**Compliant Mechanisms: Principles and Design**
**Prof. G. K. Ananthasuresh**
**Department of Mechanical Engineering**
**Indian Institute of Science, Bangalore**

**Lecture – 04**
**Mobility and degrees of freedom in compliant mechanisms**

Hello. In the last 3 lectures we discussed Compliant Mechanisms, all the prospective in overview and then the spirit of complaint mechanisms as well as some applications, now let us get to the details, one of the first thing that one would want to learn about whenever we consider study mechanisms is mobility analysis. In fact, a mechanism design is all about how we can constrain various bodies in the case of rigid link mechanisms all the rigid bodies connected with joints so that they move in a manner that we want them to move in compliant mechanisms it is not different that is why we begin first with mobility analysis.

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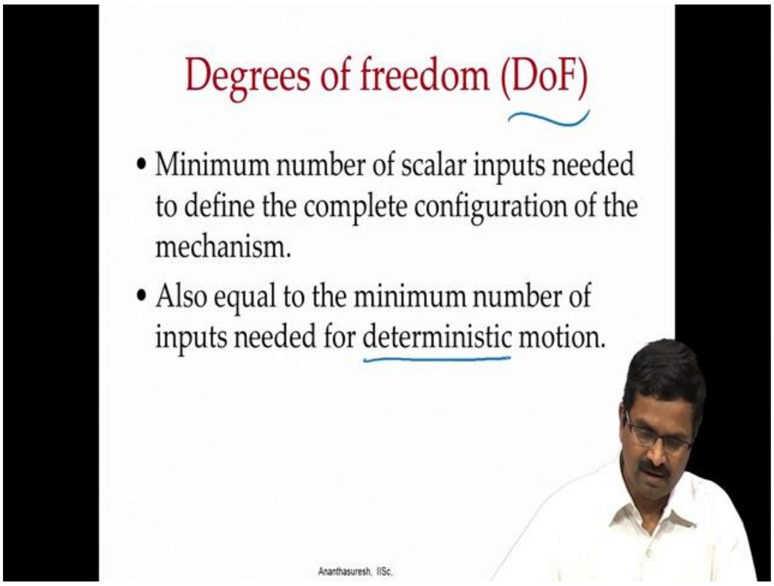


So, if we look at the mobility analysis the first concept that we need to concern ourselves with is the number of degrees of freedom or as we call DOF, we want to determine the Degrees of Freedom of a mechanism, and what is DO? DOF is the minimum number of scalar inputs needed to define the complete configuration of the mechanism is a minimum number of scalar inputs required that is if you have mechanism, if you give so many inputs and these inputs can be thought of as actuations.

So, degrees of freedom also gives you the minimum number of actuations or inputs needed for deterministic motion, deterministic word here is important because that is exactly what we want mechanisms to have there is no random motion, if you give as many deg actuations as the number of degrees of freedom then you know the position velocity acceleration of every member in the mechanism, this is what we learn for rigid body mechanisms or rigid body linkages.

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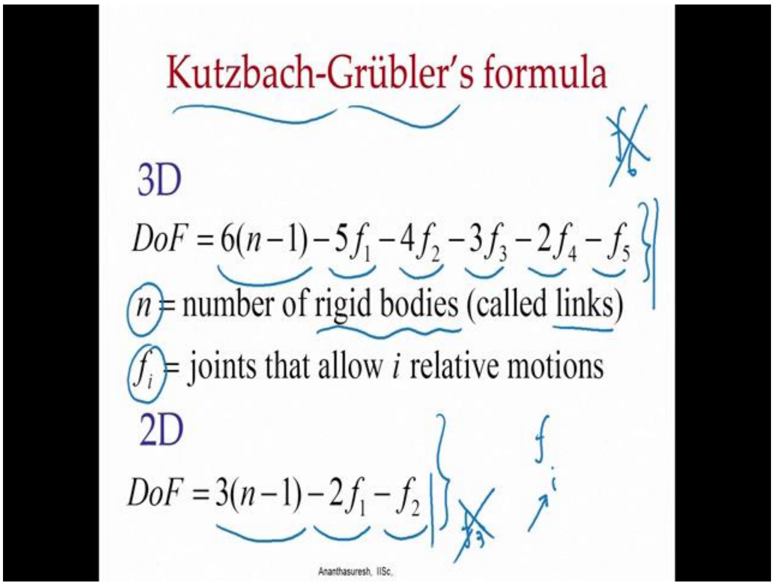


The formula that becomes useful to analyze mobility or to compute the degrees of freedom is the familiar one that all of you would have learnt in your kinematic class that is Kutzbach and Grublers formula, Kutzbach is usually given for not 2 degrees of freedom for planar mechanisms things that are no need 2 dimensions x y plain, and Grublers is given normally for spatial or 3D and those formulas are shown for the general case for the 3D as well as for 2D here, n is the number of rigid bodies and its miss number that we always use the word link normally means in plain language something that connects 2 things, but here we say it connects 2 joints. So, it is a kind of a miss number, but people normally use links when we should actually mean rigid bodies.

There is n is that and then f I, which is here is the number of joints that allow i relative motions between the 2 bodies they connect. So, every joint connects 2 bodies, 2 rigid bodies, and if I say f i then i indicates the number of relative freedoms that this joint allows between the 2 bodies it connects. So, if you have a revolute joint that is a hinge a hinge allows 1 relative rotation that is a relative motion it allows. So, that will be called f 1 its number of such joints is what f 1 will indicate similarly, f 5 and in 2D we have the formula that 3 times n minus 1 minus 2 f 1 and then minus f 2.

In the case of 3D we have 6 into n minus 1 minus 5 f 1 and 4 f 2 all these are subtracted, here there is no f 3 that is here f 3 is absent, because if you say f 3 it allows 3 degrees of relative motions in 2D everybody has only 3 degrees of freedom in a planar mechanism. So, if you say f 3; that means, 2 bodies are not connected similarly f 6 is not present here, because again it does not make sense to talk about a joint that allows all 6 relative freedoms that exists between 2 bodies. So, its stops at f 5 over here stop at f 2 in the case of planar mechanism.

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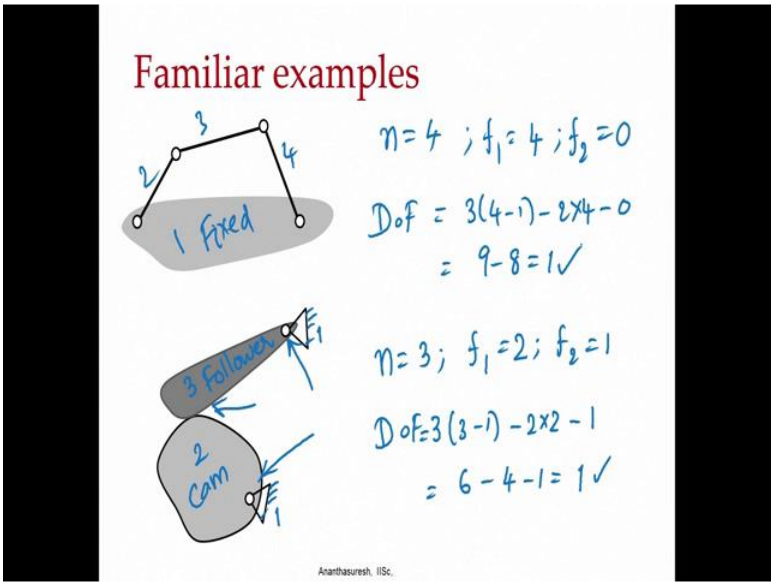


And these are some of the familiar examples for you 4 bar linkage where their n equal to 4 this is the 1 that is fixed, we call that 1, 2, 3, 4 and there are f 1 joints 1 degree (Refer Time: 05:10) joints 4 revolute joints, and there is f 2 there are no 2 degree freedom joints that is 0. So, now, if you write degrees of freedom for this it is 3 into 4 minus 1 minus 2 into 4 minus 0 that is 9 minus 8 equal to 1 is a 1 degree of freedom mechanism.

Similarly, we have a cam follower mechanism here, that also if you count n equal to 3 here the fixed one this 1 this is fixed and cam is 2 followers 3 n equal to 3 f 1 we have a pin joint there, a pin joint there that is 2 we also have an f 2 joint here, which is this one that is a higher pair with a line contact surface contact that is 1 now if you do 2 degrees of freedom for this 3 into 3 minus 1 minus 2 into 2 minus f 2 that is 1 this will be 6 here minus 4 minus 1 one in both cases we got 1 degree of freedom these are things that are very familiar to you.

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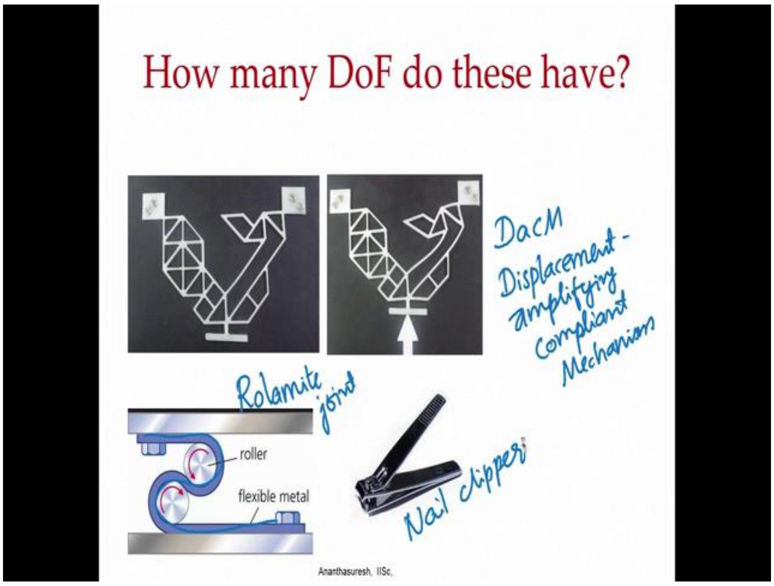


Now, let us ask the question if you have compliant mechanism such as the one shown here this is a DACM, since we are going to use it a lot let me define that it is Displacement Amplifying Compliant Mechanism; DACM. If we have such a thing it may look complex, but we want to talk about degrees of freedom of such a mechanism or what is known as Rolamite joint, this has flexible metal strip that goes around as shown here or our nail clipper how do you talk about degrees of freedom for mechanisms that have flexible or elastic segments in them, that is what we would like to address before we do that let us just understand group less formula rather we derive.

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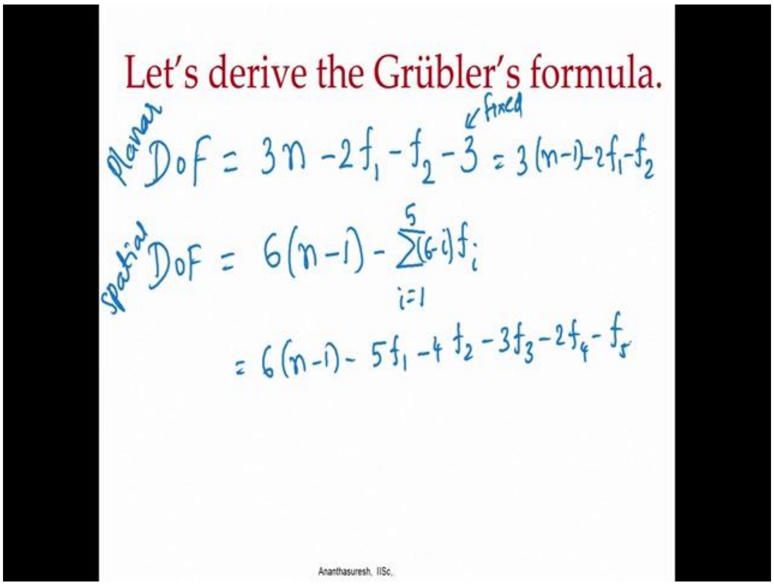


So, if I talk about degrees of freedom let us we take 2 dimensional case if there are n bodies I will have 3 n degrees of freedom, because everybody in a plain will have 3 degrees of freedom 2 translations and 1 rotation, now if there are f 1 joints that is joints that allow only 1 relative motion; that means, that they are removing two. So, I have to multiply by 2, 2 times f 1 number of 1 degree freedom joints then it will be minus 2 f 1, similarly minus f 2 because an f 2 joint allows 2 degree of freedom and hence it removes 1 degree of freedom you have to subtract all of them minus f 2 that is how we get.

Similarly, if I go to 3 dimension or spatial mechanisms we will have each of them will have 6 degrees of freedom and we have to of course, when there is a fixed 1 we have to change that we have to remove 3 here whenever there is a fixed 1 here, and that is how we get 3 into n minus 1 minus 2 f 1 minus f 2 and here in spatial case this is a 3D or spatial whereas, this is planar if there are n bodies 1 we subtract, because that is fixed and 6 is what each body has we have that and then we have I can just write it as i into f i where i goes from 1 to 5 not 6, because 6 degree of freedom joint does not make sense no joint at all.

So, whatever we have here minus 2 f 1 minus 2 f 2 depending on this should be 6 minus i. So, when i equal to 1 that is f 1 it is going to remove 5 degrees of freedom. So, that is how we got the formula 6 into n minus 1 minus 5 f 1 6 minus 1 6 minus 2, if I have f 2 that will become 4 and then 3 f 3 minus 2 f 4 minus f 5 that is a interpretational group less formula right.

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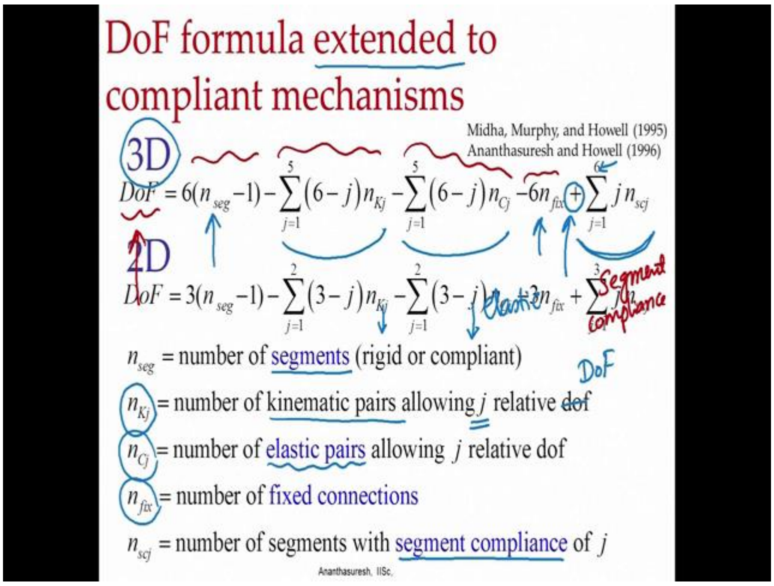


Now, let us look at that extended to complaint mechanisms. So, we understand this part now instead of calling f 1 f 2 f 3, we are using n k j that is the number of kinematic pairs or kinematic joints that allow j relative degrees of freedom, this degrees of freedom DOF, but the difference is that here we do not just have n, but we have n subscript seg meaning we have segments we do not count bodies any more we count segments and we say that the segments can be rigid that is a traditional rigid link mechanism or compliant or elastic segments.

So, this will be an elastic or compliant segment we count all of them and that is our n seg. So, that part is familiar, but then we also have this one where we are subtracting not only the kinematic pairs whatever degrees on that they remove, but also we consider the elastic pairs our n c j, n c j indicates the number of elastic pairs that allow j relative degrees of freedom, it just like a kinematic pair now we have elastic pair so that part is also clear.

But, we have this new thing called n fix. What is that, n fix is the number of fixed connections you may wonder why need to include, but then there will be cases in compliant mechanisms where an elastic segments is attached like a cantilever connection to a rigid 1 or 2 elastic segments can be connected to each other using this fixed connection. The fixed connection is one does not allow any degrees of freedom between the 2 bodies, any relative degrees 2 bodies or 2 segments in this case. And each such thing will remove 6 degrees of freedom. We are again looking at a general 3D case, 6 degrees of freedom it will remove. So, if they are n fix that many fixed connections we have to remove so many degrees of freedom.

But the last one needs even more attention in this modification, which is addition you see that this is not minus addition. The addition comes because elastic segments have additional freedoms that they provide, without that freedom compliant mechanism cannot even move or deform unless the elasticity form we will not get any motion. So, we have to pay attention to this particular one, where it says n s c j is called the segment compliance of j, segment compliance can be 1, 2, 3, it is going up to 6 here in spatial case in the planar case it will be up to 3 and it is saying that if there are let us say n s c 1 that is 1 seg single segment compliance elastic segments are there that will basically add j meaning that 1, n s a 1 we take j equal to 1 so it will add 1 degree of freedom that is why we have this plus time ok.

Let us look at the formula one more time, this part is the first one and second term this is familiar only thing is now we count the segments that include both rigid and elastic, and then part is what was there in Grublers formula and this one we add, because in a compliant mechanism there could be elastic pairs which act just like a kinematic pairs, but have elastic degrees of freedom n fix because occasionally we would need to have fixed connections among various segments in the mechanism and this segment compliance is something that we will discuss a little bit more. So, that we understand what the degrees of freedom means.

One thing to note first is that the degrees of freedom given for the compliant mechanism in this manner actually gives the maximum degrees of level that are possible degrees on definition was minimum number of actuations needed, but now we are talking about degrees of freedom that is a maximum number of degrees that can give to provide deterministic motion, it can be anywhere from 1 to the number given by this DOF. So, 2D case also one can write.

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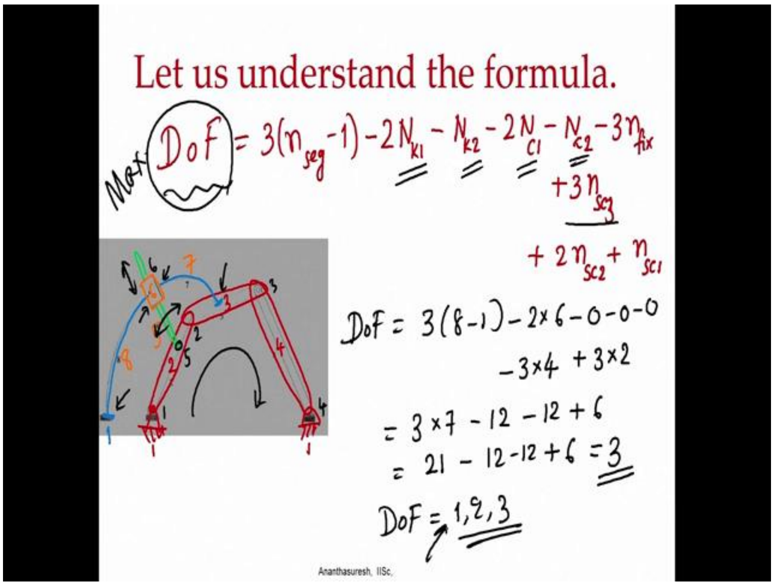


Let us write that 2D case here, DOF is equal to 3 into n seg minus 1 and then minus if I follow that notation of 2 N k 1 minus 1 N k 2 those are the once you can have, and then I can have 2 N c 1 minus 1 let me erase this the minus N c 2 these for elastic pairs, and then we have 3 n fix and then we have plus the segment compliance that is 3 n s c 1 sorry, s c 3 plus 2 n writing here 1 second. So, I am just writing the formula plus if there is a segment that has segment compliance 2 and then segment compliance 1 that is how it is, if you look at the mechanism that is there let me re draw it. So, here there is a rigid body here, there is a rigid body there. So, we have 1 and then let us call this 2, this is 3, this is 4 that is like a 4 bar linkage and that is the rigid change.

Let us use a different color to show the compliant part there is an elastic segment, these were the fixed connection cam. So, this elastic segment is kind of welded to this rigid 1 and similarly welded to the fixed frame 1, and then we also have a connecting thing here, which is this and there is also a slider let me use a different color for it there is a slider. So, if i call this 1, 2, 3, 4, 5, this is 6 and this is 7 this is 8. So, if we look at how many degrees of freedom this has how many segments are there we counted 8. So, the DOF for it is 3 times 8 minus 1, we have we are counting both elastic segments the blue once 7 and 8, and the rigid once 2, 3, 4, 6, and 5 and we had a fixed frame that is 1 so, but we count everything 3 into 8 minus 1 how many 1 degree of freedom kinematic joints are there 1 between 1 and 3 4 bar basically has 1, 2, 3, 4, and then we have prismatic joint here and there is also another joint there.

So, we have basically if I use in black 1, 2, 3, 4, and there is 5, and there is 6 over there. So, I have 2 into 6 there are no n k 2 joints that is 0, N c 1 N c 2 N k 2 they are not there in this one so they are all 0. So, n c 2 n c 1 then n c 2 minus 3 times n fix fixed connections are there is one over here, that is one over here, there is one over there, one over there, meaning the 2 elastic segments are welded to for example, elastic segment 7 is welded to 3 as well as 6 elastic segment 8 is welded to fix from 1 as well as 6. So, we have 4 fix connections I hope you see the important fixed connection compliant mechanisms.

We could have put a joint there that is also possible now in this particular example we have fixed connection, there could have been a revolute joint between 7 and 3 and then we have this n c 3 n c 2 n c 1 that is segment compliance, choosing segment compliance we need little practice, but when you are in doubt you take the full 1 let us say we have 2 elastic segments here let us take segment compliance of both them to be a full in planar case that is 3, segment compliance refers to in what ways the elastic segment moves between the 2 bodies that it connects to.

Now, elastic segment is like a joint also because it allows relative freedom between 2 things, a rigid body would not allow only a joint would elastic segment would. So, if I do that, if I take this it will be 3 into 2 that is I am taking this then n sc 2 n sc 1 does not matter. So, in this particular case I get 3 into 7 minus 12 here minus, this is 12 and this is 6. So, what I have here, this is 21 minus 12 minus 12 that is 24 that is minus 3 overall and then plus 6 so what I get is 3, and these 3 degrees of freedom can be interpreted that is the first thing we need to learn the 4 bar that we have here clearly has 1 degree of freedom that we already know that is 1 degree of freedom and after that we look at this green one, green one can rotate this way that way that is 1 degree of freedom and the slider can translate.

So, we are able to interpret the 3 degrees of freedom in this case in the form of rigid body modes that this has as a consequence elastic segments their deformation completely gets fixed imagine that I position this red or brown 4 bar in 1 configuration, and then I rotate the green link that is second degree of freedom move the slider then elastic segments their complete deformation will be determined and that is what we are saying.

So, the formula here gives the degrees of freedom to be 3 and the other hand, if I apply less than 3 let us say 1 2 for this mechanism it will still be able to give you some motion just imagine that it is there, and then I take this green one and rotate elastic segment will give you enough stiffness and they will move the 4 bar linkages as well. So, degrees of freedom for this linkage can be 1, 2, or 3, but not more than 3 that is interpretation it gives you the maximum freedom as possible it can be 1, 2, also.

So, when you apply Gublers formula directly to this mechanism by treating 7 and 8 as rigid bodies, let us say whatever degrees of freedom you get that will be the if it is less than 1 we say 1 degree of freedom is what is the minimum freedom for a complaint one, if it is more than 1 whatever that has will be degrees of freedom, but this formula that we have for modification for compliant mechanisms will give you that many maximum number of degrees of freedom, degrees of freedom defiantly still hold that is minimum number of inputs required, but degrees of freedom itself now we have a upper bound on it or maximum value in whether you give 1 input to this mechanism or 2 inputs or 3 inputs it will give you deterministic motion that is what we want to have in terms of mobility analysis.

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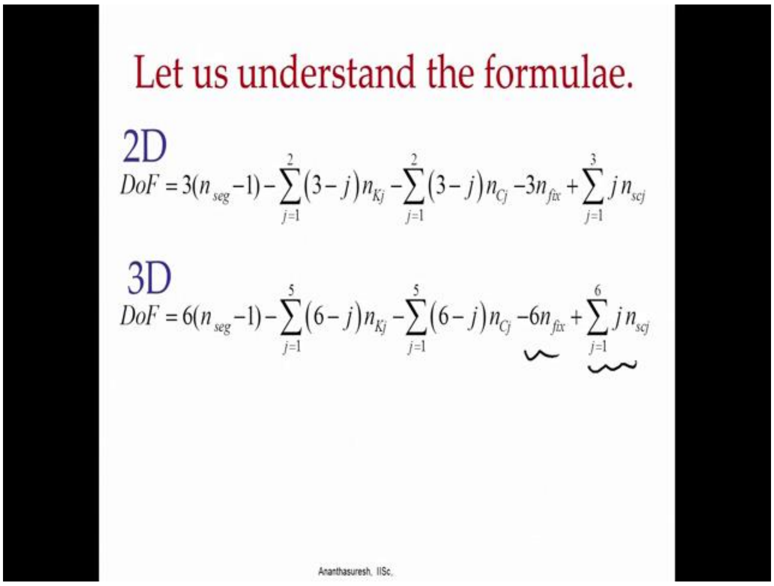


So, 3D also the same thing we have n fix and now the segment compliance. So, 2D and 3D now we have understood.

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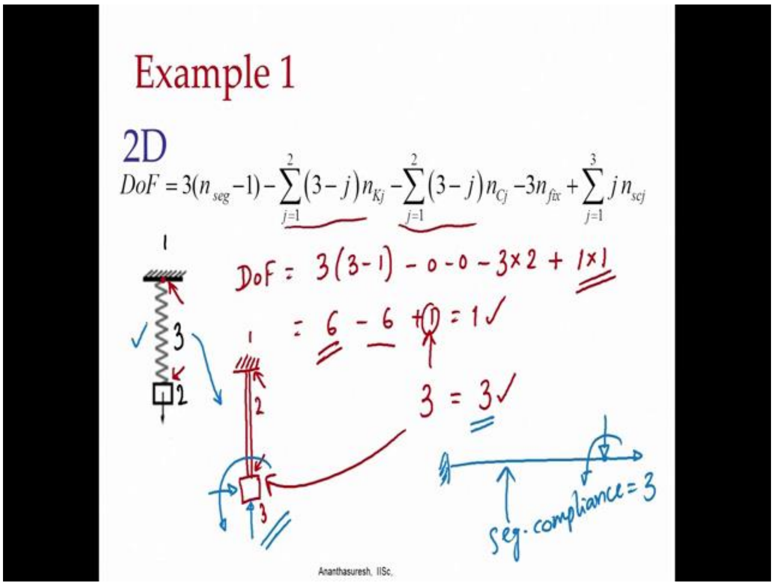


Let us look at a simple example, here are the fixed frame 1 and this moving body 2 and there is an elastic segment in this case is spring 3, when I have that here let us apply this 2D formula and then we see what we get. So, degrees of freedom here 3 into 3 segments minus 1 and then there are no kinematic joints. So, this thing will not contribute this 1 also not there whereas, this 1 is there. So, minus 0 minus 0 fixed one. So, elastic segment is connected over here and over here those are the fixed connections. So, what we get is 3 into 2 and then we have this segment compliant we are understand, if you take a spring what is the possible deformation for it can extend and contract we do not normally use springs to bend them.

So, I can take this as j equal to 1 so it will be 1 times how many such segments is there n c j that is 1. So, if I do this, this will be 3 minus 1 2 that is 6 this is 6 plus 1 so I get 1 degree of freedom. I am happy because this has 1 degree of freedom right, but if want to take the segment compliance of this as more than 1, here I took 1 that is why I have 1 degree of freedom instead if I have let us say a bar with this 1, I do not want have a spring now, but I have a bar if I count I will have again 3 segments. So, that does not change this number and the fixed connection over there and over here that does not change this number 3 minus 3 times 2 whereas, this number will change instead of 1, if I now say it has segment compliance 3 it will suddenly will have 3 degrees of freedom for this one, and we can interpret that if I have a bar it like a cantilever beam where I can apply forces in that direction I can also apply a moment there. So, those are the 3 degrees of freedom.

So, if I take a cantilever beam, here I am showing vertically it going from spring to this takes plane segment compliance instead, if I have a cantilever beam like this we tend to say that a cantilever beam will have infinite degrees of freedom that is not true it will have if I say the force is being applied only at the free tip the force can be that way this way or that way and that is basically what we have just discussed. So, a cantilever beam has a segment compliance of 3, segment compliance of this is equal to 3, when in doubt you take the full when you know like in this case of a spring you take it has 1 because the spring can only go up and down right.

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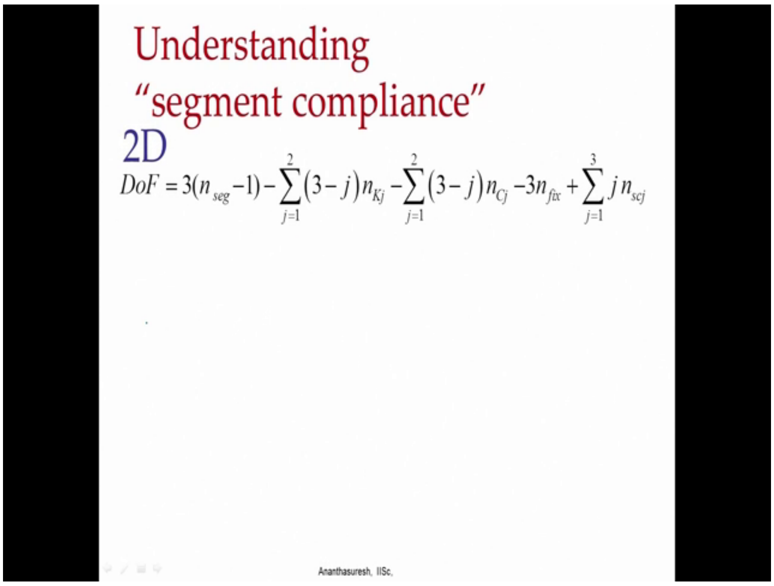

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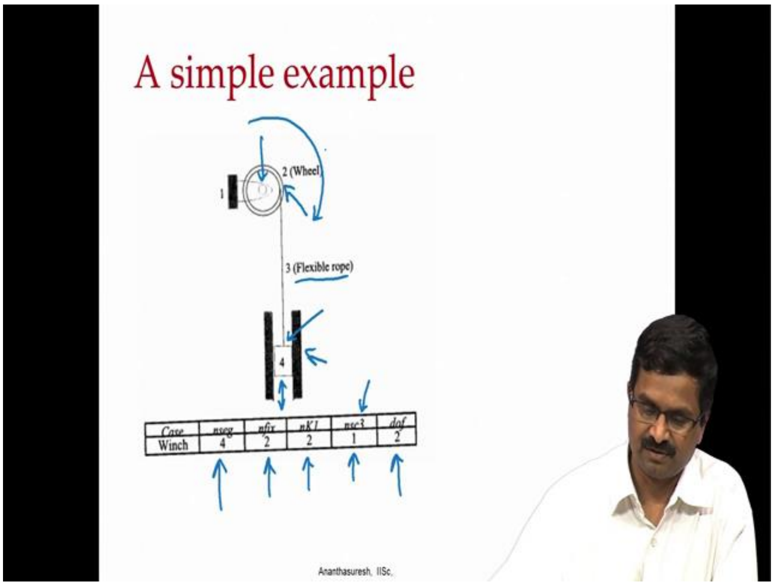


So, to understand segment compliance let us take a simple example, here the number of segments is 4 that indicated in the figure the fix frame 1, and the wheel 2, and the flexible rope is 3, and the block is 4, number of fixed connections that is over here and over there and n k 2 there is a prismatic joint that is over here and there is a pin joint there and then I have taken n s a 3 here as 1 the flexible rope has segment compliance of 3, because it should be able to extend here as well as go round pulley if you take that you will get degrees of freedom 2 again there is a maximum freedom possible for this mechanism.

One freedom you have to interpret one is the rotation of the pulley after you do that the block will be positioned somewhere and then we can apply force here, we can contract it or extend it either way we have 2 degrees of freedom for this and that is it if you give this freedoms everything about this mechanisms is fixed including what have flexible rope instead of flexible rope even if you have let us say a flexible strip, which could buckle when apply force upwards that still counts as 1 degrees of freedom here 1 degree of freedom for the wheel 2.

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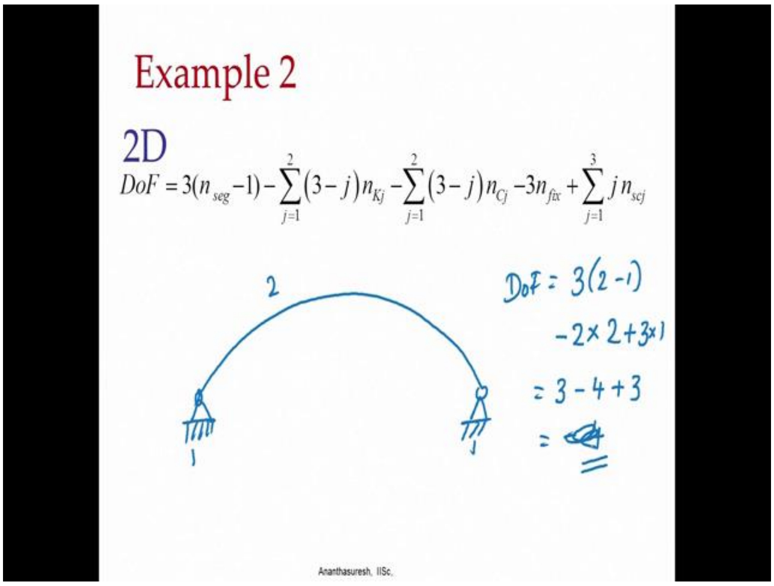


So, we can consider several examples you know. Let us actually take a notch like that with pin joints on either side; we have 1 and then 2. Right now if we say if you apply this 1 we have 2 segments, so degrees of freedom for this is 3 into 2 minus 1 and then we have kinematic joints 2, 2 of those that will become 2 times 2. There are no fixed connections here. And segment compliance, if I take this as segment compliance 3 that will become 3 times 1.

So how many degrees do I get? I get 3 times 2 minus 1 that is 3 and we have minus 4 plus 3, this turns out to be like minus 4 degrees of freedom that it has now. If I just take segments only 2 segments here the fixed one and this and then I have 2 kinematic joints that give c minus 4, and then segment compliance 3 if I take I get minus 4; that does not sorry, this is not minus 4. I am going to stop here.