Compliant Mechanisms: Principles and Design
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Lecture – 53
Aspects of Mechanical advantage of compliant mechanisms

Hello, we have discussed design methods and also compared them in the last lecture. From now onwards, we will look at some final aspects of compliant mechanisms one of the very important ones is mechanical advantage, and that's what we will discuss in this lecture and the one after this. So, what we will discuss is a set of aspects that one needs to worry about in terms of mechanical advantage for mechanisms, compliant mechanism is a mechanism the most important thing is transmitting force and of course, motion when you are transmitting force you need to have mechanical advantage. In fact, the principle use of a machine any machine is actually to give an advantage to the human being. So, here compliant mechanism has tools to do exactly that.

So, we need to discuss how much mechanical advantage that is very little force you want to accomplish a task of handling a big load that's what mechanisms do all of them or machines in general, in this case also we look at how compliant mechanisms mechanical advantage can be looked at or can be analysed or even develop an optimization method to get a mechanical advantage that is good for and given application or even maximize mechanical advantage.

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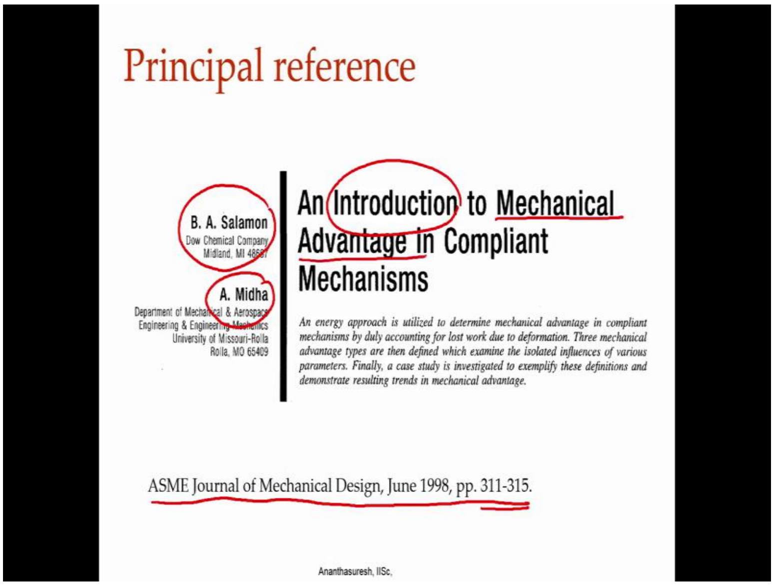

So, let us look at the aspects of mechanical advantage in compliant mechanisms. So, the principle reference for this work is master thesis at Purdue University by Bren Salamon and Professor Ashok Midha was the advisor of this student and they had a very nice paper. In fact, that master thesis itself is a an excellent thesis that looked at as it says very humbly just an introduction to mechanical advantage, but all that one wants to know about mechanical advantage is in this, I would say 4 page paper 311 to 315 and that's what we will discuss today, what was in this paper what are the nice things that Ashok Midha and Salamon Bren Salamon had come up with.

In this paper has more detail, but it is a 4 page paper we would highlight the important aspects of mechanical advantage that they discussed in this one, as you may recall because Ashok Midha is the one who coined that term compliant mechanism and had done some very fundamental work at a very beginning of the field at the inception of this field one of the great thing he has done is to analyse mechanical advantage in a way that is very insightful as we will try to discuss today.

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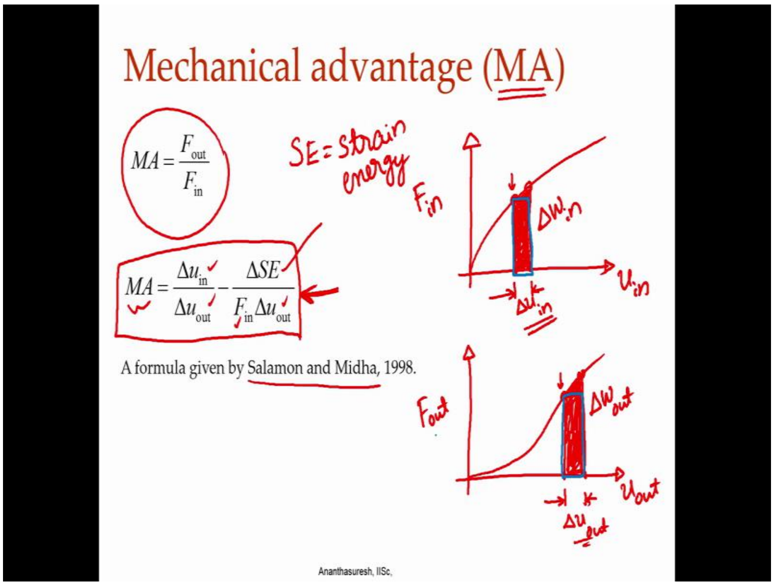

Lets first define mechanical advantage, mechanical advantage acronym MA mechanical advantage is simply output force divided by input force as simple as that, that everybody knows even high school you are taught when you talk about levers we will say what’s the mechanical advantage output force divided by input force. Now Salamon, Midha gave a formula for mechanical advantage of compliant mechanisms. So, that is this one. It is main advantage is, there is some delta in by delta out minus delta SE by F i n and delta out, how does this come about. So, that’s what we will first discuss. So, before this let us actually see what this delta in delta out and all that is. So, let us look at 2 plots let us say there is input displacement and then input force F i n u i n and then F out and u out.

So, if you have a compliant mechanism you have input force and input displacements there could be some curve there similarly, F out and u out just so it looks different I will draw a curve differently, it can be anything, it can be stiffening, softening whatever. When you have this, when you look at this formula what we mean by delta u i n is that, this mechanical advantage of mechanisms in general they are not constant throughout the range of motion see any machine and mechanism that has motion it is going to move at different configurations we will have a changing mechanical advantage it is not going to be constant for a simple lever with fulcrum it will be constant, but not in general. So, if you have a situation where things are changing, we should look at let us say a particular point let us say I take a point here, there is some u n and correspondingly there is let us say there is some u out the corresponding point over here. So, this point and this point correspond to each other. So, you have some f n and some u n.

Now, if you want to define mechanical advantage then what we will do in the derivation of this particular formula consider what if there is a small change in the input displacement that is our delta u in. Similarly, there is small change in the output displacement that is we are moving this is delta u out; this is delta u in delta u out which is what we have here. And then we have delta u out again of course, there is F i in there is no F i F out in this formula. There is also delta SE, delta SE SE for us is strain energy because, in a compliant mechanism deforms there will be energy stored in it elastic energy which you call strain energy.

So, when you go from 1 point to a new point there will be some extra energy stored and that is this delta SE. SE is in general strain energy when you just imagine that from a particular point that you are at if you slightly go to another configuration, there will be additional energy that comes in it is positive or negative depending on whether unloading or loading, but there will be a delta SE that is; what is shown here, and all of those define the mechanical advantage of a compliant mechanism as was done by Salamon and Midha.

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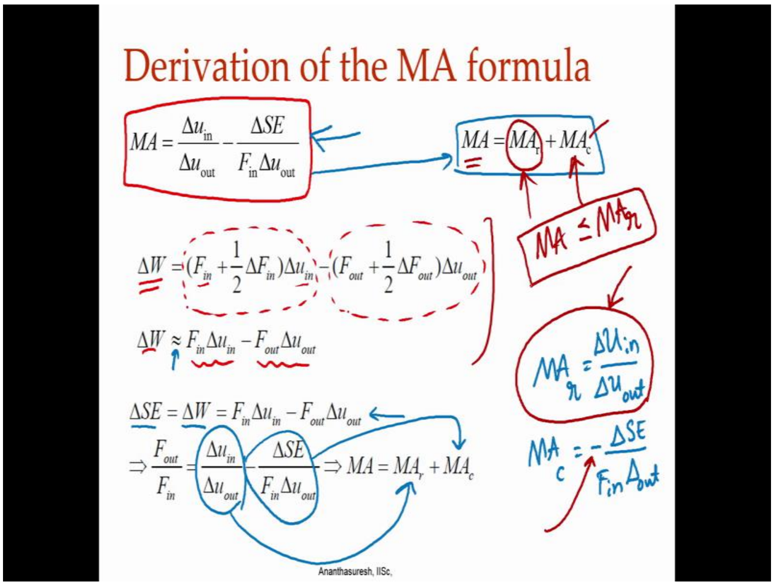

The way that they derived it was as follows: you this is what we need to derive. So, the first thing you do is look at the change in work both the input work and output work. So, I will circle this part which is the input work and then circle this which is output work. So, input work has a positive sign output has a negative sign that makes sense because, output is going out of the system, input is coming into the system where a system is compliant mechanism. Why is this F in plus half delta F in u in for that we need to go to this charts here the plots that we get, we are basically talking about this. So, this portion this rectangle will be F in times delta u in. So, what we have here F in times delta u in the other part, half of delta F in times delta u in is this little portion. So, I will just fill it, basically area under this; is what we are looking at when we say we have the input work. So, this entire area if I take that that will be delta W in.

Similarly, if I take this other one and take this entire thing here, this will be delta out. So, the energy balance if you do work and energy joule balance if you do this delta W here, now, we have this and we have that. Now we are actually making a small approximation that is delta F in and delta u in is a second order quantity same thing delta F out delta u out then we just check F in into delta u in minus F out delta u out, in other words, we are only talking this portion, that is we are taking only this much rectangle and leaving out that little part in both cases. So, this is F in times delta u in F out times delta u out. That’s an approximation that’s why we are showing the approximation, now this delta SE should be same as this net work that you have you are putting in some work by input force taking out some, because of some output the difference has to be equal to energy stored in the compliant mechanism. In other words, delta SE should be equal to input work that is F in times delta u in and then minus F out times delta u out that is delta SE.

Now, delta SE is equal to F in times delta u in minus F out times delta u out, now from here if you rearrange this equation, F out by F in will turn out to be delta u in by delta u out minus delta SE by F in delta u out. Basically, the formula that we have here is easily derived in this manner. So, here there is a notion of looking at mechanical advantage that is this one has 2 components they are both indicated here, we can call this MA r that is the rigid body kinematic component of mechanical advantage, and there is including the minus sign that’s an important thing to say including the minus sign is compliant and elastic part of the thing. So, we have MA rigid body which is simply delta u in by delta u out instantaneously, the mechanism is the reverse of forces. So, F out by F in is mechanical advantage. So, delta u in by delta u out is the mechanical advantage also that is a rigid body part. And then MA c that's a compliant part is minus delta SE divided by F in and delta u out, it is a compliant part. Notice that the compliant part is actually negative.

So, the compliant mechanisms mechanical advantage is going to be always less than or equal to MA r that is one observation Salamon and Midha made, that is actually clear because, part of the input energy is stored in the compliant mechanism. So, both the efficiency and mechanical advantage will be hampered compared to the corresponding rigid body linkage if there is one. But then within compliant mechanism itself the kinematics is clearly present, that is something we have been saying in this course. So, compliant units are not different from rigid body linkages they are not different from structures also they are in between. So, here inherent kinematics exists and we have that coming. So, we use simply energy arguments here, and to show that Mechan advantage has a kinematic part and elastic part is actually quite interesting and we will see more of it in this lecture how it comes about. There is a rigid body or a kinematic part and there is a compliant part which is negative. It looks like a disadvantage right now, and then in the next lecture we will see how we can overcome this seemingly disadvantages position that compliant mechanisms have their mechanical advantage will always be. So, I can say because, second 1 is negative at this point MA is less than or equal to MA r for a compliant mechanism because, this is negative. So, we will see how we can make that positive also later, but for now let us say that this is an insight in general, this is true unless you do not do something extra special. That is the first thing we have to understand.

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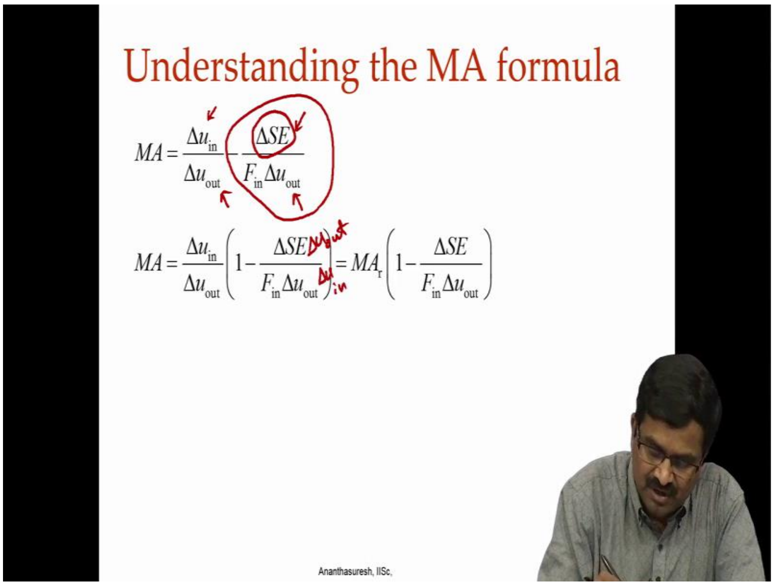

And let us understand this formula more right, we have this rigid body or kinematic part and then an elastic part and look at the terms what are they delta u in delta u out that changes from position to position because, that's why we have taken this incremental displacements here and incremental strain energy as well. If you want to make this second part also positive F in and delta u out they will be positive, because you would have assumed a direction for input force and output displacement they cannot be negative. If they are negative your main function is not being served delta u out if you make it negative it is not going in the direction you want it is going somewhere else. So, that's not good.

So, there signs cannot change what sign can change the sign of delta SE can change that is as the compliant mechanism deforms, it actually has it is energy decreasing. If you have such a situation then you can make the compliant part also become positive. So, overall mechanical advantage can increase that is the modification that we will discuss in next lecture, but right now just remember that unless you make delta SE negative, your mechanical advantage compliant mechanisms always going to be less than the kinematic part of it or rigid body part of it.

So, we can also rearrange to understand if you think. So, delta in by delta u out if I just take that as you know out common between the 2 I can rewrite this as delta SE by F in delta u out I think there are some more things I have to do delta SE F in delta u out I have taken delta. So, I have to actually multiply right. So, this will become delta u out and then delta u in, that now when you do.

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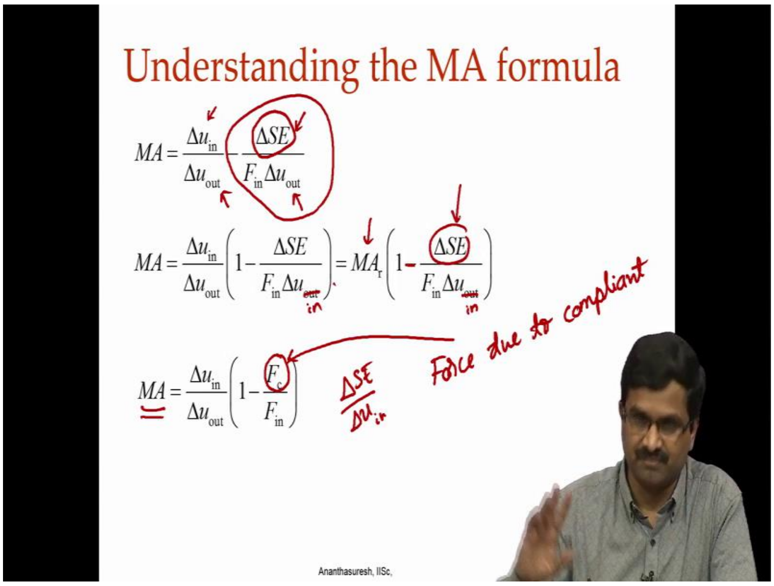

So, basically this is at hypo this should be in and this should also be in then it will be correct, when you multiply these things go and then you get delta u out there. So, we can take basically MA r and show that 1 minus delta SE by this when you do that again you see this is a negative quantity, it will always reduce unless you make this negative that's something to note here, and Salamon and Midha defined this something called F c what they call force are comp force due to compliant, that is when you have a compliant mechanism unlike rigid body linkage the input force is used partly to deform the mechanism to store energy basically motion comes because, velocity formation force is needed that's what they called F c, and F c if you see from the formula here is delta SE by delta u in from here. That's what they define as F c force that is needed to just deform the compliant mechanism.

So that we can make contact with the work piece, that's not really helping in the mechanical advantage to apply force and the work piece or an object that you want to hold. So, that is actually 1 minus that it comes the more F c is the less the mechanical advantage is going to be that's something that one is to understand, but then again we emphasize that that can be made negative in some cases.

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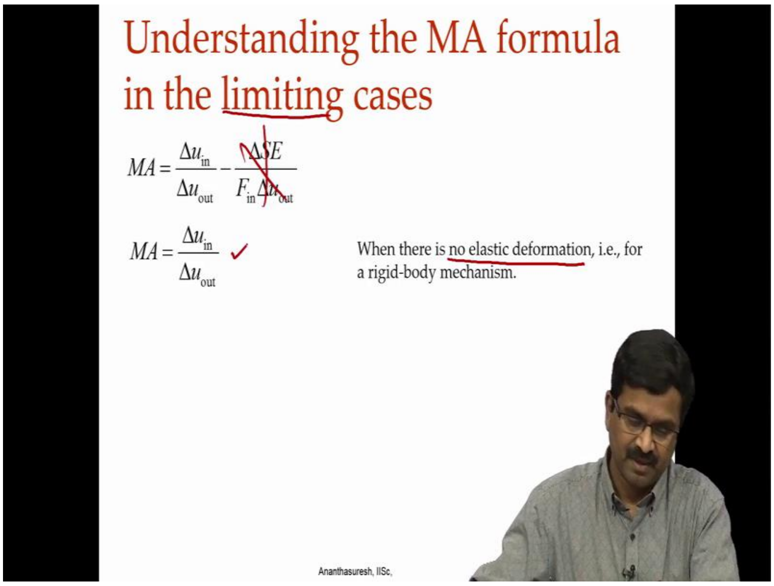

Now, also let us understand what happens in the limiting cases, we will consider 2 limiting cases one limiting case, when there is no elastic deformation. So, whatever we derive for a compliant mechanism we should be able to generalize it or make it specific to rigid body linkage in which case there is no elastic deformation. So, this goes you get some delta u in by delta u out that is essentially make an advantage of rigid body linkages. So, our formula is correct, again compliant mechanisms lie in this continuum spectrum between structures and rigid body linkages. So, in that sense this formula holds good. What about the other extreme which will be interesting to look at.

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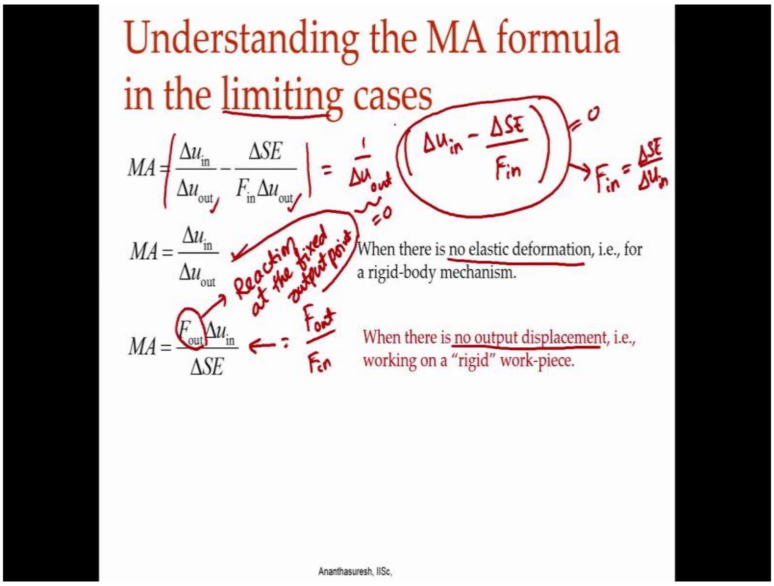

So, what if there is no output displacement what does it mean you have a compliant mechanism which I have here let us say it is a gripper.

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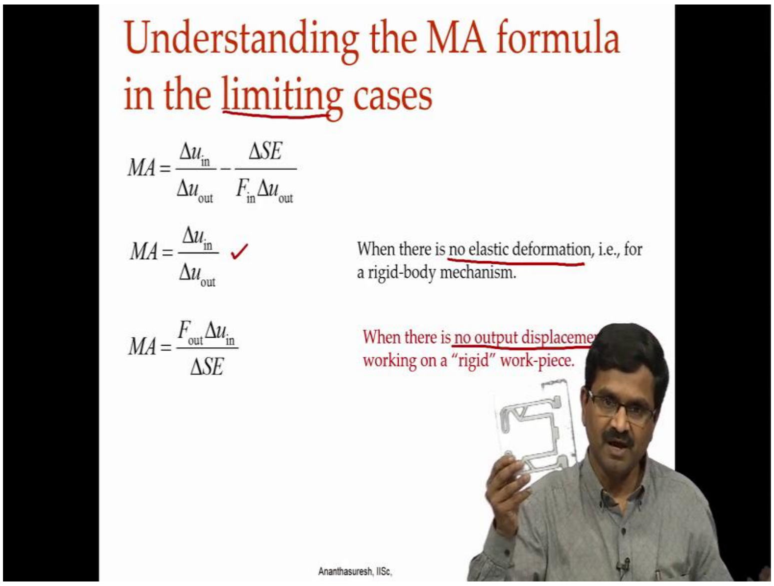

So, when apply it is fixed at these 2 locations.

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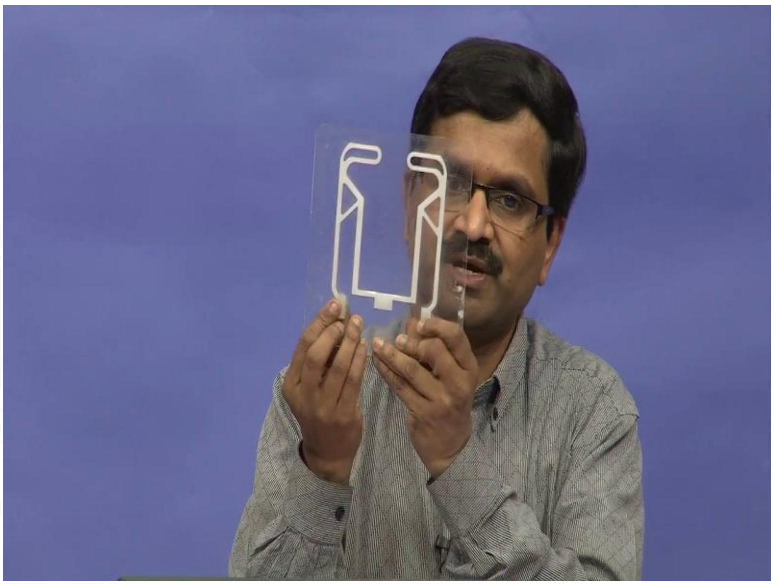

And this is like the jaws that come together. So, when apply force, when apply force it goes like this. So, initially when let us say my object is a small object and until I touch I need this F c the force just to make contact with the work piece I have not done any work on the output, after it touches then I start doing it. Now if I look at this configuration just after touching just after I touch the there is more output displacement if my finger is to be what to be rigid that is very stiff object if it is there, there is no more output displacement. If there is no output displacement we have a little problem here in the formula, because we have output displacement in the denominator.

So, does it mean this formula is not applicable it turns out that in that particular case the numerator that that we have if we take out you know this thing here, if I take delta u out then what I get will be delta u in minus delta SE by F in. So, if you look at this when this becomes 0, there is no output displacement then we would say it is infinite make an advantage is infinite it is not because, the numerator here will also be equal to 0. The reason is, as we had seen in the derivation that delta SE that change in the strain energy is equal to the input work and then minus output work now there is no output work, because output displacement is 0 input work is F in into delta u in. So, this also is equal to 0. So, in the limiting case if I take the limit of this when output displacement goes to 0 it will be finite again, and that is given in this manner by introducing this F out.

So, we see this F in because it is 0, F in is delta SE by delta u in that's what we have put here again this is the F out by F in what is F out whenever you have a rigid work piece over that compliance mechanism interacting there is no displacement there will be reaction force that is your F out that is the reaction at the fixed output at the fixed because, here there is nothing like rigid, but very stiff object at the fixed output point. So, there fixed output point you have similar F out divided by F in we can talk about mechanical advantage there, so it is not like it is infinite because, both numerator and denominator go to 0 when output displacement go to 0 this is something we need to understand the formula is actually full proof.

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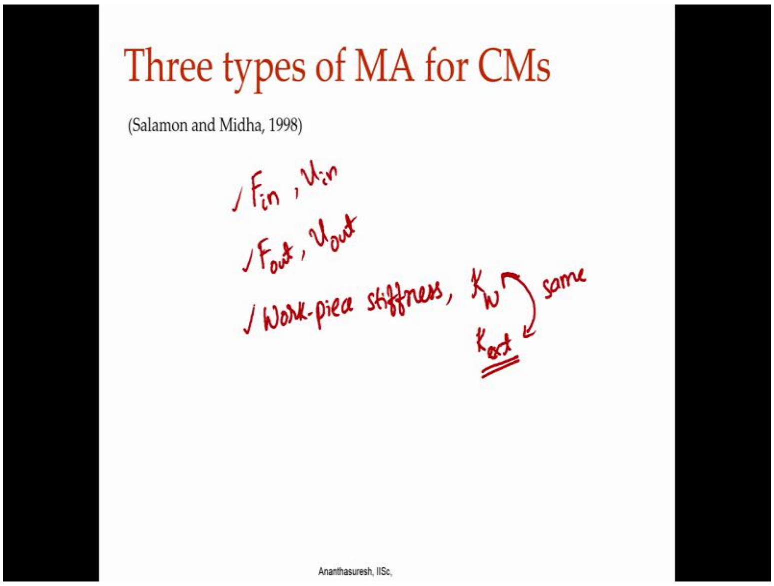

Now, Salamon and Midha defined 3 types of mechanical advantage, and the first in each type something remains constant what are the things we need to consider we have input force that is varying, we have output force that is varying in addition to that we also have work piece stiffness because, as a linkage or a mechanism we have to interact with the work pieces some of them may be flexible some of them may be stiff. So, that work piece stiffness which Salamon and Midha used the symbol K w and we have been using K external in our s l model assemble model and so for. These 2 are the same symbols 2 symbols for the same quantity. So, depending on what you fix you want to fix input force you will get one type of mechanical advantage fix output force or may be even output displacement or may be input displacement there all these parameters or for given work piece stiffness you will have our mechanical advantage take care of these things Salamon and Midha defined 3 types of mechanical advantages.

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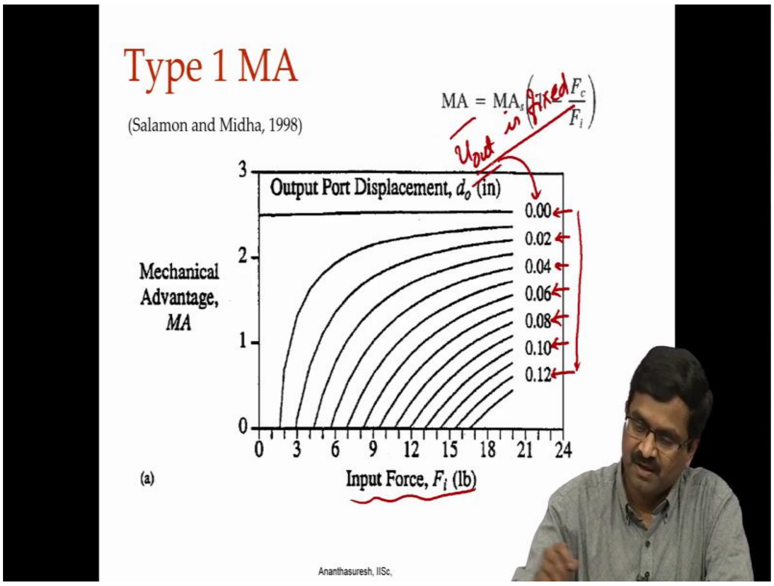

One first type, type 1 mechanical advantage is for fixed output displacement that is u out is fixed for a compliant mechanism if you want to have this much output displacement as you vary the input force mechanical advantage varies. So, here input force in the x axis for fixed output displacement for each output displacement there is a separate curve. So, this is the one, you get several curves a series of curves. These curves are always very useful because, they give you insights here what do we see as output displacement is increasing mechanical advantage is decreasing and sometimes, it can even go negative meaning that output or input they are going in the other direction output force input force. So, after that you know this is 0 already to an extent it actually goes negative, what does it mean that, instead of resisting output load it needs additional force to move by that much displacement that's what it means.

So, if we have an object you are app applying input force it should press the object and apply force on it, but here if it goes negative what means that you need additional force output load is not resisting you have to apply additional force over there. So, that's not good. So, they do not even show the negative mechanical advantage that's not the advantage that is disadvantage, but you can see that as you want more and more displacement you have decreasing mechanical advantage goes in a non-linear way as well with input force.

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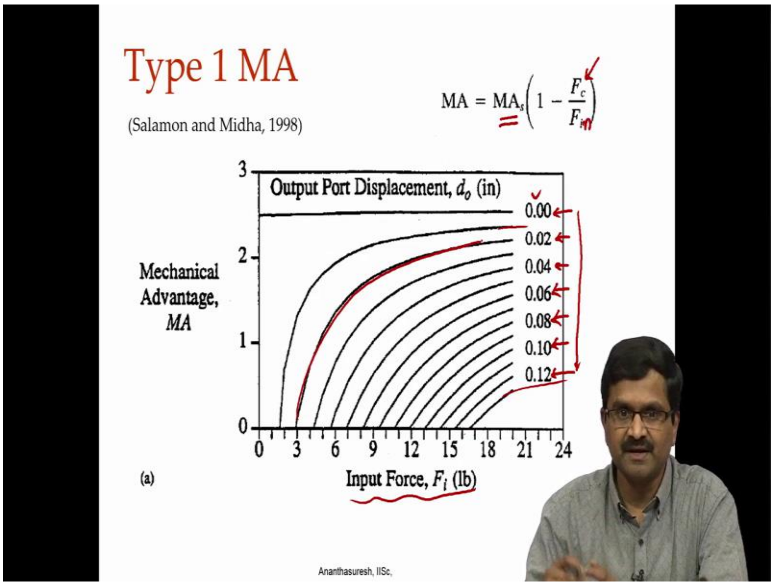

In fact, what Salamon and Midhas paper gives is that we can write it as, there is a limiting mechanical advantage that is MA s all of them are going to like this one has gone asymptotically, if you extend far enough they all going to some value they define it as bounding mechanical advantage times 1 minus F c that is the force due to the compliant part that is deform in the mechanism F c by F i n. You can actually fit that to this model, they are all numerically done, but you can fit this model because there is inherently this behaviour in the compliant mechanism

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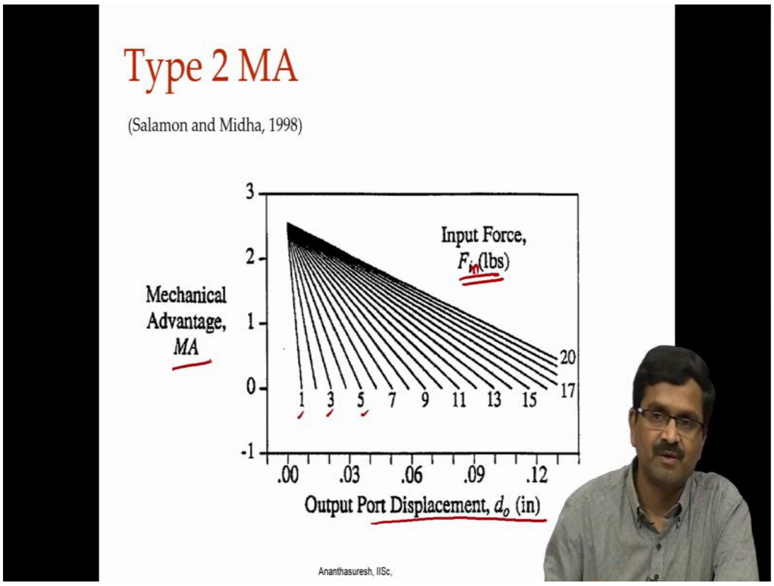

Let us look at type 2, in type 2 for different forces. So, you fix the input force. So, input force are 1 2 3 5 whatever we get different ones here, now output displacement is there not output force output displacement varying and then you have mechanical advantage.

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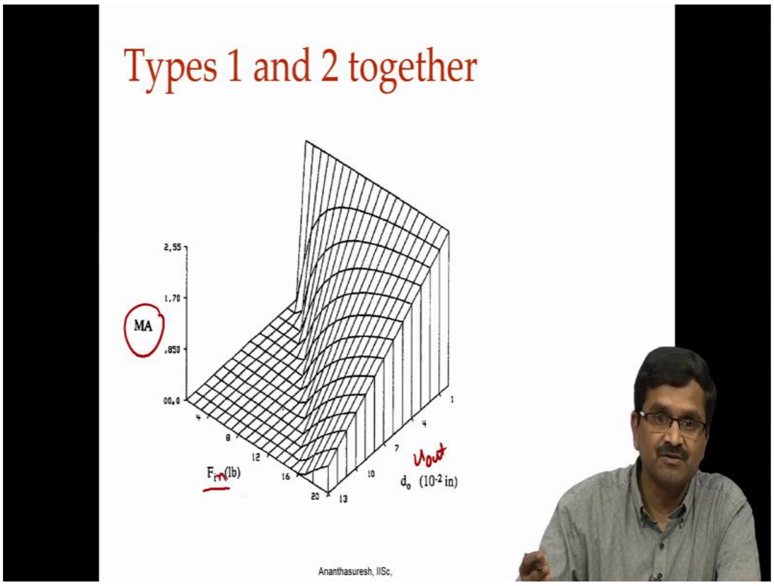

And you can look at both of them together as well type 1 type 2 3 parameters you have F in and then u out there you they use d o it is from their paper mechanical advantage both type 1 type 2 you can imagine like a surface the slices of this, that is fixing F out you will get one slice fixing F in you get another slice is what we consider type 1 and type 2.

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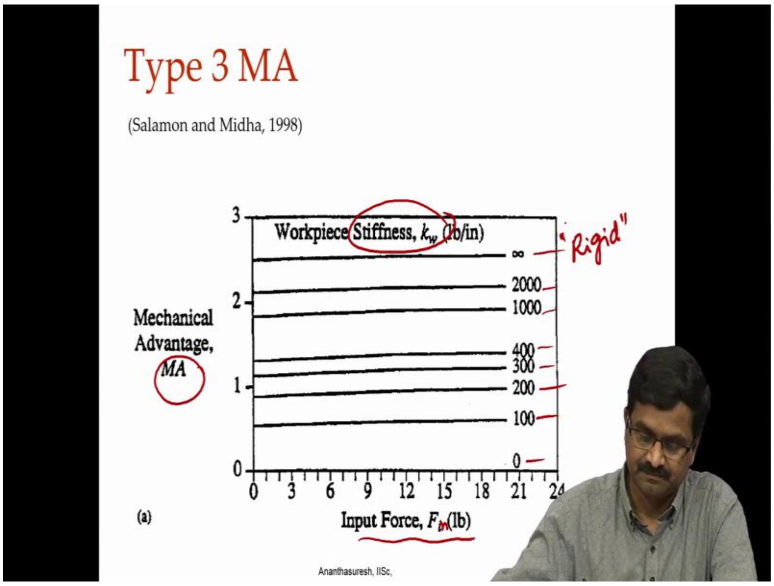

There is a type 3 where you would fix stiffness at different value 0 no work piece 100 200 some units and going all the way to infinity with regard in with different input force you can get a mechanical advantage. They look pretty much constant here, but actually they are not they slightly vary, but in this big scale we are going from 0 to infinite stiff infinite meaning it is completely rigid work piece they type 3 mechanical advantage.

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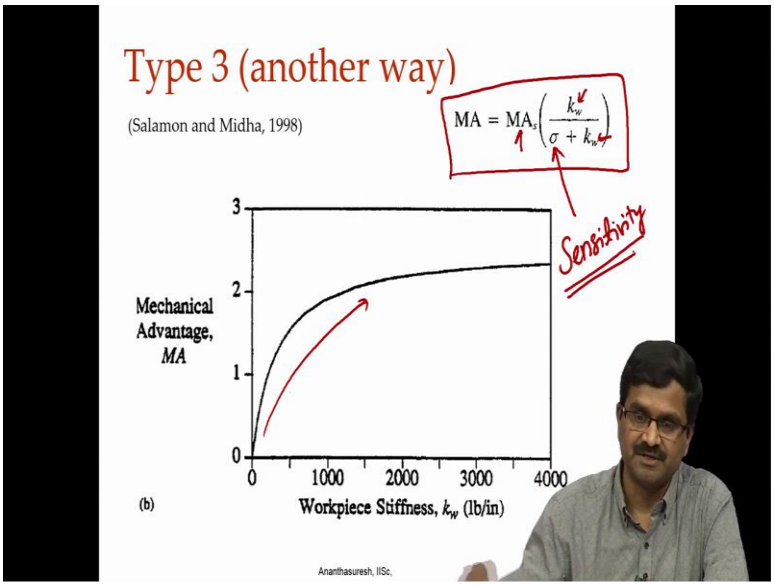

Another way to look at this is work piece stiffness you vary and get this limiting value finally, it goes to the all of these curves go somewhat like this asymptotically if you plot that value here you can see that as work piece stiffness increases mechanical advantage is actually increasing. So, these curves are also useful and this one can be put into a form which is what Salamon and Midha give that is again bounding value of MA s and then there is work piece stiffness there is a sigma that they use which they call sensitivity which we will discuss now that sensitivity. That is sensitivity of the compliant mechanisms to the stiffness of the work piece compliant mechanism mechanical advantage or for that matter even rigid body linkages kind of it will be a function of the work piece more.

So, with the compliant mechanism because, there is inherent elasticity and it is kinematic characteristics will change for rigid body linkage kinematics is defined as mechanical advantage for a compliant mechanisms there is a compliant part depending on whether the object is stiff or flexible it is mechanical advantage changes that is captured by this sensitivity.

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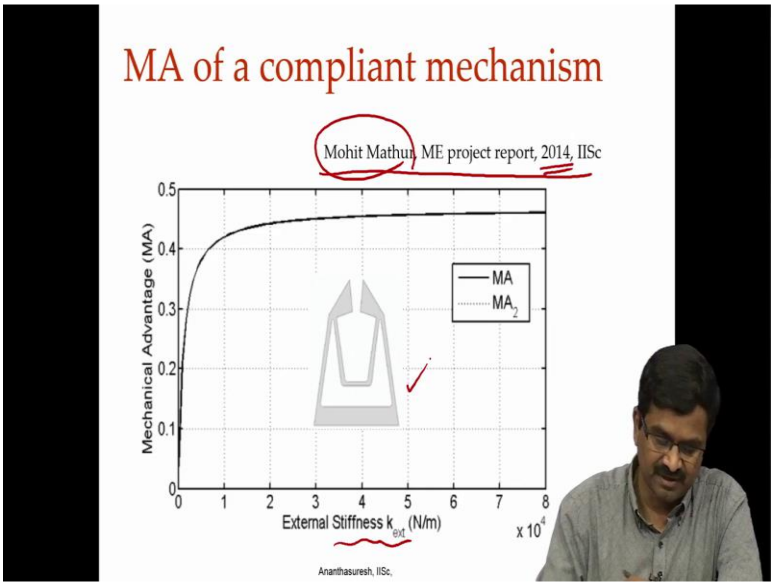

In order to understand this, we move on to another reference which is ME project at initiate of science by Mohit Mathur couple of years ago in 2014 he had done, where whatever we saw with mechanical advantage that we saw whether if external stiffness how mechanical advantage changes for a compliant mechanism that is given is shown here.

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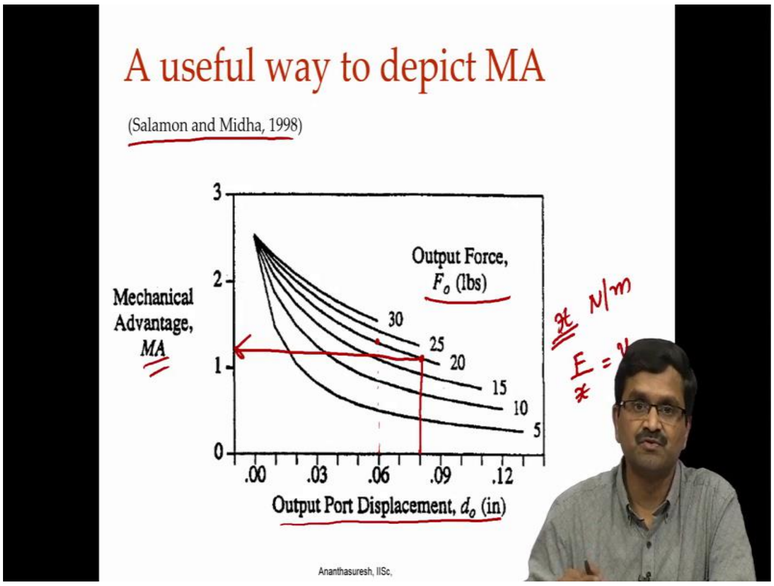

So, Salamon and Midha had done one more thing, which is to depict in a different way. So, you see mechanical advantage shown against the output displacement for various output forces now you put various output forces this is useful because, if you have work piece you would know let us say it is a linear linearly modelled work piece meaning work piece has a like a spring linear spring then you would know the force and displacement some Newton per meter.

So, let us say some x Newton per meter, then you would see if our output force is let us say 20 and corresponding displacement if you have 0.6 you would know. So, if you have x Newton per meter how much deflection. So, you have force by x this is stiffness you will get some output displacement. So, you go along this whichever let us say this value then you would know if for that output force what the mechanical advantage is. So, for various work pieces for a given compliant, if you draw these curves we can see what mechanical advantage you would get depending on the work piece stiffness, this is the another useful way to depict MA.

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Ah Mohit Mathur had done some work on understanding this sensitivity that we just discussed that is over here; what that sigma is, how does how sensitive is the work piece stiffness to the how dif how sensitive is the mechanical advantage to work piece stiffness.

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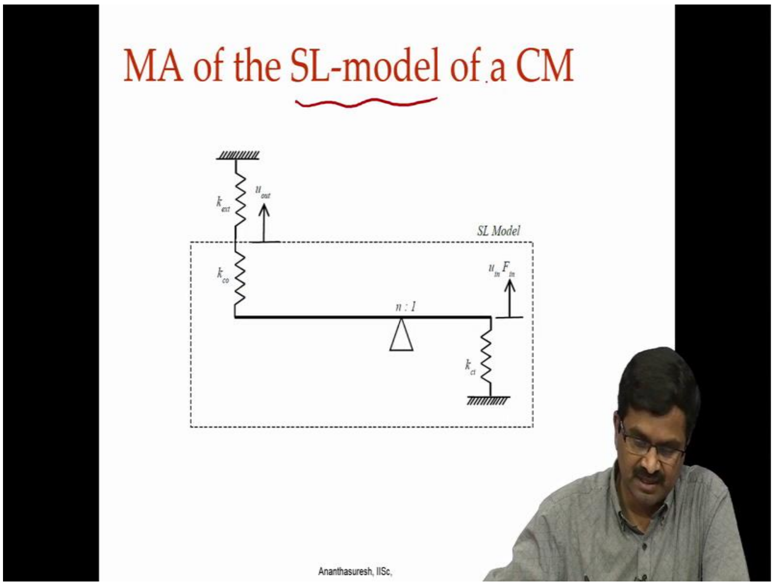

In fact, he had looked at this spring lever model because, that's a model on the compliant mechanism we wanted to see whether it captures this mechanical advantage. So, using that, because everything can be done analytically again writing potential energy, writing the equilibrium equations where we can solve for u out.

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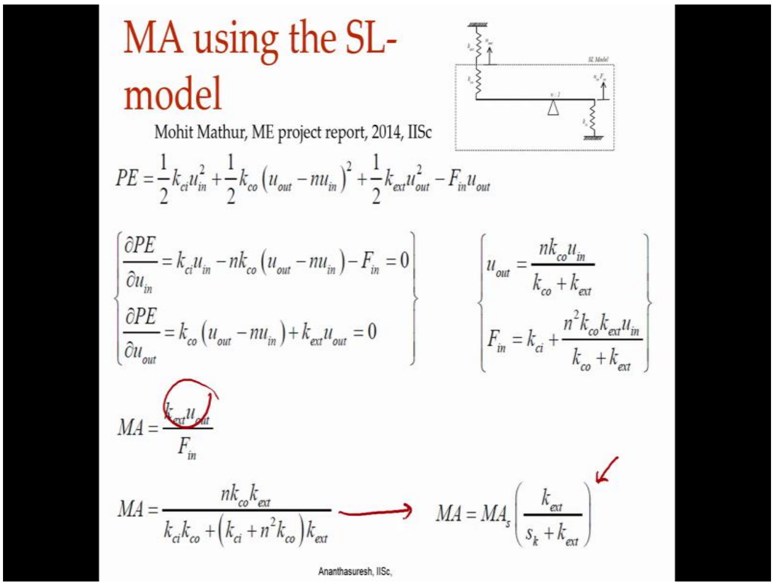

And F in put them back into the mechanical advantage formula this is F out by F in you get a formula like that, and this formula can actually be put into this fashion with just rearrangement which we will see in the next slide.

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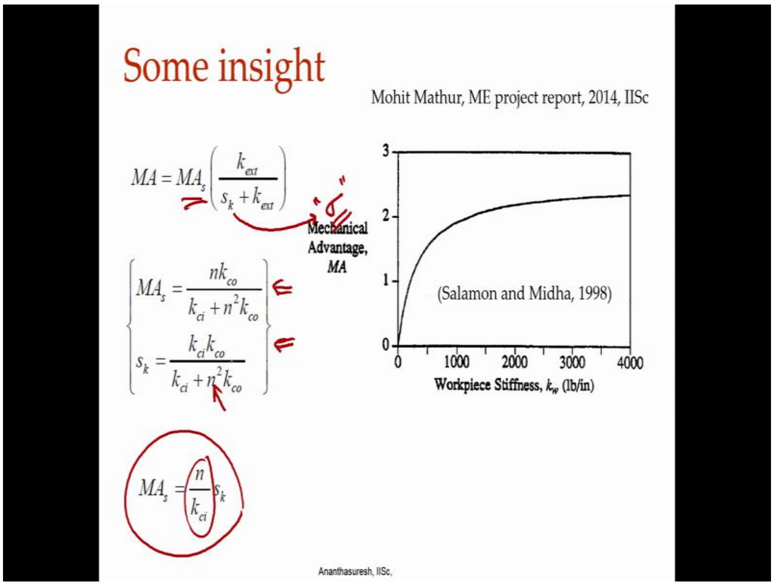

So, we if you want to get mechanical advantage in this way where that sigma that is there S k we are convo sensitive into S k, but equivalent to what Salamon and Midha called as sigma, that MA s is given by this and sensitivity is given by this. Now if you look at these 2, you will understand that how K c i K c o n need to be adjusted in order to get a sensitivity you do not want sensitivity to be high because, with all kind of work pieces you should get the same mechanical advantage you want sensitivity to be low at the same time MA s that you get basic ah bounding mechanical advantage you wanted to be high and in fact, these 2 are also related which this analytical model tells you, if you look at these 2 equations you would see that MA s is actually related to S k with this n by K c i does not mean that MA s is high when there is n is high when have n is geometric advantage inherent geometric advantage right. If you remember lever ratio if n is high you think MA s high no because, S k has in the denominator n square. So, when you plot it you would know that when n increases does not necessarily increase MA s, but you see what influences sensitivity what influences MA s.

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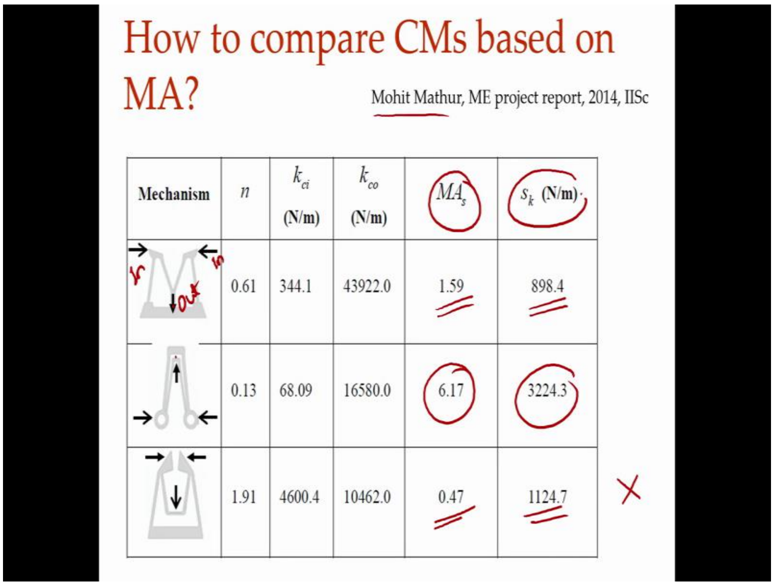

And if you do this for different mechanisms as Mohit had done you can see that a mechanism that has high MA s has some sensitivity, if I go to other mechanism that has let us say low MA s also high sensitivity this actually very bad anyway, it is a gripper does not have mechanical advantage where as these are the one that can actually crush things over here the output this the output here this is input.

So, we look at this MA s and S k can talk about not only mechanical advantage, but also we can talk about sensitivity to the work piece stiffness.

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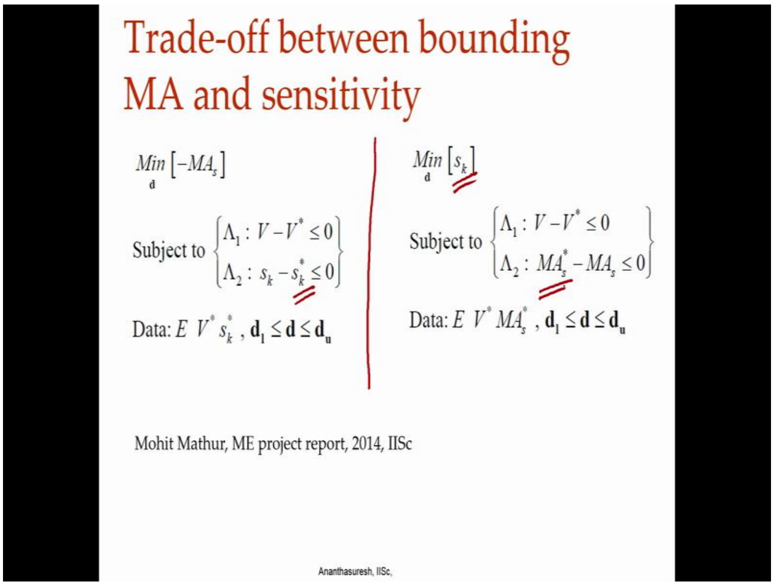

So, based on this Mohit actually had posed optimization problems one is minimize negative MA s or maximize MA s subject to a limit on the sensitivity or minimize sensitivity subject to at least some MA s star that is, some basic mechanical advantage you should have MA s star is less than MA s meaning MA s is greater than MA s star that you have two way suppose in the problem and that’s what he had done and which when you get in terms of this K c i K c o n tell you a lot about how to design it.

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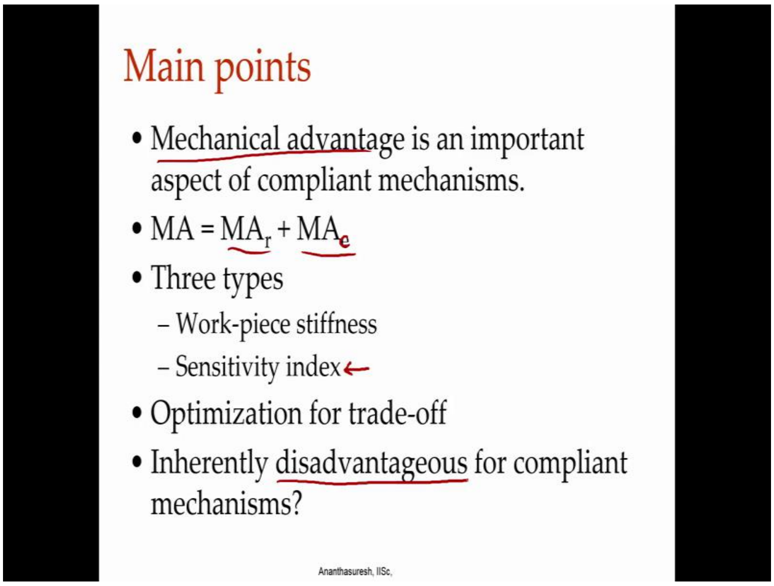

So, with mechanical advantage with the work that was developed by Salamon and Midha, one can go little further and let us just summarise. It is a very important aspect of designing compliant mechanisms that's mechanical advantage, it has rigid body and elastic or compliant party part to it and 3 types mechanical advantage can be defined based on, work piece stiffness, input force input displacement, output force output displacement more important sensitivity the work piece stiffness is an important aspect that we need to remember, and we can do optimization for the trade off and inherently it is disadvantage compliant mechanisms when it comes to mechanical advantage, but as we will discuss in next lecture it is not. So, if you take a little more care it cannot always be done, but in some cases you can actually make the mechanical advantage higher without actually that big reduce the mechanical advantage.

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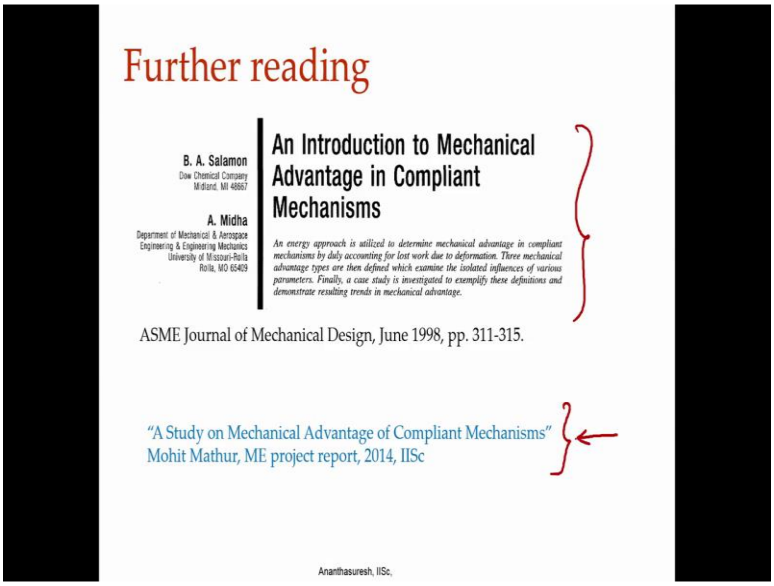

So, the further reading for this, this very nice paper and this ah report of Mohit Mathur and that little bit more will discuss in the next lecture.

Thank You.