 0 square minus 2 11 0. That is your strain energy and what is work potential? Let us go back to the previous slide this F let us say this as the reference for that now this height will be if I take the length of this as 1 from here to here that length is 1 this will be 1 sine theta. So, 1 sine theta that is a distance over with this force as moved force is this way it is going up. So, they are in the opposite directions and then work potential negative or the work done. So, it will be negative of negative positive.

So, that will be m g l sine theta because force and the displacement are in the opposite direction. So, that is negative sign. Work potential is negative of negative of the work done by negative of the work done is negative here. So, total positive. So, potential energy if I write then I will get k by 2 for the strain energy part there is b square cosine square theta b square sine square theta that makes it b square and then I have h square I have taken care of that and then let us take care of these constants as well.

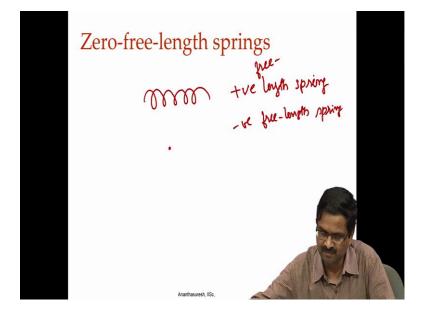
So, I have 10 square minus 2110 and then I have 2 b h sine theta and then I have plus m g l sine theta which is the work potential now, what are we saying potential energy has to be constant for all values of theta. So, one way to do this is I will write k by 2 b square is

constant, h square is constant, 1 0 square is constant, 1 1 0 is constant then I get this 2 b what we had. So when I have 2 b sine theta and then I have m g l sine theta; in order to make it constant the sine theta coefficient this 2 b k there and m g l.

So, what I have; if I do not want it to depend on theta then what I need to do is I will make 2 2 gets cancelled, minus b k that is what I get here plus m g l, this is what we will multiply sine theta if I do not want it to depend on theta I just need to make this thing 0 so that I will not have that those two terms at all; if I do this, if I choose my b k in such a way that b k is equal to m g l that will become 0. So, what I end up will be k by 2 b square plus h square plus 10 minus 2110. So, that is gone.

So, now we have if I change the color this term, this term get cancel because I am making this to be 0. I need to satisfy that that is b k should be equal to m g l if I do that then I will be fine that like a design. So, if go back I have to choose the spring constant case as that that b k equal to m g l is satisfied now, if you come back and look at it these are all constant; b, h and 1 0, but then this l is not because if you see l there is theta in it what you do. So, that is where this Lucien LaCoste's cleverness came he said let us make 1 0; 0. What does it mean? 1 0 is the free length of the spring if I have a spring when there are no force is acting on it that is 1 0 free length; The idea that Lucien LaCoste had was let us make the free length zero that looks nice then I have this and this go away then I have potential energy constant as static balancing.

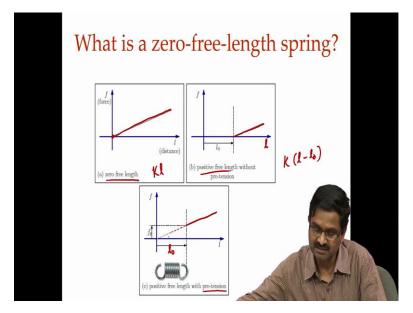
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But how do you achieve it in practice, he had ways of doing it also. Zero-free-length spring is one which has no force when a length is 0 and applies a force when a length is positive, that is if I take a spring like this it is a positive length spring; zero (Refer Time: 26:43) length is something if you really compress it compress it there is the force.

So, it should be a spring which has zero-free-length and then you can stretch it the way you want or compress it if you want, but something like this you cannot do it is just (Refer Time: 26:57) length. It looks like a very radical concept in fact, that is a radical concept in fact, in this 1934 paper he also talks about negative free-length spring this is positive free-length springs, we can have negative free-length springs and you can have zero-free-length springs also; what does it mean? It means the following.

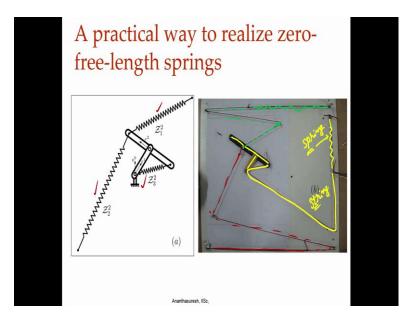
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If you have normal spring and a positive free-length spring, positive free-length spring if I say 1 0; 1 is its length at any time, 1 is the length there is 1 0 it will go like this if it is a linear spring. A zero-free-length spring will have a force characteristic like that length is zero it is zero, which is if we connect this spring between 2 points the force exerted is the spring constant times the distance between the 2 points that is the zero-free-length spring.

That is not true here because length you have to subtract 1 0. So, if I write positive freelength spring I write k times 1 minus 1 0 whereas, here I would write k times 1 that is the zero-free-length spring. So, you can get such springs if you pre-tension them that is if it goes like this is a 10 here for a spring if there is a pre-tension in this spring such springs are available then if you extrapolate it then it will go to zero-free-length such that is one way.

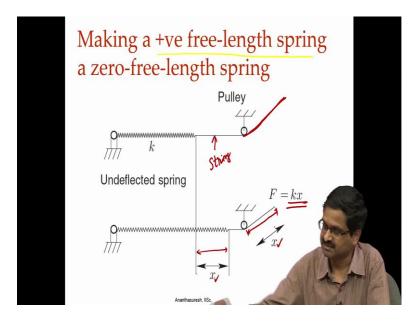
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There is another clever way; you do not need any special spring you can take a normal springs and make them zero-free-lengths, in this particular case these springs are all zero-free-lengths how do you achieve it here? You see the diagram here it exactly follows what is indicated there.

So, there is this one and this one and there is a spring here between these 2 points there is a spring, but actually there is no spring what you have is actually a string that goes like this and goes like this and there is a spring attached here there is a spring here this is a positive free-length spring, but there is a string attached this is string that is a spring, this is a string attached to this there is a positive free-length spring there is string attached the goes over pullies and comes like this, it is done in such a way that they are assembled in such a way that the force exerted here is proportion to this length. Let us see how it is done.

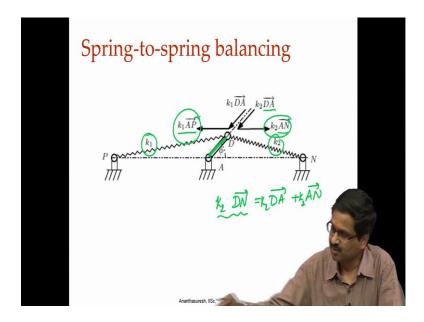
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It is done like this we have a positive free-length spring and you take that positive freelength spring attach a string inextensible string presume only inextensible you attach that and then you take it like this.

Now, you position the pulley in such a way that this length is what you want here that is what shown this x this x here; that means, that when you bring it back here it will go and the spring will not exert any force. The moment the string is long enough and you take it like this whichever way you want then the force proportional to k x that is what we want x is measured from here, there is no length when there is no force that is how you can realize zero-free-lengths in practice and that is what is done for all the spring. Those springs are here and then the string goes like this spring here and then the string.

So, between these 2 points from here to here we have a zero-free-length similarly, between this point and this point let me do with a different color between this point and this point there is a zero-free-length spring that is attached to a string and then there is a spring which is be (Refer Time: 31:21) here. So, you can realize them using positive free-length springs like this.



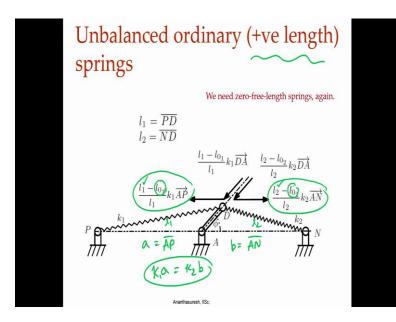
Now, we move on to another concept can you balance one spring versus the other; that can be done. So, you have two springs one of spring constant k 1 other spring on k 2 here is a rigid part and this one will be balanced perfectly for any angle. So, one spring balances the other. It can be seen clearly. So, if I were to resolve the spring force if I look at this A D N; I can say D A vector D to A that force that is can be written as I (Refer Time: 32:09) come from D to A; I can say I have D to N.

So, I will go like this and then come back N to A or later write it this way; let us actually write it, the spring force which is k 2 if it is zero-free-length spring k 2 D N in D to N or N D whichever way I can write it as summation of this A N. I want to go from D to N. So, I can say I can go to D A and then I can go to A N as a vector without k 2 it is satisfied.

So, I can multiply k 2 everywhere meaning that the force of the spring which is k 2 D N; I can resolve in terms of D A and A N. Likewise the force of the second spring, the first spring here can also be resolved as k 1 A P from k 1 A P and then I have D A here that will be the spring force here now, if I choose k 1 k 2 such that these two are equal. They are already opposite to each other, their magnitudes are equal, then the resultant goes through this and that will stay where it is because there is no torque.

This way of working with forces is one way to do balancing static balancing. It is a Spring-to-spring balancing one spring is balancing another spring.

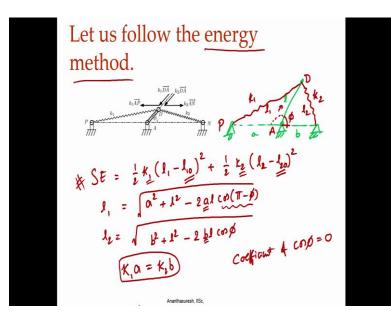
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So, instead of that we can actually do this with energy method which we just did for explaining Lucien LaCoste's method. So, force way if you do if I do not have zero-free-length spring instead I have positive free-length spring then this and this have to be made equal in all position which is not possible because 1 1 and 1 2 that is length from P to D that is 1 1 and D to N that is 1 2 will not be the same in all positions of for various values of phi. So, they would not be statically balanced.

So, the best way to do is you have to make it to be zero-free-length spring; if you make that we can make this these to equal in all positions, if it is 0 and if this is 0 l by l 1 cancels and you have static balancing all the time if you choose k 1 A P, A P is this length let us call this a and let us call this A P the length A P and then this is A N. Let us call this b if k 1 a is equal to k 2 b it is balance provided we do not have this l 0 1 and l 0 2 that is free-length springs. You have positive-free length it is not be possible. So, spring-to-spring balancing can be done it is an important concept because compliant mechanism can also be thought of as a spring it will not be linear where as the spring nevertheless, can you balance the compliant mechanism which is a spring with another spring. So, spring-to spring-balancing is what we are talking about here.

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So, this can be done with energy method if you will. So, we can do this let us say that I have what is here. So, let us say this length and this length; so, this length let us call it a this length let us call it b now, I have some length let us call it 1 and then now, I put 2 springs both are they have to be zero-free-lengths that is I have this k 1 and k 2 let us call this length 1 1 this is length 1 2 in that position when I have this angle phi.

So, now, let us write strain energy for (Refer Time: 36:36) force is here that is a thing it is not gravity balancing or force balancing; it is spring balancing one spring balances the other. If I write strain energy here for the first spring let us say half k 1, 1 1 and there will be a free length for it 1 1 0 square and then half k 2, 1 2 and its free length square now we will argue that unless 1 1 0, 1 2 0 are zero; strain energy in this case will not be constant for all values of phi. Where does phi lie; phi will be there in 1 1. What is 1 1 if I use cosine rule for this triangle P A D if I (Refer Time: 37:20) use that then that will be a square plus that 1 square that is 1 over there a square plus 1 square minus 2 a 1 cosine pi minus phi that is this angle square root that will be 1 1.

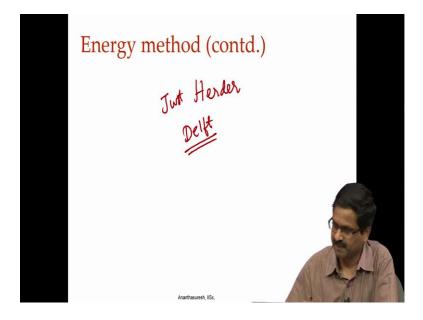
And then 1 2 will be square root of b square plus 1 square minus 2 b 1 cosine phi; there will be 1 1 and 1 2 now, if you were to substitute here and then see you will find that unless you make 1 1 0 and 1 2 0; zero you will not have static balancing possible here.

So, another way to say that you need to have zero-free-length spring, one more thing you will see is that there is cosine pi minus phi cosine phi when you substitute here when you

expand this thing here you would again find that if you want this strain energy to not depend on the angle phi that is any configuration (Refer Time: 38:35) should be the same then the coefficient of this cosine phi that will come when you expand that should be equal to 0. That will give you because there is a 1 here b 1, 1 is common there is a and multiplied by k 1 there and there is b multiplied by k 2; what we said that is k 1 times a should be equal to k 2 times b they will have opposite signs because cosine pi minus phi is minus cosine phi, this is cosine phi both have negative, but this will have negative cosine phi.

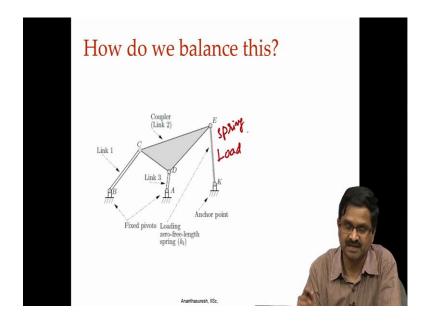
So, in order to make this coefficient of cosine phi 0; we need to have k 1 a equal to k 2 b.





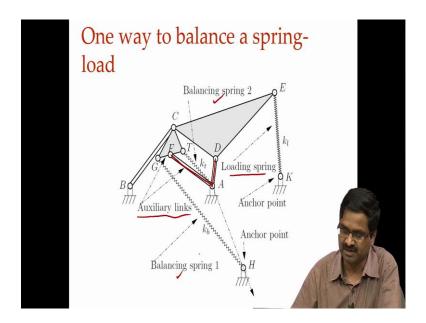
So, energy method allows you have to come up with the condition for static balancing, that is exactly what was there in Lucien LaCoste's work and spring-to-spring balance also the same thing holds. This work was done by Professor Herder quite a bit he is at Delft University; you can look up his papers where he has used this spring-to-spring balancing.

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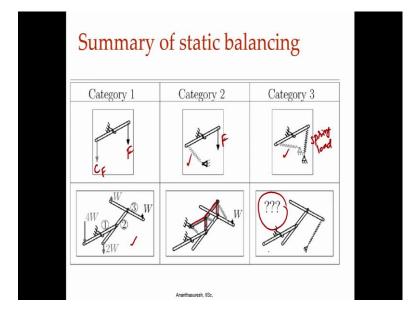


Moving on this is one of the things that Professor Herder had done; if you have 4 bar linkage now you have a load here which is actually a Spring Load. This could very well be a compliant mechanism if want to balance; that is what will discuss in the next lecture.

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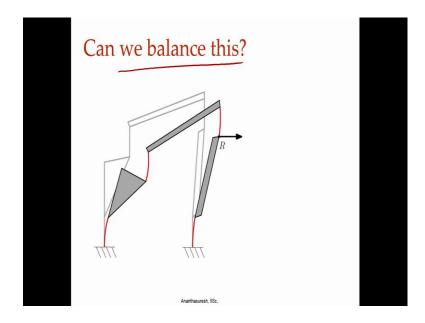


So, what Professor Herder did is added a number of springs and also additional bodies. So, original one did not have these two, he adds those and adds the Balancing springs and Auxiliary links are there and this is the Load spring, but he added two auxiliary bodies and two balancing springs then he gets a static balancing for this; there is a method that he developed for that.



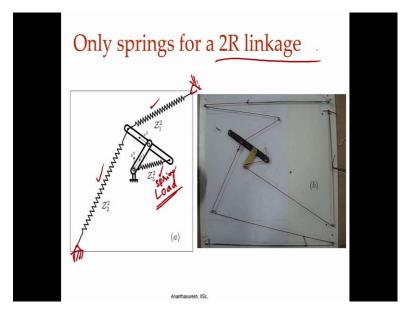
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Here is the state of the art in static balancing of linkages with springs. First if you have linkage if there are forces like this we can do this with counter-weights or with the spring the way Lucien LaCoste did. Here, there is a force and there is a balancing spring; now here, there is a spring load and there is a balancing spring, that is a Category 1 is force with counter-weight force and here spring with a force you are balancing here balancing spring-to-spring as we discussed and as you take more and more complex linkages this also becomes complex and you need to add additional bodies like it is done; this gray ones or the additional things that are added along with this springs whereas, is it possible to balance with springs alone that was what was done by Doctor Sangamesh Deepak in his PhD thesis.



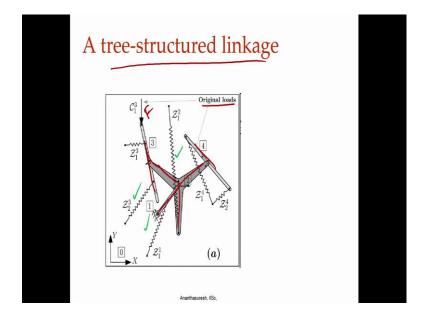
And he also extended this if have a compliant mechanism like this can we balance this by simply adding more springs to it meaning that, can I add some flexible segments and get that all those springs have to be zero-free-length springs that is why we talked about zero-free-lengths here.

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So, just somehow the thoughts here which we already saw this mechanism, if this is the load spring for a linkage like this he just adds two more springs balancing springs like

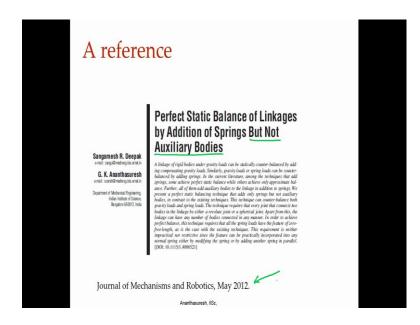
this which are anchored to the frame like this and all are zero-free-length springs that you have balanced everywhere under this spring load that is one thing he did or a 2R linkage.



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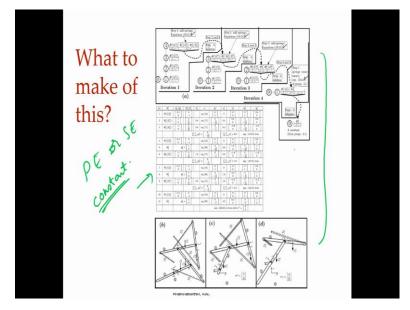
(Refer Time: 42:38) general method for any linkage or it can have a tree-structured linkage now there is here and in branches of this way and that way and this way and there could be spring loads more than once. What is the original loads, there can be forces, there can be springs then he would add this balancing springs I will keep them with a different color you have balancing springs only balancing springs no additional bodies it can be done.

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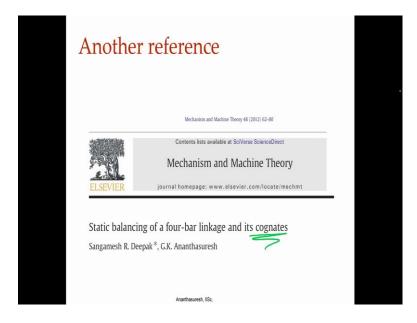
Here is a very important method that he did without using Auxiliary Bodies you can look this up in this paper in Journal of Mechanisms and Robotics published in 2012; Perfect Static Balancing using only springs, but no bodies, no additional bodies it has complicated figures like this in the paper.

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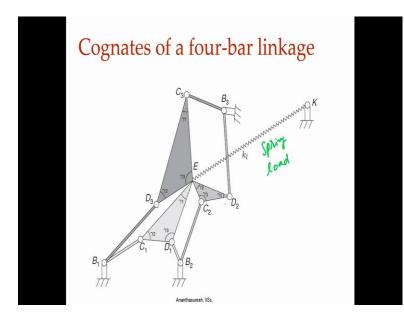
But you do not have to worry all it does is whatever in this table that you cannot see if you have some configuration variables like, theta angles or sliding positions or whatever coefficients of those variables have to be made zero to make potential energy are since, and there is no work potential it will be just strain energy for it to be constant any dependence on the configuration variable like angles, sliding positions and so forth; those coefficients you make zero. That is the essence of it even though it may look very complicated essence that we discussed is exactly that. If you have expression first energy potential energy written will you make sure that coefficients of the terms that depend on the configuration variables we make them zero, solve a set of equations we get static balancing condition.

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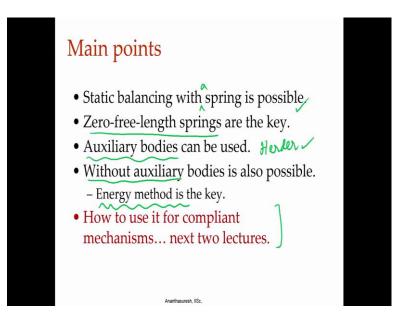


Here is another reference where he had used the Cognate if you have four-bar linkage there are two other four bars that have cognates if there is a spring load his method actually said that if you balance statically balance one for four-bar linkage other two (Refer Time: 44:51) can also be statically balance.

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If it just a settled result in his PhD thesis; to summarize we discussed that static balance of with a spring or with a few springs is possible and for that you need Zero-free-length springs that is the important thing, and (Refer Time: 45:11) Auxiliary bodies that is what Professor Herder had done and Herder's group what Sangamesh Deepak did what without using auxiliary bodies we can do that in that the energy method is the key which we discussed two examples the Lucien LaCoste's example with a force load or a spring load both ways we did, we discussed now, in the next two lectures what we will do is we will use these concepts in order to statically balance a compliant mechanism. Thank you.