Complaint Mechanism: Principles and Designs
Prof. G. K. Ananthasuresh
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 45
Kinetoelastostatic maps

This is the third lecture in this non-dimension analysis, I can say that this is a settle concept and probably it was not entirely clear the first 2 lectures of this non-dimensional analysis. In this third lecture I would like to begin with a few questions that, my students here 5 of them are going to ask I am sure that will you heard the other 2 lectures you might have the same questions in your mind, may be there different in which case you can always contact me. Let us go with the question that my student have on this they answers to those may clarify some of the question that you yourself might have in the first 2 lectures on this non-dimensional analysis. So, I have one student Nithesh asking questions, Nithesh.

Student: Suppose a complain mechanism has 3 segments and for example, change alpha d for each one differently alpha d 1, alpha d 2, alpha d 3, can I put the average of those 3 in the alpha d in the equation?

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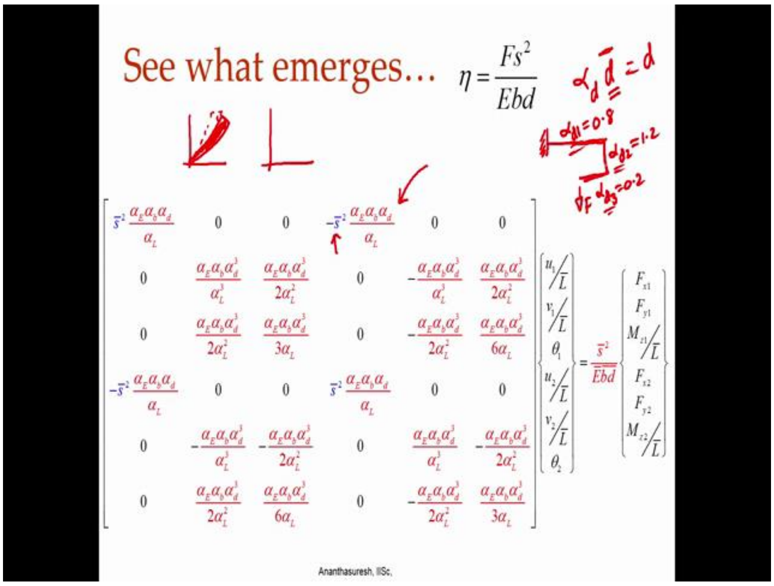

So the question is we have if we look at this screen now, we have this alpha d alpha b and so forth. If you recall what alpha d is, alpha d times d average d will be equal to d.

What this means is that, the d vector for let us say there are he said there are 3 segments. Let us I have one segment like this, another segment, another segment. Let say I have a situation like this a 3 segmented beam like this, let say force acting on this is this. So, his question is when he takes this alpha d when I say d for this will be something and the something and something which of them has some 3 alpha d.

Can he taken an average of alpha d? That is not the intension here what I would say is that, if you take this there will be an alpha d for this segment. Let us call this alpha d1, this will have alpha d 2, this will have alpha d 3, meaning that there is an average d bar there and then d of each one of them is some fraction of it, fraction of it or it is a multiple of it.

Now, if I take one set of alpha d1, alpha d 2, alpha d 3, I will get one map. Let say the map look like this. Now if I change let say let first say this is 0.5, this is 1.2, this is 0.2, let us say I get this map. Now if I change this from 0.6 to 0.8, then the map for it will change it may become something like this, we do not know how it will become it will change. Similarly if we change these proportions for every one of this proportion, there will be a different map. That means that, in this stiffness might if you see here is s and all this alphas, when you fix them the map is fixed, if you change them map changes. So, it does not makes sense to take average of alphas of 3 different mechanisms because one set of alphas will fix a particular complain mechanism, when you change this alphas will get different mechanism, where the proportion are changed. Next another question is going to be asked by Safwan.

Student: From the bucking ham pi theorem we saw that there were 4 non-dimensional quantities required for expressing the entire Kinetoelastostatic maps, but it appeared that we could capture the entire behavior from using just 3 parameters, this slenderness ratio and the ratio of the geometric members and also the eta. So, what is the relevance of the fourth parameter here?

So, the question is we have used as a post facto analysis of what we have done, we use this bucking ham pi theorem. In that for our case four non-dimensional parameters came, so if we look at this slide now where we had said, that we have 4 non-dimensional parameters.

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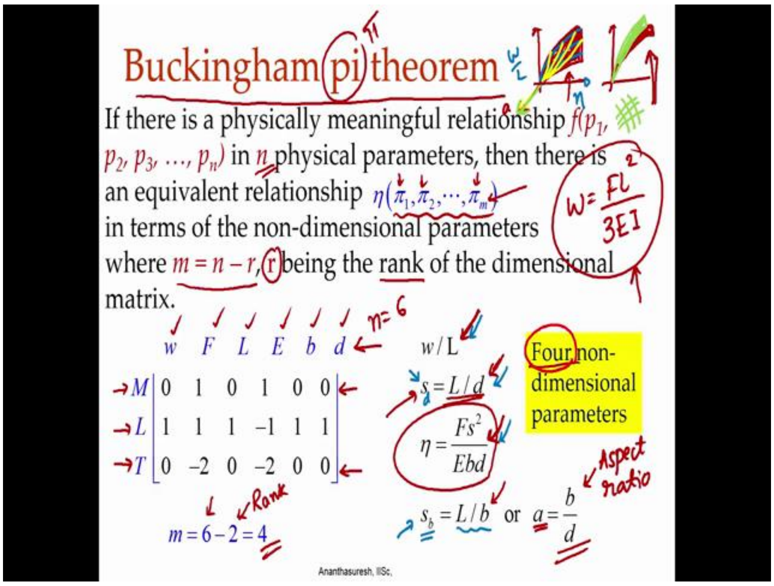

So, in our analysis so for up to this slide we had of course, the non-dimensional response there and then we have this s and then we have this eta. We said that instead of taking s as L by b, we could have called it S b, if I call this S d we would have taken S b which is L by b we could have. If we took that what would have happened why we need this fourth one because the 3 are enough for this.

So, if I look at this think here we said that if this is the w by L and this is eta axis, then by varying the s value you are able to fill the whole map, we would parameter the whole map. Now instead of this S d that we have used, if I use S b the parameter is may be different. So, let me use a different color here in this little think here, if we use S b let me change that to L or something, there could be a different parameter curves like that, it could be or let us say that we imagine another axis.

So, let me choose a color that you can see. I put a third axis out of plane if I call that the aspect ratio or Sb. Let us call that aspect ratio or the cross section because we said that for a given value of eta there is no unique value of w by L. In order to (Refer Time: 06:29) uniqueness, you fix this S d are what we simply called s which is L by d, but if I change what is s fixes is? It fixes the overall size L and the cross section size g, but cross section has if you have seem rectangular cross section, there are 2 parameters breadth and depth b and d we are fixing s means that we are fixing the cross section size to overall size, but what about change the cross section itself. If I change cross section that is aspect ratio, ratio between b and d if I change, the map is going to change. If a map is like this if I change b and d the map might change to something else let say I will draw here it may become like this. So, what does it mean?

As I it going to change that means, that if I take that as a third axis imagine that there is a map for certain b by d aspect ratio, let us have 1, it will be 1 plane map. Now if I draw another map where b by d is 0.9 and then 0.8, 0.7, I get several maps. Let us take all this maps stag them together that is equivalent to indicating a third axis, you look at a volume of this map it is not a planer any more we have that. So, when you have 3 parameters we can show the relation on them easily parametising like we did.

But there is a fourth parameter, one way to is when you have a map I can have one set of parameters like this, another set of parameters like this. That is one could be s, other could be aspect ratio a. Other is, go to 4 dimension, 3 dimensional rather than 2 dimensions. 2 parameters are use for the axis and the third parameter is use for the third axis and then you will have parameterization of the whole volume it could be done that way also. So, Harish Shankar is going to ask the question.

Student: Why is eta considered as an index of bending?

So I think I had when we looked at this early on in this discussion of the last one of the last 2 lectures.

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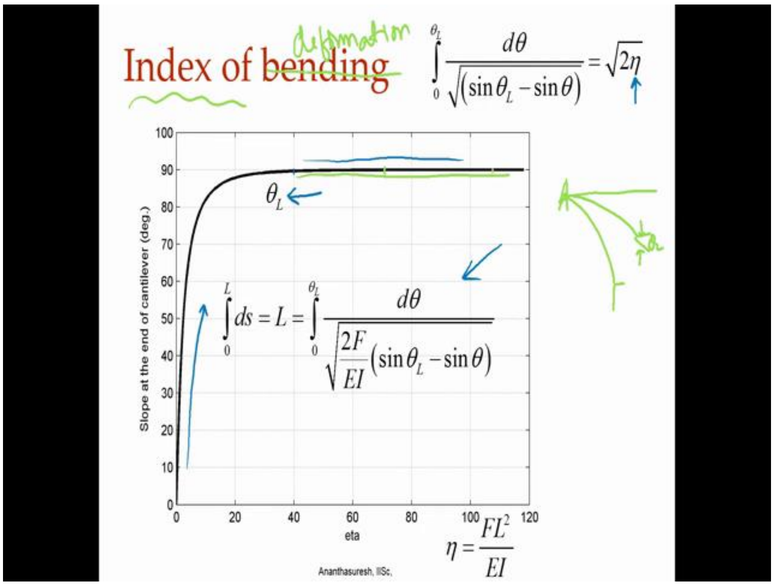

So, we had called this eta as “Index of bending” that was made in the context of a cantilever beam because if I take cantilever beam when I apply force is going to bend like this. So, this angle is the theta L. So, we saw that as eta increases, theta is increasing very rapidly and then it saturates also. So, the question is why do we call it index of bending? Because here in this portion eta is changing, but there is no change in the bending of that cantilever here.

So, is it right thing to say call it index of bending I think that correct you should not call it index of bending, maybe you should call it Index of deformation. Something is deforming the larger the value of eta, the larger the deformation because if I take some complaint mechanisms they may not be a lot of deformation or the elastic elements, because of the boundary condition it had or the force that is applied on it.

As eta value increases, there will be more and more deformation you said deformation for planar, complaint mechanisms that are made up of slender beams either there is axial deformation or bending deformation those are the true. So, it is not just bending, but let us call it Index of deformation rather Index of bending.

But most complain mechanisms depended on bending more than axial deformation. So, we just called Index of bending, but I think the objection is correct we should call it Index of deformation. I think Shamanth has a question.

Student: Why is a slenderness ratio is why it is above, why is it always about 10.

So, the question is when we showed a number of examples, let me take this one. So, you always started with s equal to 10 and went up to 100 or you know some value like that 80, 100 and so forth. So, let me remove all the written parts its clean now. So, we have shown let us look at the slide.

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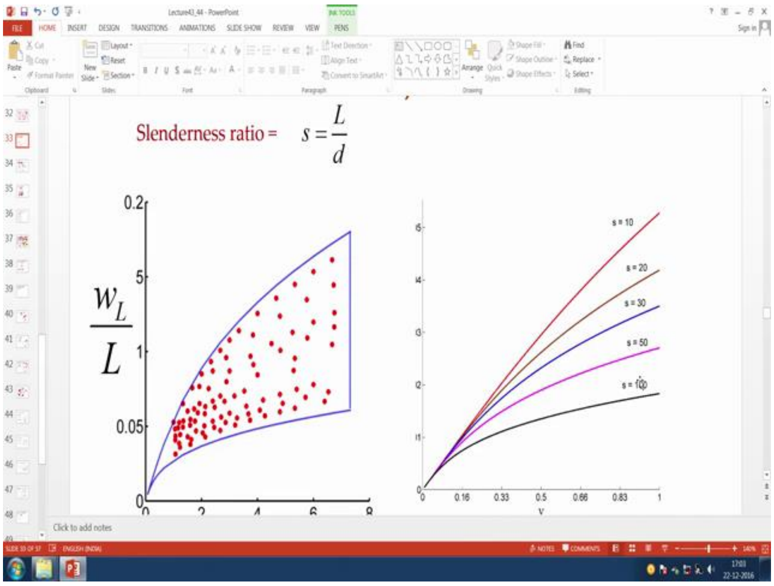

We see that s equal to 10 and 20, 30 the question is why do we start with 10? Why not less than that? The reason is we start with 10 is that, if s is less than 10 because you related L and d the beam theories applicability starts here because (Refer Time: 11:51) beam theory wants because it does not account for share affects, its slenderness ratio has to be at least 10. 10 or 15 there is a narrowed that one can establish analytically. So, anything less than that you can draw, but it does not make sense from the beam theory as fro we start with 10. Why do we stop at 100 or 2 100? Because by the time you reach a large value like 100, your beam is 100 times larger in the cross section. So, very very flexible beam or slender beam, lots of bad things would have happened by then because it might break or it might over stretch or bucker something might happened.

So, that is why we start from 10 and then go to a large value as I already explain, as you increase the value of s there is a notion of convergence here, the curve does not change the whole lot initially from 10 to 20 there is significant change and then 20 to 50, where is in this case, but 50 to 100 when you do the change is similar to what you have from 10 to 20.

Things get crowded together. So, over all if you see on the left hand side the map becomes end of that when you do this s equal to 100, 200 we will little bit more and more, but the difference will be very small, but you can go as long as you as large value of s as you want, but there is no point because a practical utility decreases when the beam becomes very slender. Let us take another question from Anusha.

Student: In Buckingham pi theorem while choosing the repeating variables, we usually take geometry parameters and material parameters. So, is there such a systematic procedure here also? How do we choose the variables to non-dimensional the parameters?

So, the question is let me remove all the written part. So, you see Buckingham pi theorem here. So, when we look at these parameters.

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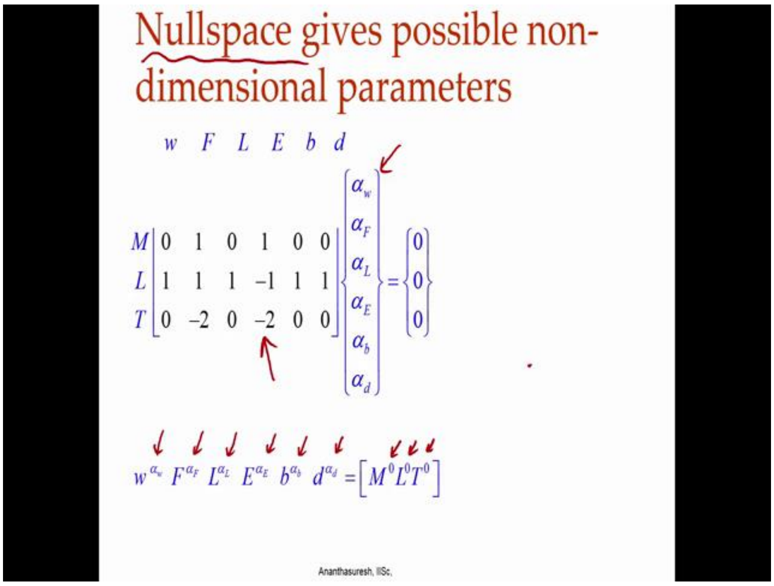

So, here we have these 4 parameters and there could be many more parameters which we also said if we look at the null space of this matrix, this is rank deficient by 1. So, we have infinite values of this alpha.

So, that is we are trying to come up with non-dimensional number where w F L E b we need to decide exponents we needed lots of values, where we taking this particular ones or instead of this displacement if I take stress or some other parameter how do (Refer Time: 14:24) non-dimensional thing normally in fluid mechanics you have a way of doing it, but here which parameter works we cannot clearly state which one will work, we can generate a lot of parameters, but then whether they lead to a single curve or not has to be experimented and found out as the knowledge stands today.

So, we will be discussing that in a later lecture as to how to non-dimensionalise stress, natural frequency and a few other elastic responses of interest to us, so some are this question that the student asked might have been on your mind. So, that is why we have included this.

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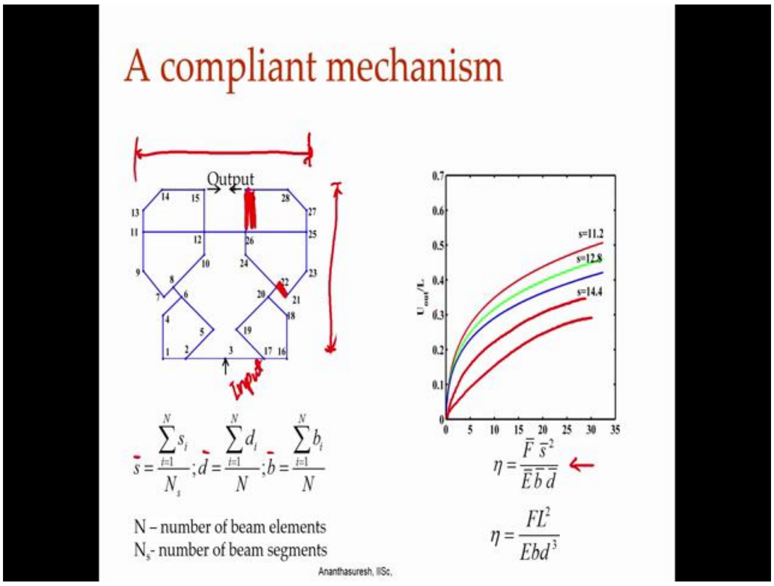

Now, let us move with the remaining discussion of this third lecturer non-dimensional analysis how is it useful? You can for a complaint mechanism you can find this I we do not put bars anymore after sometime, but you this is s bar, d bar, b bar and that is how we get this theta that we have essentially FL square E b d cube, but is F s square by E b d when you take this averages for complaint whether lot of a elements it will be fix somewhere know all that boundary condition will be there, there applying input here and there is an output there. In this case s value is 11.2, 12.8, and 14.4. If we increase s more and more curves will come like that.

So, will (Refer Time: 15:53) entire map then you would see what you get? For a complaint mechanism where you fix its size again we do not change proportions meaning if I take this mechanism, it has some over all width here and the height we do not change the height to width ratio. Similarly if there could be a wider thing here and then a narrow one here when I make this wider one that much more this is also made that much by the same factor. So, the proportion we retain in drawing these curves and ones you this will know what be the displacement that is how we can draw this curves? How are they useful?

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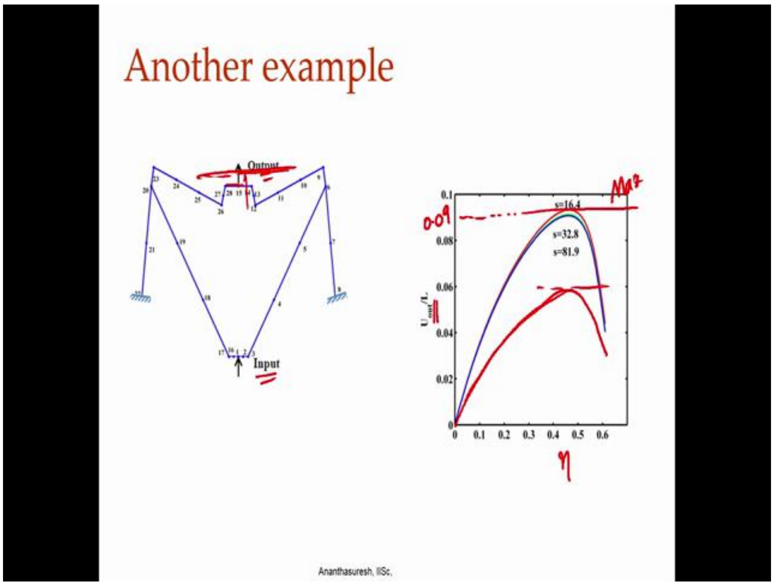

Let us look at this one here is a mechanism where there is some input, there is some output. In this case output by L is drawn for eta, we can see that it an increase it is a maximum value and then comes down so there is a maximum value. Different s values here s is 16 32 and 82. So, the spared is small that indicates that this has little in influence of axial deformation this particular complaint mechanism more than that that it has a maximum. If I look at the value it is let say 0.09.

So, u out by L is 0.09 as you apply the force here this one move some point after it actually retracts. So, this the limit it cannot go we could be applications where you want to handle delicate objects as you push at input more and more, output goes to some value after it only comes back it does not push further. It is good one for gripers that we to handle the delicate objects and the nature of the complaints is captured in this, ones you have these kind of Kinetoelastostatic maps drawn, you understand something about the capability of the complaint mechanism and that is a useful one of the uses of this. By looking at this map you know how it behaves earlier ones you just had things that just go like that, but here it goes like this and comes back that mean there is a maximum here there is a limiting output displacement.

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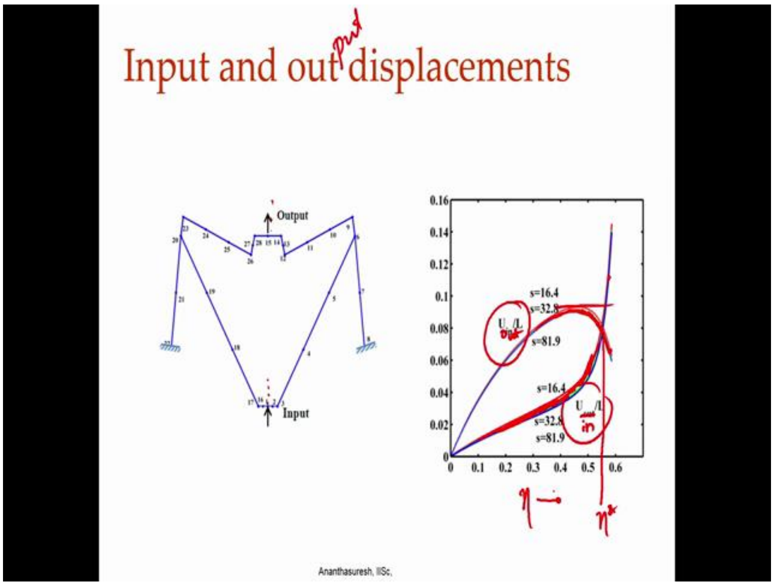

Now, let us a take the same thing there is a small thing here this is input and output, but this is actually out and in are probably interchange here, the 2 points are applying the force here, this will have some displacement this will have some displacement; when do you use it as a displacement amplification complain mechanism, this the input and the output displacement.

You can see that beyond this value of eta this is again eta axis let us call this eta star, until the time it is amplifying meaning that output displacement is more than input displacement at some point then meet and then input actually has larger value compare to the a output. So, we can see these things what you might think as a amplification mechanism after why will be applying more and more force, it become the other way it becomes force amplification, when input displacement is smaller than output displacement, such things also is that in front of our eyes and this is very descriptive of the topology and shape and proportions of the complain mechanism. Again note that these maps can be drawn, when you have a given topology, given shape and given proportions of both cross sections as well as the overall size.

Because s value is the one that parameter that links the cross section size to overall size, you can vary as again for the mechanisms that are bending dominated the maps 10d to be very narrow both for output displacement and input displacement both of them the spread is very small. So, that is all for this here and that is all for that here, there very close to being curves of course, the spread is different for different degrees of freedom, but overall their spread is limited; that means, it bending dominated that is some you can understand.

If things like output displacement is kept or taken for that matter appoint and also how to points one is more than the other as increase force, that is increasing value of eta you can tell that. So, by looking at this curves for given proportions and shape and topology you can say what that is capable of doing.

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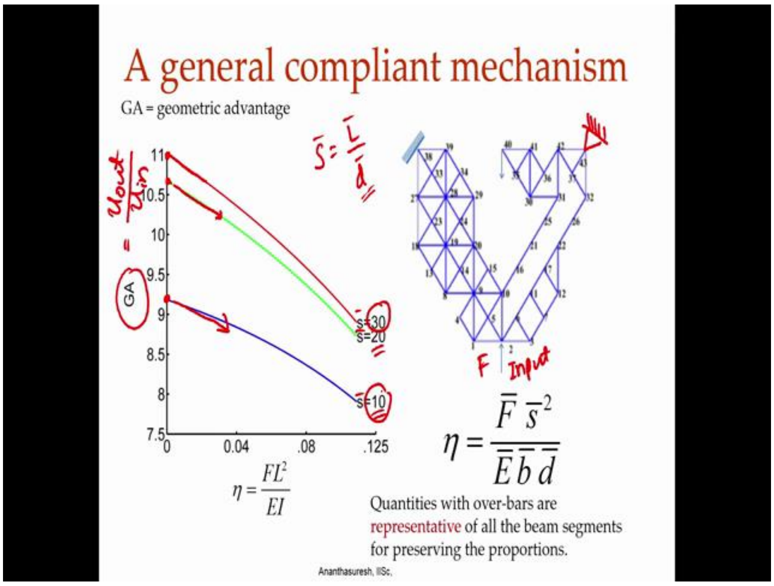

So, this also is useful to look at geometric advantage. Geometric advantage is output displacement divided by input displacement that is non-dimensional number naturally. If you see for this particular complaint mechanism, which is you know fixed here also and fixed there if apply input force here this is our input force it amplify displacement. You can see that for different slenderness ratio that is, I take the same topology same size I vary S, S if you recall is L bar S bar, all these are bars where not putting everywhere, but implicitly they are there L by d. If take a 10, I get a geometric amplification and beginning that is in the force is zero instantaneously if you do that will be 9 point something, as you apply more and more force, the first thing you notice is that geometric amplification actually decreases, initially it is 9 point something after it drops.

If you want more geometric amplification, when you increase s to 20 then it becomes 10 point something and then 30 11 point something, you can see that this particular mechanism is increasing means that cross section relative to the size of the mechanism has to be small because S has to be large in order to get more amplification here. So, L by d has to be large means that L has to be larger than d, the larger it is the more amplification you can get.

But you also see that as S increases, the drop the rate of drop is more compare to S equal to 10 output we do not go less than that as we already said, but if we see the drop the rate of the dropper 11 is much more than it is for this 9 thing, that is rate of dropper equal to thirty is more than S equal to 10. Such things also you can figure out that is when you are designing, these kind of insights are useful, how do you get how much amplification, but these things.

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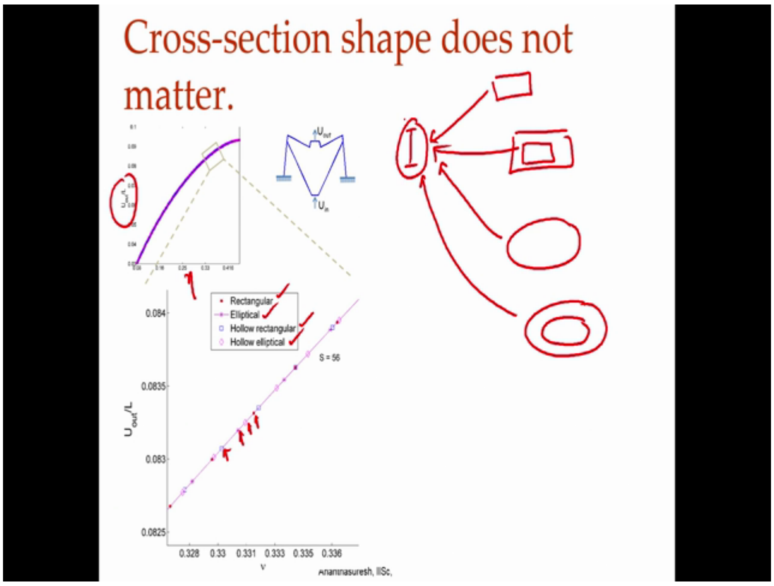

Another thing we can do is that what about shape cross section shape, we did the entire analysis in the last lecture based on rectangular class we talked about b and b d q by 12, what if I change? This particular thing shows that when you have certain. So, this is eta and this is again (Refer Time: 23:06).

So, when we put this little widow here that is shown here, there is Rectangle cross section, Elliptical cross section, Hollow rectangular, Hollow elliptical, 4 different cross sections are taken here, what we say is that they all have different symbols thus red dot and star and square, blue square and a magenta rhombus, they all lie on the same curve because what matters here is second moment of area I, from where it comes, from what cross section, whether it is rectangle or it comes from rectangular annular cross section or elliptical cross section or it comes from an elliptical cross section with an annular thing it does not matter, this is move around this curve. So, it is independent of cross section also, but of course, for cross section where it lies. So, whether it is here or here or here that much it matters, but the map itself is invariant to cross section that is clear because it only takes I value, not how I value comes. So, cross section also can be chosen for these things ones you indicate value of I.

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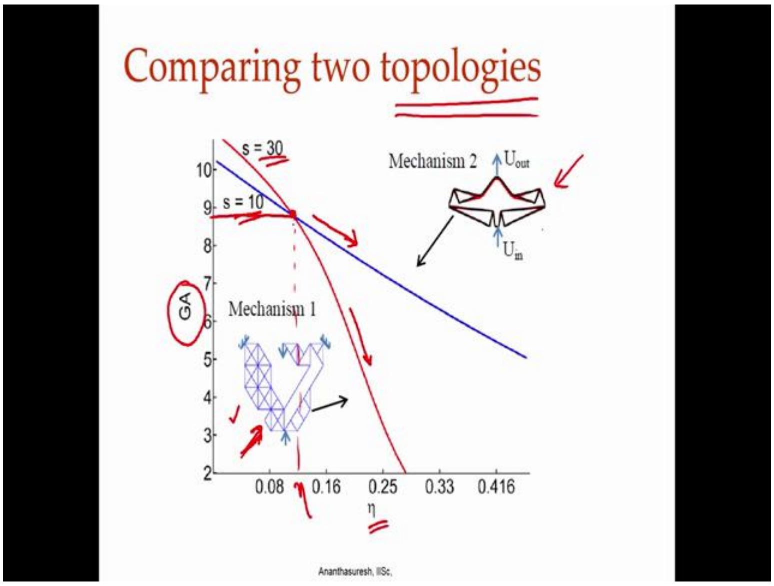

Using these maps if you do it for 2 different mechanisms and put it in the same plot, here of course, the blue one is done for s equal to red is for s equal to 30. So, that they are in the same eta range.

You draw several red curves for different s values, several blue curves for this mechanism for different s values, but what you see again if it is geometric amplification that you are interested in, you can look at both mechanisms at the small values of eta that both are comparable. Start from 10 little more than 10 and then they come down what you can see that, geometric amplification of this mechanism it drops more or less linearly compare to the other mechanism which is here.

So, we had seen this mechanisms geometric amplification, in the in previous slide here. Here we had see that this little curving, but now this little curving compare to the other mechanism seems pretty linear because this has want a much higher value of eta here, this is pretty much linear where as this one. This is more rapidly decreasing geometric amplification compare to this. So, if we are using this in a sensor which will discussed a case study later in the course, we see that geometric amplification decreased linearly is more palatable than decreasing non-linearly and rapidly decreasing.

So, we can compare to topologies now and infer which is good both are amplifying, both have let say at this point we are taking eta, both of them have 8 point something. So, which one would you choose; you have to choose that while designing you has to choose one where it decreases, rapidly or slowly or smaller rate that is how would you choose? In this particular case we would choose this mechanism rather than this red mechanism if did not want geometric advantage to drop rapidly when the force changes. So, we can choose topologies like this.

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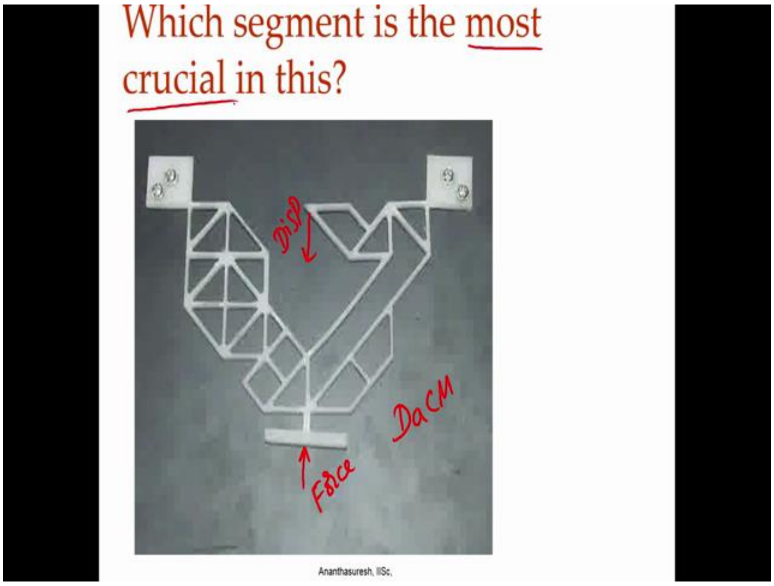

Let us say we have the same mechanism n apply a force, which going to blue like this with a lot of output displace which a Displacement amplification complain mechanism, these what was given by topology optimization lot of beam segments are there, which of them is the most crucial, if you ask the question, what we could do? When you draw the maps is draw all of them.

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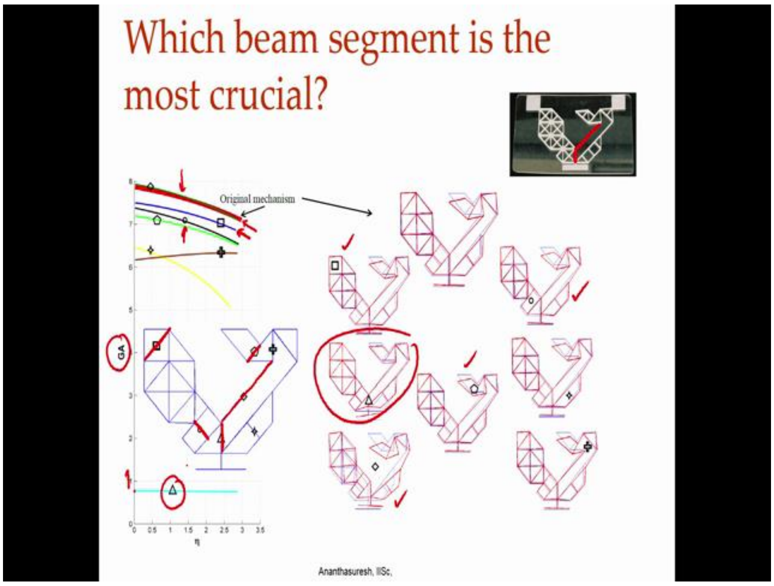

So, what we have done here is we have taken this mechanism and let say I look at this thing here, what happened is? This particular thing is missing, red color red for this one. So, that led square color that is the thing.

So, when you remove that particular segment, the original mechanism is the red curve here, that is this red curve original mechanism with all the elements intact; if I remove this it becomes blue one, your geometric amplification decreasing little bit; it may not be to that crucial compare to others.

Let us take this circle, where is circle? Circle is here, if I remove this element where is that here I remove that element then that circle one is the black one here, these another one diamond here. So, that is this one where diamond here, is if I remove this does not matter. So, we look at this mechanism, would you believe that by removing this you actually do not change the behavior that much? We can see it here one can also do (Refer Time: 28:05) and find out, but drawing this curves for crucial elements, is very useful for desire may be they do not want that for whatever reason. Now we see that diamond curve does not matter at all this is useless, this particular thing we will remove it, thing will be are exactly the same way.

Let us come down to other one green, the pentagon that is this one that is this segment. If I remove that well that becomes a green more or less is they are there is not that much drop. Let us look at this psi and one is triangle that is this. So, where is the triangle? The triangle is here, this particular mechanism that is missing here. If you remove it, it comes down its not geometric amplification any more it is less than one; this is one here less than one. So, in this mechanism this particular element here is a crucial one if you remove it does not what as you want.

So, by drawing this curves you tell the user, let say these curves are pre drawn for a given complain mechanism if user wants to get away with one element, they can do that, and then see that which is the crucial element.

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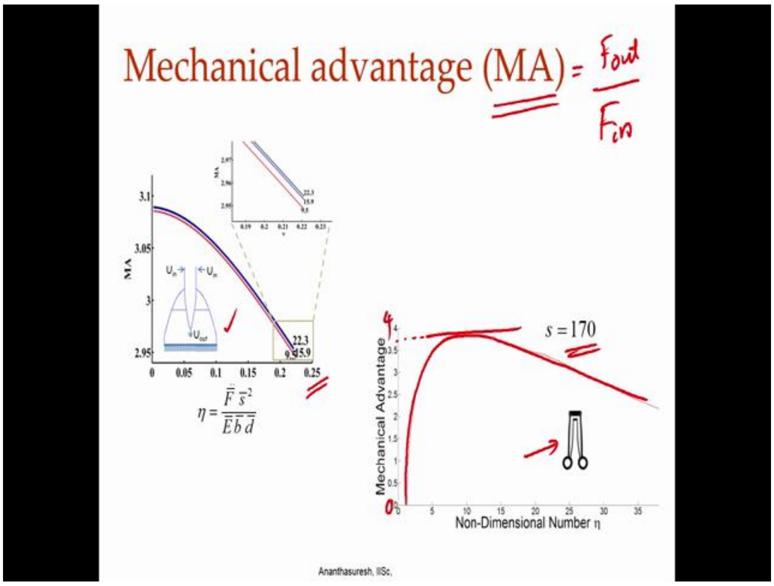

So, there are things like that you can store for a mechanism, we can also do with mechanical advantage that is also non-dimensional number, mechanical advantage we define it as output force to input force, ratio of input force to output force.

If you draw that for this particular mechanism, we can see how mechanic advantage changes starts from 3.1, 2.95 pretty much it has not changed up to eta equal to 2.5, constant mechanical advantage where as this one this is 0 here and 4, again this goes to maximum and comes back; that means that, with this topology that is this topology no matter what you do you cannot get more than 3 point something as your mechanical advantage. For s equal to 170 here, we have to see what the curves for other is values, but the behavior does not change like we saw for geometric amplification also.

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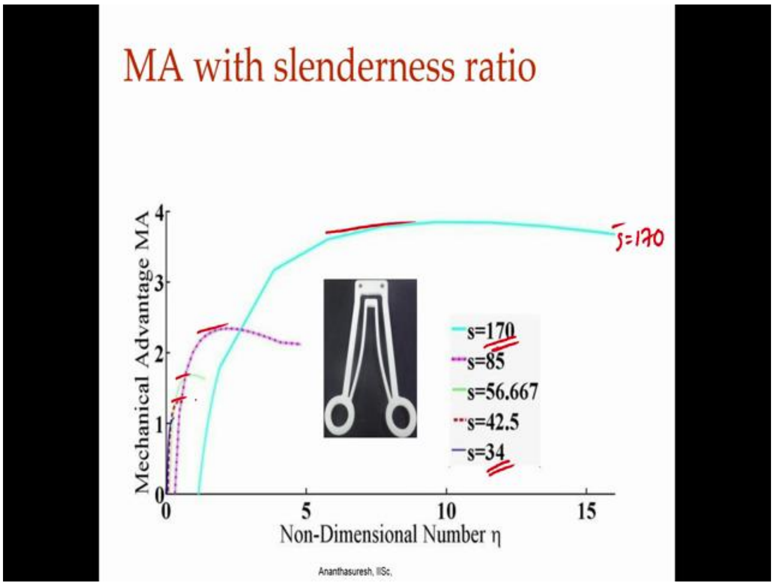

And we can also plot for this mechanism that is the same as what we have here. For different values of a starting from 34 to about 170, this that there is a maximum that things does not change, it is there and you can draw that and see as you increase s value this is s equal to 170 or S bar, it goes to 4 otherwise it is smaller. So, we can get all of this information here.

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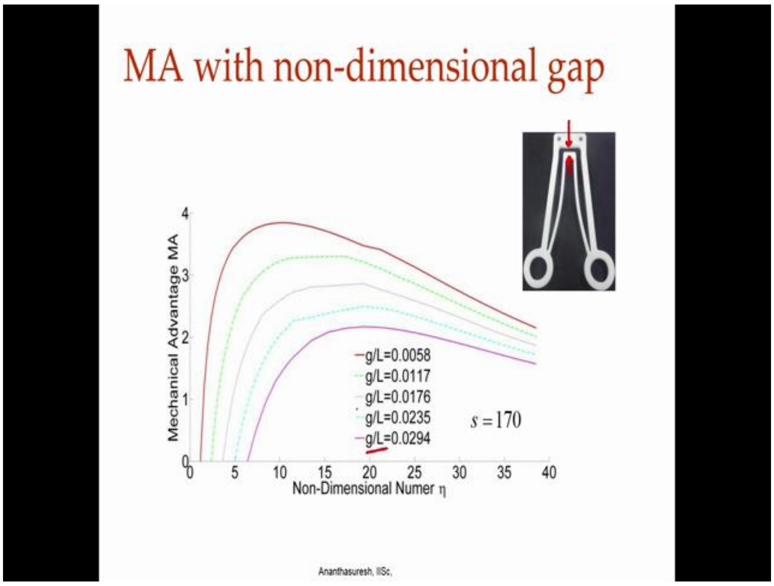

We can also non-dimenentialise the gap that is it mechanism, there is gaps that also we can non-dimensionalise and draw this for different non-dimensional gaps. So, we can choose the object here, we will come back to mechanical advantage analysis later in the course for few lectures devoted to that and we can do different work piece stiffness also that can also be drawn, right now it is given absolute values for that can also be non-dimensional.

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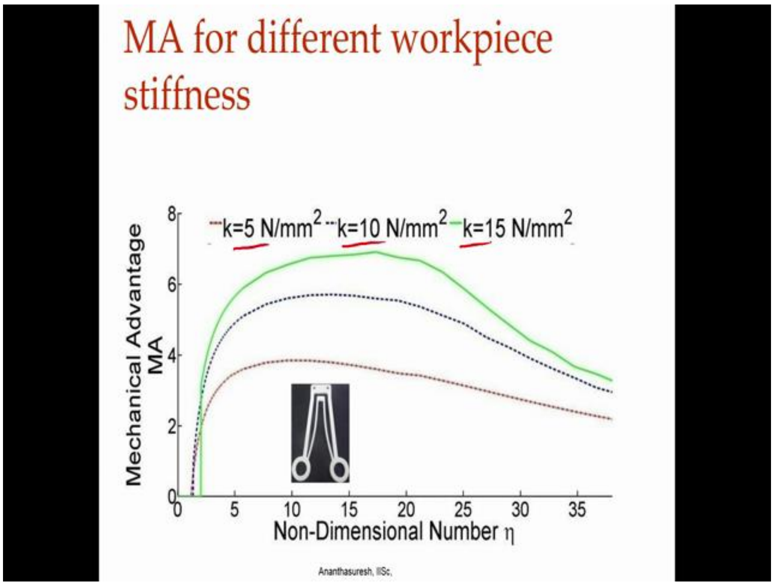

So, you can look at (Refer Time: 31:14) work piece stiffener, let say how does mechanical advantage change with respect to work piece stiffness.

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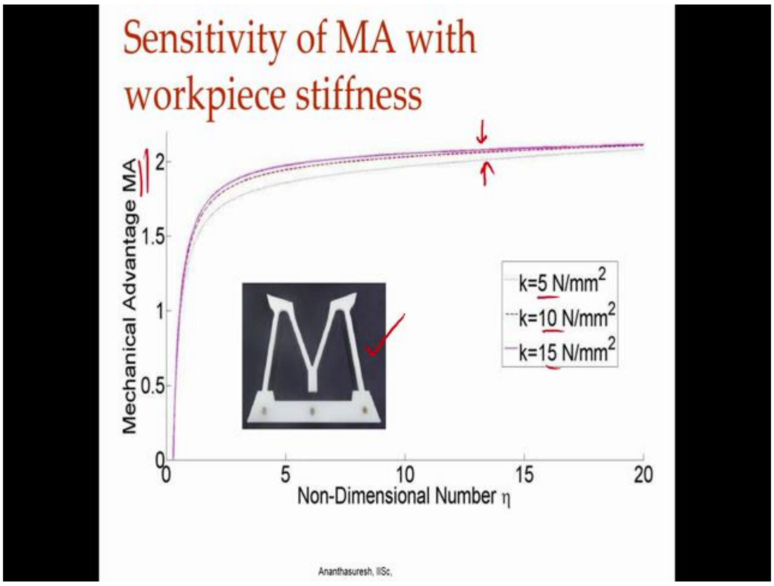

In this case it is not changing a whole lot, such things also we can make out for some complain mechanism it may not be sensitive, other ones it may be, so such things also are helpful what designing.

So, this design technique is different from the other tool, here we are talking about some qualitative features which will not be captured with the computational intensive methods because we are doing, the calculation inputting just to end this lecture. These are the third lecture in this and then will see some case studies and others in the next remaining 3 lectures on this topic. We further reading there is a conference paper 3 years ago and there is also a (Refer Time: 32:01) of Harish Verma Indukuri, where details are there about this method. So, in the remaining lectures we look at some design case studies to see how these maps can be used.

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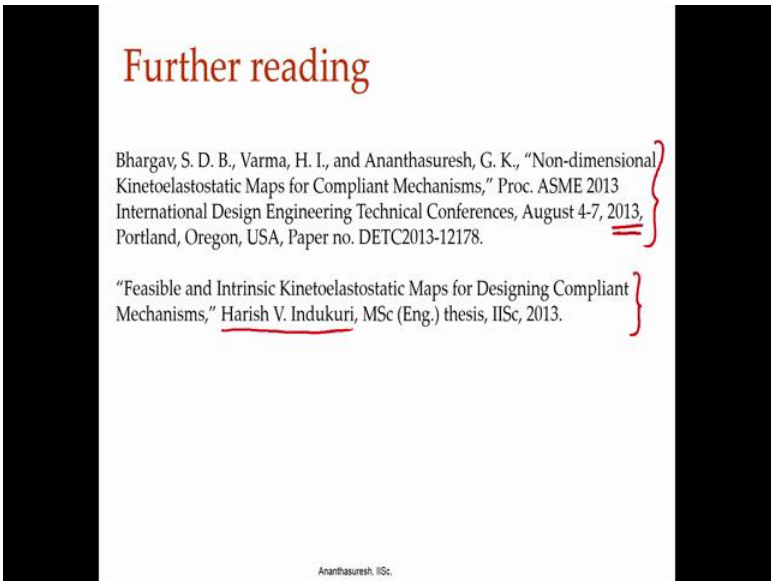

And then how we can non-dimensionalise other parameters like maximum stress, natural frequency and few others and look at few more (Refer Time: 32:26) that exist in this non-dimensional analysis based design technique.

Thank you.