

**Compliant Mechanisms: Principles and Design**  
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**Lecture - 54**  
**Mechanical advantage of rigid-body and compliant mechanisms**

Hello, we continue with the previous lecture when we discussed mechanical advantage issues in Compliant Mechanisms, today we will quickly review what we discussed in the last lecture and then discuss somewhat can be called advanced concepts with mechanical advantage relating to compliant mechanism. So, this is the lecture number 54.

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**Highlights from Salamon-Midha (1998) paper**

- MA formula
 
$$MA = \frac{F_{out}}{F_{in}} = \frac{\Delta u_{in}}{\Delta u_{out}} \frac{\Delta SE}{\Delta u_{out} F_{in}} = MA_r + MA_c$$
- Three types of MA
  - Effect of input force/displacement
  - Effect of output displacement
  - Effect of the workpiece stiffness
- Sensitivity coefficient
 
$$MA = MA_s \left( \frac{k_w}{\sigma + k_w} \right)$$

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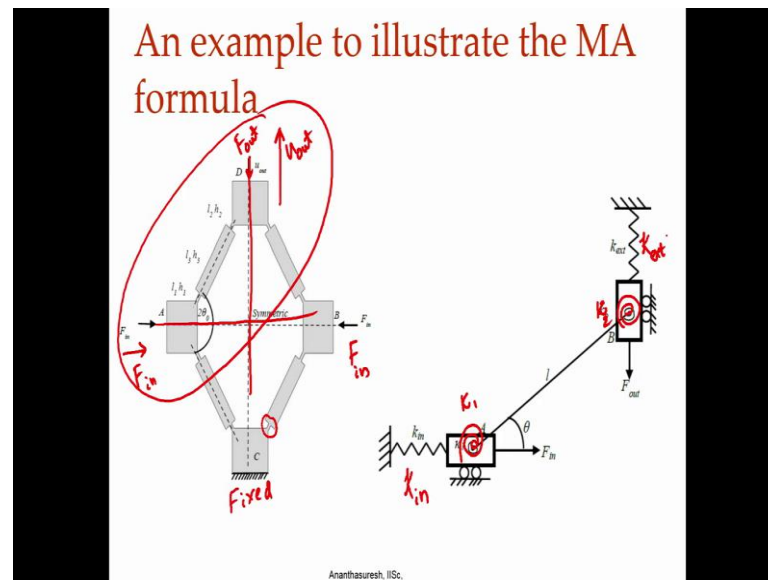
So, we were talk about mechanical advantage and there is a formula that we took from this paper of 1998, from Salamon and Midha. So, where they gave the formula for mechanical advantage where I have conveniently color coded the here blue one is the kinematic part or what we can call rigid part, and then the red one is the compliant part are the elastic part. So, mechanical advantage of compliant mechanisms is a rigid or a kinematic part compliant or the elastic part two parts are there and the definition of course, is output force divided by input force that is the mechanical advantage for compliant mechanism, because they have inherently some kinematics in them intrinsically there is some kinematic behavior which is capture as. In fact, this is delta u in by delta u out not u in by u out that a incremental change in the displacement divided

by incremental changes in the output displacement that is input and output that is the rigid or kinematic part, the other one including the minus sign that is some need to notice include in minus sign that part is  $M_{Ac}$  the compliant or the elastic part, delta change in strain energy these also delta that you refer to the previous lecture is  $\Delta u_{out}$  and then  $F_{in}$  input force.

So, this is the mechanic advantage formula that we have and we also discussed that there are three type of mechanical advantage one can define depending on what we are fixing. So, that we have the effect of input force and displacement, output force and displacement, and most importantly the work piece stiffness. Because depending on the stiffness of the work piece that is the object, which we are going to apply force on to the mechanic advantage actually changes for compliant mechanisms, because there kinematic effected by the forces because there is elasticity in them whereas, rigid link, linkages or mechanism, during mechanisms if you take their kinematic is not altered by the forces, but in compliant mechanisms since there are elastic members the forces the input force, output force, influence the kinematics inherent kinematics mechanism and hence, mechanical advantage actually changes and that is what we are discussing both  $M_{Ar}$  and  $M_{Ac}$  would change depending on the elastic component, because this incremental displacement  $\Delta u_{in}$   $\Delta u_{out}$  are changed by the forces.

So, we have to consider the effect of stiffness of the work piece and for that there was the sensitivity coefficient that is  $\sigma$  as defined in the paper by Salamon Midha, where is symbol  $\sigma$  in our rotation we have this as  $s$  and  $k_w$  is  $k_{external}$ , that is external that is work piece stiffness what we are talking about into the work piece is here, how sensitive the mechanical advantage that mechanism is given by this  $\sigma$ , if  $\sigma$  is 0 then we just have what is known as bounding value of the mechanic advantage  $M_{As}$ , because then it will become 1 when  $\sigma$  equal to 0. So, our aim is make to  $\sigma$  as small as possible.

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Now, one other point we need to remember is that the compliant part of mechanical advantage is negative, because there is a negative sign here including the negative sign we are calling that MAC. So,  $\Delta SE$  is there  $\Delta u$  out and  $F$  in with negative sign is MAC. So, what this formula tells you is that mechanical advantage of a compliant mechanism is actually hindered by the compliant component, because rigid body component or kinematic component is simply the inherent kinematics there  $\Delta u$  in  $u$  out. So, now, to that one you are adding a negative quantity adding meaning we have a put a plus sign here adding a negative quantity which is MAC. So, from the view point of mechanical advantage there is actually disadvantage for compliant mechanisms that is clear, because part of the input force and hence energy is use to deform the mechanism. So, there is like a cut taken here as compare to rigid body linkages and hence, both mechanical efficiency as well as advantage are effected when we have this and then we in the last lecture we had said is there a way to overcome this, and I am going to discuss that now, how do you make that disadvantage disappear? That is mechanical advantage of compliant mechanisms can it be more than the kinematic as a rigid body part that is more than this blue part.

And the way to do this is make the  $\Delta SE$  negative, because in that formula minus  $\Delta SE$  divided by  $\Delta u$  out  $f$  in,  $f$  in direction we cannot change that is input force and incremental change displacement it should be in the desire directions so  $\Delta u$  in also cannot be negative. Then what can be negative is  $\Delta SE$  the change in strain

energy to illustrate that, I am taking a simple examples, it also will be an illustrative example to see how we can evaluate or derive the mechanic intended formula or a given compliant mechanism, the simple compliant mechanism which is fixed at the bottom here and there is F in here it is a symmetric one. So, there is F in and both sides you press like this and the top one grows up so the output displacement here. So, when we press like this you should go out and F out can be here u out is shown with an arrow that is to see that you know F out is in that direction that is not u out F out will be resisting output displacement downwards that is this way and u out is going up.

So, such a compliant mechanism here we have taken something simple were there are this flexural flu wards which is not a great thing, but an example here taken this. So, for this we can make a module that is I can take the symmetry. So, I have symmetric axis this way, and that way I can take only this portion. If I take only that quarter, then I can show that there is some k in here, there is a transnational stiffness and there is also a rotational stiffness here, that is kappa 1 similarly there is a kappa 2 here and then there is k external, k external is like work piece. So, we are acting against that or towards whichever way look at.

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**MA formula**

$$a = l \cos \theta \Rightarrow \Delta a = \Delta u_{in} \approx \frac{da}{d\theta} \Delta \theta = (-l \sin \theta) \Delta \theta$$

$$b = l \sin \theta \Rightarrow \Delta b = \Delta u_{out} = \frac{db}{d\theta} \Delta \theta = (l \cos \theta) \Delta \theta$$

$$MA = \frac{\Delta u_{in}}{\Delta u_{out}} = -\tan \theta$$

So, these models for that and once you have the model, we can write the displacement. So, if I call the distance from here to here as a, and then here to here as b. So, a equal to this is l this is theta the angle. So, a is l cosine theta if I want delta a that is going to be

my delta u in how much even I apply input force in that direction, how much this it displays we have to take that d a by d delta into d theta to first order, these an approximation determine the Taylor series, and d a by a is equals to l cosine theta. So, d a by d theta is minus l sin theta, because a equal to l cosine theta times delta theta will give you delta u in similarly, when I have b that is this here that is l sin theta delta b will be my delta u out incremental output displacement that is d b by delta theta since, b is l sin theta d b by d theta is l cosine theta.

So, this is l cosine theta delta theta so now we have delta u in b delta u out. So, kinematic component delta u in by delta u out is minus l sin theta delta theta divide by l cosine theta delta theta delta theta get cancels to l l get canceled it will become minus tan theta, that is the rigid body component or kinematic component. So, that is what we get this is actually MAr it will be minus tan theta and then we have this formula for this.

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**MA formula (contd.)**

$$\Delta u_{in} = -l \sin \theta \Delta \theta$$

$$\Delta u_{out} = l \cos \theta \Delta \theta$$

$$SE = \frac{1}{2} k_{in} \Delta u_{in}^2 + \frac{1}{2} k_{ext} \Delta u_{out}^2 + \frac{1}{2} \kappa_1 (\theta - \theta_0)^2 + \frac{1}{2} \kappa_2 (\theta_0 - \theta)^2$$

$$\frac{dSE}{d\theta} = \frac{1}{2} k_{in} l^2 \sin \theta \cos \theta - \frac{1}{2} k_{ext} l^2 \cos \theta \sin \theta + \kappa_1 (\theta - \theta_0) - \kappa_2 (\theta_0 - \theta)$$

$$\Delta SE = \frac{dSE}{d\theta} \Delta \theta$$

$$MA = \frac{F_{out}}{F_{in}} = \frac{\Delta u_{in}}{\Delta u_{out}} = \frac{\frac{dSE}{d\theta} \Delta \theta}{\Delta \theta} = \frac{dSE}{d\theta}$$

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So, which also has the complained component for that we need by the strain energy, we need the strain energy for this the k in the spring times delta u in square that is the incremental displacement in that, and then this spring that has delta u out square for which we have the expressions, these two and then we have kappa 1 times theta minus theta not square there will be one position which is the free state of the torsion spring that is denoted by this theta naught for both springs that is kappa 1, which is over here and then kappa 2, which is over there is some free for state for it with theta naught and then

when it displays is to another angle  $\theta$ ,  $\theta - \theta_0$  will be the strain energy in that multiplied by  $\kappa_1$  than half of that and then likewise the last term as were.

So, we have a  $c$ , if I want to get  $\delta a_c$  and into the derivative. So,  $dSE$  by  $d\theta$  we take the derivative with respect to  $\theta$  we get this and, then I can write  $\delta SE$   $dSE$  with  $d\theta$  times  $\delta\theta$  once I have this I can go here again the  $\delta$ s are missing cut and paste if you miss one place that miss everywhere so  $\delta u_{in}$  by  $\delta u_{out}$  we have those. So, would  $\delta u_{in}$   $\delta u_{out}$  you have now we have  $\delta SE$  as well if you do that you will get the blue part, which is the kinematic part you get the compliant part again compliant part  $\delta SE$  if you see you have to look at this term. So,  $\delta dSE$  by  $d\theta$  into  $\delta\theta$  if you would say  $\delta\theta$  is positive then you have to look at these expressions to see if there is a chance for them to become negative, because unless  $\delta SE$  is negative we are not going to have a positive contribution towards the total mechanical advantage.

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Because,  $M_A$  what it is and then you have to plus add something positive. So,  $M_A$  has to be positive that can be positive along with this negative sign here, it is possible only if this is less than 0, that is what we want that we have to look at this  $dSE$ , but  $d\theta$  that is what will be controlling the sign of it, because likely we have  $\frac{1}{2} k_{external} \cos^2 \theta \sin \theta$ . So, where ever this becomes let us say positive and then depends on relative values of  $k_{in}$  and  $k_{external}$  and then we have  $\kappa_1$  and  $\kappa_2$  one of them has minus another is plus sign, over all we have to see how they all become positive or negative, but is it gives a hope that first some parameters of  $k_{in}$   $k_{external}$   $\kappa_1$   $\kappa_2$  you can make  $\delta a_c$  negative for part of the motion of this device.

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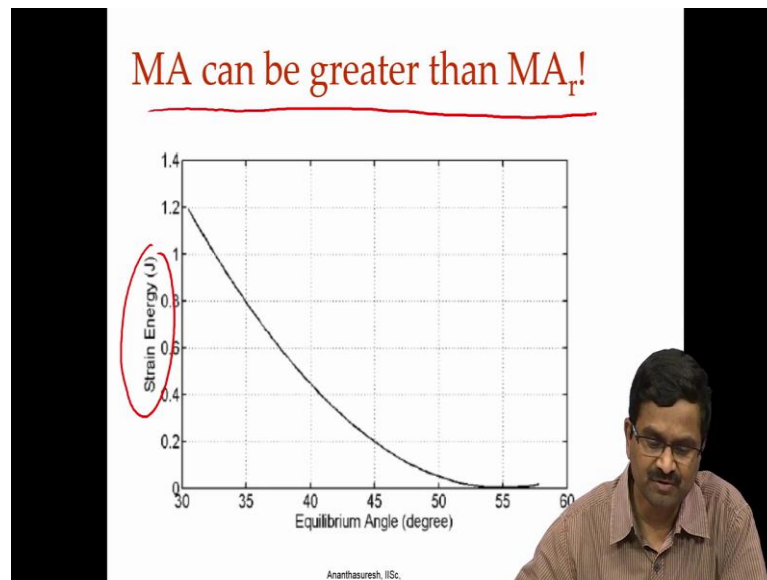
If you do that we get mechanical advantage that is actually more than the kinematic part are what we call rigid point, that is idea we have make  $\delta SE$  negative meaning strain energy of the device should decrease as you deform it that is counter intuitive, because whenever something deforms we know that is energy is going to increase, because is this deformation. So, how do you make a decrease?

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Before we discuss that let's look at for these examples, which some parameters variable to get that this is the total mechanical advantage this is  $MA_T$  this is  $MA_C$ , we can see that in this particular case  $MA_C$  is positive because as we can see here this is  $MA_T$  that is very little here, this is  $MA_C$  total mechanical advantage of course, is more than the rigid body that is dash line above that we have the dotted line, which is over advantage because it is positive for a long range that is from 0 to about 55 degrees at the angle here it is positive after that it goes negative, which I have not shown here with the point is that we can actually design compliant mechanisms, where we can have the compliant component positive for the mechanical advantage, that is the point that we need to have the strain energy  $\Delta$ , strain energy should be negative, strain energy cannot be negative, but  $\Delta$  strain energy can be negative or rate of change of strain energy respect to the state variable of the device that is one particular variable degree of freedom we can call it that should be decreasing, then it is possible and that is exactly what you shown here. So, strain energy here should be decreasing as you deform the mechanism.

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So, in that case mechanical advantage can be greater than the rigid body or kinematic component of the mechanical advantage of the compliant mechanism.

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How to make SE decrease?

- 1) Snap-through or bistable phenomena
- 2) Pre-load

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So, how do we do that, how do we make strain energy decrease as you deform the mechanism again I emphasize that this is non intuitive, whenever you deform something you think or which actually correctly if you think that strain energy will increase, but how do you make it decrease. So, one way is to have a compliant mechanism that has snap through or bi-stability, another way is to have pre load in the compliant mechanism



that is you put in some strain energy to begin with and design the mechanism such a way that when it deforms instead of it taking energy from the input force it will take from the stored energy, that is you give compliant mechanism a savings deposit that is it will have the saved energy unit pre loaded it will take that to deform and then whatever input force or input energy putting in that will be use to serve the output needs that is the principles here. So, you need to have snap through or bi-stable usually these things also can be done with pre load or without pre load.

So, if I say this is one I would say another is pre load that is you save some energy in to the mechanism, when assembling mechanism itself a compliant mechanism will always have some fixed points so you move the fixed points. So, that forcibly they are tightening and fix to a frame. So, that there is some energy without any external forces acting if you do that; that means, there is a pre load; that means, that already some strain energy in the mechanism now when you move it, it will use its shown energy to deform rather than using additional energy for that you have to carefully this in the compliant mechanisms, that is the way it would move would make it possible for the mechanism to use energy that is stored just about anything you design may not what that you have to do it carefully that one simple example, we just discussed enables to do that you can pre load it and then use that energy to come will be discussing this bi-stable phenomena in complain mechanism in the next few lectures next three lectures actually.

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**MA analysis with SL (spring-lever) modeling**

- We get some insight into what decides bounding MA and sensitivity to workpiece stiffness.

$$MA = MA_s \left( \frac{k_{ext}}{s_k + k_{ext}} \right)$$

$$MA_s = \frac{nk_{co}}{k_{ci} + n^2k_{co}}$$

$$s_k = \frac{k_{ci}k_{co}}{k_{ci} + n^2k_{co}}$$

$$MA_s = \frac{n}{k_{ci}} s_k$$

Workpiece  
 MAS  
 SL  
 SL Model  
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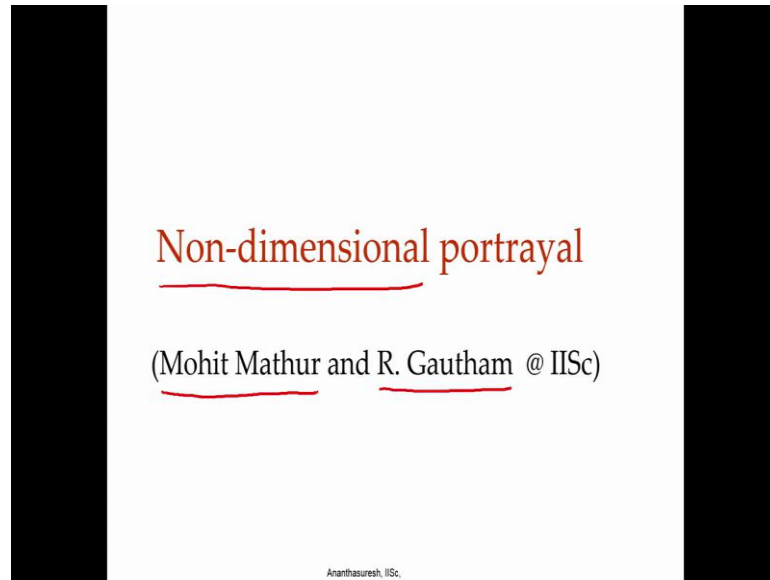
So, another point I would pick mechanical advantage, there is switching nears now we also discussed that if use spring lever model we can get some inside this what we have discussed in the last lecture, we just now read writing that that this mechanical advantage has a bounding component and then there is a sensitivity take coefficient to the  $k$  external that is the work piece stiffness that is in two places this is the work piece. So, work piece has some stiffness its nice if you have compliant mechanism whose mechanical advantage does not depend on the work piece stiffness is nice to have, because you do not have to design one for handling very flexible objects one for rigid objects. So, if you want to have one mechanism in that works for several or a large range of work piece stiffness that will be good for that you have make sure that is  $s_k$  is small, if it 0 that is ideal otherwise you make it as small as possible the spring lever model where we have this lever ratio  $n$  and there is input state stiffness  $k_{cI}$ , output state stiffness  $k_{cO}$  these 3 parameters how to the influence this bounding mechanical advantage that is a synthetically go there, when  $k$  external keeps on increasing then it  $s_k$  becomes irrelevant then the it becomes 1 so get to MAs so just goes a synthetically. So, what influences we have the formula here  $n$   $k_{cO}$ ,  $k_{cI}$  influence all 3 parameters influence, how they influence also is here and then what influences  $s_k$  is also here and the relation between MAs and  $s_k$  is also here.

So, our aim is to make MAs rather bounding mechanical advantage as larges possible at the same time, we want to make the sensitivity as low as possible and between them there is a relationship MAs and  $s_k$ , if you say that you want to optimize. So, that I want to maximize MAs by  $s_k$  then it amounts to maximizing  $n$  by  $k_{cI}$ , because MAs by  $s_k$  is that because I want to maximize MAs minimize  $s_k$  so I put in the denominator. So, I have to make  $n$  by  $k_{cI}$  it is not as simple because, MAs also depends on  $k_{cO}$  as we see in this formula. So, all in to related we has discuss optimization problems here that, but s l models spring lever model gives an idea designing a compliant mechanism in view of mechanical advantage also throughout the motion because these 3 parameters  $k_{cO}$   $n$  and  $k_{cI}$  change from position to position.

See if we can determine them we have choose discussed earlier will also know how mechanical advantage changes were complaint mechanism, these are the things that are not accounted for when you do the synthesis the way we have discussed, because those synthesis techniques mostly focus on the functionality that is the direction in put force

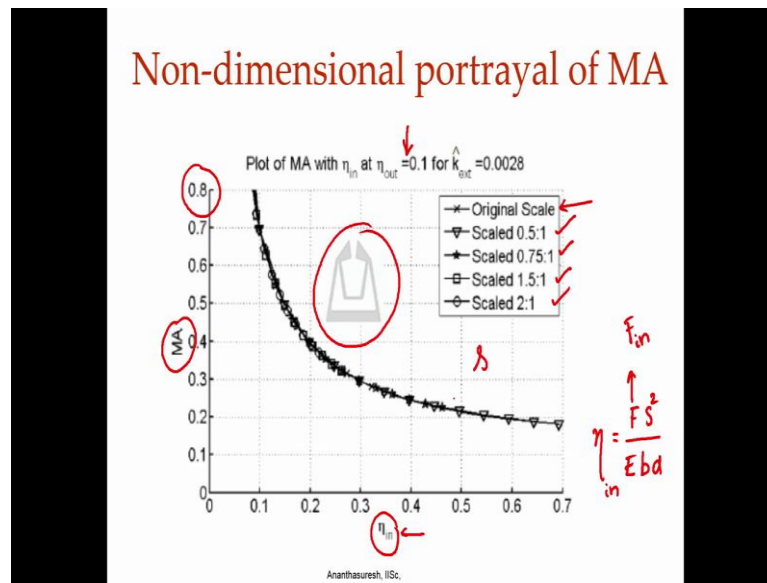
moves input displacements happens and then how output force, output displacement to that is what we have discussed, but now it is a final point of mechanical advantage which also can be accounted for with this spring lever model parameters.

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We will also now discuss the Non-dimensional portrayal which you had discussed earlier in general how to use this as a design techniques as a forth design techniques non dimensional maps, which you had call kinetio-elastic statics maps, and kinetio-elastic maps. Now, we discuss for a few minutes how we can actually use mechanical advantage also and portrait in non dimensional fashion, this is the work done by one project assistant my group R. Gautham and Mohit Mathur, he was a m tech student at IISc is their work.

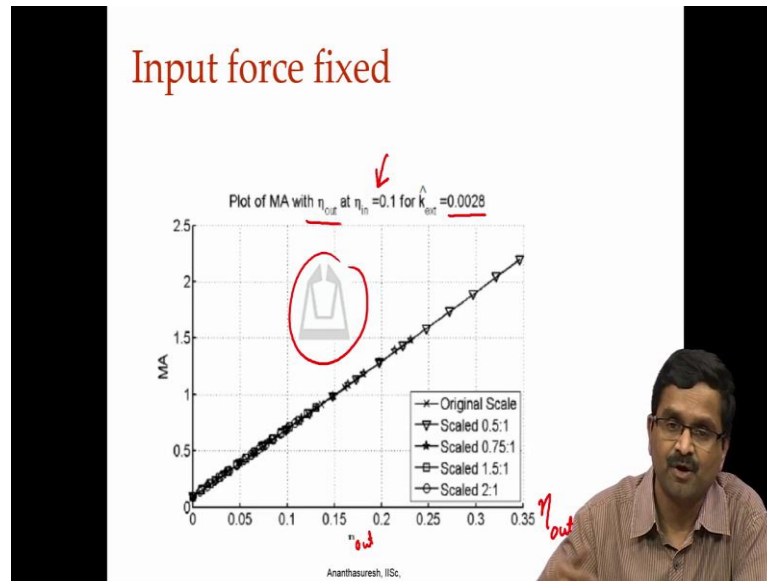
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So, what is shown here is that mechanical advantage which is on the y axis its non dimensional because,  $f_{out}$  to be  $f_{in}$  output force the input force has non dimension and then we have here  $\eta_{in}$ , if you remember from the lectures on non dimensional, non dimensionalization of compliant mechanism performance parameters was  $F S^2$  by  $E b d f$  here is taken as  $F_{in}$  for some fixed output.

If you think about mechanical advantage type one type two type three, either output displacement or force are fixed here output force is fixed in that is also non dimensional fashion  $\eta_{out}$  is point 1  $\eta_{in}$  is here. Where that becomes  $f_{in}$  that becomes  $\eta_{in}$  if that is  $f_{out}$  becomes  $\eta_{out}$ , and what is shown here is for different sizes right the scaled 0.5:1, 0.75:1, 0.5, 2:1, and then there is a original scale that is what we had said that they all line a curve this is for this mechanism a compliant gripper the all line a curve, we can see how mechanical advantage changes. In fact, here it is 0.8 meaning that is actually not an advantage know  $F_{out}$  is smaller, if it is mechanical advantage smaller than 1  $F_{out}$  is smaller than  $F_{in}$  is not within advantage, but just to show that we can actually do it against this then as we have discussed once we have this curve that will be an  $s$  value slenderness ratio for it and for that value it is fixed, we can draw it for different  $s$  values as 1.

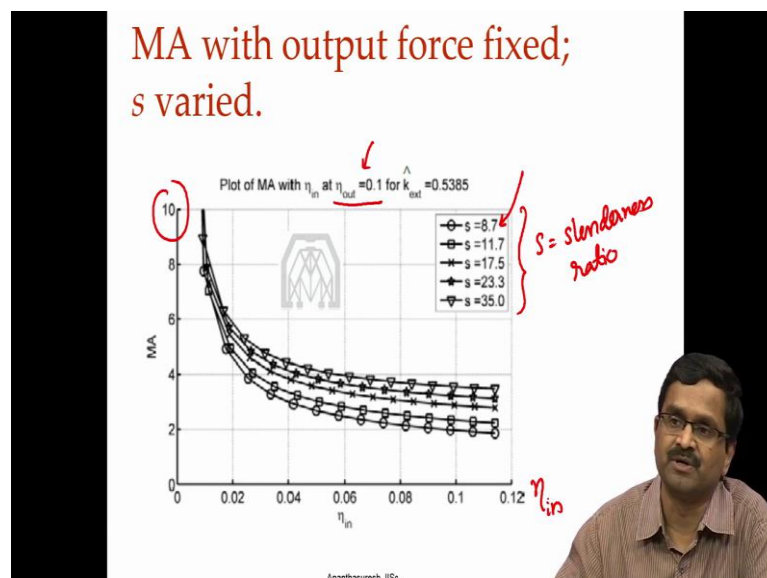
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And this is here where we have taken it for particular output stiffness again for you know this mechanism we have drawn this for different scales how it goes here we have input force fixed whereas, output force this varied. So, what we have here is actually eta out so this is the how the mechanical advantage changes with eta out.

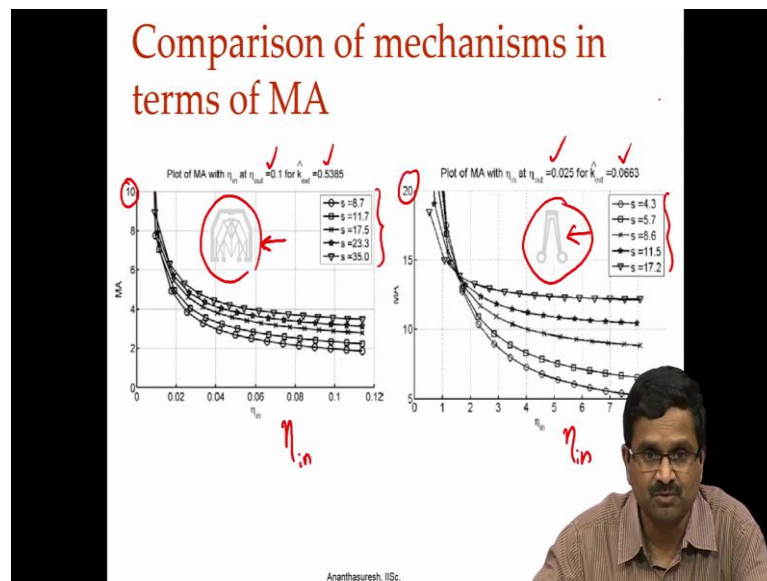
So, output force here input force is fixed previous one was that because, there parameter are as we saw that is why is necessary to define different mechanical advantage depending on what is fixed.

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Here, is one that is down for deflects slenderness ratios. So, again it goes from 8.7 to 35. In fact, anything less than 10 one should question it, but 8.7 are close enough to 10. So, how mechanical advantage changes goes up to 10 here and again output force is specified here if you change that the curves will change. So, I have to draw all of this for different parameters, but most importantly first slenderness ratio,  $s$  here is slenderness ratio here drawn that we can see what happens and on the x axis we have and  $\eta_{in}$  because output is fixed.

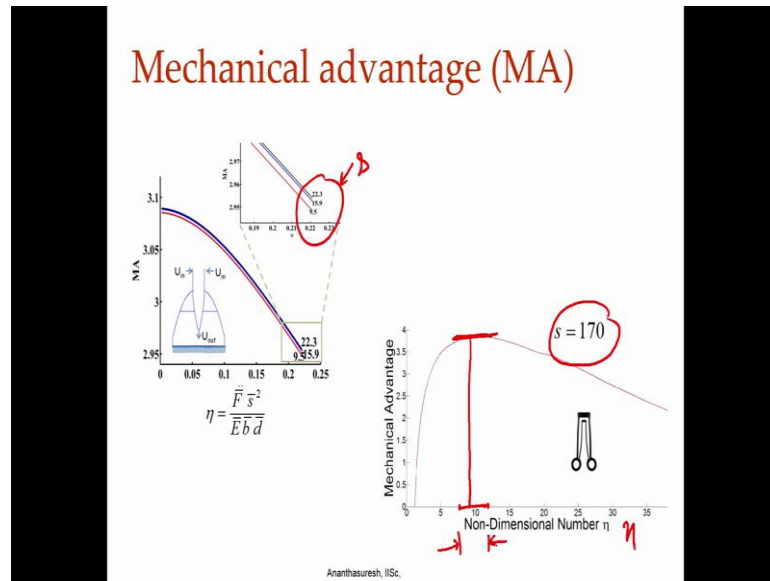
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And when you have this plots, now this is drawn for two different mechanisms and of course, this goes up to 10 this goes up to 20 and different  $s$  values, here you can 4.3, 5.7, should be actually discounted by the just drawn here. So, if you look at them we can see how the mechanic advantage changes with a  $\eta_{in}$ , with basically has a input force see how it changes for fixed output force we have to draw it different output forces as well and also  $k_{ext}$ , that is work is stiffness you can do that and looking at it we can see how your mechanic advantage changes and you can choose the appropriate range for you to work with. So, non dimensional one once you have if you want to use this mechanism or this mechanism you want the further decide let us see that this mechanism already satisfies work in kinematic requirement or functional requirements, now you want to go to the next level that is mechanical advantage wise is it helpful or not.

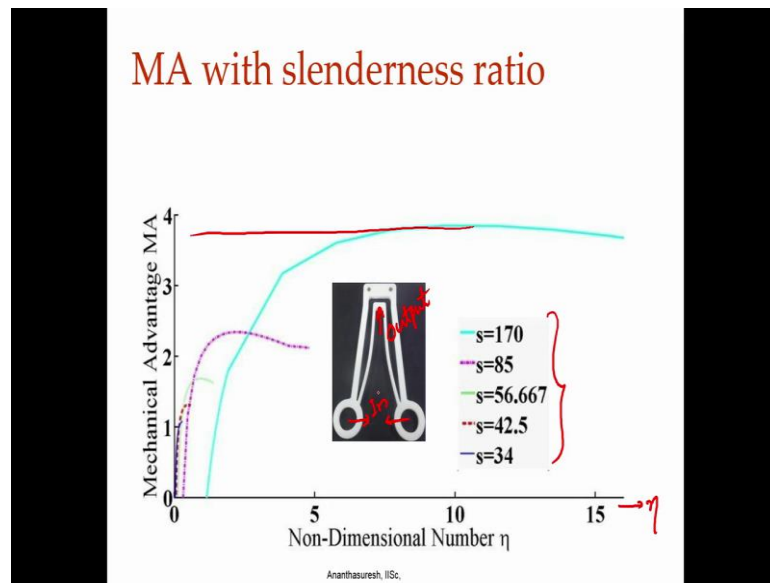
Now, to do that if you have this charts you can quickly assess what mechanical advantage you get right. So, that is how this charts you can be used and you can compare different compliant mechanisms as it is shown here. So, you have a data base of 85 mechanisms if you have these charts for all of them these are kinetic-elastic maps here. So, we can also wonder about which how the mechanical advantage changes, which one to choose.

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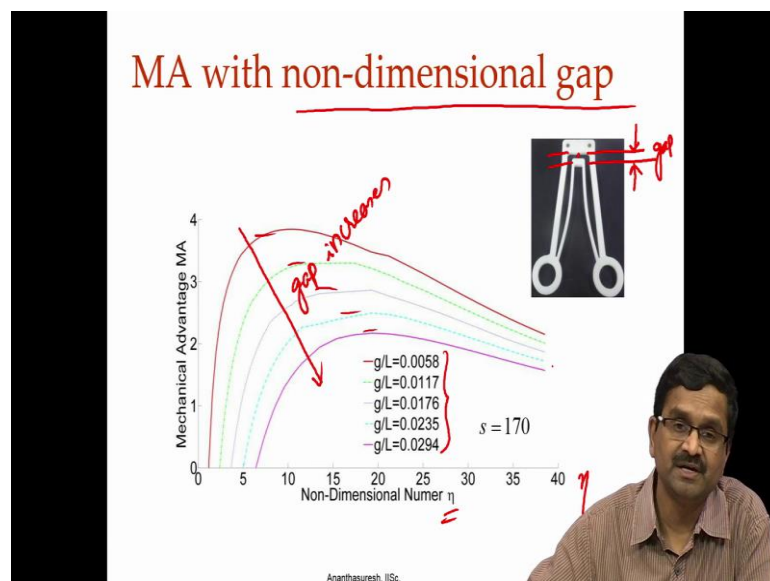
So, sometimes as we had already discussed mechanical advantage can you have a peak a maximum and you can see at what value of this eta, and it is s equals to 170 here you can read for different access also as it is done for this mechanisms s equal to 22, 60, 9.5, it is done if you have this you can actually compare and then see what you want to choose and where you want to operate, that is where do you want to apply the force what range of input force where you get the most mechanical advantage you can this design, because non dimensional you can adjust your size and other things to operate here because, that is where the mechanical advantages largest we can actually operate your mechanism in that appropriate range.

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And you can do it for different slenderness ratios as we had already discussed it. So, goes from 34, 42, 56.67 and 85 and 70 you can look at all these and then what you get and you can see that you can (Refer Time: 28:26) as you increase the value of eta meaning that force as you apply, you can apply the force in this case in this input force is like that and then output force is like that, this is the output this is the input you can see how mechanical advantage changes. So, all that you get non dimensional you can resize at material whatever, because it is non dimensional.

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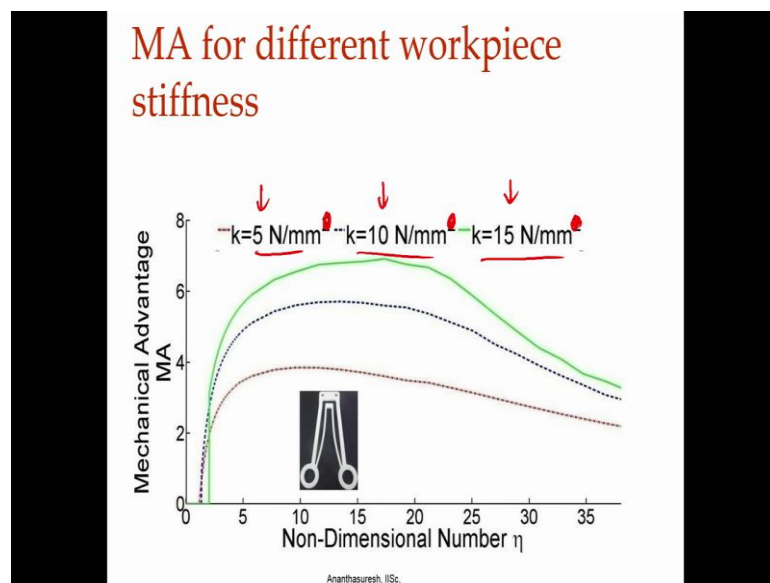




And you can also non dimensionalize the gap here, in this particular one is this lets a this the gap which have there is a little objective, which is not touching not fitting in between the two edges here then you have to see for different gaps how does it work then looking see how that changes mechanical advantage changes based on the gap.

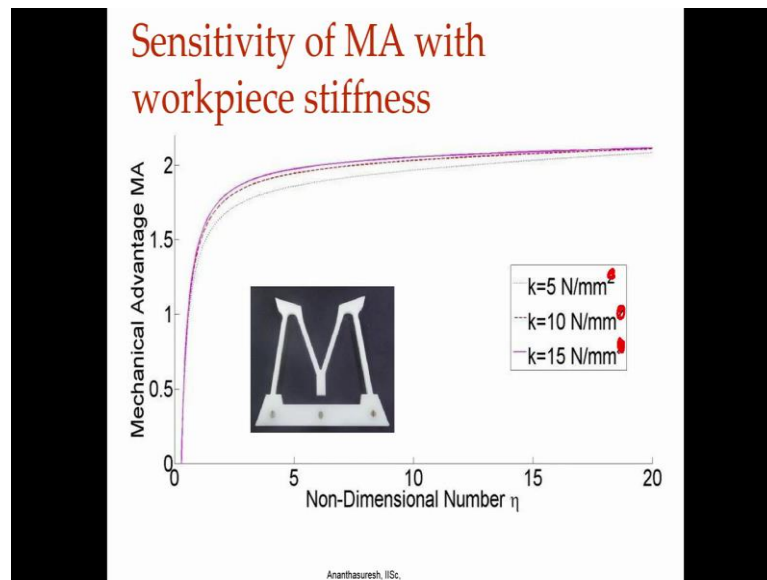
So, the gap is increasing like that because, the red one is 0.0058 non dimensional g with the gap divided by the characteristic length of the mechanism and then goes all the way down. So, actually gap this is 0.2. So, gap actually increases this way we can see how mechanical advantage changes, gap increases it is mechanical advantages decreasing with eta meaning that that has a F in. So, how that changes apply more or more force it goes there, there is a maximum for all of them somewhere and then it decreases also at a less steep rate initial to increases much sharply then and decreases have towards we have that will be helpful in designing from the wave point of mechanical advantage.

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And you can also do it for different work piece stiffness; right now it is shown as absolute by that can also be non-dimensionalized. So, that you can use the same chart for many more situation then specific values of work piece like that 5 Newton per meter square or actually it should be k.

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So, there should be only that  $k$  naught meter square newton per millimeter. So, we can also look at this for this again not square so this is state work piece stiffness you can drop mechanisms and choose the mechanism that you would want to use.

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### Main points

- How to obtain MA for a given compliant mechanism.
- MA can be larger than  $MA_r$  provided that there is pre-load or negative-stiffness
- Non-dimensional portrayal of MA helps in comparing different mechanisms.

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So, the main points of that how to obtain mechanical advantage for a given compliant mechanism which discuss with a simple example with this lecture and then you also discussed how to make mechanical advantage of a complaint mechanism larger than the kinematic part this  $MA_r$  and rigid part that can be done with pre load or bi-stable

mechanism. So, basically there should be a negative stiffness for some region that is when strain energy can actually decrease it can never be negative, but it can decrease with deformation of the mechanism.

And we also discuss in non dimensional portrayal at the fiction of mechanical advantage. So, that we can compare different compliant mechanisms and choose the one the source are purpose not just from functional requirements, but also for mechanical advantage requirements, because mechanical advantages more like a secondary thing first you have to have a mechanism that serves the purpose of the function its intended to after that is a luxury you want to have it mechanical advantage, but if you think about compliant mechanisms are rigid body mechanisms mechanical advantages is the very important characteristics, but design method we have discussed do not directly address that, that is what we need to remember, that is what we talked about in this lecture.

Thank you.