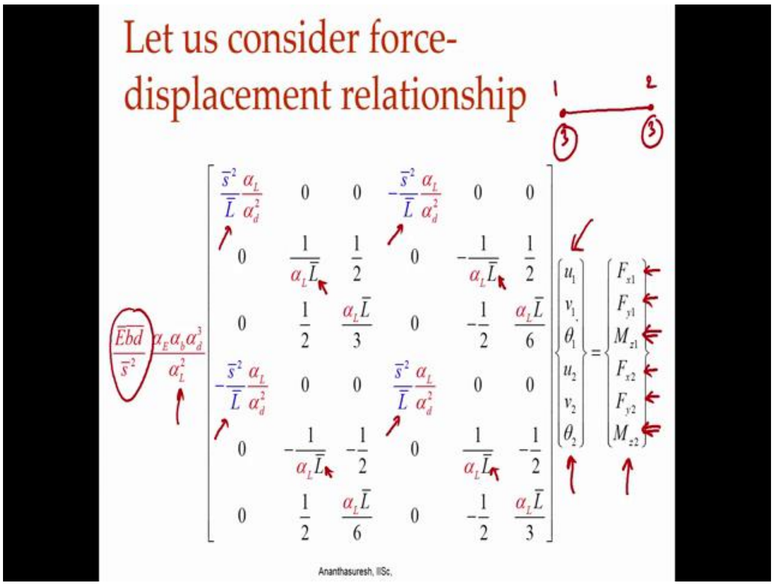
Compliant Mechanisms: Principles and Design

Prof. G. K. Ananthasuresh
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture - 44
Deformation index and slenderness ratio of complaint mechanisms

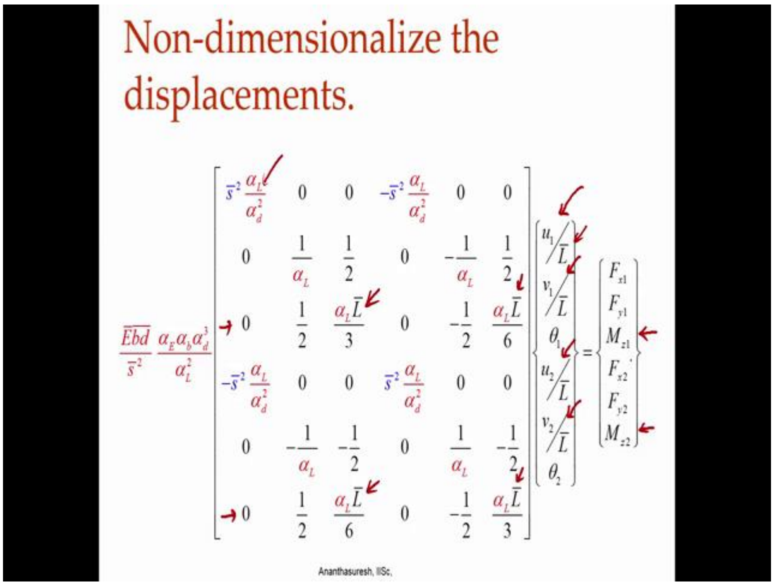
Hello, this is a continuation of lecture number 43 we stopped in the middle, when we are discussing some non-dimensionality for a fix beam or a general beam. So, you have to watch lecture number 43 to makes sense out of this lecture. So, let us continue where we left of in lecture of 43.

(Refer Slide Time: 00:36)



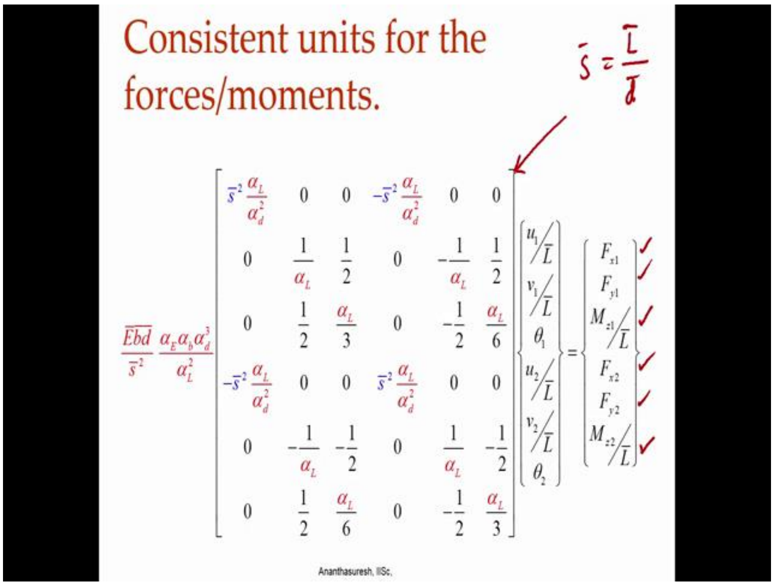
So, we have here the force displacement relationship where in the 6 by 6 stiffness matrix, we use proportions that is this alpha l for length proportion, alpha d for depth proportion, alpha b for breadth proportion, alpha E for youngs modulus proportion, meaning that what if you have multiple beam segments in a beam and you are looking at 1 beam segments or beam element, how does it look is what we are saying here. So, we have the for each beam there are 6 degrees of freedom let say this is node 1 and node 2 there are 3 degrees of freedom here and then 3 degrees of freedom here that is what is shown here.

(Refer Slide Time: 01:25)



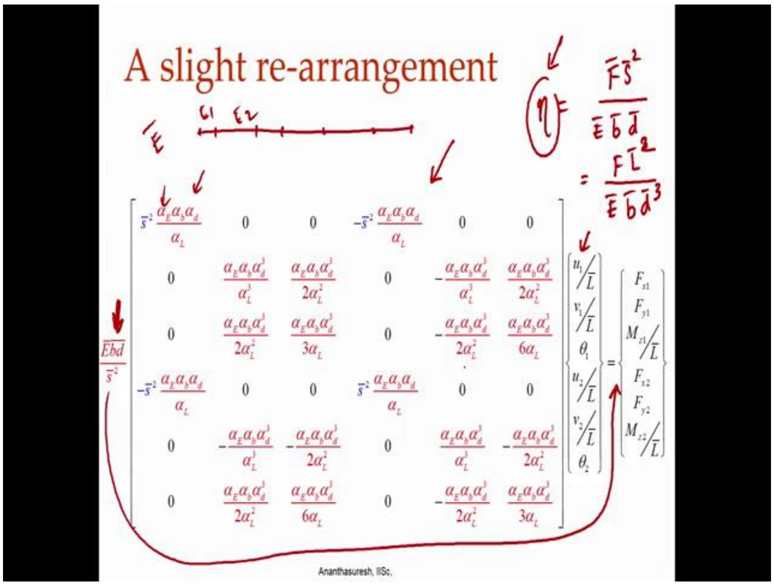
So, we take this now we try to non-dimensionalize our displacements now. If we see u 1 by L, v 1 by L, u 2 by L and v 2 by L, anyway theta 1 theta 2 are already non-dimensional we do not touch them, when we do that the L z were there here come back and some place L's it is still there. Now the L bar that average length is only in four places. Other ones if we just see what was there we had L's here also right. So, now, they are gone in these cases, but they are gone the moment we put them into the displacements like we have done here. So, they are gone, but these since are there we take them to the right hand side that is this third row and sixth row correspond to the moments here, because they correspond to rotations, what we do is these L's we take the other side. Because what we have a matrix equation is 6 equations and we try to look at what happens. So, we try to get that L bar that was in here we got it here, so that it non-dimensionalize, now the right hand side will it has the same units meaning that these are all newton this is newton, this is newton, this is newton, this is newton, where as this was moment. So, we divide this by L bars that also becomes unit of that becomes Newton.

(Refer Slide Time: 02:44)



So, now if you see over here in this matrix what we have are all proportions meaning alpha l, alpha d alpha b and alpha E and s bar, s bar is also; s bar this slenderness ratio, this is L average divided by d average. So, we take average is representative thing and we have this. So, this one is completely dependent on the proportions of the beam it is cross section and the length and everything. So, this is applicable not just for 1 beam, but any beam that will be there in a thing that one would consider in a complaint mechanism or for that matter beam with different boundary conditions.

(Refer Slide Time: 03:46)



So, what we do is we will try to do a little re arrangement. So, that we take this we keep that outside take all these in inside become little smaller font, because the quantities became larger. So, when you look at this and then take this to the right hand side that is. So, what we get is well that is not in next slide if we take another side we have F here, and then we have s square here, s bar square and then we have E b and d that is exactly what will call it eta, because if we remember s bar is L bar by d bar. So, s square will be L square by b square and these already there it basically amounts to f L bar square divided by E bar b bar d bar cube.

So, again we got our non-dimensional thing. So, when you have a matrix system like this where both bending and axial effects are considered, axial deformation effects are considered there is axial strain energy as well bending energy has considered we get this stiffness matrix, here by introducing this proportions what we mean is that if proportions are the same this stiffness matrix does not change, it can be a long beam where we have smaller larger longer segments probably made of different material properties also different cross sections whatever. Finally, if you look at this, this thing as long as this alphas are the same it remains the same, and all of these when it goes to the right hand side, when this goes to the other side when you take this s square by E b d and then force is there are the moment is there M by L if you do it depends only on that that is non-dimensional displacements then will depend only on eta as long as a proportions are fixed.

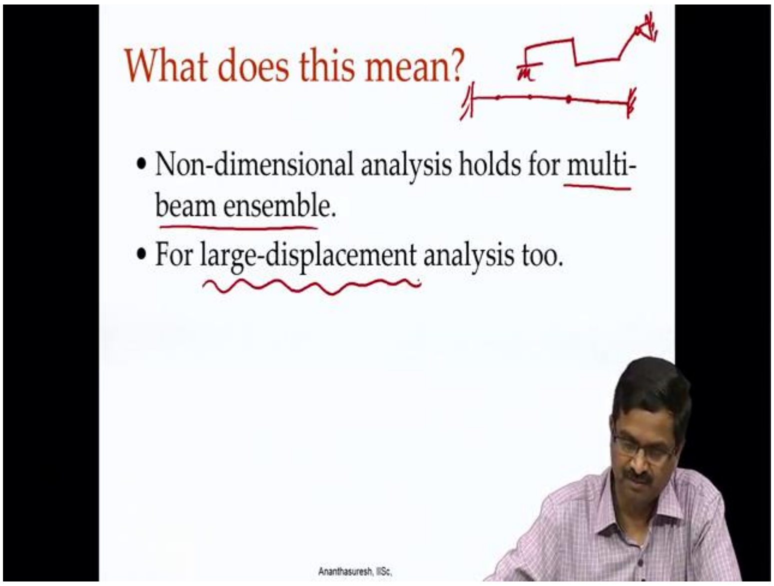
That is your alphas, now you just saying that alpha E means that Youngs modular this may be E 1 this may be E 2 or whatever, then we will have average E and correspondingly there will be alpha for this one or that one or that one. So, whatever is in this stiffness matrix if the proportional keep the same we do not change them, if we keep the same then the non-dimensional displacements depend only on eta. Now we assemble all these into a bigger stiffness matrix for each beam element if you do that it is still will be a function of only eta and not anything else this we took a stiffness matrix for small displacement analysis.

Instead if you do large displacement analysis and take the tangent stiffness matrix it would still hold tangent stiffness matrix also will only depend on area cross section, Youngs modulus, force length and second moment of area i, if you take that and do this kind of analysis it will still hold. So, what we are saying is that non-dimensionalizing if you do in the manner that we have done then the non-dimensional displacements depend only on eta, eta is for the entire thing because when you say force and again there are multiple forces we will also take average force if you well F bar we can have.

So, when you do that F bar s bar square by E b d all with bars meaning average values are E representative average. So, only one representative thing, but it can be any other it depends on that whole thing that is the entire beam that you have will have 1 eta just like cantilever beam entire thing had 1 eta entire beam, whatever boundary conditions, whatever multiple beams making of a complaint mechanism it will have 1 eta associated with it into one single characteristic for given set of forces acting on it, given material properties. It could be different from segment to segment and complaint mechanism, different cross sections different lengths for the things entire things depends only and eta because, everything we can take it to the outside the way we want to separate it out.

As long as proportions are the same that is important because, they are in the stiffness matrix if we take the proportion the same then response whether it is small displacement large displacement depends only on this one single value which is eta.

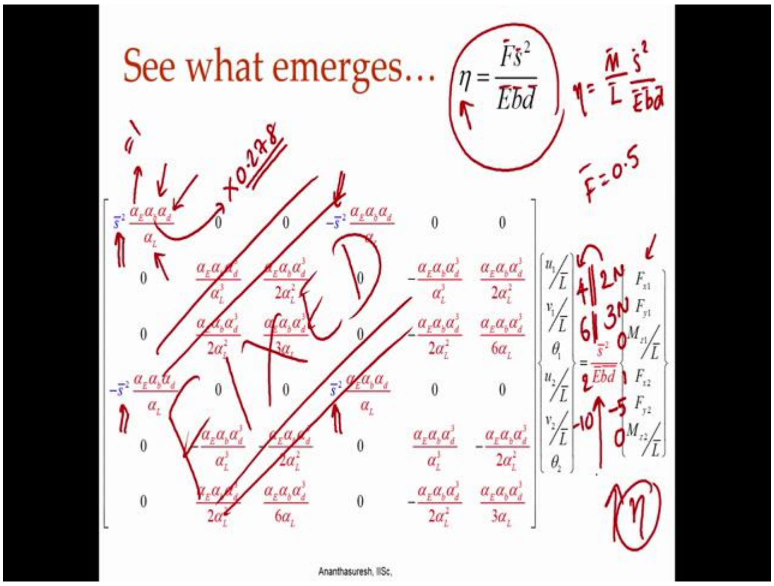
(Refer Slide Time: 08:39)



So, what does this mean? So, non-dimensional analysis holds for multi beam ensemble. So, we will see with examples what we mean by that I can take a long beam let us a fix, fix beam we can take the entire beam as one and this definitely holds as we saw for a 1 beam element. When we did this analysis of non-dimensionalizing and non dimension displacements and the output force the their input force that we have that is forced moments if you again make them to have same units what you get the right hand side is eta.

So, left hand side we said depends only on proportions hence non-dimensional displacement function of eta alone. So, when you have that it applies to multiple segments if I have in here it applies to that or a beam can be like this whichever way fix it here and I put a pin join here, whatever I may draw for all of those things this will hold and this definitely true for large displacement analysis s bar now let us see to make more sense out of it what we have here, what emerges here is our non-dimensional factor.

(Refer Slide Time: 09:50)



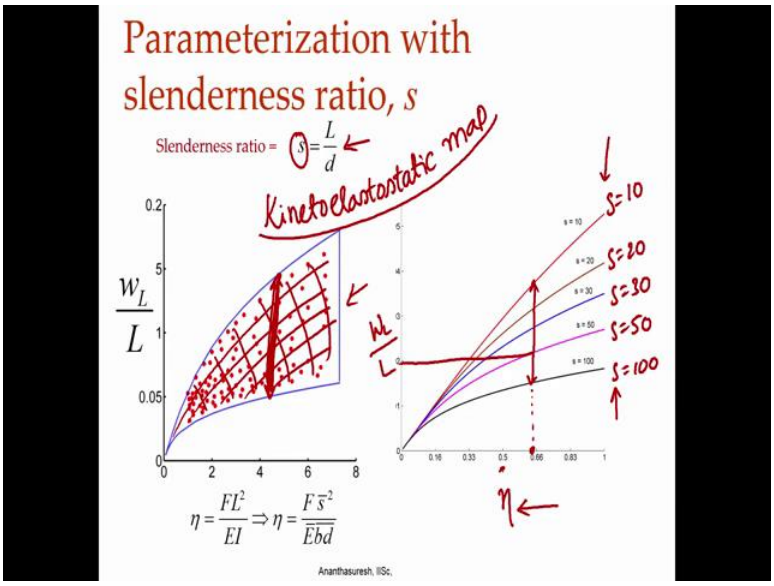
When you have something like this what I said let take to other side, if you see F s square by E b d. So, if you have force at this node or that node all of them of are going to give you apply moment instead of eta being like this we will have M by L s square by E b d, when you write like this without bars I am just not putting bars, but what I mean is that all this should be average values for the entire beam segments if there are multiple moments applied as well have M bar also.

So, similarly this will be F bar s bar E bar b bar d bar. So, that you get this eta value whether it is force or moment, when you have this again emphasize that if just whole thing let me write it, if you fixed that is you do not proportions in complaint mechanism or a ensemble of beams if you fix it. And that the forces the way you apply there are all of them you apply that is all f x 1 y 1 m z 1 f x 2 a f i 2 m z 2 if you do all of them they will take some kind of an average in the sense of F bar and M bar by L if you take eventually what you get here will be one single eta I have to change this eta.

So, when I say proportions, proportions are the forces also have to be followed meaning that you have F here, and then let say this is 2 newtons this is 3 newtons now when i say average let say there are no moments let us a 0 and let say this is 1 newton this is minus 5 newtons this is 0 for the sake of discussion, now let say we do not put moments now average you take. So, you have 5 5 get cancel flow so over all get 1. So, F bar in this case you take it as 0.5 and compute your eta here.

Now, for this analysis to hold if I make this 4, instead of 2 if I make this 4 I have to make this 6, because I am multiplying by factor 2 and this will become 2 this will become minus 10 that is what we mean we had maintained proportions, if there are multiple forces, similarly since your (Refer Time: 12:18) have defined this alphas that alpha e. If I do not change youngs modulus alpha E will still be equal to 1 that nothing changes here, if I increase the breadth by a factor of let say 0.278 you just put alpha b equal to 0.278 all the breadth of the all the beams in the mechanism will change by the same thing same thing with alpha d and change the length and so forth.

(Refer Slide Time: 12:51)

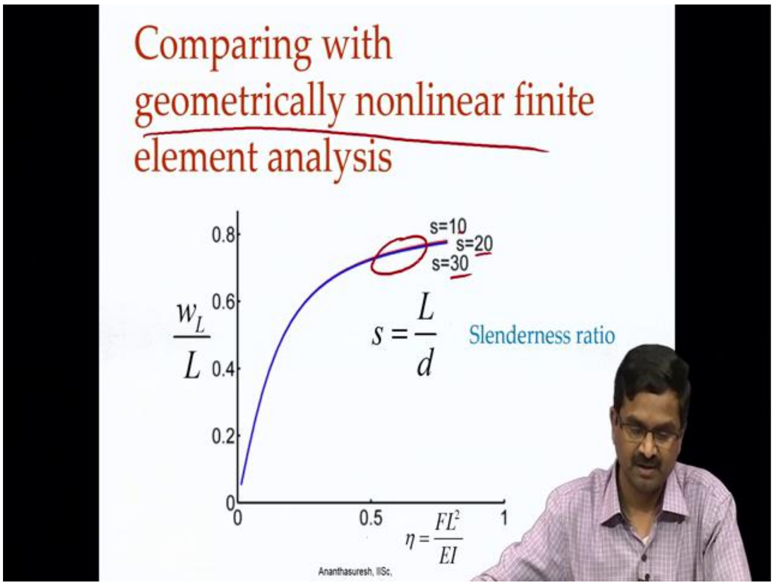


So, you have to maintain the proportions then this will hold the breadth. So, if you go back our fix, fix beam we said that we have this entire space of this to be our kineto elasto static map, since we have use this word a lot let me write this one we called kineto elasto static map, because there is some kinematics there is definitely elastic deformation here and the static this we can call kineto elasto static map so that is what it is. So, that were did not make sense because we take an eta and we said that it can be anywhere here. So, we did not get a unique value of non-dimensional displacement.

So, now we say that by using slenderness ratio because if you notice when I said fix their proportions, I forgot to mention that there is also s bar here right, that mean that also has to be fixed not just the red ones are proportions s if you have a fixed value of s then it depends only on eta if s changes it does not that is exactly what we see here, that when this long range are possible what actually it means is that let say I take a particular value of eta here this is eta this range actually means that something is changing, that something is actually this s. So, this is s equal to 10 this is s equal to 20 s equal to 30 when I say s is actually s bar 50 s equal to 100.

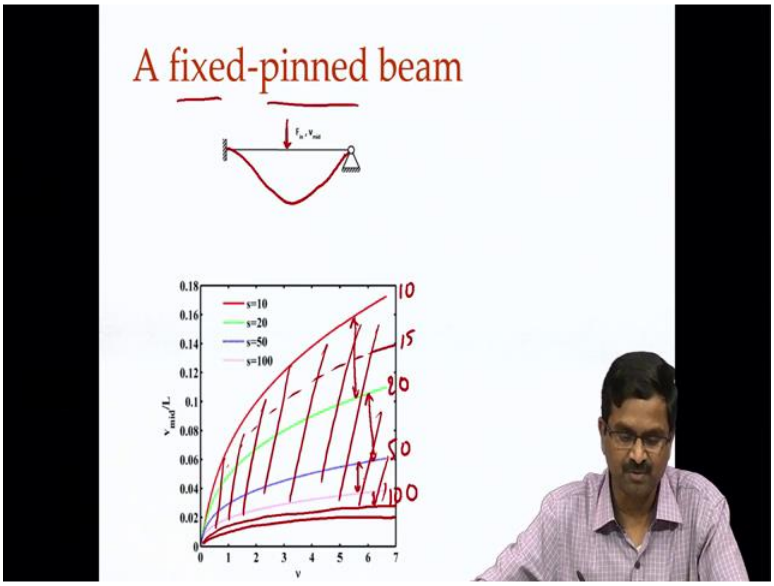
So, what it means is that knowing eta is not enough for a fix, fix beam for a cantilever it was close enough at least there was a very little deviation from the curve that we had when you did finite element analysis whereas, here it is all about the place as you can see there is a complete map here. So, what it means is that when you have eta your non-dimensional displacement that is W made by l some transverse displacement it is not unique value because there is another parameter which is s. So, you can actually parameterize this map using this second parameter which is slenderness ratio s that is what it means that is what did analysis now here we have not one non-dimensional number which is eta, but you also need to have another one which is slenderness ratio we have two then it is very unique because I go near and then it is s equal to 50 then I know the W l is so much. So, now, when you have map like this we need to have 2 parameters

(Refer Slide Time: 15:43)



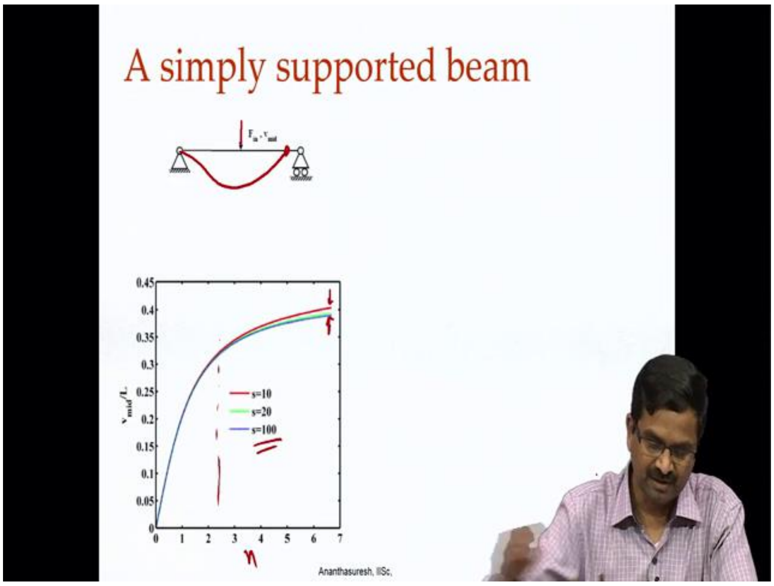
So, now let us look at this for some beam conditions we have geometrical analysis done for different s value s equal to 10, 20, 30, for some condition I do not remember which boundary condition it is, but it is definite not cantilever because we see s equal to 10, 20, 30 there is little difference there is red curve and a blue curve.

(Refer Slide Time: 16:11)



In this case, the boundary condition indicated it is fixed and pinned. So, fixed and pinned beam if you do then we see that s equal to 10 here this is 20, 50 and 100 you should notice something as I am increasing the slenderness ratio 10 to 20 the gap is. So, much when I go from 20 to 50 not even doubling it is more than double the gap has reduced 50 to 100 you know I from here it is a factor of two from 10 to 20 here 50 to 100 the gap has decreased.

(Refer Slide Time: 17:50)

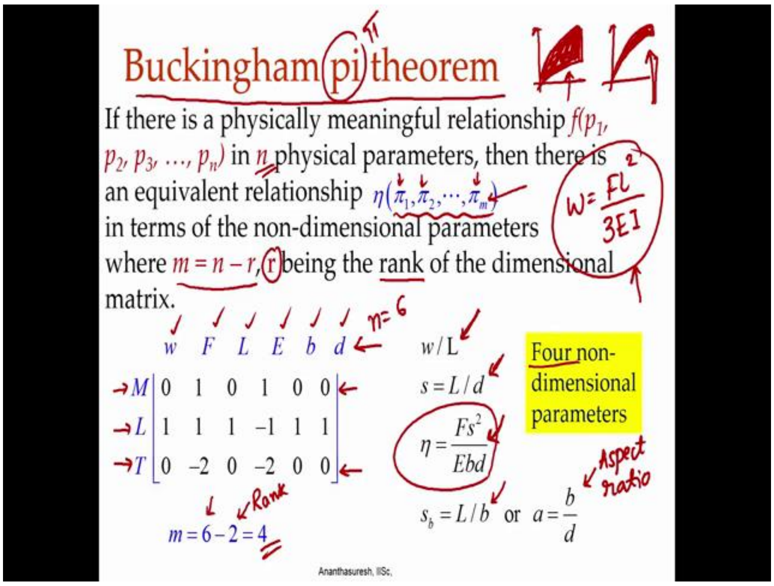


Now, if you go 200 it may be more like this the gap will keep on decreasing meaning that eventually we will be you know as you go this map here, the whole thing can be parameterized for a few curves here in between if you want you can also interpolate let I have not done 15, but I can roughly estimate what 15 is if you want you can draw to draw each curve it takes 1 finite analysis non-linear 1. We have to do that if you see when the map is very large here for this boundary condition which is fixed and pinned when I apply a force here as shown this can deflect like this only by stretching. So, whenever stretching is more the map will be bigger or more spread out a cantilever beam stretching is limited it is very closed to a single curve for a fix, fix beam there was something for a fixed pinned beam also there is a spread here.

So, let us look at some other boundary condition now this is a simply supported beam meaning that it is pinned here and pinned here with a slider you can see that the map s equal to 10, 20 and 100. So, very little spread the whole thing is more like a small (Refer Time: 18:06) fact for lower values of this eta, we do not even see a difference for different s values. So, this dependence on s comes in only because of axial deformation see you simply supported beam when it when apply the force like it is shown here, simply we can slide it can actually become I am talking about large displacement analysis you can go and can still maintain the same length or rather it will not go that way when it has to move in to keep the length same it can move in like this.

So, length can still be the same you guys are going like this. So, if you go back and look the our matrix there is s here write when s when things are not changing much with the dependence on eta, which means that influence of s is small, whenever there is not much of axial deformation as it is true for a simply supported beam, fix fix beam there is a lot of a influence of axial stretching then this map will be a spread otherwise it wont be.

(Refer Slide Time: 19:16)



So, with all this now before we illustrate this for complaint mechanisms let us we call how we arrived at this. In fact, it was arrived at by doing this non-dimensional portrayal for displacements transverse displacement of a point in a complaint mechanism by taking this average l average b average d we material this same everywhere. So, there was no average E this E by plotting again is eta when we do this we notice that for some complaint mechanisms the map was quite large meaning that it was something like this. So, the whole thing was there for some of them it was narrower you know it is non-dimensional.

So, as long as this units of this y axis are the same and this also we take the same we can actually compare them this was something like this for some of them it was very wide, and if you actually noticed we actually analyze those complaint mechanism. We saw that those things where it is narrower seem to bend a lot more than stretching axially things that have wide spread map seem to have a lot axial stretching and comparatively compare to this one limited bending. So, that told us that there is something happening here. So, we did this analysis that we discussed.

Now, let us go and look at it more formally by looking at this Buckingham pi theorem, that again one would study in fluid mechanics where you have lots of non-dimensional numbers. So, what this theorem says is that let us read it if there is a physically meaningful relationship in n physical parameters, let us call it p 1 p 2 p 3 this pi has nothing do with this our pi by the way it just that people denote this parameters later with non-dimensional thing pi. So, there is a physically meaningful relationship among a set of parameters then there is an equivalent relationship, whatever relation you write instead of putting them in the physical parameter that you have, you can make them non-dimensional if there are n parameters here there are only n m parameters here where m is less than n and that is given by this that is what theorem says this m is n minus r where r is the rank of the dimensional matrix that is if you take that physical relationship, in this case we had physical relation should be transverse displacement w is f l cube by 3 E i and i equal to rectangle cross section is a b d cube by 12 you get something what it is involved are transverse displacement force, length, youngs modulus, breadth of the cross section, depth of the cross section.

Now, we write the dimension. So, there is mass dimension, length dimension and time dimension, w there is only one and no mass no time f this is m l t minus 2. So, 1 1 minus 2 1 0 1 0 E has Newton per meter square. So, we have newton has mass time acceleration.

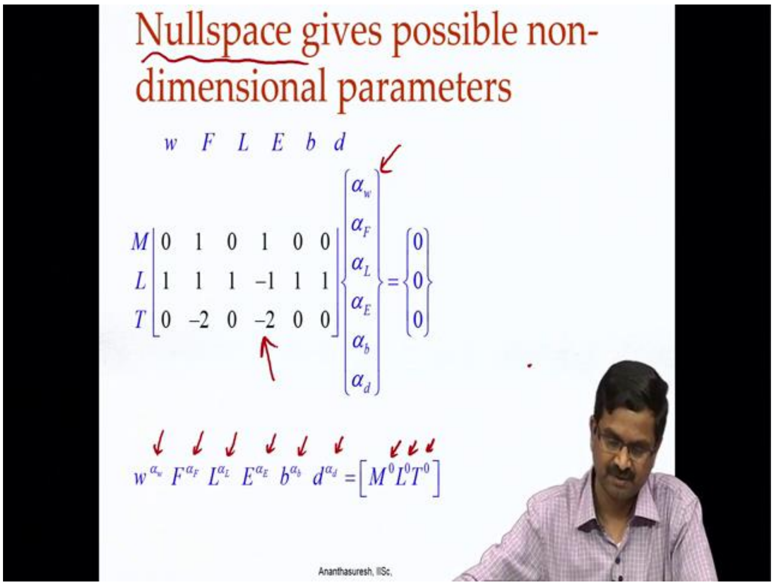
So, it will be mass 1 l minus 1 force is 1 divide by area square minus 1 minus 2 breadth 0 1 0 depth 0 1 0 these are dimensional matrix, clearly you can see that rank of the matrix is not 3, but only 2 because the first row and the third row are identical because minus 2 minus you multiple by minus 2 so rank of this is 2.

So, what this theorem tells us is that in these particular case how many physical parameters we have including our transverse displacement 1, 2, 3, 4, 5, 6. So, our n here is 6 and the rank of this matrix is 2 this is our rank. So, what this theorem tell us is that your relationship that is we have our relationship w equal to F L square by 3 E i where i again is b d q by 12. So, that relationship if you want express non-dimensionally you need 4 non dimension parameters is what this theorem says whatever 4 non dimension parameters there are 4 non dimension parameters. So, 1 is non-dimensionalizing w itself and then we introduce this slenderness ratio we have this eta and then we have also because we needed slenderness ratio why do with d we can also do with b that is you can change that independently or you can call it aspect ratio of the cross section.

So, these are the things that you need to express relationship non-dimensionally this is a subtle concept, but you have to pay attention to see what we are saying here that is a relationship it is not just for small displacements once again i emphasize even for large displacements if you have 4 non-dimension parameters we have w by L is there. And then s either with d or b or both and then eta if you have then you can draw this curve that is relationship that need not be linear relation it be non-linear relation also we can do this that is what is Buckingham pi theorem says, but how do we come up with.

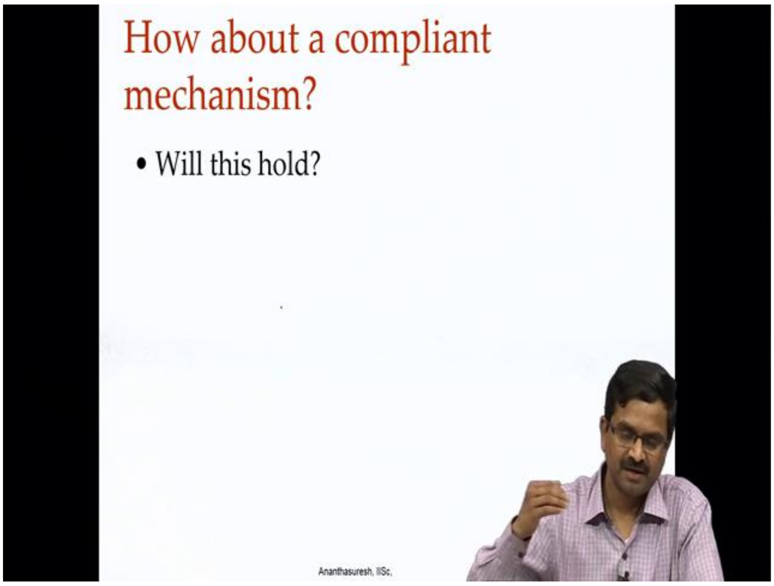
For example, if we started Buckingham pi theorem knowing that there are 4 non-dimensional parameters could we have arrived at this I would say it is not easy it is not straight forward we had to do that cantilever beam analytically and from there get this there is this eta, which is the parameter E when in the case of non-linear that how we arrived here knowing this that 4. There are 4 non-linear parameters is not enough to arrive at this and also we saw that s was there we did finite analysis s was also there in this stiffness matrix unless you fix as you do not get a single curve s changes the curve changes, rather the curves spreads in the to create this kineto electrostatic map as we have called it.

(Refer Slide Time: 25:57)



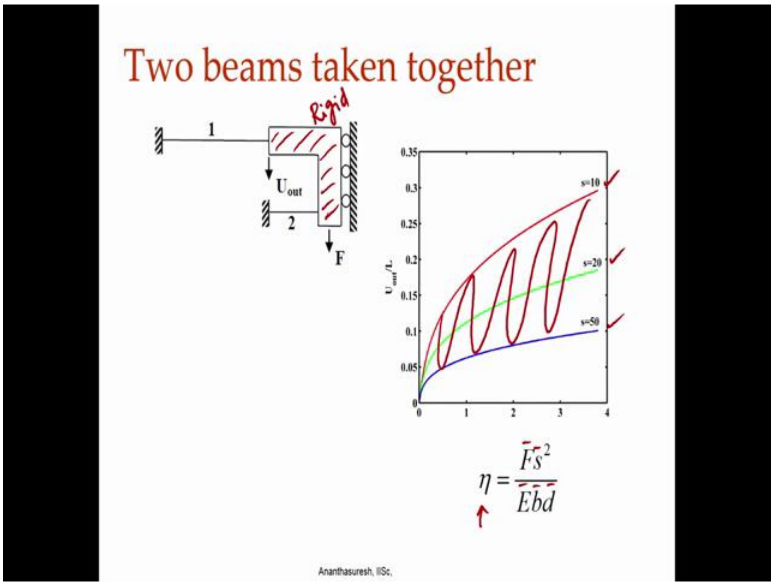
So, how do you find this you can always find them using the null space concept you have this if you say that I want to create a non dimension number. So, there will be 0’s for M L and T and I say there is alpha w alpha f alpha l alpha E alpha b and alpha d, I want to find those I put them here I want to find these things. So, that I get 0 for M 0 for L and 0 for T and clearly this is rank deficient we know. So, if we find the null space of that then we will be able to get a number of non dimension parameters any of them will be non dimension parameters the, but they may not give you what we are saying here what we are saying here is that if you know eta and s there is a unique response for non-dimensionalized displacement.

(Refer Slide Time: 26:53)



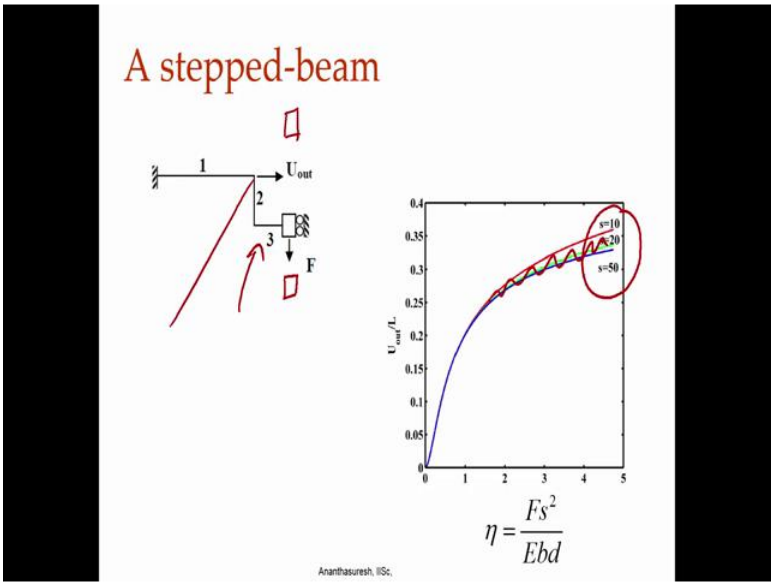
So, will this hold for entire complaint mechanism is the question we ask, we say that were 1 beam it works will it be true for entire complaint mechanism for that what will do is take it 1 by 1 and then see.

(Refer Slide Time: 27:09)



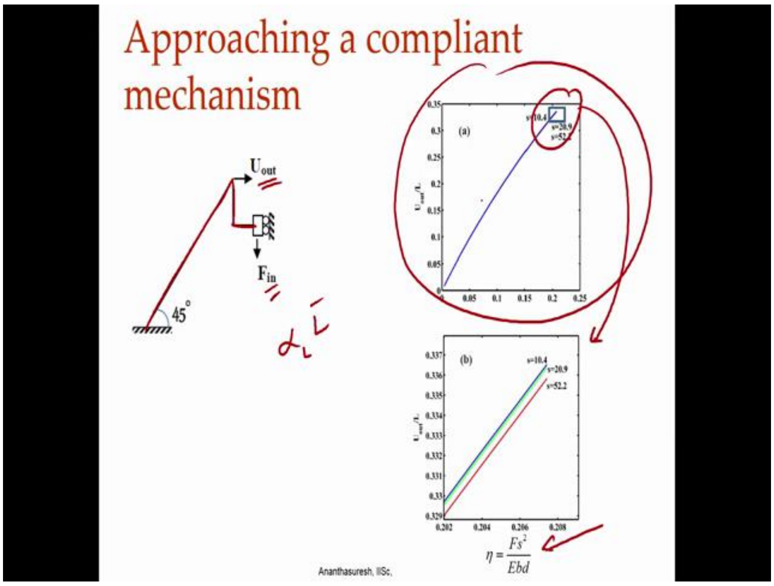
Let say first we take 2 beams together. So, I have a beam here and this is a rigid body let say we connected 2 beams with a rigid body that is allow to slide in the vertical direction now there are 2 beams. We can get this eta with average force average s E b d after while we remove this bar you see that s equal to 10 s equal to 20 50 there is a spread and you can see that as they move this rigid body up and down then these 2 beams have actually d form in order to move this rigid body block there is axial deformation that is characterized by spread out kineto electrostatic map.

(Refer Slide Time: 27:58)



Now, what we will do is will change this a little bit. So, we take a stepped beam. So, instead of have a rigid body we all remove that and make a step beam we have 3 beam segments again we do this now you see when s E changes from 10 20 and 50 the spread is very little. So, it is spreading very little and we can find out here that when this is being moved like this we have 3 beams they can bend and they do not have to rely on as much stretching to move this block from there to here or here where ever things can bend. So, it is bending dominated if it is bending dominated we have a very narrow map these are non-dimensional does not have to do anything with material property or the size or cross section. So, over all is what we are looking at now we will modify little bit to go closer to a compliant mechanism. So, whatever the 3 beams we had there now i am just making this beam go like that.

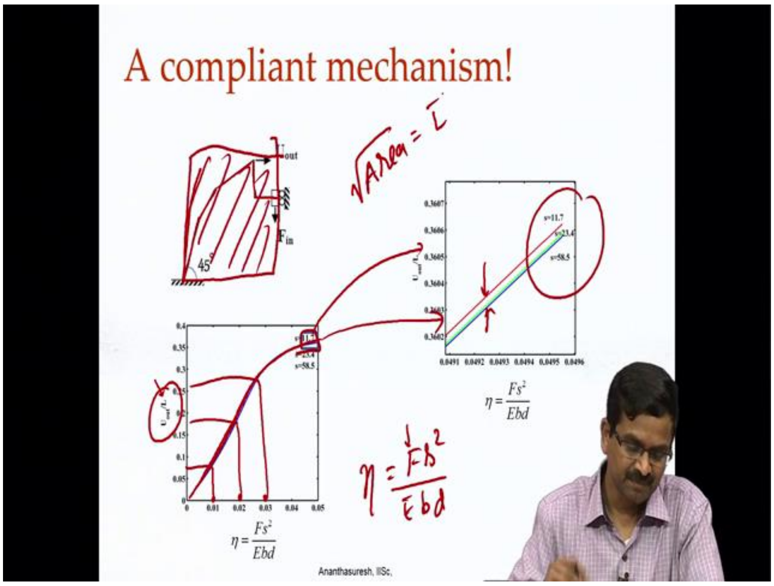
(Refer Slide Time: 29:00)



So, now you see the whole thing has become almost a single curve. So, you do not find anything in order to see something I have to zoom in here which is to zoomed inversion here for s equal to 10 20 and 50 2 there is 10.4, 20.9 whatever where as when you take this beams like this you can purely get s bar to be a nice number like 10.0, 10.4, 20.9, 52.2, these are all s bar. So, we do not write it after a while it just the average things there.

So, you see that there is very little difference it is a single curve and you can see such a complaint mechanism is bending dominated these very little axial stretch. So, that is what we see for this complaint mechanism once I have this curve, I do not need to the another finite analysis if my interest is only to apply force here and look the displacement there that is nice thing about this kineto elasto static map, but again I cannot change this angle I cannot change the proportions of these lengths and their cross sections and so forth. Remember our argument we do not change any proportions, but we can change the force we can change the overall size that is we can change that L bar we have you can multiplier whatever number we have alpha l alpha anything for that it will depend only and this eta and get there.

(Refer Slide Time: 30:28)



Now we modified a little bit more by putting 5 different segments and their shape changes once again it is a single curve we do not say anything, but zoom in if you do we have little box around that portion we say little bit again s is 11.7, 23.4, 58.5. So, only when you zoom in this is a difference. So, it is also bending dominated complaint mechanism and it is a single curve that is what we see that once you have the curve you do not have to do another finite element analysis the way this mechanism moves everything is captured here again if you see eta as F in it.

So, F s square by E b d is what we have and s value here does not matter, because this is bending dominated all of them are almost go coinciding as we see over here in this zoomed in portion, now once you write this eta as F increases you may have here double that becomes 0.02 and triple it is 0.03 and so forth. We can actually get what happens the displacement for how long it is linear we can find out it is non-linear, but you can see how long it is linear and what is the maximum it can have.

So, if it is going like this our eta is only is 0.05 here, I could have increase it to a larger value we can see where it goes for how long how much of you out can you get relative to the size the mechanism size here L which could be the average lengths of all the beam segments or if you want you can also do it like as is only representative it can be square root of area occupied by the in case, I take this whole are like a bounding box and we take this area take square root that can be L bar if you want it can be anything. So, once we have it we can see how much displacement you can get. So, we will discuss a lot more in a continuation of this lecture we will pass here.