Compliant Mechanisms: Principles and Design
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Lecture – 05
Maxwells rule and Grblers formula

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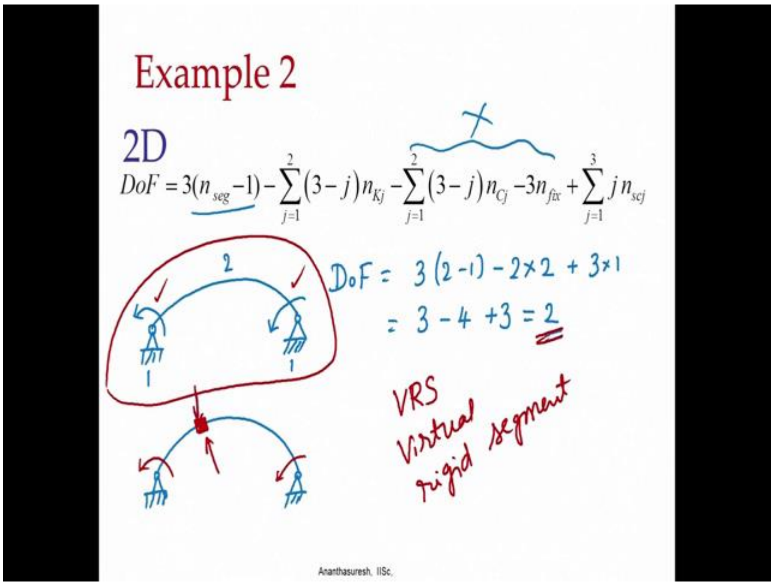

Let us take another example. Let us take an arch like that with a pin joint there another pin joint there, how many segments are there we have fixed flame; that is one segment and then two here. Now if you do degrees of freedom for this 3 times 2 minus 1, we took care of that and then kinematic joints we have 2, 2 pin joints that will be minus 2 times 2; there are no compliant or elastic pairs here and there are no fixed connections. These 2 are not there and now we come to the last one. If I take because I do not know what to take segment compliance for this arch so, I take 3. (Refer Time: 01:13) we take the full one then it will become 3 times 1. There are no other elastic segments here only one. If I do this is 3 minus 4 and then plus 3 that gives us 2. There are 2 degrees of freedom, we can interpret because there are 2 joints here so, I can apply a torque there, I can apply another torque there. So, 2 degrees way can be explained now let us take a small variation of that. So, we take the same arch and let us say that now I am applying a force here ok.

Now, what happens to count a degrees of freedom; we have basically not introduced any new segment we introduced a force; that means, that when I have an arch like this I have 2 degrees of freedom which we have already indicated as 2 torques here, there is this one degree of freedom another degree of freedom, but now I am saying in addition to those since it is an elastic segment. I can apply an additional load here; that means, the degree of freedom has increased. That is where the confusion to say that if you have elastic segment, you can apply as many forces as possible and hence that infinite degrees of freedom, but that will be incorrect and also confusing so, what we say is that whenever there is force acting we will introduce that there is kind of a virtual rigid segment. That is virtual in the sense it is not there, but we use it for our convenience then the degrees of freedom formula holds; we can also interpret as to how many are there, when there are not any like in this example there only two, there is no question the movement we have a force at that point we introduce a virtual rigid segment.

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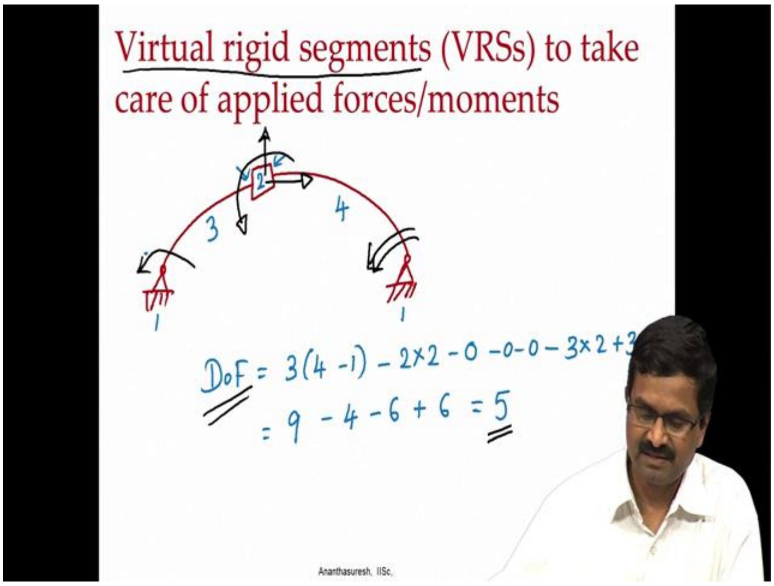

When we do that for this one; I have now the segment where I want to apply the force and then this, we can work it out saying that we have one here and then the 2 virtual rigid segment; we have to count that that has broken our elastic segment into two; there is a third and forth.

Now, if you do degrees of freedom formula at this one. We have 3 into 4 minus 1 minus 2 joints are still there that is 2 times 2 there are no compliant ones, that is 0 number of fixed connections there are no compliant pairs that is all 0 here and then fixed connections 3 times we have 1 and 2 here, that is 3 times 2. Segment compliance again when in doubt we say segment compliance of 3 there are 2 such things that is in segment 3 and 4, let us see what comes out now this is 9, 4 minus 1 is 3, 3 times that 9 minus 4 and then this is minus 6 and then plus 6. What do we get 9 minus 4, 5, 6 and plus 6 minus is cancel, we have 5 degrees of freedom where are the 5 degrees freedom, 5 degrees of freedom like what we had before we have this torque, that torque that is 2 now that we have a rigid body a rigid body in a plane can have 3 actuations x force y force and a moment, again 5 comes out fine. Wherever there is a force we introduce this virtual rigid segment. That we can interpret degrees of freedom meaning that again it can have maximum of a degrees of freedom in this particular one, if I give only this torque it will move deterministically only this will little move deterministically both are them it will move or a number, that is 5 degree that we identified any number you apply from 1 to 5 will give you deterministic motion. That is a characteristic of the degree of freedom formula for compliant mechanisms.

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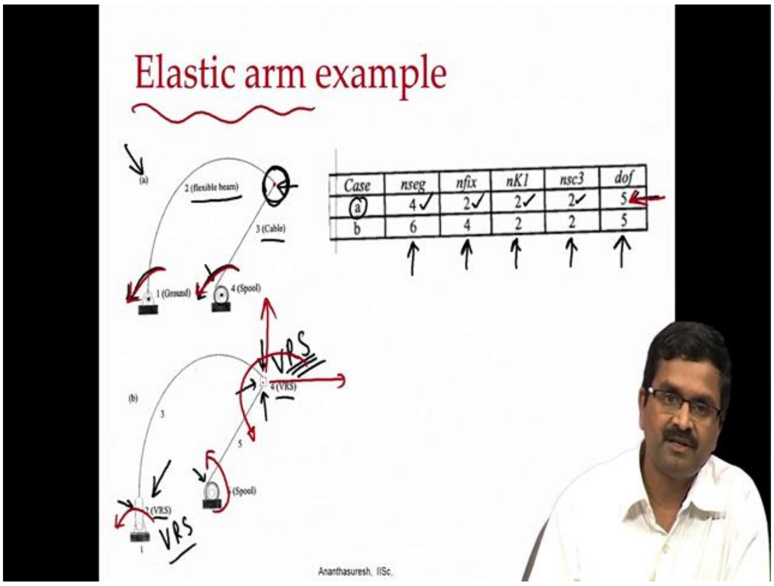

Now, let us take an example reported in the literature called an Elastic arm. What you have here is a flexible beam and there is a cable that goes over a spool here (Refer Time: 05:53) and then there are the motor at this thing as a kinematic joint. We have shown it with Virtual Rigid Segment and without it. This is a VRS over here this is a VRS over there. There is one here Virtual Rigid Segment and there is another one. Let us look at this formula computed both ways number of segments here in the first case, case a that is this one, we have 4 segments, there are 2 fixed connections where are they? There is a fixed connection between 2 elastic segments and the last segment and the spool and we have nK1 2, that is over here and over here nsc3 2 because I have the flexible beam as well as the cable like when in doubt we take it as 3; if you do that we get 5 degrees of freedom. How do we interpret this here 5 degrees of freedom; it is clear that I have one degree of freedom there another degree of freedom here, but what about the other 3 that it formula gives, that is where we have to say when we are taking elastic segment and another elastic segment when they both are connected like it is happening here; it is useful to have Virtual Rigid Segment when there is a pin joint to the ground you can take it or not take that does not matter, but it is useful whenever elastic segment is connected to rigid segment another last segment taking a VRS Virtual Rigid Segment makes it easy for our analysis. If you do that by adding this VRS over here and over here, number of segments increases to 6 from 4 to 6. Number of fixed connections also goes up to 4 because we have two additional ones that come there and here between VRS and the flexible beam VRS and the row; the other ones over here so, we have that and then the earlier whatever we had that is still there.

Number of fixed connections has gone up and number of kinematic joints remains the same, the same old things and nsc3 elastic segments are still two with segment compliance 3, we have that if you calculate again it will give you the 5 degrees of freedom. Now it is easy to interpret in the sense that we have 5 degrees of freedom that we can have as rotation over there at and then rotation over here to torques and then since a Virtual Rigid Segment I can have force in the x direction, y direction and the moment on it. In fact, this elastic arm has point a robotic hand attached to it, that is the end effecter that can have 3 degrees of freedom and then you have two torques and that is how it works out. We can interpret degrees of freedom and you see the now 2 cases where you would use virtual rigid segment; one is wherever there is a force and elastic segment and the other case is whenever 2 elastic segments are attached to one another or an elastic segment is attached to another rigid segment you put Virtual Rigid Segment then interpretation becomes easy. In the case of a where we got 5, we did not know how to interpret the extra 3 freedoms other than the 2 torques here, the moment you have Virtual Rigid Segment then we know that at that point you can apply 2 forces and a torque.

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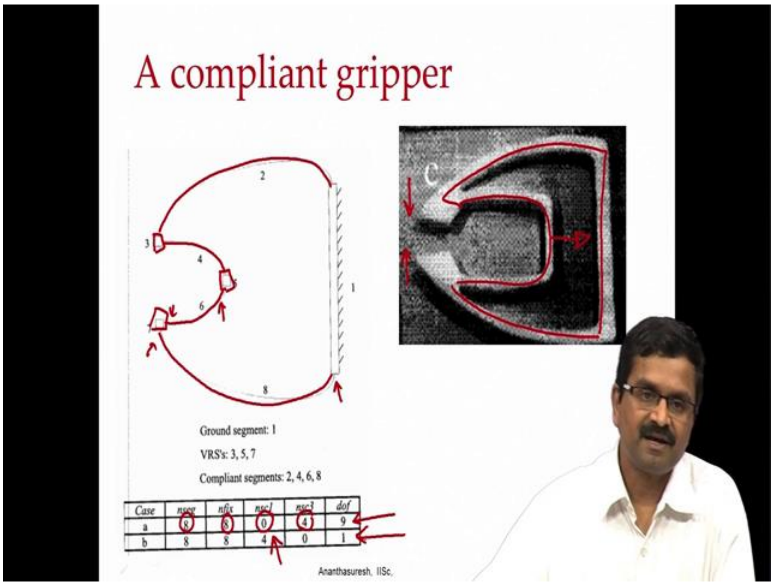

Once we have this if I take a compliant mechanism such as that we can see that there is an elastic segment here and here and then there is this one which is attached to this; we can interpret that as a VRS there that is shown there on then another segment, another VRS here because this mechanism where you apply force in the direction and this gripping action happens here. There is a force you take a VRS and then you take another segment and of course, the symmetry you have this. In this particular case you have number of segments 8, number of fixed connections 8, that is 1, 2, 3, 4 and then 4 more and nsc1; we are not taking any, we are taking nsc3 as 4 the segments 8, 6, 4 and 2. degrees of freedom then come to 9, since there are 3 virtual rigid segments each of them can take 3 degrees of freedom, we can say it has total 9 degrees of freedom, but looking at this particular gripper you think apply of force is only one degree of freedom. If you want to do that then you have to make a little change as shown in this case b the case b we are saying that instead of nsc3 being 4, you are saying that nsc1 is 4; that means, that each of these segments have segments compliance only one, what is it mean in this particular case these segments only bend, they do not stretch or move in 2 dimensions know when you say elastic segment in planar case as 3 degrees of freedom, it can bend in more than one way to cause x and y displacements and rotation impossibly. Those are the 3 freedoms, you say that each of them simply bends only one mode in which case you take segments compliance one here for each of those then you get degree of freedom one and that is all we can interpret. You have to have a little understanding of segment compliance depending on the situation.

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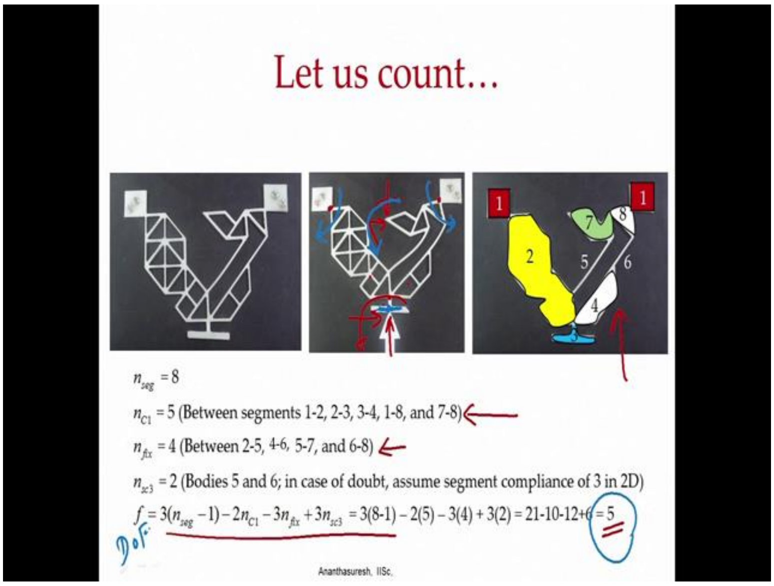

When you take a complicated case such as the one shown here, which I have a real mechanism of what you see on this screen, where you apply force we had seen that earlier.

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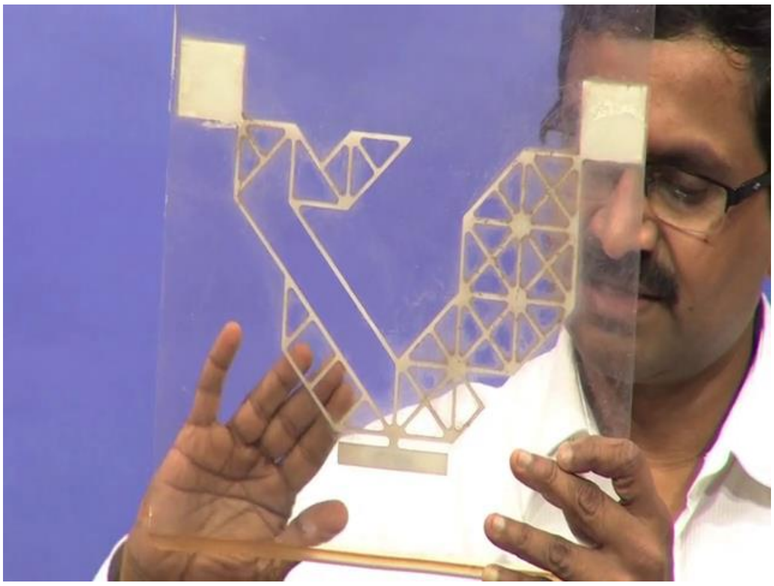

It is going to move for this portion for this mechanism. If you want to do it there are too many beams here, there are a lot of beams. Where you have so, many beams you keep on counting each one of them, instead what we says that all this thing is more or less rigid and so, is this little portion and so, is this portion and so forth. The whole thing can be one rigid segment; that is what we have shown our interpretation here. Let us go back to the slide and then interpret this entire thing as one rigid body, this as another rigid body this portion is another rigid body and in this one as another rigid body and then interpret and then we have elastic segment there, elastic segment here, these 2 are elastic segments. With that interpretation we can count the degrees of freedom. Counting is the strength of (Refer Time: 13:01) formula. Here if you do, we have 1, 2, 3, 4, 5, 6, 7, 8 segments when you do the calculation and between segments 1, 2 and 2, 3 and then 3, 4 and then 1, 8 we have flexural pairs; there are flexural pairs over here and the way we have shown over here between that and this and then over here. There are 2 between this and this that is this and this and that as shown in this one over there.

Then you have fixed connections also between 2 and 5, 4 and 6, 5 and 7, 6 and 8 and then nsc3 we have taken 2 that is segments 5 and 6 again in case of doubt you take the full so, we have 2 such things we put that into the formula which you can check slowly, we get 5 degrees of freedom and we have 5 degrees of freedom, we can actually count them. How do you get 5 degrees of freedom for this if supposed to be the force we apply here this moves in that direction in amplified way that is clearly one degree of freedom, but then nobody prevents you from applying a force in that direction and movement also.

You can hold it and rotate, those are 3 degrees of freedom after that since we have the output here we can also apply a force over there and we can apply a force over here what it tells you is that, then you cannot apply or you put that in a different color. If it tells you that there only 5 degrees of freedom, then you cannot do this independently. That are because you have flexible one, you can also apply those, those blue ones you cannot. When you do that we have to disturb the other degrees of freedom over here.

The red one that we are written 5, 5 degrees of freedom that is a maximum freedoms possible for this mechanism when you cannot do this, when you do that when you apply a torque at this point. That let us say this joint, if we apply a torque there you will be holding at an angle; if you hold at an angle and try to apply this blue actuations, it would move and; that means, that whatever you have specified before now will not be valid and that is what this number says. Compliant mechanisms; however, complex they are, there will be maximum degrees of freedom for it as predicted by our formula. This is our degree of freedom; that is a maximum possible for this is 5. Once you specify well we have 8 possible freedoms here, that we can apply out of those only 5 can be applied independently other ones you cannot independently do it, because then you have to disturb the remaining that the first 5 that you have apply. That something we can understand by simply counting.

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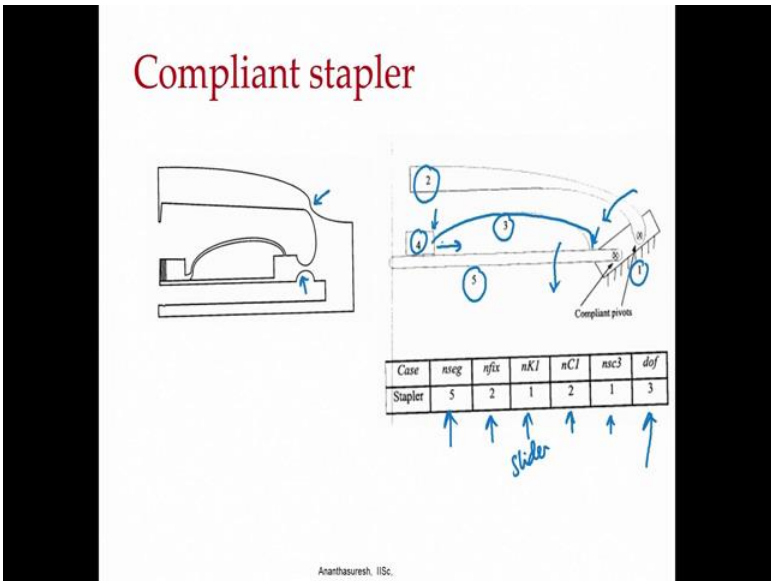

We can take example of a Compliant Stapler that we had seen. Here there is an abstraction that is shown as the frame one and then top portion of this as 2 and then there is a elastic segment 3 and then this block 4 and then lower one where we hold the staples that is 5, number of segments is 5 fixed connections, you can see where they are that is one over there for this one, l it goes to the other side so, there is this nK1, we have a kinematic joint which is a slider here, that is the slider; 4 moves relative to 5 and then we have nC1, there are 2 these are the flexural things. These are the flexural or elastic pairs we have 2 nsc3 the elastic segment that we have is 1, if you do that degrees of n terms will be 3. You can interpret that meaning that I can have one freedom there another freedom there and I can move this block those are the freedoms.

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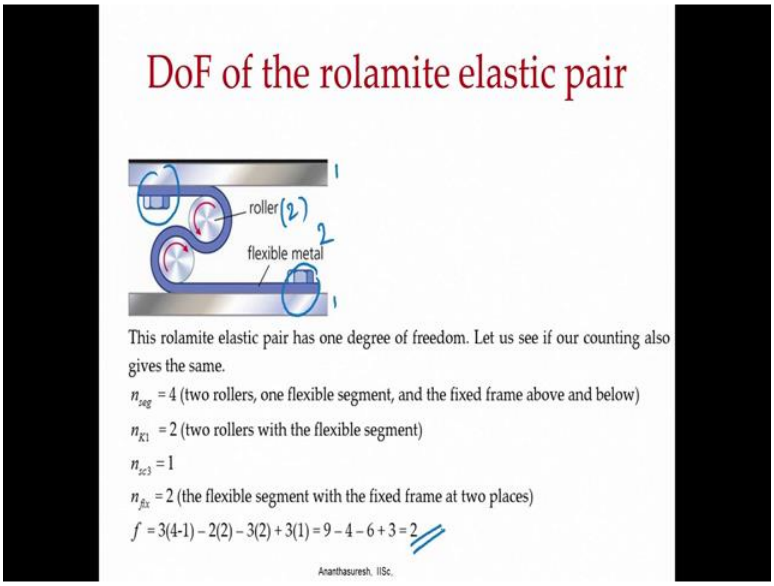

You can interpret degrees of freedom for anything that you like and we come back to rolamite and here we have shown that number of segments are 4, we have top one and bottom one is fixed frame, that is the same thing under flexible metal is 2 the strip and then the 2 rollers, here the 4 segments and in nK1 you can see what revalue joints we have 2 rollers and in (Refer Time: 17:45) 3 there is flexible metal has segment compliance 3, one such thing n fix where it is fixed we have 2 of this fixed connection. In fact, fixed connections are literally shown here in bolted connections fixed connections. If we do that degrees on transfer to be 2 for this; each roller can be rotated or you can move this back and forth and the rollers can be one another can be rotated other ones rotation will be automatically fixed. So, maximum freedom possible for this is 2.

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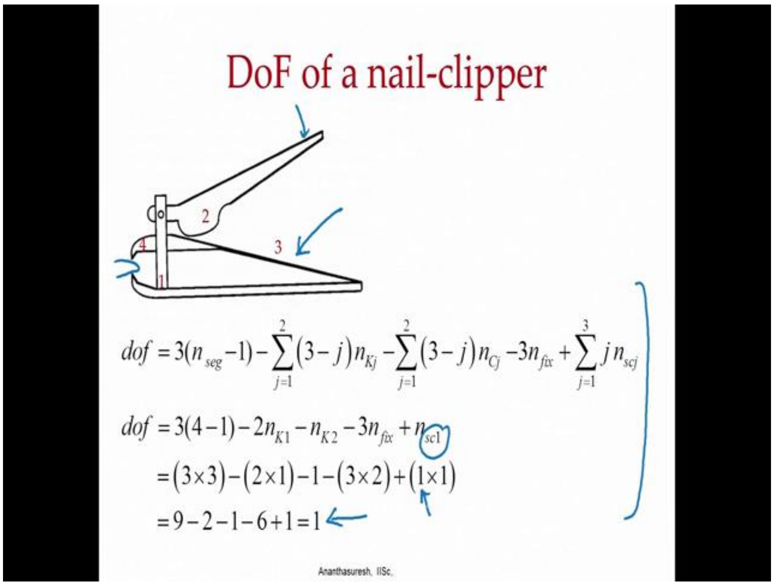

We can go to nail clipper, again all the information is there you can go through it and count for yourself and then see that when it shows that degrees of freedom is one for a nail clipper; that means, it is actually well designed one. Because you apply the force here your nail is here to clip and that is how it works. It deterministic motion even though it has elastic segment here, what segment compliance you have to take here; if we notice we have taken it to be one nsc1 because this particular segment in nail clipper only bends, it does not stretch it does not bend in a way that between the 2 bodies it connects it does not create x and y motions independently. That segment compliance 1 that comes with practice when you try various examples, you can use the correct segment compliance, but when in doubt take the full one in 2D it is 3, 3D it is 6 and then try to interpret in what ways elastic segment is deforming.

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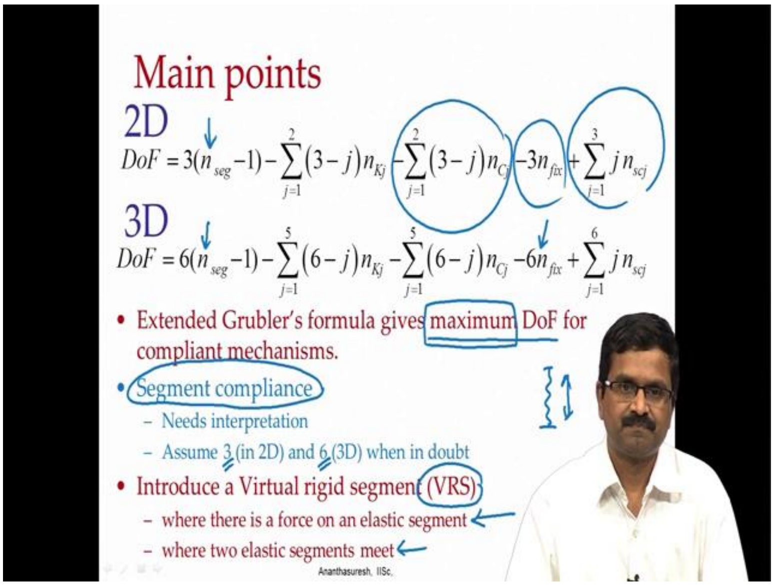

The main points of this is that both in 2D and 3D we have a formula which gives you again the maximum degrees of freedom possible for a compliant mechanism and we need to understand this concept of segment compliance, there are 2 conditions in which we have to use or this segment compliance that it needs interpretation and assume 3, when you are in doubt in 2D assume 6 when you are in 3D. This interpretation is one way and the other way is assume the maximum and then interpret what comes out like in a, if I have a spring that we took we know that spring only stretches this way or that way that is 1 degree of freedom does not bend, then if you can interpret right away take segment compliance 1, 2, 3 in 2D or 1, 2, 3, 4, 5, 6 in 3D. When you are in doubt take the full one and then interpret when you get the degrees of freedom for the mechanisms as we have done in a few examples in this lecture.

Next important concept you note is this Virtual Rigid Segment. That also has 2 conditions one is that wherever there is a force or a movement introduce the virtual segment, if you can apply a force and elastic segment then you can apply a force in other direction as well as a movement the degree often exist, but you are not exercising you do not have to because again the formula gives you the maximum degrees of freedom possible for a compliant mechanism.

Having a VRS at a point where there is a force or a movement enables you to interpret these degrees of freedom properly and then you would also use a VRS when 2 elastic segments meet; we have taken an example illustrate that. You can also take a VRS when elastic segment connects to a rigid segment, but more can be unnecessary when you do that you will introduce a fixed connection and a segment so, the segment is 6 with a positive fixed connection with negative. They both canceled it does not hurt you; it will become clearer for you. When you are first practicing you try to take a VRS whenever elastic segment connects to a rigid segment, but that is not really needed, but if it helps you can take, but certainly we need to have a VRS when 2 elastic segment meet as we have seen in the example.

To recap, we can interpret degrees of freedom for compliant mechanisms by extending the Grublers formula that we are very familiar with in rigid body linkages. The additional terms are this elastic pairs and then the fixed connections and the most important segment compliance and interpretation of segments as opposed to just bodies segments includes both rigid as well as elastic ones.

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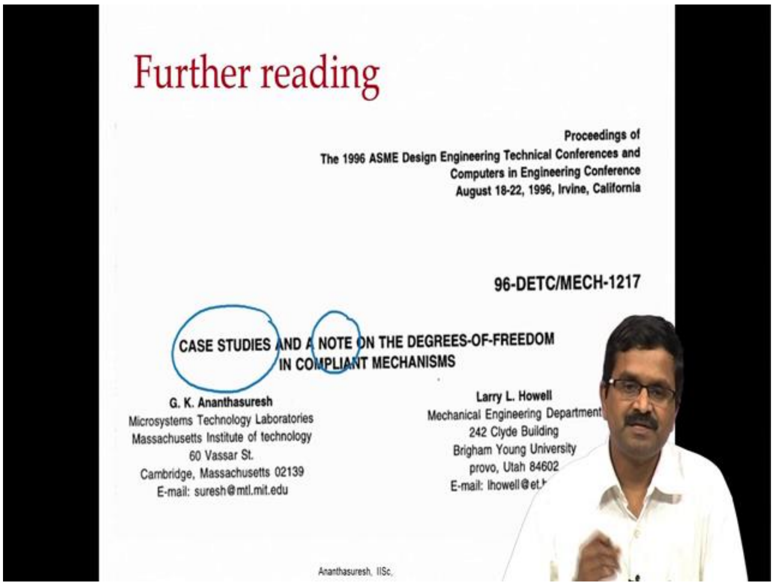

If you want to know more about this there is a paper, that will made available as a supplementary material for this lecture, which includes a number of case studies we have used some of those case studies in this lecture; there are many more examples in that and the references that are there, which was what was done by professor Ashok Midan his students, where they had define the concept of segment compliance degrees of freedom. What this; note of this paper says is how to apply those concepts to several compliant mechanism that we see around us in fact after listen to this lecture and reading this paper and the paper there are cited in this will be able to interpret degrees of freedom without any doubt or any compliant mechanism. That will be the most important thing for us to design compliant mechanism or even analyze them because without knowing degrees of freedom it will be very difficult to understand the determinacy of the motion that mechanisms have.

Thank you.