Complaint Mechanisms: Principles and Design
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Lecture – 41
SML Model for Compliant Mechanisms for Dynamic Response

Hello, we are into the 5th lecture on the third design technique, which is based on spring lever model and feasibility map we discussed how to design, how to redesign, how to select all of that using this, but so far we have focused on static applications only.

Today will extend the spring lever models for dynamic applications in which case, we have to bring in mass also. So, it becomes spring mass lever that is SML model and that can also be constructed for a given complaint mechanism and the selection procedure and redesign procedure use in feasibility maps can also be done. Here instead of having only stiffness maps, which suffices for static applications we need to have stiffness maps as well as inertia maps for dynamic applications as we will see today.

So, let us look at spring mass lever model and how it can be use for selecting a complaint mechanism as well as redesigning it for dynamic specifications and that is the key word today and the additional key word is the mass.

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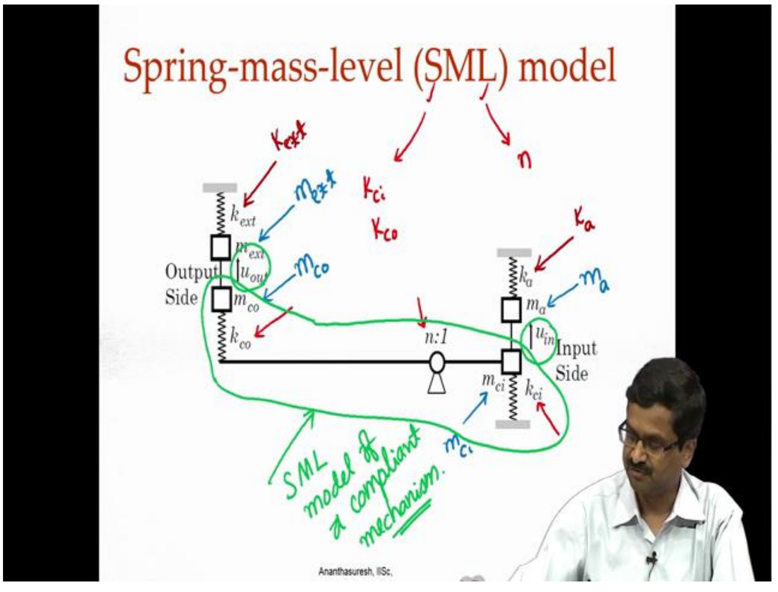

Now we have an SML models, spring mass lever model. This is how a spring mass lever model looks like. So, you have the lever as we had earlier. So, we have K ci, K ci and then we have the lever ratio n intrinsic one and output side stiffness K co. K ci, K co n for only it there in the spring lever model, it is we had spring and the lever. So, spring relates to K ci input side stiffness and K co which is output side stiffness and lever relates to intrinsic amplification between output and input which is n here.

Now, when you say mass we have shown that now let us use a different color, so we have m co and then we have m ci. So, these are the mu parameters for the sake of dynamics you will consider dynamics, we must consider mass and we have that here in terms of a m ci and m co, and likewise actuator earlier had ka. Now we also have m a that is the actuator mass because actuator also moves, so it has it is own inertia so m a will be there, likewise we have m external these are new one, because for inertia if have an object that object moves and that also we have inertia which is what we are showing. So now, earlier we had this ka, now we have m a also and we had k external, we have n external. So, these are the parameters pertain into spring lever model. Of course the actuator and the external object are not part of the complaint mechanism and hence they are not part of the spring lever model. So, spring lever model will just a circle is only this, whatever we have put in this green part is the spring lever model; that pertains to the complaint mechanism, SML model of a complaint mechanism.

So we have this to represent complaint mechanism abstractly just like s l model did for static response. Now we can also handle dynamic response, if we notice this is a 2 degree of freedom model meaning that we have one input which is the input side which you are indicate U i n and output which is here which is U out, U in U out 2 degrees of freedom.

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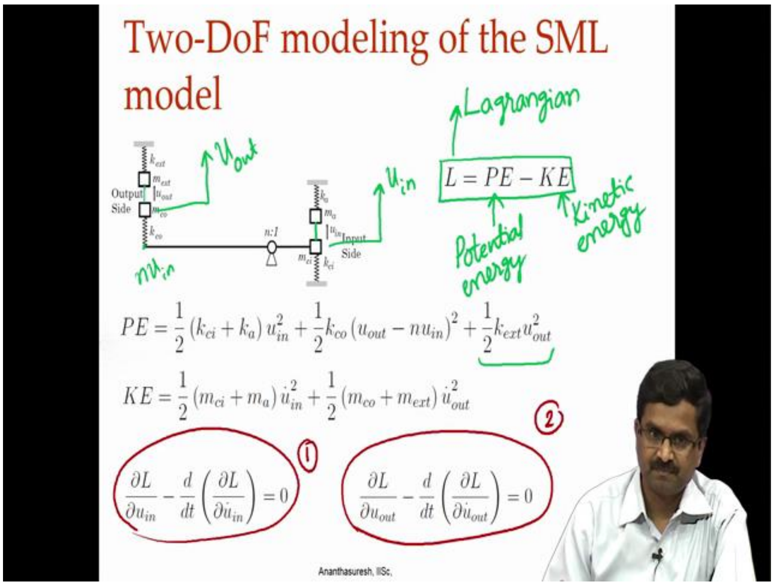

So, when we write equations of motions for this model is a dynamic system now, we have input side displacement U in and output side that is m co displacement which is U out and of course, external force will have it is own, but we assume that they are connected together with something, that can be a rigid rod or they can another spring and so forth. Depends on how it interacts, so but we have that also indicated here and similarly actuator, again there is connection between output set of mechanism and this, we are just partitioning the inertia into input and output just like we did for the stiffness earlier. Is a 2 degree of freedom model will write equation of motion we know the there will be 2 equations of motion, one governing U in, one governing U out, that does not mean that U in and U out are decoupled. In fact, they are not decoupled here will see that they will be a coupling term between the 2, in other word you cannot solve for U in alone or U out alone.

We have to solve for them together because their equation of motion are coupled with each other. So, in order to write equation of motion the first thing we do these we start with Lagrangian. So, this L here we call is Lagrangian as in the Lagrange in the Lagrangian dynamics and PE here is potential energy of a system, this is the potential energy and this is as you would have guessed is Kinetic energy, this is the kinetic energy.

So, we take this Lagrangian and write that potential energy here will have due to the springs K ci ka, they both will have U in square in their Kinetic energy term and K co will have U out minus nu in because m co how much were it moves, for that spring this moves this moves by nu in, this point will move by n into U in.

So these moves by U out, U out minus n U in square will be that. So, all this is stain energy in the mechanism and the external spring also has energy you put half k external U out square that is the potential energy here due to mainly strain energy, all of it is strain energy. And then we have kinetic energy which is m ci m a U in dots square and then likewise output m co an external U out dot square this means that, the inertia of the input side is combined with inertia the actuator, inertia and output has combined with inertia of the external object.

So, we write the kinetic energy. Once we write these two, you can write this Lagrangian expression and then write Euler Lagrange equation, with respect to U in as well as U out that is dou L by dou U in minus d by d t of dou L by dou U in dot that is expect U in and then U out we have this equation dou L by dou U out minus d by d t of dou L by dou U out dot, that is equal to 0 this is equation 1, this equation 2, and as we will see when we evaluate this we get coupled equations of motion which are shown here.

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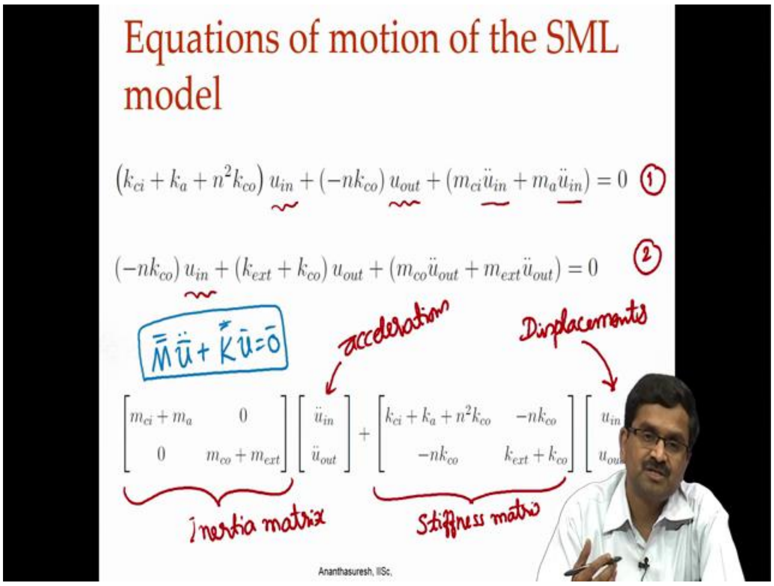

So, why do say this coupled the first equation has U in double dot, but it also has U out here U in here is, but U out term is there likewise U in term is there in this; that means, that these 2 equations what we wrote in the last slide as 1 and 2. These two equations are coupled equations, but there simple enough to solve, the reason we say there simple is that there linear in the sense that we can write they in the form of a matrix.

So, what we get here is same as what we had done for the static response earlier with spring lever model this we call it a stiffness matrix and what we have here where this no coupling is inertia matrix.

This is the inertia matrix, these are the accelerations of the input output points is U in double dot, U out double dot these are accelerations and these are displacements, that easily solvable is a 2 degree of freedom linear equations, ordinary differential equation you can easily solve. Right now there are no forces this should also be 0 0 it is not like 0 because is in here 2 degree of system these are the external force is 0 if there non 0 also you can solve, but idea is that we can constructed to if I have model by this spring mass lever model SML model.

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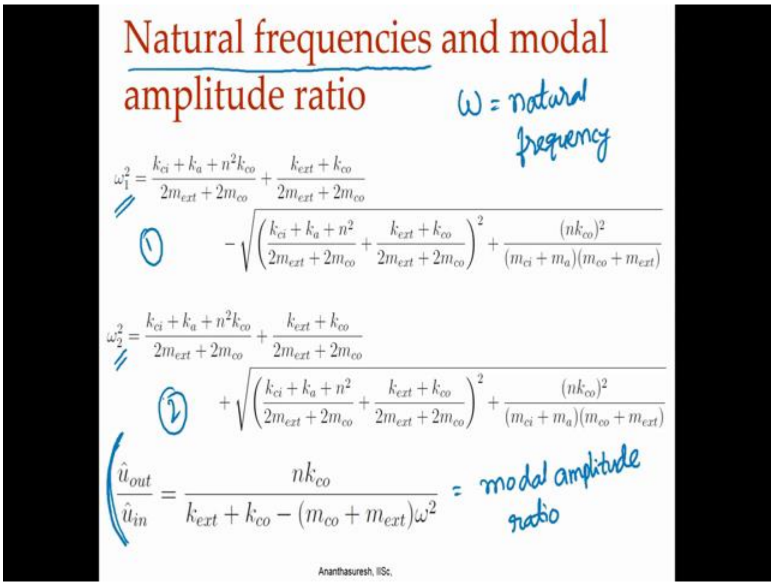

When you have this model we can actually compute symbolically the natural frequencies. So, we have these natural frequencies over here is a 2 degree of freedom model, so there is first natural frequency and second natural frequency, omega is our natural frequency by doing the Eigen analysis of the system, we showed in the last slide which is this m, m times U, if I say m is a matrix, U double dot, plus K U is equal to 0, these a matrix system we can do the Eigen analysis of this problem which I say Eigen problem in a way. If assume U is harmonic function, U double dot is harmonic function of U are rather U is the harmonic function double dot will be omega square U with the minus sin, that gives you Eigen value problem if we do that you can compute this frequencies omega 1 and omega 2 long expressions, but not a problem because they are simple enough.

We can also derive symbolically U out by U in the modal amplitude ratios, if these are modal amplitude ratio because correspondently each frequency they will be a mod shape and the mod shape they will be a magnitude for U in degree of freedom, U out degree of freedom. So, here we have written this modal amplitude ratio because this is important when somebody wants geometric amplification, we need geometric amplification between output and input then this expression that we have here is useful.

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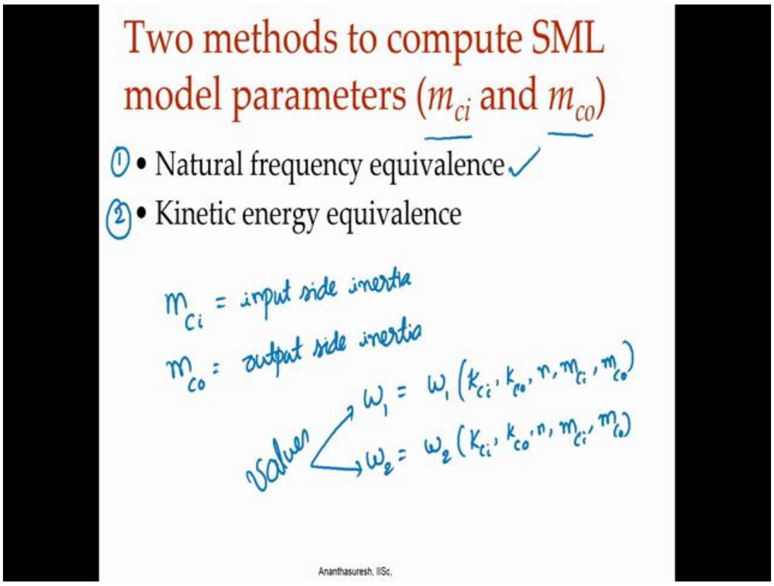

Now, let we have this next thing we need to understand is how do we compute this parameters? We all ready discussed how do compute K ci K co and n. Now we also need to find out how to compute this SML additional model parameters which are m ci and m co input side inertia output side inertia.

So, we have m ci as we have all ready indicated it is input side inertia, and then m co is output side inertia; how do you compute them? There are 2 methods for it, there is method one and there is method two, in method one we use is natural frequency equivalence colons meaning we have derived expressions in the last slide omega equal to something it is going depends on. If you go back and look at what is there will have K ci, K co, n, m ci and m co, k external those sins will be there similarly omega 2 will also be the same parameters it would depend on, the same parameter same 5 parameters. By using the numerical values of these, so here we get values from model analysis so we complete complaint mechanism and then solve this non-linear equations which we have over here, that is the first equation, second equation and determine m ci and m co, this is one way where frequencies are of importance for the problem that is under consideration these a method one.

There is second method which tries to equate the kinetic energy, the first one equated natural frequency you have 2 equations you can solve for that, kinetic energy equivalence is another method.

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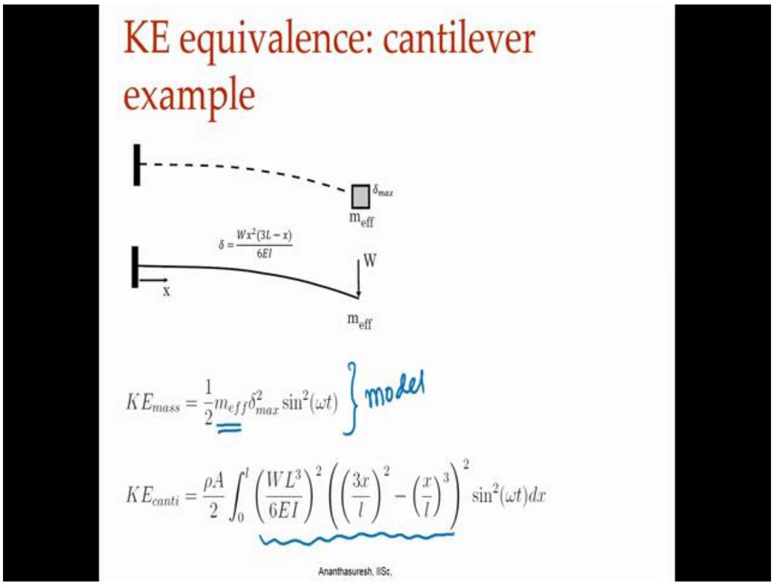

The weight works is you want to mixture that the kinetic energy of the SML model is equal to kinetic energy is entire complaint mechanism, this is how we also do for a structure (Refer Time: 15:29) the cantilever, others in vibration course you would have learned that if have cantilever, when it fixed at this end and this is deforming you imagine lumped model one degree of freedom which has displacement equal to the displacement of the free tip, when I fixing here and this is vibrating, you say that degree of freedom is your lump model. So, we discussed all ready how to gets the equivalents spring constant. Now we want to get the equivalent inertia, clearly this point is going to move, but is going to move the maximum compare to all other points.

So, taking the entire mass and lamping it here does not make sense because when in the beam bands, only part of the beam is deflecting a lot other part is not deflect. So it is mass for or inertia is not going to be the entire mass of the beam, but only a fraction. But what do we do, we try to make sure that the lumped mass that is put at the end of the cantilever beam, should have this same kinetic energy as the entire distributed kinetic energy of the beam.

So that is shown here this is the model, where we are indicating there is a m effective and then delta max square and then sin square omega t, for the kinetic energy if you have seen harmonic motion cantilever, you take the equation for static deformation of a cantilever beam with a load at the free end. Then you will get an expression kinetic energy of the cantilever distributed kinetic energy and the lumped model equate the two, and try to find m effective in a similar way here with our 2 degree of freedom model we get the entire kinetic energy of the complaint mechanism and equate to kinetic energy of the spring mass lever model.

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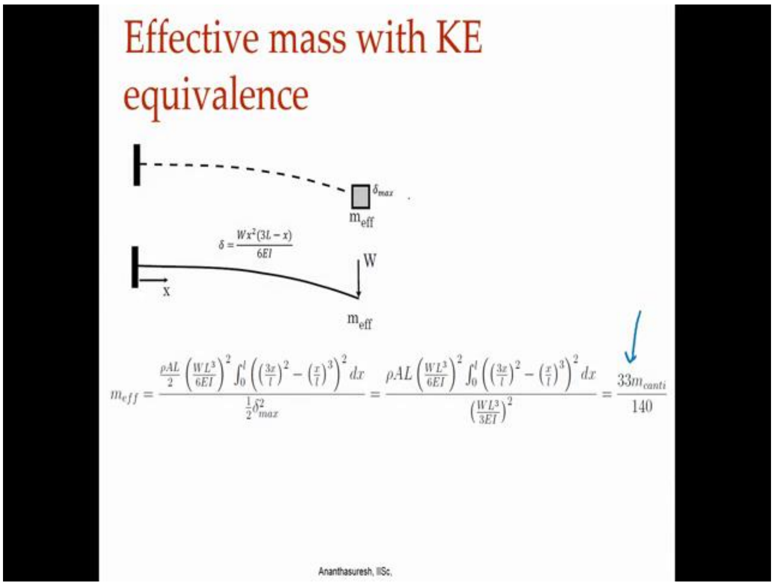

So, if you write the kinetic energy you will get an expression, which equate to the kinetic energy mechanism. So, based on that you will get something like this with effective mass of this are 33 by 140 into mass of the cantilever.

So, there are 2 methods and between the 2 whichever is more accurate one who choose, but actually depends on what you want. If natural frequency is a criterion under problem under what you are considering, then use in the first method where you get kinetic in natural frequency equivalence is more applicable. But on the other hand if you are trying to do the time response let us say you have evolve and how much time it takes for the evolve to close or a switch to open for such things you need to have kinetic energy equivalence because that is a better measure for that problem.

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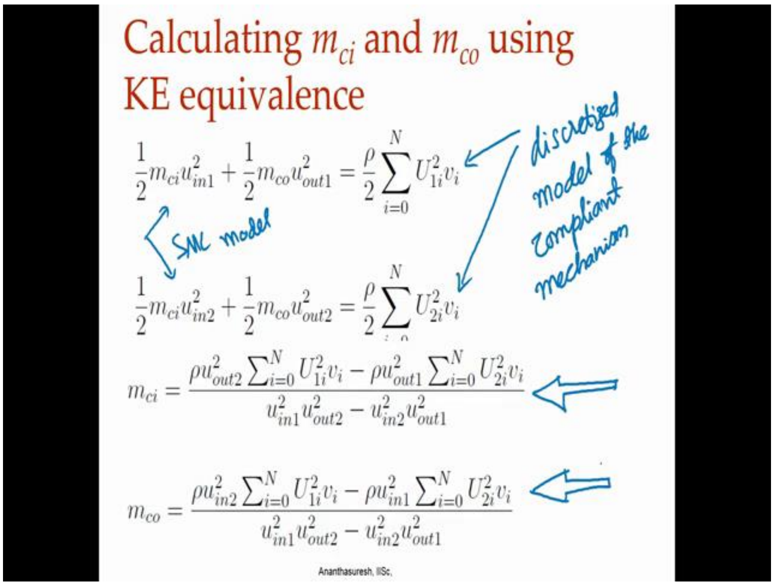

So, we have two methods, sometimes you can use method one sometimes you can use method two, in order to validate the methods you can do first the kinetic energy equivalence what I just said if you have discretized model, this is for the discretized model you write the kinetic energy for the discretized model of the complaint mechanism, this is the discretized compliant mechanism and this is SML model, we equate kinetic energy of these two and that. So, we get to calculate m ci and m co. So, that is the m ci expression and this is m co expression.

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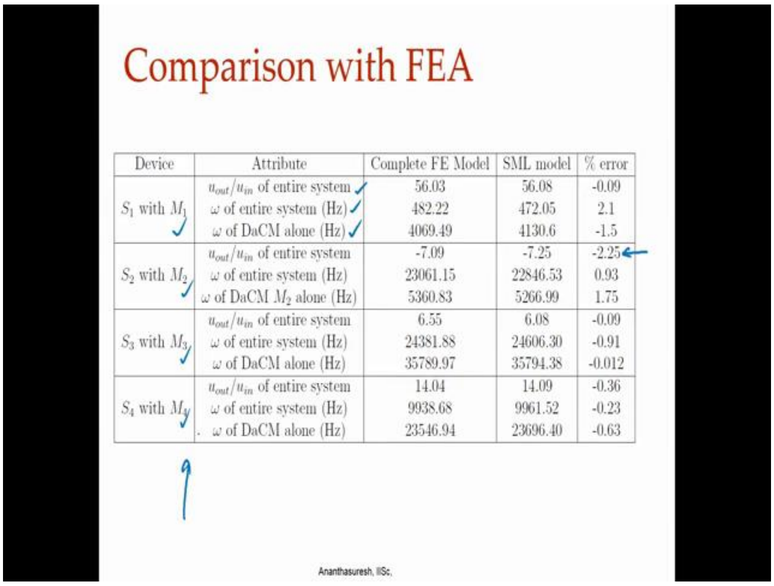

Device	Attribute	Complete FE Model	SML model	% error
S_1 with M_1	u_{out}/u_{in} of entire system	56.03	56.08	-0.09
	ω of entire system (Hz)	482.22	472.05	2.1
	ω of DaCM alone (Hz)	4069.49	4130.6	-1.5
S_2 with M_2	u_{out}/u_{in} of entire system	-7.09	-7.25	-2.25
	ω of entire system (Hz)	23061.15	22846.53	0.93
	ω of DaCM M_2 alone (Hz)	5360.83	5266.99	1.75
S_3 with M_3	u_{out}/u_{in} of entire system	6.55	6.08	-0.09
	ω of entire system (Hz)	24381.88	24606.30	-0.91
	ω of DaCM alone (Hz)	35789.97	35794.38	-0.012
S_4 with M_4	u_{out}/u_{in} of entire system	14.04	14.09	-0.36
	ω of entire system (Hz)	9938.68	9961.52	-0.23
	ω of DaCM alone (Hz)	23546.94	23696.40	-0.63

Here is an example there lot more examples in the PhD thesis of Sudharshan hedge and couple of paper that are written on a. So, here by taking four mechanism M1, m 2, m 3, m 4 where comparing the SML model with a complete finite element model, in terms of this U out by U in omega of the entire system omega of the DaCM alone we can see, what kind of a comparison exist. And in fact, comparison is pretty good that is the spring mass lever model captures complete finite element analysis quite accurately in modeling here and the error is as you can see maximum is 2.25 everywhere else is pretty close to 0 and the m 1, m 2, m 3, m 4 that are noted here are basically the data base of complaint mechanism that we have, a number of them are given here four of them for use to draw that. These are called DaCMs because they are displacement amplifying complaint mechanisms long name and say acronym name DaCM.

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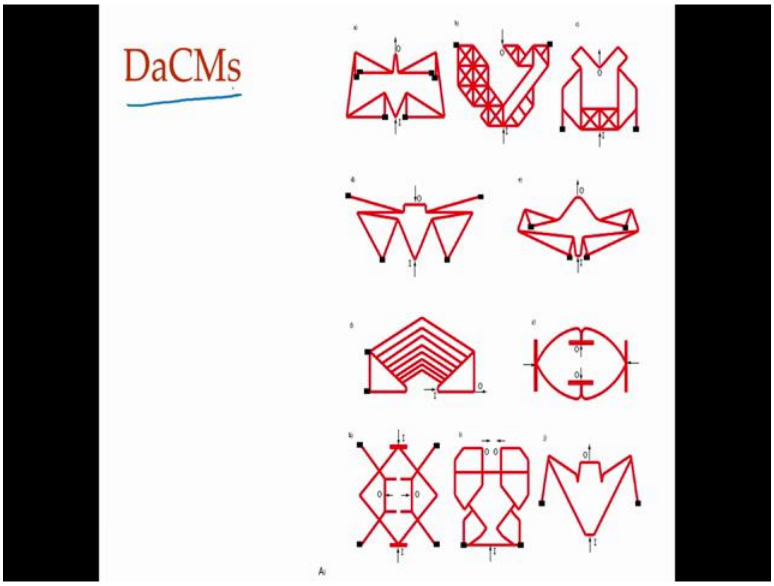

So, here for these we have all the parameters K ci, K co and these are static and then we have 2 dynamic parameters m ci and m co. The other 5 parameters for the spring mass lever model or SML model we know how to compute that.

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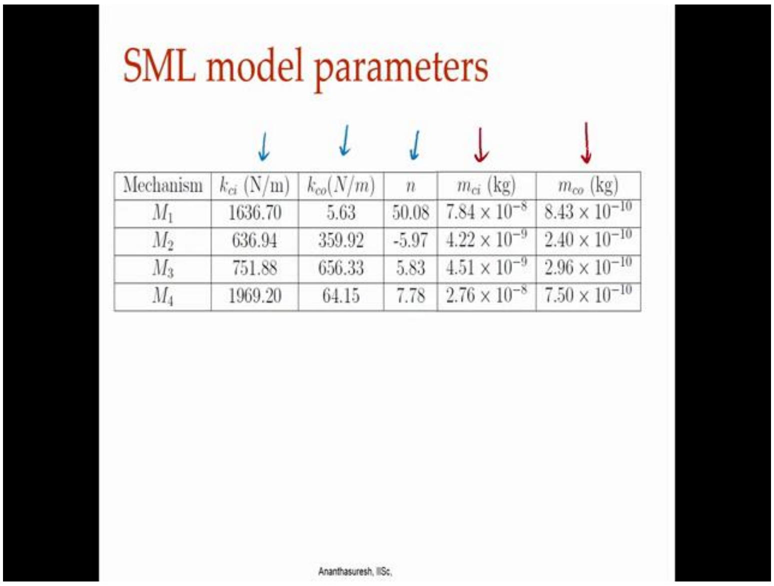

Mechanism	k_{ci} (N/m)	k_{co} (N/m)	n	m_{ci} (kg)	m_{co} (kg)
M_1	1636.70	5.63	50.08	7.84×10^{-8}	8.43×10^{-10}
M_2	636.94	359.92	-5.97	4.22×10^{-9}	2.40×10^{-10}
M_3	751.88	656.33	5.83	4.51×10^{-9}	2.96×10^{-10}
M_4	1969.20	64.15	7.78	2.76×10^{-8}	7.50×10^{-10}

Now, we go back and then see what we can do it in terms of validating this model.

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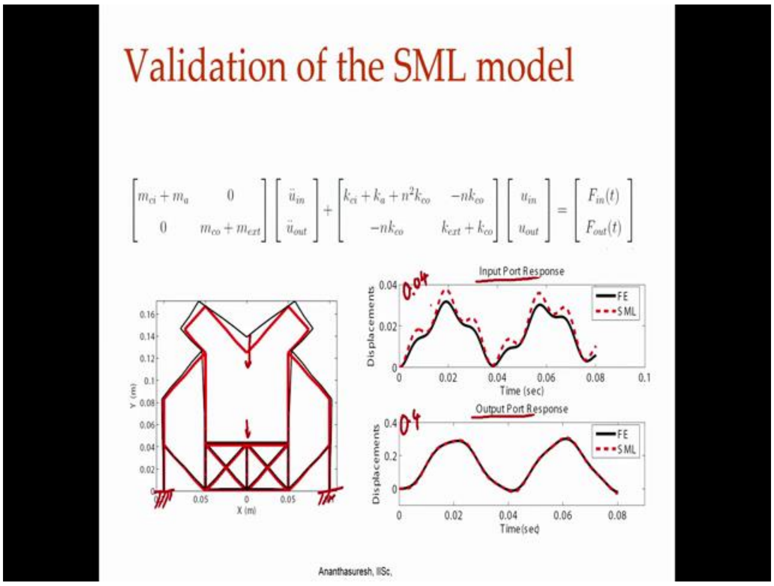

Here is any model in the approximate, here we have a 2 degree of model entire comparison such as this let say this is fixed over here and fixed over here, when we apply force over here like this, this should also come in the same direction in this particular one. We can take this and then give a step input at the input or outputs somewhere and then observes the other things and it gave pretty good match in the case of input displacement output displacement, output appears more accurate input because here the number is 0.4 whereas, here the number is 0.04.

So, when it quantity small comparison another quantity that has error, will be quite visible when the quantity is large ten time larger than 0.04, then they appear to be on the same curve that is the full finite element analysis and SML analysis.

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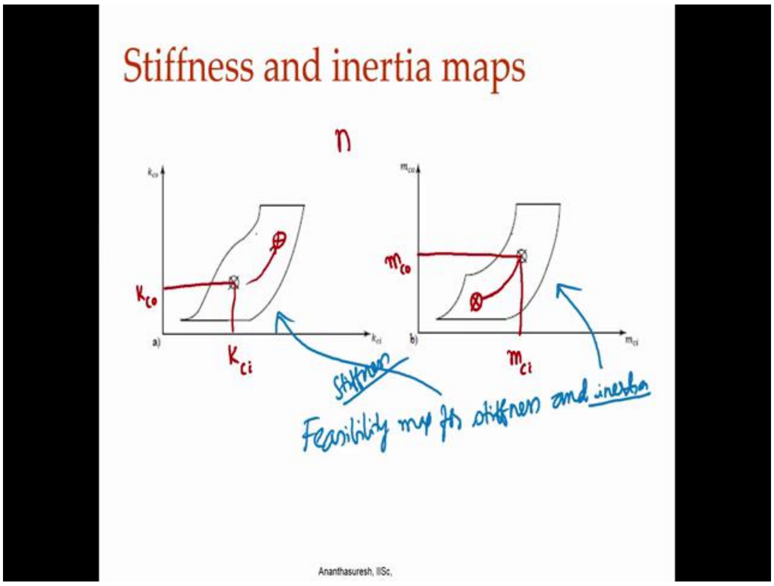

Having done that, next thing we need to do is in addition to having stiffness maps; we need to now have inertia maps. Stiffness maps showed K co verses K ci, inertia maps will show m co verses m ci. So, there will be 2 maps and then we can have n value as a gray scale in both and there is a curser indicated, there is curser for a particular point if I click there that will also m ci, m co which were not shown, but they are shown here there shown. So, this will be m ci value, this is m co value, this is K ci value, this is K co value.

So, as you move the cursor let us a from there will move somewhere else, you would see that this cursor also will move in some fashion, in some fashion I am saying not suggesting in particular way it actually depends that now, the feasibility volume is not a tridimensional space of five dimensional space. Now we are projecting on to 2 dimensional space, K ci K co space, m ci and m co space, that is important remember that now we write a feasibility map you will have feasibility maps for stiffness and inertia.

So, our feasibility map here this is for stiffness and we also have inertia and that is for the inertia, so this for stiffness, this for inertia. So we can take these and get these 5 parameters of the mechanism then want to selection procedure.

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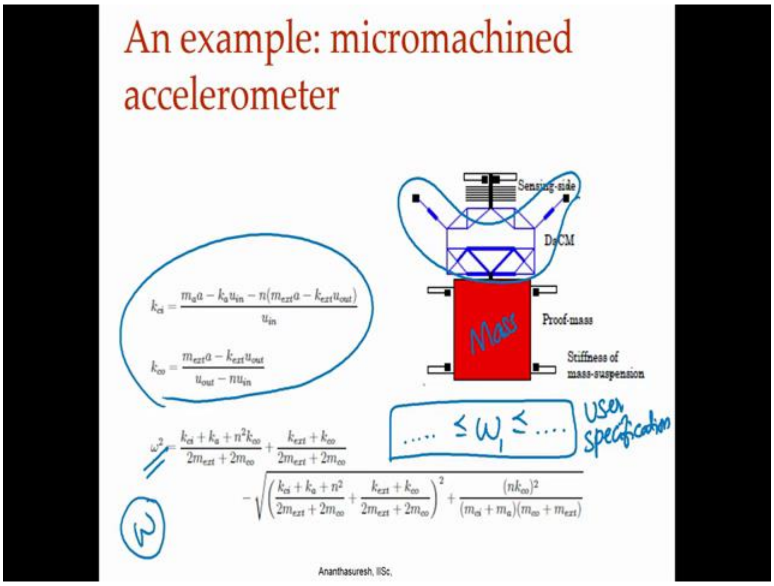

So, let us say we take this example of a micro machined accelerometer where we would like to pick this mechanism, this complaint mechanism there is all ready proof mass and suspension given, now for this we do not know the design yet we do not know it is stiffness characteristics inertia characteristics. We develop this model by using one or the 2 methods, first K ci K co and have has to be done, the way it was done for static applications spring lever model.

Now spring mass lever model you are, you can use one or 2 methods that we discussed today and get all the parameters and go ahead with stimulation of this and compare this with finite element analysis to see that there equivalent, but then we do not have a design; first we need to see based on users specifications, if natural frequency omega is of interest to the user, user will specify that natural frequency let say fundamental one should be more than certain value less than certain value, this also becomes a user use specification in addition to input force, input displacement, output force, output displacement and so forth. Whatever we have, are actuator stiffness, external spring and external inertia, actuate inertia, we can also have omega to be certain limit then you can use is equation in conjunction with the 2 static (Refer Time: 25:49) equations. Now will also have equations of motion, we need to solve for the parameter space, such as the one shown here.

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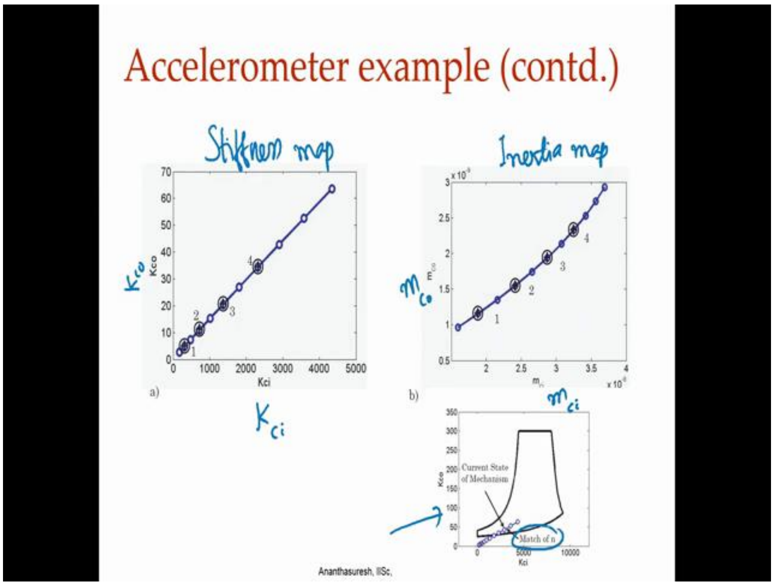

Now, we have K ci here this is stiffness map and K co here this is the stiffness map whereas this one is inertia map because which shows m co verses m ci and these are the parameter curves, what is a curves here In fact, feasible volume is quite small around here, there are parameter curve we can see how to choose a mechanism where the point has lie to inside the feasibility map been stiffness map as well as inertia map and the cursor are will curve, when you move somewhere here it will assortomatically move and once you do that, you can focus on the stiffness map and try to get it inside and match the value of n also because that is important; because n is not feasible here we only match K ci K co m ci m co, but we also need to match the n value which can do just like we did in the static application case.

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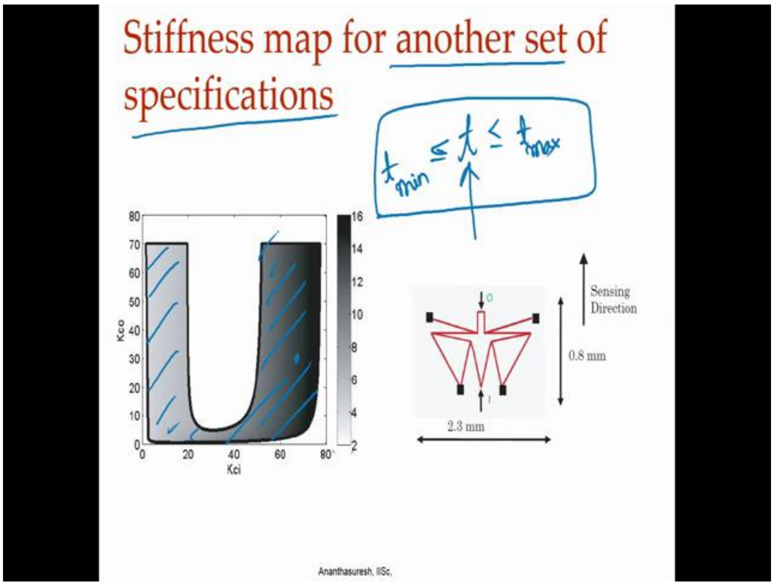

So, typically the stiffness map when you consider frequency or another set of specification here like time taken to move to a certain distance like a you have valve, how much time does it take to open the valve. That can also be input, in which case we can say time to open some time they can be a t min because that say user specification are t max same thing for a switch, they can see my switch should open let say in 20 microsecond, then you say anywhere between a d into 22 if your microns that becomes a constraint for constructing in the feasibility map.

So, one can derive in expression for t in terms of the SML model parameter is that of five of them and a external spring, external inertia, actuator stiffness, actuator inertia all that you can put and try to construct this feasibility map. And a point here satisfy the everything we can occurred with that we had done, if it is not need to the redesign using this parameter curves like we did for this static applications, in that what is being shown here that stiffness map that is all of these area.

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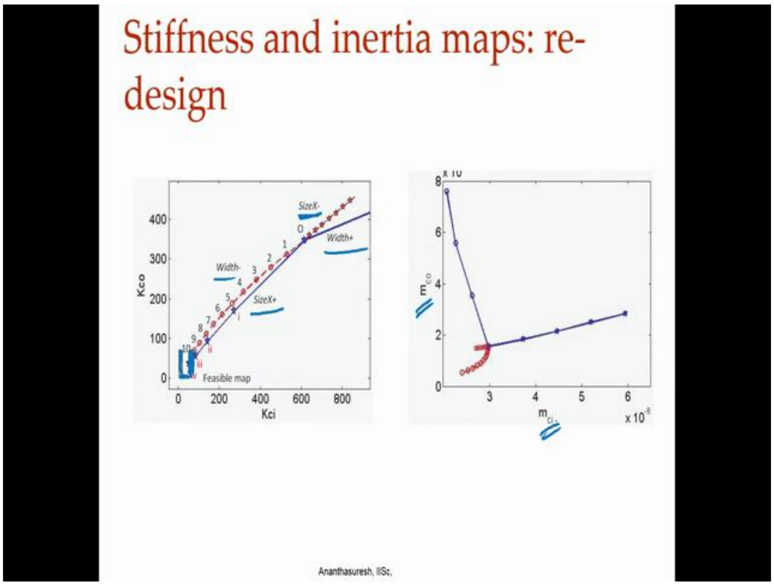

Now, is being shown very little in the next one that is here, this a map and how we can move from where we are by increasing width, decreasing width, increasing x size are, this is decreasing x size, this is decreasing x size, this is increasing x size we can actually constructed. Finally, get it in to the feasibility map both in K ci K co stiffness map as well as m ci and m co inertia maps.

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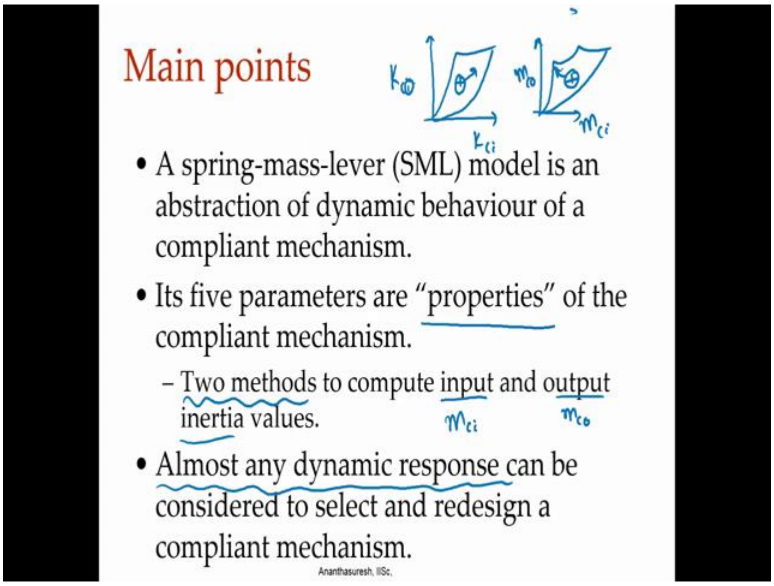

To summarize what we can do in terms of reselecting redesigning a mechanism for dynamic application, where to first decide on what perhaps specification user wants to put.

These are usual input force, input displacement, output force, output displacement and then we have actuator stiffness and actuator and inertia also here now and then external stiffness and external inertia we have those where SML model. We will have an input side inertia, output side inertia m ci and m co and then n remain as it is specification will be natural frequencies time to close are some maximum stroke during the dynamic application, any of those any dynamic response you can put and then try to take then to account in the 2 degree of SML model.

So, that you can be use to draw the feasible t map, for that dynamic specifications again this 5 parameter that we have can be thought of as properties of complaint mechanism, K ci K co n and m ci and m co. We discussed two methods to compute input and output inertia values, either other methods is fine if natural frequency is a performance specification in design then we have to use a first method where we use natural frequency equivalence, otherwise we can use kinetic energy equivalence, the second method to compute input and output inertia values, that is m ci and m co parameters and almost any dynamic response can be incorporated in selecting as well as redesigning a complaint mechanism. The key here is to have two maps, we have the stiffness map K ci K co verses K ci will have (Refer Time: 31:07) likewise m co versus m ci. So, this will have a stiffness map will be something here and they will also be inertia map. So, we put a cursor here, cursor here they both correspond each other, when I move this, this will also move to corresponding point in that map. So, the selection it is an (Refer Time: 31:30) procedure works for dynamic application as well.

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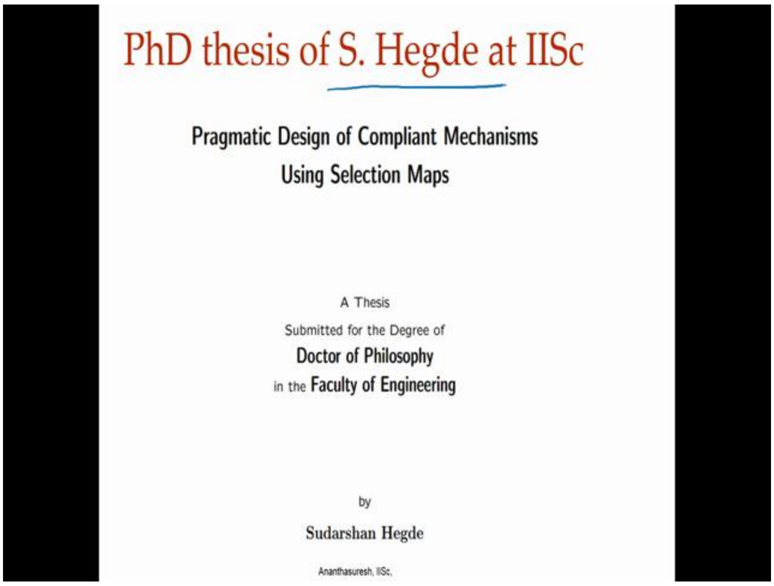

The material that we have covered is taken from the PhD thesis of Sudharshan Hedge, which is available to you as part of for the supplementary material for this lecture and also 2 papers that have been coating for this part of the course, for the spring lever model as well as spring mass lever model.

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In the next lecture we will look at an implementation in MATHLAB as well as a java based script, where we can do this feasibility map being drawn as per your specifications and how to select the mechanism redesign and so forth.

Thank you.