Compliant Mechanisms: Principles and Design
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Lecture – 47
Non-dimensinalization of stress, frequency, and other measures

Hello, we are looking at non-dimensinal maps based design technique for compliant mechanisms. Today we will look at non-dimensinalization little bit systematically and then see how we can non-dimensinalized any response of compliant mechanisms, when I say any response we have to see what are all the things that are of interest was will take a few that are have interest and then see how we can dimensionalize and still be able to get the so called Kenetoelastostatic maps. Today we are going to introduce some dynamic related parameter, so you can just generally call them Kenetoelastic maps rather than putting static or dynamic.

So, let us look at this non-dimensinal or non-dimensinalization of mini responses, all of them are related to only elastic responses of compliant mechanisms. So, let us look at what elastic response we can consider and how we can non-dimensionalize them so that we can capture the behavior of compliant mechanisms easily in a 2 d chart or a look up table either one.

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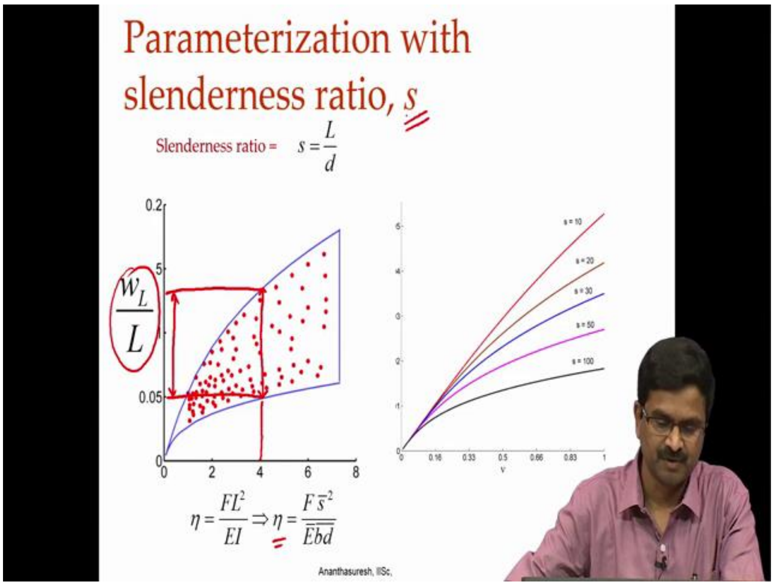

Now just to recap what we had discussed in the last three lectures, parameterization with slenderness ratio that as along with this eta.

So, we have eta here in this axis and then we can put in any non-dimensinal response. So, far we have discussed only displacement based once and we thought that if there is a map like this. So, we have a number of points here again the problem with this was that if I take particular value of eta, our range of values and what will be my non-dimensinal response I would not be able to I get a range of values there. So, we thought that we should parameterize it and parameter is this slenderness ratio we had done this analysis in the last few lectures.

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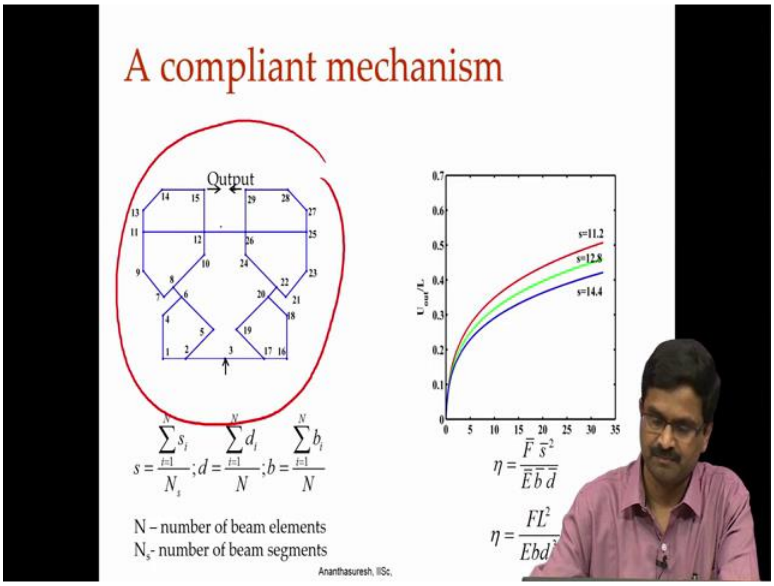

So, what we had a map now it becomes bunch of curves which (Refer Time: 02:28) So, any compliant mechanism we can define this average depth, average breadth, average young’s modules, average force and get our eta and in the process we also get average slenderness ratio and we can plot this and once we have it, it captures the entire behavior of compliant mechanisms for input output data specified. If I change input output the map is going to change for sure.

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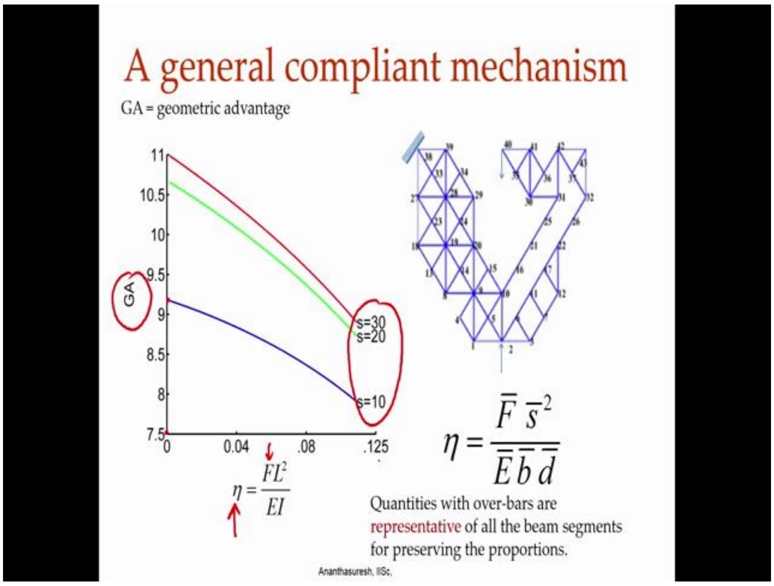

And the if you look at what we do with this, we thought it if have a compliant mechanism when we draw this maps, we can see how a particular non-dimensinal from here it is a geometric advantage, how we changes with our non-dimensinal parameters eta also for deferent values of s, we will geometric advantage does it decrease at what rate does it decrease all of that.

Because, eta once we chose a mechanism, eta also indicates the force that is if eta equal to 0 if you take here, then the response will be whatever geometric advantage just before you start applying force, as you apply force how does it change that will be indicated by this.

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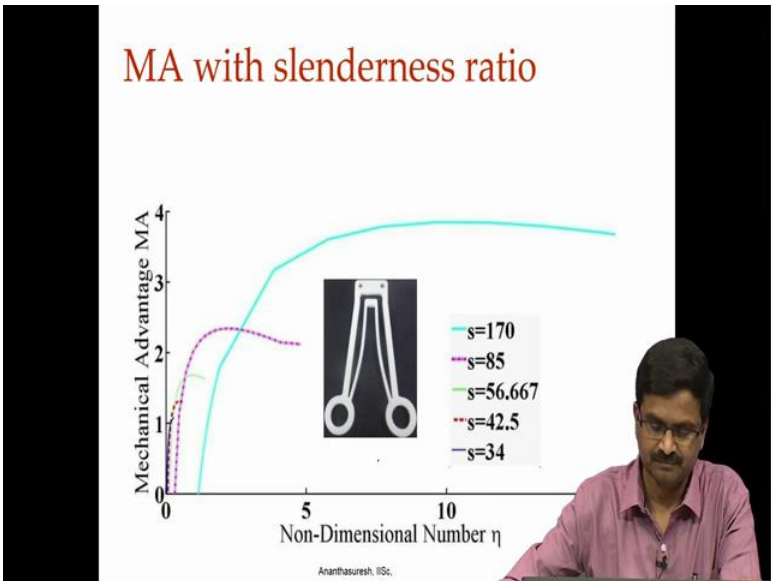

And we can also do mechanical advantage will turn to it in a later lecture as to how to look at mechanical advantage in non-dimensinal fashion for compliant mechanisms, in general with and without work piece and so forth, any one changes the gap as well.

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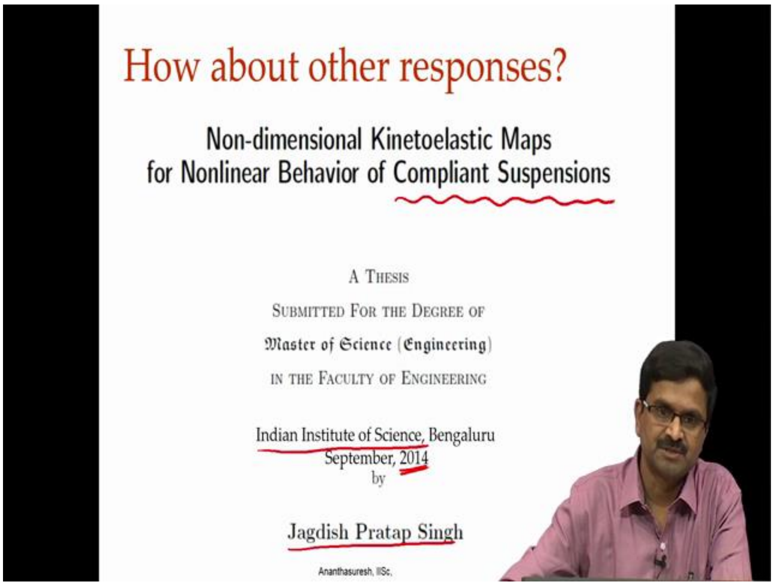

Now, let us ask how about other responses when we see other first let us look at a reference this was done by Jagdish Pratap Singh in his master thesis at the IISc, 2 years ago 2014. So, there he considered this for something that we are going to revisit or compliant suspensions which are used to widely in micro systems or micro electro mechanical systems or MEMS and we will come to that later, but some other idea that we are going to discuss today are taken from Jagdish Pratap Singh's thesis.

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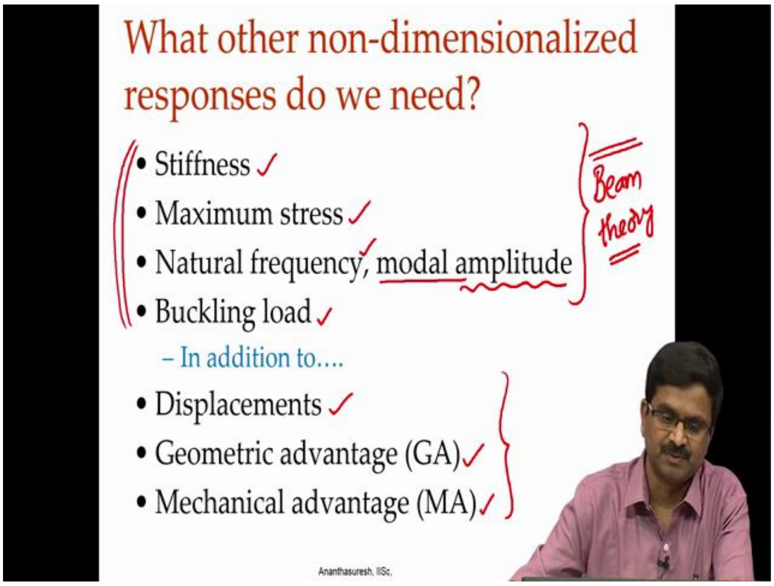

So, what other responses are we talking about? We have already discussed displacements, we non-dimensinalized displacements with the average size of the mechanisms and we also look at some maps where we plotted geometric advantage that is output displacement divided by input displacement. So, that we can look at how that changes and something with mechanical advantage which is output force divided by input force, we already show these.

Today we are going to see some other responses like stiffness, maximum stress from strength consideration if you want to design we need to know that and dynamic considerations, normally we are interested in natural frequency or model amplitude by that what I mean is that the mood shape of the structure, which has really no amplitude, amplitude will come only if you consider dynamic where you multiply the displacement vector if it is discredited or displacement function, as some of Eigen functions. Similarly, if you discredited displacement vector as a linear combination of Eigen vectors or what you call mode shapes.

So, if you draw a mode shape of a structure, what will be the magnitude of that instead of calling it amplitude I can also call it magnitude at a particular point that may have interest that also can be non-dimensinalized and sometimes buckling is an important criteria and we can also non-dimensinalized. Buckling load and any other thing that one may think of can be done, all of this we do with beam theory in mind that is something that we should not forget because our basis for non-dimensinalization came from an analytical solution from large displacements of a cantilever.

Later we extended to beams of different boundary conditions and argued it from the view point of stiffness matrix for that, both linear as well as non-linear or large displacement or what we call geometrically non-linear cases, then we generalized due to whole compliant mechanisms all that hinges upon this beam theory. So, a compliant that we consider should have beam segment in it which is widely true with compliant mechanisms, very rarely you find a compliant mechanism planer compliant mechanism that does not have beam segments.

So, all this we were doing in the contest of beam theory, within that whatever other conditions are there will be able to consider here in this treatment because beam segment they either wend or auxiliary reform that is structure contract, in order to do so the properties geometry further required or the length, length of the beam segment under cross section, cross section has certain movement of area or if it axial area will be important.

So, those are the thing that will appear whether we consider small displacements or large displacements of cause, material properties are there, but geometry parameter this are the ones and since our analysis holds for simply beam condition cantilever and a few other beam beams are different boundary conditions, we argued that it extend to general compliant mechanisms as well. So, no matter what kind of performance measure that you consider want to non-dimensionalized it is actually possible.

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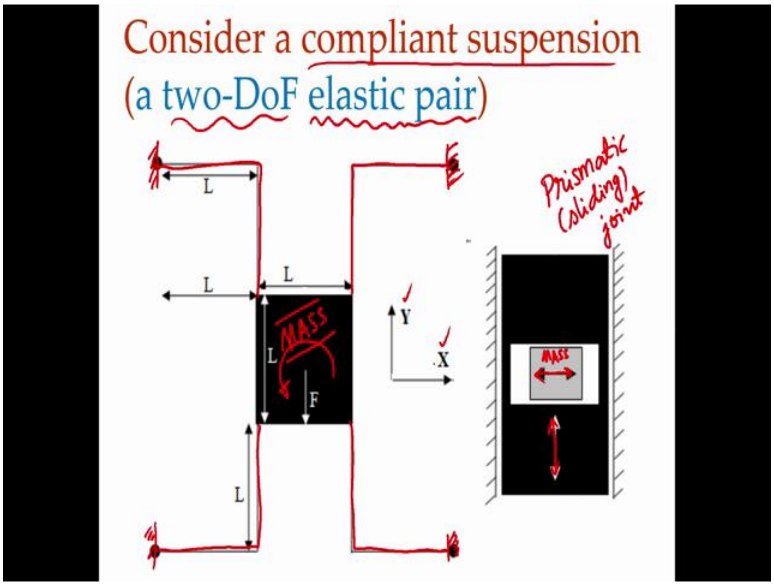

So, let us look at first a suspension because Jagdish Pratap Singh had done this work for the context of work is called a suspension. Why it called a suspension? If you look at the central mass, this is sometimes called proof mass, but accelerometers, gyroscopes, lot of different senses.

So, when you have this mass, this mass has to be suspended in order to make it move. It is an accelerometer the mass should be free to move one of the way that we do at micro scale is to use prismatic joints. So, here on the right hand side we have prismatic or what we call sliding joints, this is a 2 degree of freedom sliding joint, because you have motion in this direction as well as in this direction X and Y so to speak.

X and Y both are there, and rotation is not there because you cannot see this block if I call this the mass now you cannot rotate because in a slot. But this type of a thing is not suitable for micro systems, because the sliding here would cause two problems; one is a lot of fiction and that is going to be more (Refer Time: 09:49) micro scale because of the anything to do with surface forces is going to be more dominant at the micro scale as suppose to the large scale and the other thing here is that, they were that happens will things slider micro scale things are going to burn out and that because of three body were particle that come will get start all kinds of problems will happen. So, people want to avoid these joints as much as possible. So, here we need to suspend that this mass in a same way, were we have this compliant suspension we have this beam segments. The circle here indicates that it is fixed and this is called a crab legs suspension we will talking about it all the suspensions later on. So, we have four legs like this, that look like for some people appear crab legs and this is suspending them as that is why it suspension compliant suspension a 2 degree of freedom, what we call in this course elastic pair that is similar to kinematic pair we have an elastic pair.

When you have something like this, the elastic pair is only an approximation of the kinematic pair here this mass has zero stiffness in x and y translational directions and infinite stiffness in all remaining four directions, what remaining four directions? The translation along the z if you do not let it go out that will be infinite stiffness, rotation about x, rotation about y, rotation about z all will be infinite stiffness. Whereas, here if you imagine our mass suspended in this manner, it will have not zero stiffness in x and y, but low stiffness; that is how this beam segments are designed.

Out of plain, it will not be infinite stiffness here it will be almost as much as it is in plain. So, it is not very good 2 degree freedom joint. And rotation about z it is high, but it is not infinite. You can image in the suspension, you can take this mass and we apply of torque like that, hold it and rotate it if you rotate it you can actually do, but this stiffness will be high, but not infinite as it is for the kinematic joint case.

And similarly if I want to rotate about x axis, y axis the rotational stiffness will be high, but not be infinite is an approximation. So, it is important design the suspension, so that they have proper stiffness in the desired directions. So, what we do for that is to look at this multi axial stiffness, what do you mean by multi axial stiffness?

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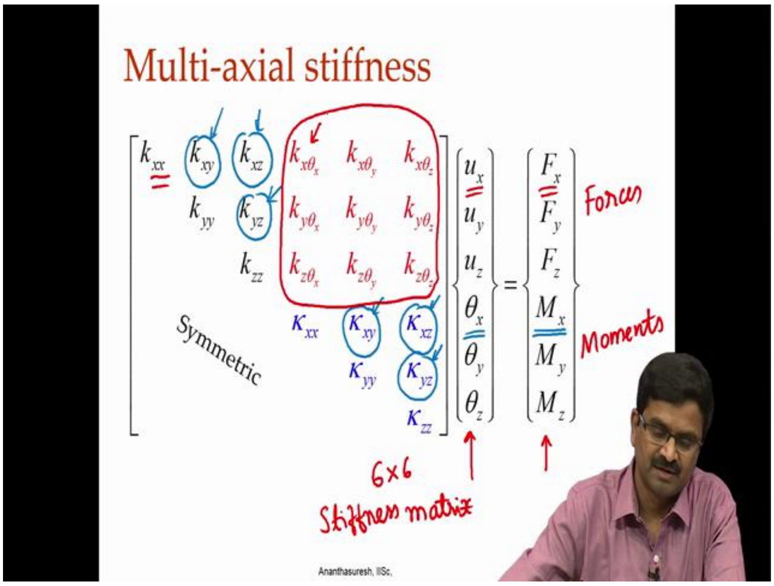

We look at the degrees of freedom, u x u y u z that is translations along x y and z directions, theta x theta y theta z which are rotations about x y and z axis corresponding forces and moments. So, first three are forces and then moments. So, this forces and moments are to be related to the displacement degrees of freedom, three translation u x u y u z and three rotations theta x theta y and theta z and that is given by what we can call stiffness matrix.

So, here this what is 6 by 6 matrix is the stiffness matrix, this is not stiffness matrix in the sense of finite element analysis, although if we take one element it will be true there as well as, but looking at a body that is suspended with some beam segments that is what happens in compliant mechanisms also, were it has is multi axial stiffness about x y and z that is why call it multi axial. So, symmetric matrix k x x will relate u x and f x that why uni axial translational stiffness.

Similarly, k y y relates to u y and F y, k z z will relate u z and F z and kappa x x what is in blue the black ones are translational and kappa the blue ones are rotational that is if I apply some M x that we use the blue color to make it clear. So, if I apply some M x make a torque or a moment I will get some theta x, that is given by this kappa, that is theta x if no other forces or moment exist only M x exist, theta x will be given by M x divided by this kappa x x and then we have kappa y y kappa z z, when you notice that we also have the cross ones k y x, k x z, k y z as well as kappa x y kappa x z kappa y z that is within the axis we also have co couple that is if I apply a force in the x direction dose it cause displacement only in the x directions or its going to cause displacement in y direction as well as z direction there are capture by this cross terms, that we have 3 in the translational case then we have 3 with the rotation case also.

That is important because the cross axis sensitivity of the suspension is an important measure it is also important in general compliant mechanisms, if you want output to move in certain direction you probably sometime intent you should not move in the perpendicular direction.

So, there will be stiffness in the intended direction and very are flexible in intend direction and stiffness in the perpendicular direction. So, that may be there, but then we have these red ones. So let me change to red color. So, this red ones are also interesting that is some suspensions when you apply force in x direction, it may also rotate it depends on how that is design.

So, that is captured for example in k x theta x, that is if apply force in the x direction that may create a rotation about x axis it depends on the type of suspension you have so you have to look at this. So, all of these will be of interest will designed compliant suspensions. So, when you look at the array of applications of compliant mechanisms they start from very small micro sometime even nano and then miso and then to macro, you can design number of things here.

So, that is where this non-dimensinal analysis helps. When you have suspension, such as the one we consider here, if I want to use it micro scale you would have to evaluate with (Refer Time: 16:49) specification or not may be you would use it macro scale for like a machining bed or something. So, then how does it work at the large scale? That is true of many compliant mechanisms also because they can be used at micro scale, miso scale and macro scale and so forth that are where our non-dimensinal analysis would help.

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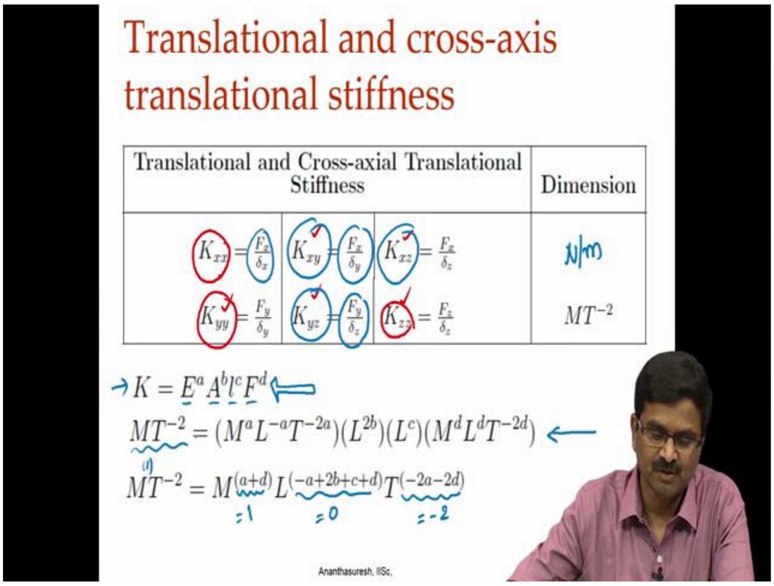

So here if I take translational stiffness, both translational stiffness as it is as well as cross axis translational stiffness. So, I will have k x x, k y y and also I will consider k x y, k y z k x z, k z z. So, this is still direct once k x x y and so forth and these other once which I will circled in a different color, these are cross how do you non-dimensionalize? Basically this definitions are F x with delta x, this is F x by delta x, this is F x by delta y K x y and K y x also will be equal the symmetric matrix and the likewise K y z is F y divided by delta z and so forth. If you want to non-dimensionalize, first the dimensions for this will be MT minus 2, unit will be Newton per meter. So, Newton is M L T minus 2 divided by meter which is L dimension, the dimension will be MT minus 2.

So, if I want to non-dimensinalized the stiffness k, I put it in terms of the parameters I have, here we are taking 4 parameters hence models area of cross section, length and F, we could have take a certain moment of area also i , but it just now taken here because let us see how much you can work with a itself because if you take rectangular cross section and keep the depth of the beam constant, then it only becomes breadth that can always be taken as any areas well. So it is taken as area Young's modulus length and a characteristic length, meaning its average length and force. So we going to non-dimensinalized in such a way that, this exponents a b c d small a b c d which are over here, we need to find this values in such a way that units will match to MT minus 2, in which case this E raise too small a, A raise to small b, l raise to small c, F raise to small d will become a non-dimensinalization factor, that is k divided by this will have no dimensions because we are trying to write this equations.

So, that K and E a l F will have the same unit or same dimensions; that means, that we have to equate this a plus d which is the exponent for M here, should be same as one here because there one for M and then for L it is not there. So, that should be equal to 0. So, this equal to 1 and minus 2a minus 2d should be equal to minus 2. So, we have 3 equations here. So, that E raise to some exponent, a raise to some exponent, l raise to exponent, f raise to exponent becomes a integrating factor.

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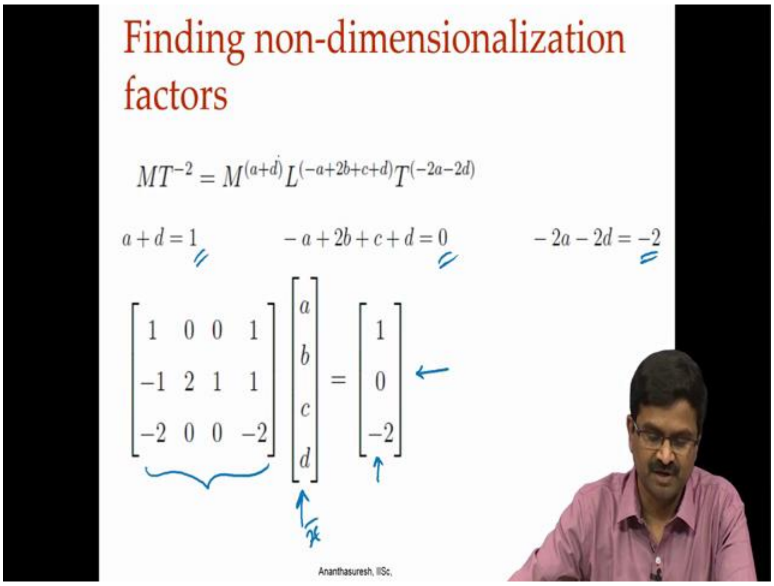

So, we have here three equations as I said a plus d equal to 1, minus a plus 2 b, c plus d equal to 0, minus 2 a, minus 2 d equal to minus 2. You can put that in the matrix form like it is shown here and the we see that is rectangular matrix, right hand side this 1 0 minus 2 is there and then we have this matrix over there we cannot immediately invert, but before that let us also see whether it has full rank or not, the full column rank for this can be 4, but this on if you see the first, and the third are the same equation.

Essentially we already see that there is a problem with it is the row rank here, where the column rank then you would find that this particular thing will be rank deficient, but we will find that. So, our aim is to solve this.

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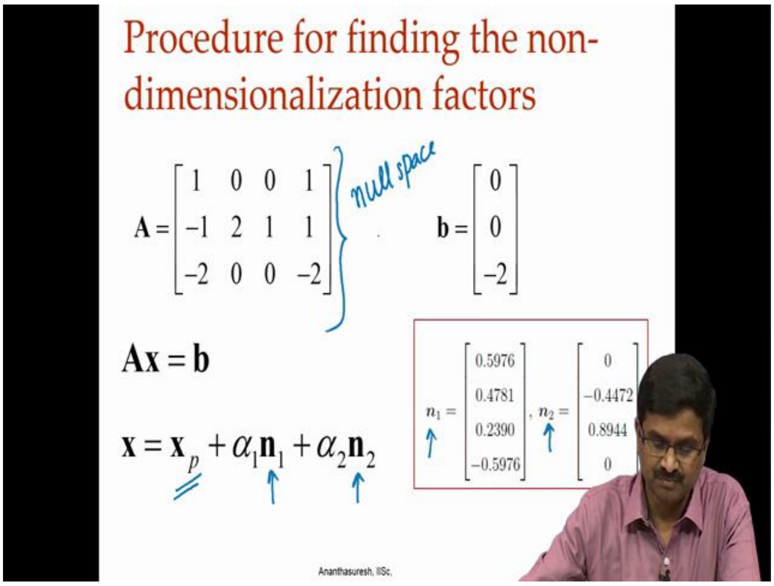

So, we have A and B. So, we have A x equal to b here because it is a rank deficient will have null space, in this particular case I put only 2, but it can be any number alpha 2 n 2 plus, alpha 3 n 3 and so forth. That is this matrix if it is rank deficient there will be a null space associated with it; null space meaning which are the values of this vector, which will give on the right hand side zeros that is original space. So, what will give us lot of null space vectors and then there is a particular solution in a way, x equal to x p plus alpha 1 n 1, alpha 2 n 2, alpha 3 n 3 depending on how many our null space vectors are there in this particular. So, that is how we try to find that x, x here are these things here this a b c d; that we want to find has exponents to the parameter we considered which are e a l and f. So we have that.

So, in this particular problem null space has a dimension of two, there is n 1 over here and n 2 over there. That means, that if I multiply a matrix with n 1 I get zeros for b; 0 00. So, this is 3 by 4 and this is 4 by 1, when multiply I get 3 by 1 that is all of this will be 0, that is what null space means here there are dimension of null space is 2 because this matrix is rank deficient by 2.

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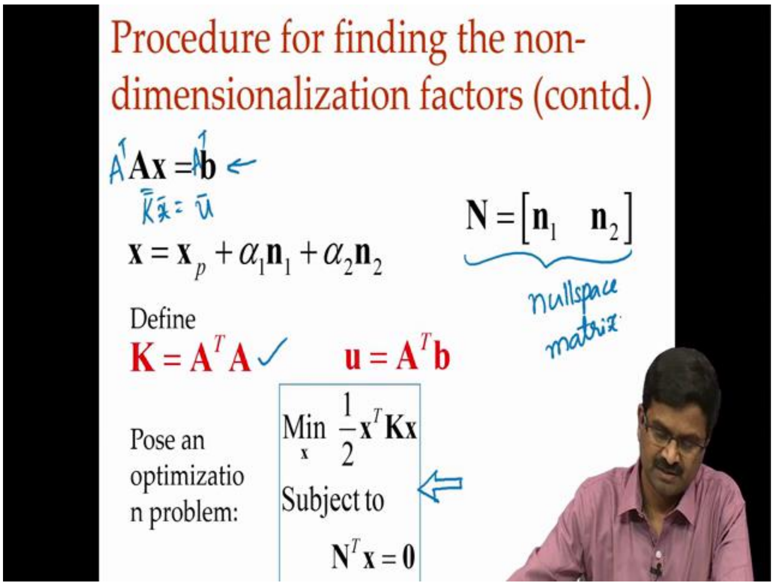

So, have these n 1 n 2, once we have this we want to find this x p and assign some alpha 1 and alpha 2, get an number x meaning that we get number of non-dimensinal non-dimensionlization factors for stiffness here. So, solving this equation to find this x p is equivalent to solving in optimization problem that is post over here, we have to post this problem we have define this K as A transpose A.

So, that will becomes symmetric and then we also we did pre multiplication transpose that is we have taken this multiplying by A transpose A transpose, that become K x here equal to what we are calling u, that u is defined as A transpose b. And solving this problem equivalent to this optimization problem, where we minimize half of x transpose K x subject to n transpose x equal to 0.

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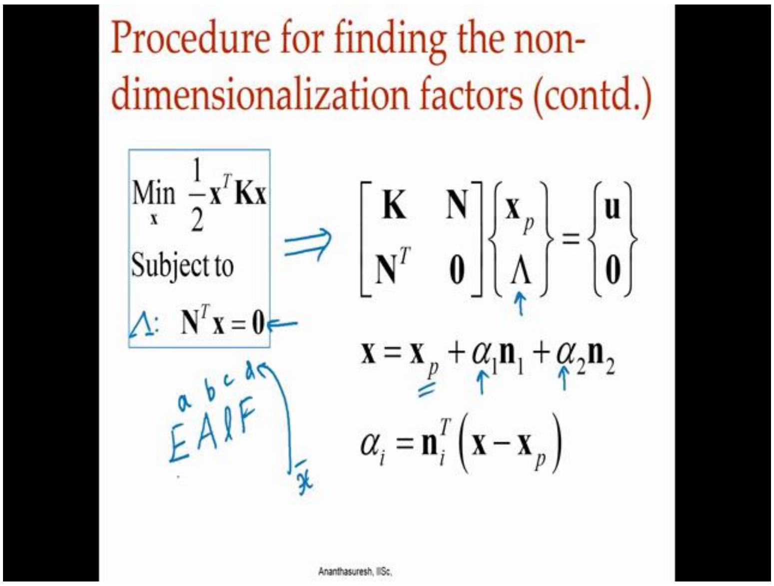

So, if you solve this problem what you will find is that, the necessary conditions where this one is Lagrange multiplier corresponding to this constraint you might as well write it here, where N is the null space vectors that is we have N 1 N 2, this null space matrix second call (Refer Time: 24:38) matrix whatever. And, where you put each column is a null space vector. So, we get from this problem we get this set of equations or unknowns are x p and this lambda and you can actually solve it if this a full rank it will be most often.

So, you can find x p and lambda once you get x p and lambda, you need to get x where we have to guess the values of alpha 1 alpha 2, whatever values you put alpha 1 alpha 2 because they two null space vectors, we have to get x we have freedom to choose alpha 1 alpha 2, we choose them in a way that x becomes vector, that gives a proper non-dimensinalization factor we had E A l and what is other one E A l. And we had one more, so f the force itself that is external.

So, I thought its outside for us, but that is all. So, there will be some exponent we call it a b c d and that comprises our x vector here, once I get that I get particular solution I can chose alpha 1 alpha 2 whatever I want and try to make this.

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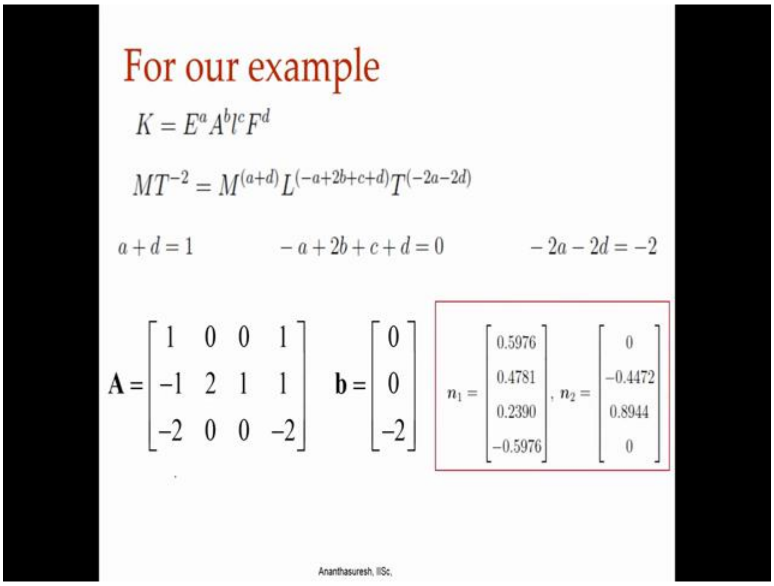

So, if I take this problem for our example, if I take this over a matrix and these are the null space vectors we can take that.

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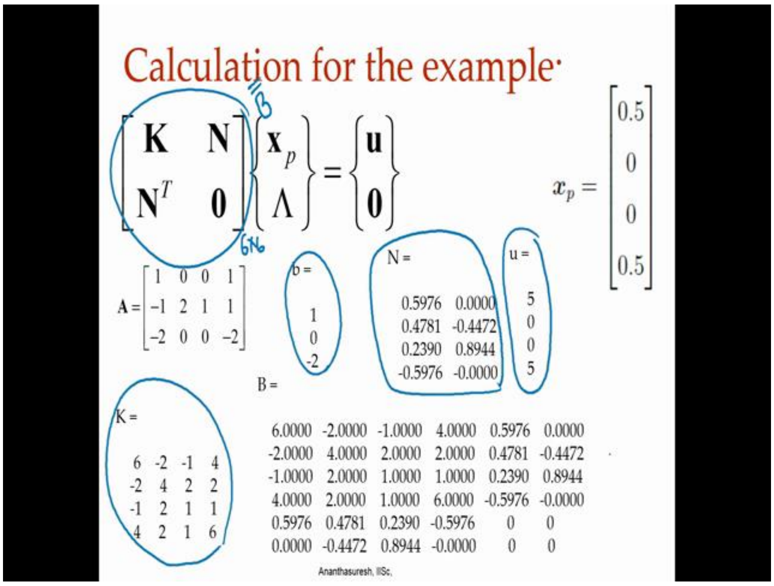

And try to construct this K matrix, this was all done in MATLAs or just pasted that, K is a transpose a became now a square matrix from a rectangular and b is here and then u is a transpose b that we have u here and null space vectors of a and then b which is this matrix this whole thing I am just calling it B that this 6 by 6.

So, here lambda we have corresponding to the two constraints we have that and then here we have two zeros correspondent to that. So, we have 6 by 6 matrix, from which we can solve for this x p. If we do that x p turns out to be 0.5 0 0 0.5; that is our a b c d for E A l and F is 0.5 0.5 one particular solution by choosing different values of alpha 1 alpha 2 here.

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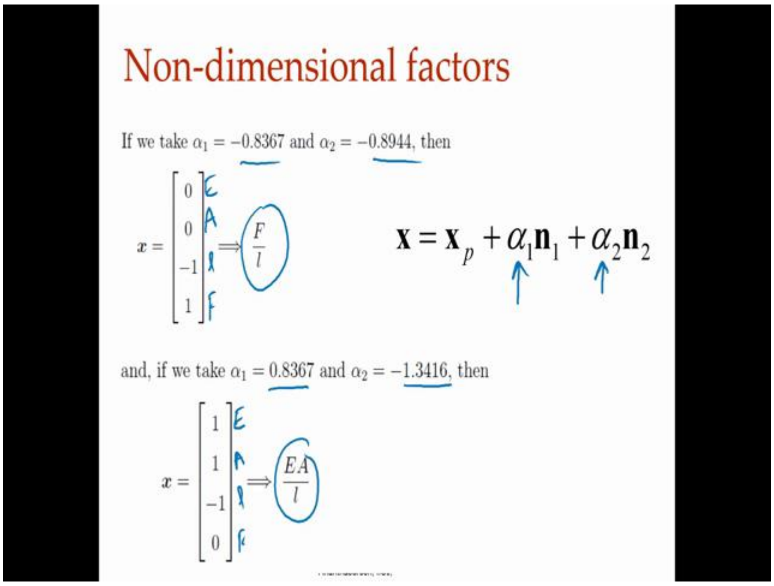

If you chose alpha 1 alpha 2 to be this values you get x equal to 0 zero minus 1, a b c d if you take then that becomes F by l. So, our thing was this is a E A l and F you see, F is refers to 1 that is F is the numerator, l is denominator because it minus 1 E are absent in this non-dimensinal factor, that is obvious one because stiffness force for unit length a Newton per meter if just taking F by L and if you take a different set of values for alpha 1 and alpha 2, you get 1, 1 minus 1 that again E A l and F that becomes EA by l that also known for axial stiffness.

So, you can find non-dimensional factors by varying the values alpha 1, alpha 2, you have lots and lots of them. Which one of them work and that is an important question here, how do we chose this alpha we can find x p systematically has we showed here in this slide, but alpha 1 alpha 2 are still open.

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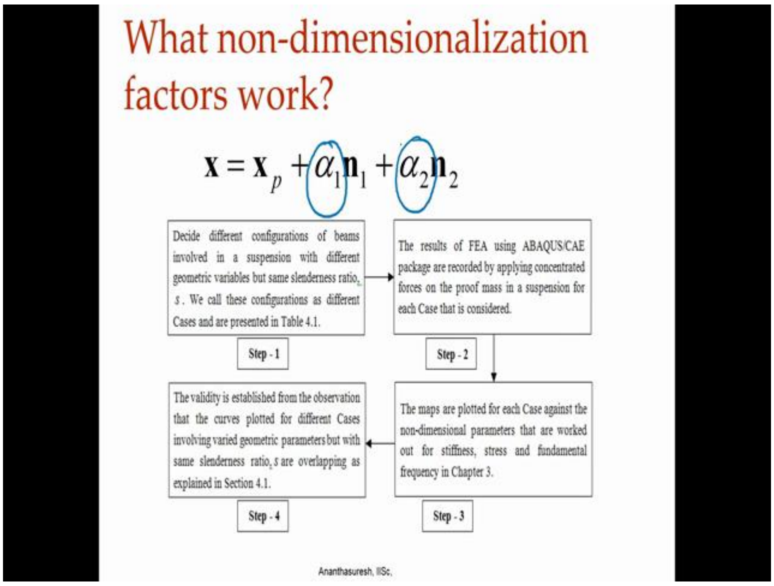

So, the way you would do is, you have to follow a procedure what non-dimensinal factors actually work? So for that that is a procedure that Jagdish Prathap Singh's m s c thesis developed, you start with a number of configuration beams that are involved and consider a few cases this is small print so you can pause and read.

You consider a lot of instances of whatever you are looking at compliant mechanisms or a suspension or whatever and then do finite element analysis and get this data and fix the value of s that is important slenderness ratio and then see they all line up on a single curve for this non-dimensinal factor, if it does not you have to change.

There is a little bit of non systematic thing there, but there is no better way at this point you have to consider whether something is correct or not by taking non-dimensinal factor, there number of has to do it because you can choose alpha 1 alpha 2 in many different ways or them lead to different values, now with the non-dimensinalization we have to plot.

So, finite element analysis you do not have to repeat, that is a same when you plot on the y axis for a fixed value of s that you make sure when you create the for finite element analysis, for the same s they all have to line a curve and we have to plot them in different ways analysis results are already there, we just need to try different non-dimensinal factor and then see when you plot them along the x axis with that non-dimensinalized factor whether it will line upon a single curve. That is what this says my systematic procedure dose, but the still a lot of thing that you need to try.

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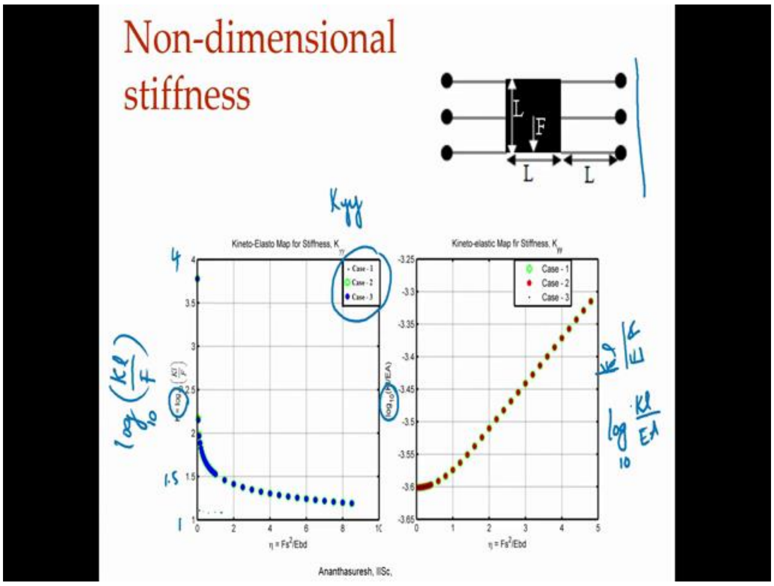

So, if you do that you will get something like this, let say I take a suspension of the kind that is shown here, we have suspension with this beams here. So, now, there are 3 cases are considered for K y y for y y stiffness using 2 factors that we have here non-dimensinal factor is this K l by F that is one thing that we got here we have K l by EA.

So, K l by EA and K l by F iswhat we have and both of lining up for all cases, then we can say for those cases it is actually working meaning that your non-dimensinalization actually gives you this plots, also notice that their log log plots because stiffness actually changes quite a breadth. So, here it is 1, this is 1.5 and 4 and so forth, log log plot. So, stiffness as you vary this eta here, stiffness varies quite a bit because you want to cover that much because you want to be able go from very small stiffness very large stiffness when you non-dimensinalize this that is what you will get, otherwise this whole thing will be just you know few points here.

So, your range is a quite a large. So, we need to do the large scale here. Log of so what we do is log of K l by F here we take a log to base 10. Similarly, here it is log of base 10 K l divided by EA and then see that the actually line up. So what we will do is we will consider how to do this for other stiffness values also in the next lecture.

Thank you.