Compliant Mechanisms: Principles and Design
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Lecture - 38
Feasibility maps for complaint mechanism

Hello, this is the second lecture on our third design technique, which uses spring lever model and feasibility map. We are just mentioned the feasibility map in the last lecture, today we will discuss how to construct it and how it is useful for designing, rather selecting and redesigning a complaint mechanism.

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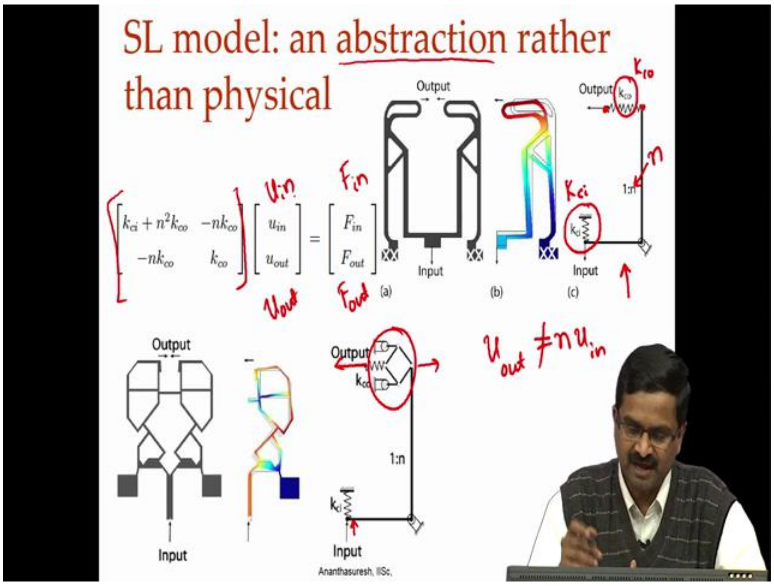

So, let us look at this technique for constructing the feasibility map and this is done with only uses specifications and no other information, than we are of course using the spring lever modeling. So, it is different from the other 2 techniques that we have discussed so far.

So, the spring lever model as you understood from the last lecture it is merely an abstraction rather than a physical model, although in this particular example we are showing as if there is a physical model of the mechanism like a Lump model, where there is input sights stiffness K ci and then output sights stiffness K co and then there is lever ratio which is n.

So, these are the three parameters which we set or the ones that one needs, when you want to have input output model for a 2 degree of freedom system. Is a terminal system more like resistors and capacitors that we have electrical circuitry, here we can actually think of this as a terminal model or port model, your input model or input terminal, output port output terminal, what happens in between you capture using f u parameters, here those parameters are this K ci, K co and n. Again the idea is it is only an abstraction and not a physical model. To emphasis that we took this example in the last lecture there when you have input or output, this particular spring lever if I were to apply input like that, then you would accept this point to go that way.

But here the output direction is this. So, there is an inverter mechanism just to think out of a physical model, but then these parameters all of that is actually unnecessary, it is not a physical model it is only an abstraction. Abstraction in the sense of a input output model, here we have U in this is the first one and then we have U out and there are forces F in and F out and we have this matrix 2 by 2 matrix that connects displacements and forces is like a lump model, but 2 degrees of freedom, which separates out input and output in a way that output displacements is not n times input displacement. For a normal lever if you take, you would have U out to be n times u in, but here it is not so.

That is how it is constructed this point here would have n times u in, where as this point will not have n times u in and that is what will be there in a complaint mechanism, that is what we are capturing. Likewise if you choose this point of output is there, when you apply some force and displaces something, let us say there is U out, input is not going to be U out divided by n it is going to be something different. Because is a 2 degree of a model you have to give actually 2 inputs to complete the data minute and that be captures any complaint mechanism.

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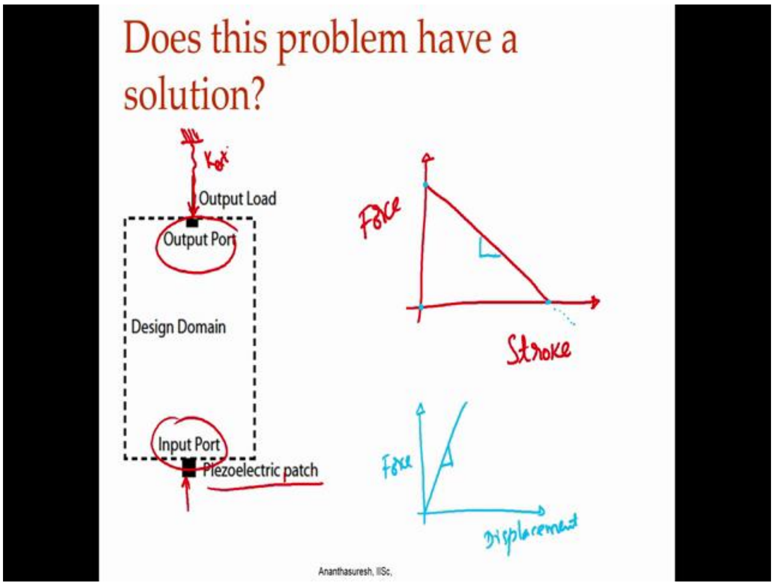

So, we go by this and now look at a problem any problem that one may want to design a complaint mechanism fault, we can say that there is a input port and there is an output port and clearly in this one, let us say input is in that direction, output is in that direction opposite, you cannot think of a actually lever. So, any lever that you think of here is not going to be anything physical is just abstract.

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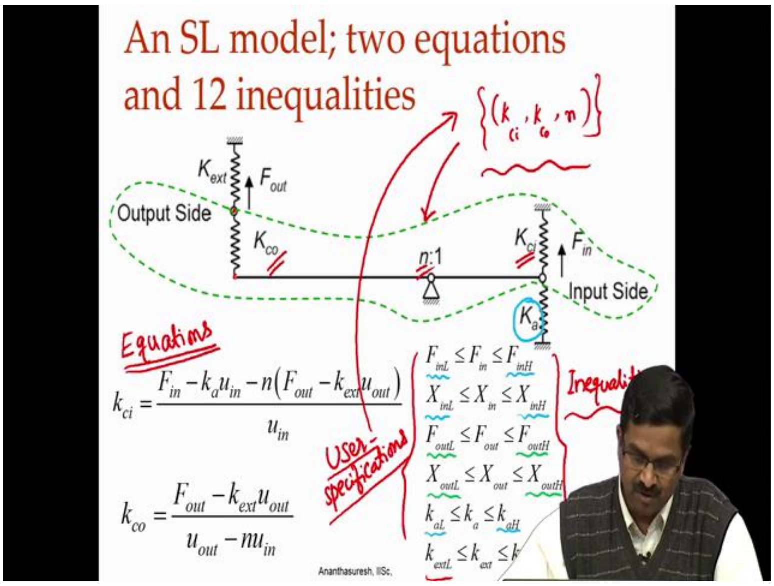

So, we can just make a model such as this, where we have discussed all these equations these are 2 equations here, these are equilibrium equations which you had derived in the last lecture by simply minimizing potential energy and then we have these inequalities these are important because these are the ones that are specified by the user, all of these are user specifications.

These are easy for the user to specify because user knows, let us say we take the previous example, if they have piezoelectric patch they would know how much force it can give and how much displacement or stroke that actuator can give. Most often piezoelectric patches, if I what to plot force verses stroke, the how much actually In fact, we can do that for any actuator, any actuator will have force verses stroke curve which is characteristic for the actuator. Piezoelectric patches normally will have a characteristic that is linear because the displacement is very small. So, that will within the linear regime of that material. So, this is solid Piezoelectric patch, when it deforms it will be like a solid deformation and it is deformation is not much you can assume it to be linear it will be like this. What does this mean? When stroke is a zero, you have the force the highest at that point actuator has a lot of force to give to whatever it is connected to.

But they will be a limit beyond which the force cannot be provided, beyond which it acts like a spring. So, it now we will force verses displacement if you think of it has a negative slope here. So a normal spring would be if I what to do this for a normal spring linear spring, if there is force and there is displacement which is our stroke here, what we call a stroke and actuator it will have something like this. This is a positive slow whereas, here we have negative slow that is the meaning of an actuator actually looking at it, is an actuator because it is slope is negative it is able to give the force in the direction of the displacements whereas, a spring will offer a force opposite direction of the displacement.

Than that is how it will be a linear actuator will be and beyond certain point, you need negative force meaning it is not an actuator any more. You have to supply the force by some other means. So, these the limit that is all actuator can move, that can push on some object for that stroke length. So now what is happening the force is decreasing as the stroke is increasing, which means that there is actually a spring in the actuator. If I take piezoelectric patch, the patch of the material is a stack has to deform. In deforming the material is being deformed there is the spring, so there is an actuator spring. That is why in the model that we show that connects to our SL model we show Ka that is actuator stiffness.

So, when a user has an actuator in mind, the user would know what is the lower limit of the force and the higher limit of the force and likewise there will be an actuator stiffness one can provide lower and upper limit is. That is if you have bunch of actuators, then the user who wants you the actuators and wants to design a complaint mechanism, will know these values. How much force can be delivered by the actuator? How much stiffness will be there and also the displacement and the input port x in, how much displacement actuator would have.

And similarly user would also know what they want at the output? So, they can specify the output force how much they want, if they want. And how much displacement they want, if they want so, those things are easy to specify. Additionally we also have k external where that comes from the object which is going to be there for the complaint mechanism deal with. For example, in the previous case there could be an output load here or there could be a spring there, which is k external there can be both a spring an output load also.

So, these are the things that are natural for the user to know and that are what we take. These are the user specifications, from these and using the 2 equations we have this spring lever model we want to construct, rather we do not want to construct the model we want to see what values of K co and K ci will satisfy these inequalities and these equations that is what we are asking.

That is the crux of this s L model and also the feasibility map concept. We are not worried about a mechanism at this point, we just to want to deal with user specifications and then inform the user, if this is what you want we say this is what will be the parameter space of complaint mechanism K co n and K ci is a triplet you will have 3 values there will be some value for K ci, some value for K co and n.

So, how many such sets will satisfy the inequalities and equalities, that is what you want to inform the user. If such a set does not exist in the feasibility map will not exist, that is the idea here. Once you have this, then we can talk about k external and k which again are already there as part of the user specifications, in the context of those we need to determine these 3 parameters, there could be several values if exist and that is what want inform the user, that is what we said last time also and I am repeating now that K ci, n and K ci, K co and n if I have the 3 axis.

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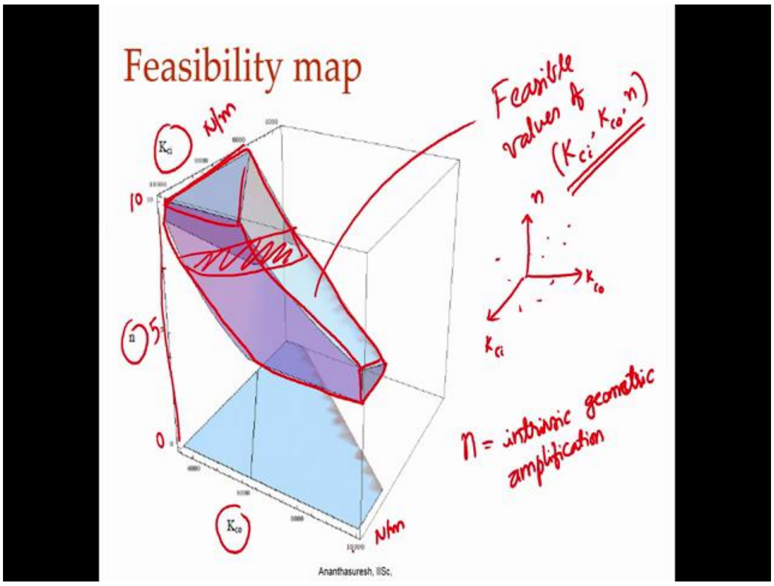

Let us say this is K ci here and K co and n these are the 3 parameters. So, they will be a bunch of points, you can take lot of them only some of them will be satisfying our equations and the inequalities, we need to find that and that is shown here.

So, the volume that is shown here this entire volume, here this volume is the one that contains all the feasible values, feasible values of the triplet, feasible values of K ci K co and n, these are the one that we want to calculate and show to the user. Now there is no mechanism as yet, it will come right now, we just want to tell the user; how the specifications relate to these values.

So that is the crux of constructing the feasibility map. It is a map in 3 dimensions, but three dimensional map even though you can draw with little difficulty you can draw it, but or a computer screen if you want to see it is not as easy to manipulate and then see that is if you want to see this is something, but. In fact, even here, we do not know how the actual three dimensional shapes are. In one image of 2 dimensional like it is taken now we will be able to really rotate this map and then see how the values of K ci K co n change. N can be positive or negative in this particular case and only has positive value because this is zero and this is 10 and this is 5, this is the n axis in this case, but sometime could be negative also. So, what does that negative positive mean? It actually shown here, if n is showed in this manner if I apply the force there this because it lever it will come down, n will be negative in this particular way.

But if you wanted to positive meaning that input force and input displace output displace the same direction, it can be positive also. So, n has that thing where it inverts the direction of the input force in order to produce output force a rather the force at the point where the lever is connected, but sometimes it could be positive also, for that you do not have to worry about again a physical lever just understand that n can be positive or negative, you will know what kind of a intrinsic amplification that is what we call.

N is intrinsic amplification it is nearly kinematic or geometric amplification, we can see the range that is entirely a consequence of user specifications, but also in this particulates whatever the unit is they are they use to be 10000 here, that could be let us say a Newton meter, same thing here 4000 for K ci.

And we can see that range for the feasible map and then say this is how stiff the complaint mechanism should be, whether specific that user has. So, when there enter some values if we construct this map, you would immediately know what kind of a stiffness and intrinsic amplification that complaint mechanism that we have not yet designed or we do not know what it is we know what it should be? This is similar to the way we choose materials for any design problem, we will have some specification in terms of stiffness strength stability whatever and based on those values we say this steel will be applicable, polymer will be applicable, a ceramic will decide.

In a similar way here before we think about a complaint mechanism, we want to see the space. The space equivalent of little properties we have properties of the complaint mechanism here, which are K ci K co and n and that is the thinking that one should have with this third design technique.

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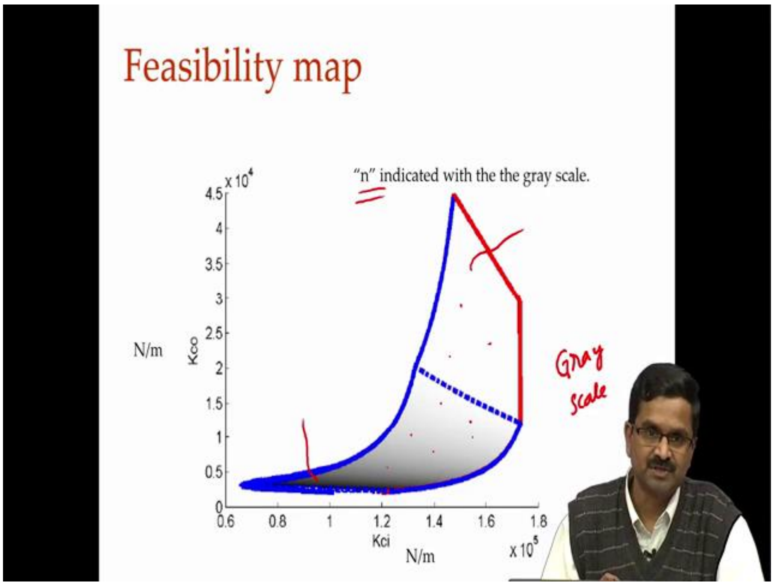

Once you have the tridimensional map that we saw, if I take a projection of that we can do projection on to in this particular case, we are showing K co and K ci. One can do K ci and n K co and n any other three combinations that is up to the user and a particular problem, we will be doing case studies later there will do K ci verses K co will also do n verses K ci as well.

So, we take those projections which are in 2 dimensions, we will do 2 dimensions of course, we lose 1 dimension because the from 3D to 2D we are coming, in which case what will happen is the volume that we have we take a slice. So, in the previous one if I go and look at this one let us say I want to choose a particular value of n let us say that one, then I will be taking a slice there; whatever the shape that is I will be taking the slice. So, that slice is what we are looking at in the next one for a particular value, but then we do something, you see this gray scale here, so somewhere it is completely white and somewhere completely dark that indicates the n value you can show that if you want or when you have this in a computer graphics interface use interface as you mouse over on this immediately n value can proper.

So, as you put here, here, here will tell you what n value is. Low value and high value not necessarily zeros, some low value high value it can indicate that everything gray scale.

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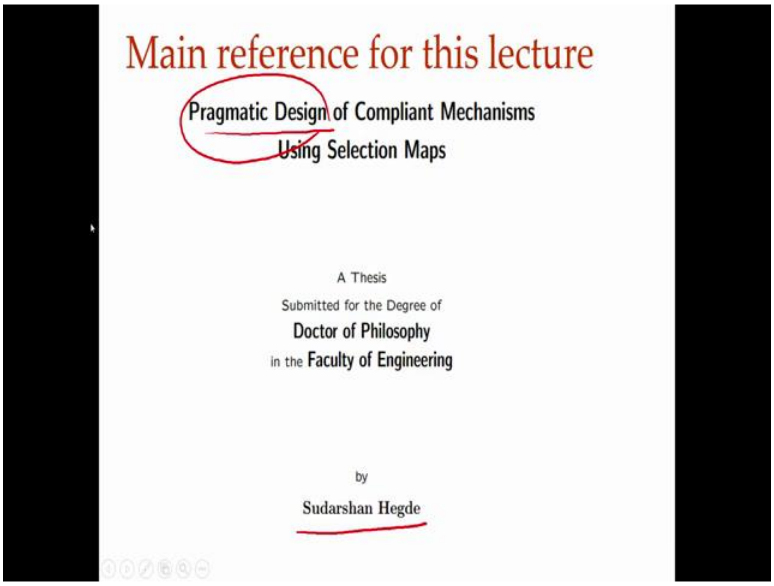

So, we can have that and see this, but idea is to be able to construct this map first. So, constructing this map one may not actually need to understand or know in or to use this design technique. Design technique is you enter user specifications, as a user we just enter what you want that is all you do not need to know anything more, but since it is say discussion of the technique we need to understand the intricacies of how this map is constructed.

So, here I am referring to the PhD thesis of Sudarshan Hegde at Indian institution of science here, is a word the title you can see that is the important to notice it is a pragmatic design. With regard to digit body linkage based design or (Refer Time: 18:24) based design one has to learn a lot in order to design using those methods. Even if you have a software you need to still know what equations you are solving, here we call it pragmatic design because we are actually making it completely user oriented, user enters the values upper or bounds for that user does not need to know about complaint mechanisms almost anything.

Here she will simply enter the values, and then we show the feasibility map. If the feasibility map let us say is null meaning that there is no set of K ci K co n values the satisfies the equations to equations we have and a 12 inequalities the 6 variables each are the upper and lower if there is nothing that satisfies, you simply tell the user and feasibility map is zero null.

Then user would know that what he or she is asking is not possible. Is not possible with any complaint mechanism is not possible at all with a any mechanism, with there has this characteristic of the complaint mechanism that is spring lever characteristic it is not just zero spring constant, it is all which springs and there is a input output geometric amplification ratio, when is not possible.

So, when you have equations you can have solutions, sometimes which maybe imaginary, but here some other real solutions are also not valid because K ci and K co cannot be negative. N can be negative, which case is just inverting the direction of the input where gives the output.

But K ci and K co cannot be negative that is actually one criterion that we use to actually generate this feasibility map and that is what Sudarshan Hegde did in his thesis. So, will switch to the thesis I work you to the chapter four of the thesis, where you can see how it is done, it is not straight forward, but at the same times it is not complete difficult there are certain things. So, I showed you the map whose continuous volume singly connected volume. Is it singly connected volume? Or they can be 2 separate disjoint sets. So, we are taking about K ci K co n values, they can be all continuous as one blob or it could be multiple blobs also.

So, what Sudarshan Hegde in his thesis has proved that, it cannot be discontinuous, everything is continuous it can be whatever shape clearly it is not a convex shape, it is non convex. But it is singly connected that is what he has proved. We cannot discuss all those details in that time that we have allocated for this particular topic, but I will give you the pointer as part of the supplementary material you will be able to have access to the pdf file of this PhD thesis and you will look at it.

But what you need to understand first is the pragmatic aspect of this technique. So, will switch to that and then come back to this.

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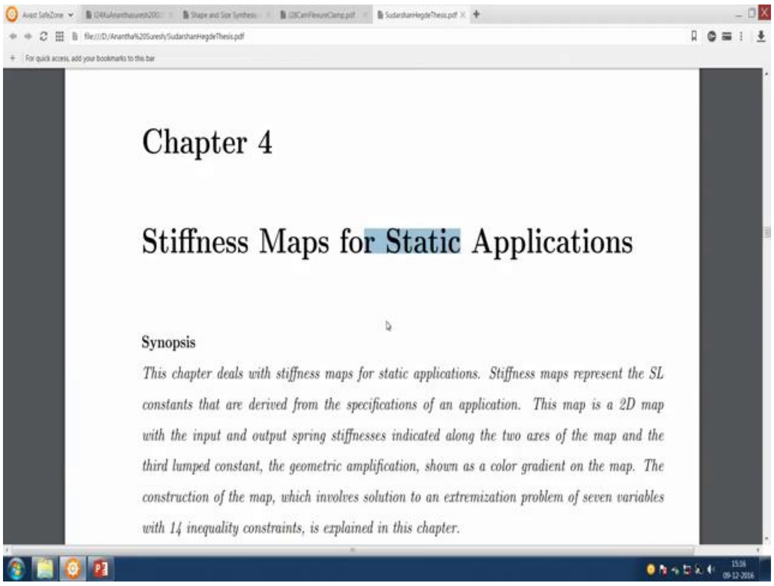

So, let us go to the PhD, thesis I have already at chapter four, where he talks about the stiffness maps because we are focusing on the stiffness inputs sides stiffness K ci, output as stiffness K co first static applications, later on will be talking about dynamic applications also, right now it is only Static. This chapter gives you complete details I will just work you through the important aspect.

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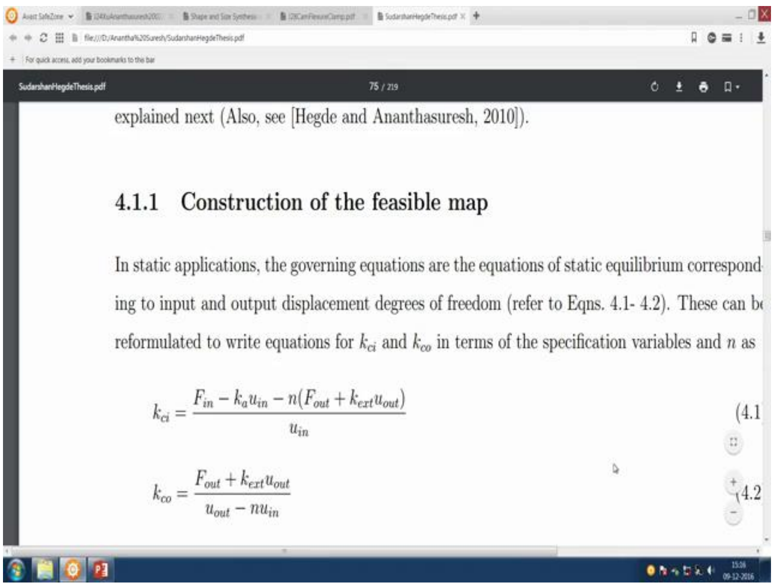

One important aspect here is that we have 2 equations. So that we have inequalities are more difficult to solve, that is where the difficulty of this problem lies. So, equations we know we have lot of different methods and many of them implemented in various software programs, but the moment you have inequalities the tabularizes and that is what I will tell you how it is done here, we start with this equations we have K ci some values there K co some values there. Now on each of the variables that are contained in it F in, k a u in, F out, k external U out for all of these things you will only have a range of values given by the user and based on that we need to get range of values for K ci, but then n is also there.

If you want to get a range first thing to know is what is the minimum value? What is the maximum value? What is minimum value of K ci? What is the maximum value, but then n is inside which is also an unknown here. K co we have F out and k external U out and U in and again are there. So, n is here and n is there and that is what makes it difficult because we do not know the range of n. We have to derive the range is feasible ranges of K ci K co and n even though we have 2 equations, if I know the range of n value then I can get these 2. For each value of n I will know K ci K co because I know these ranges, but then it is not just specific values of F in U in everything has a range. So, everything here works within the range values.

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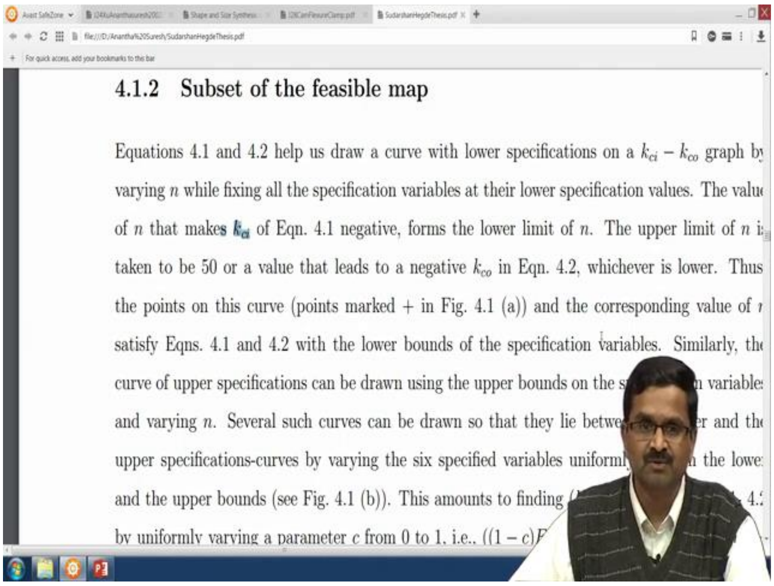

So, what we do here is pose a set of optimization problems, where we try to make K ci or K co only positive, we prevent them for becoming negative that is we try to find the values of n, such that the resulting values of K ci K co for given range values for the 6 parameter that we have, they do not become negative, they need to be positive that is the thing.

So, by solving optimization problem in each case we are assuming different values of n, we need to find the corresponding minimum value of K ci as well as maximum value of K ci. Likewise we get K co minimum value and maximum value. When you do that what you get in between there are some details that we a parameter at like here it is c that will have a range that is generated, within that you define the extreme values.

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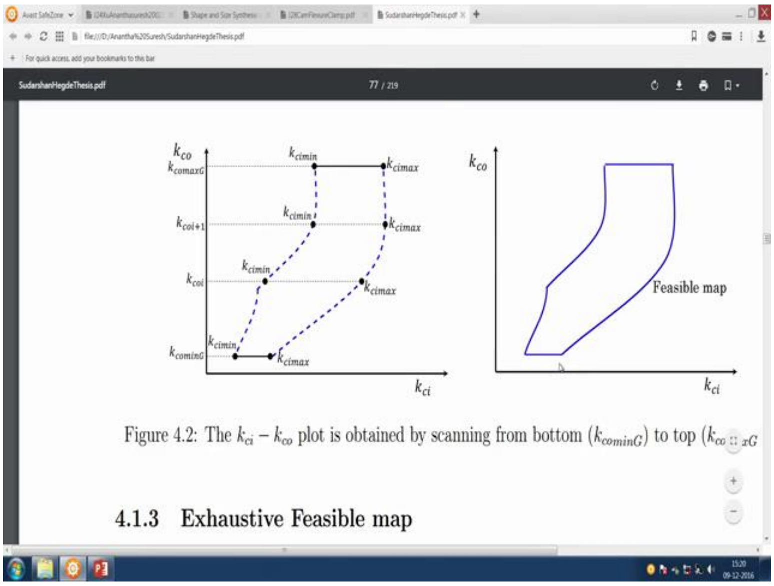

A typical problem that we would pose here is like this, let us say I want to find this blue boundary curves that are shown here, let say this is the feasibility map that is could be computed, but we have not computed yet.

So, what do we do; we take for a given let us say K co value, this is the K co axis, this is the K ci axis let us say I prescribe a K co value. Let us say this line once a prescribe a K co value, I still have n that is we do not know, but I fixed K co, then I try to find whether values of n exist at all and once I have values of n exist with our inequalities and equations, then we look at the K ci value.

In the K ci value is positive, then I take it if it is negative I discard that K co value meaning that if I what to take a K co, this feasibility map if I take K co that is over here, there is no corresponding K ci and n. That is what we decide by solving this inequalities and equations by solving optimization problem, which I will show you. So, we can take slices of K co and scan it from bottom to top or top to bottom whichever way, then for each case we try to determine the minimum value of K ci and then the maximum value of K ci, then we generate this curve and that curve, in between we just joining this that may not be the case for which we are to again do this reverse, that is we assume a value of K ci and generate extreme values of K co this is the K co lowest for that K ci and the highest and that is how we construct this maps. So, a series of operation problem they are not difficult operation problems they are simple enough, but they have to be done several times.

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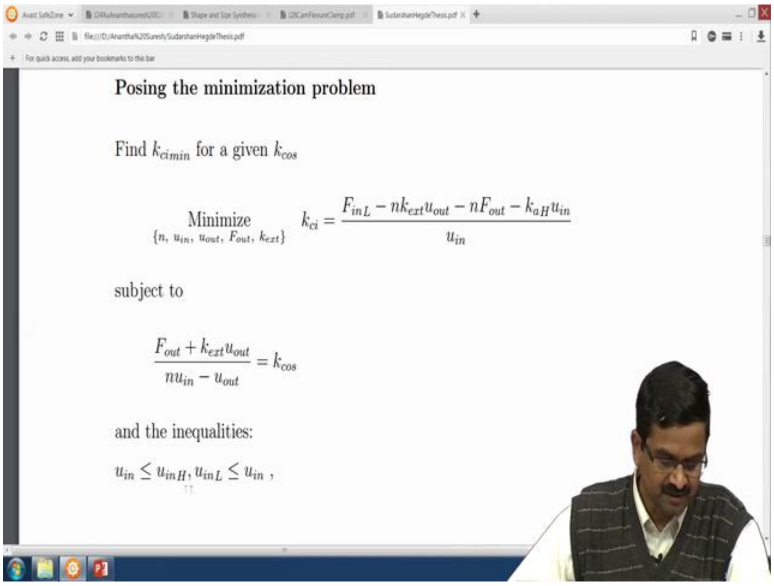

So a particular operation problem that I will show you looks like this. So, for a chosen value of K co here we have put that as K cos mean that specify a value that one level we have taken, then we have to minimize this K ci for which we have an expression, but it is all in terms of the variables which have their own upper and lower bounds and then we have subject to the constraint that, the same values that I take should have the specify values of K co. K co is what we are taking we want to find the extreme value, minimize we have to solve once, maximize we have to solve once.

But then these are simple enough because these are all polynomial equations you can actually write down the Karush Kuhn Tucker conditions for the KKT conditions and look at them analytically we not doing all numerically, we look analytically and the we have the inequalities on U in and U out and other things we put them all or these other ones.

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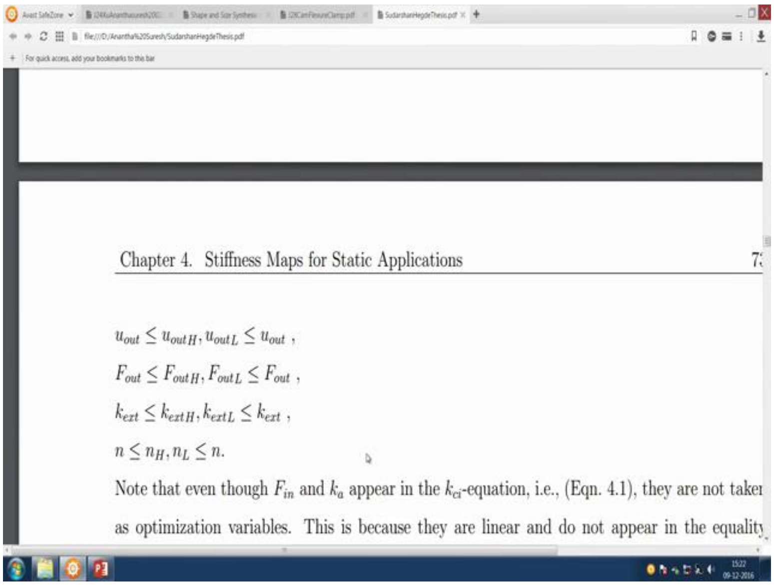

And this n values also, there is an extreme value of n and highest and lowest that you can determine by looking at the equation little bit more again all the details or difficult to discuss in a short lecture that is allocated for this topic you can read the chapter 4, you can get that the gist is that you have to assume K co and generate the side curves that I described in the figure above.

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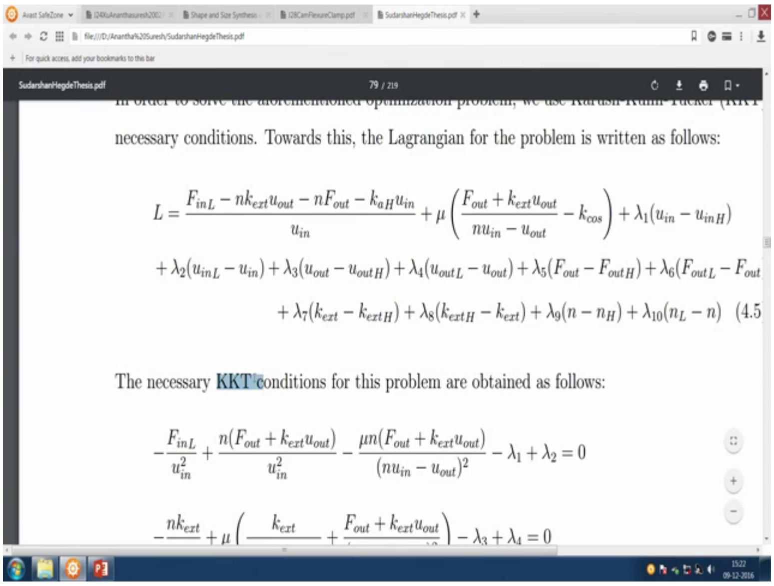

And then do the reverse you can do, you can assume K ci and get K co and like Lagrangian the usual way we write this Karush Kuhn Tucker KKT conditions and solve this problem analytically, when you do that you when you solve it analytic the other problem that is posed, you can actually find a number of conditions under which there will be a particular solutions.

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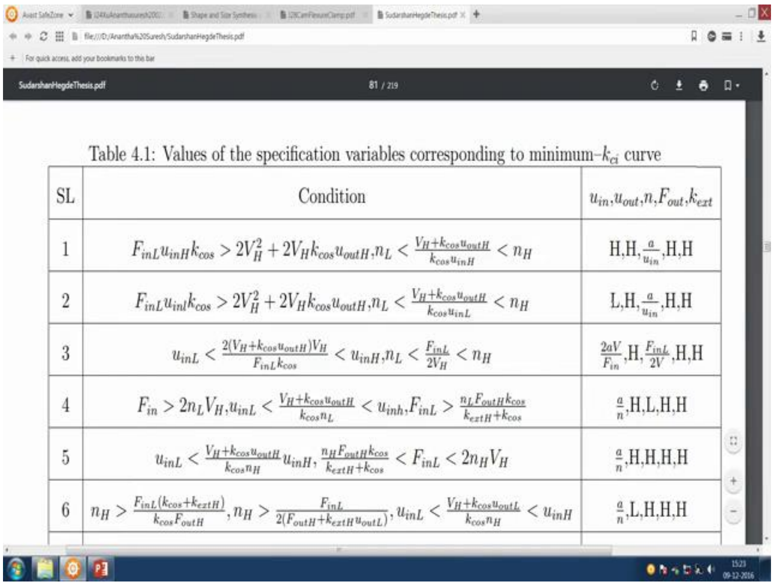
Table 4.1: Values of the specification variables corresponding to minimum–k_{ci} curve

SL	Condition	$u_{in}, u_{out}, n, F_{out}, k_{ext}$
1	$F_{inL} u_{inH} k_{cos} > 2V_H^2 + 2V_H k_{cos} u_{outH}, n_L < \frac{V_H + k_{cos} u_{outH}}{k_{cos} u_{inH}} < n_H$	H,H,$\frac{a}{u_{in}}$,H,H
2	$F_{inL} u_{inl} k_{cos} > 2V_H^2 + 2V_H k_{cos} u_{outH}, n_L < \frac{V_H + k_{cos} u_{outH}}{k_{cos} u_{inL}} < n_H$	L,H,$\frac{a}{u_{in}}$,H,H
3	$u_{inL} < \frac{2(V_H + k_{cos} u_{outH}) V_H}{F_{inL} k_{cos}} < u_{inH}, n_L < \frac{F_{inL}}{2V_H} < n_H$	$\frac{2aV}{F_{in}}$,H,$\frac{F_{inL}}{2V}$,H,H
4	$F_{in} > 2n_L V_H, u_{inL} < \frac{V_H + k_{cos} u_{outH}}{k_{cos} n_L} < u_{inh}, F_{inL} > \frac{n_L F_{outH} k_{cos}}{k_{extH} + k_{cos}}$	$\frac{a}{n}$,H,L,H,H
5	$u_{inL} < \frac{V_H + k_{cos} u_{outH}}{k_{cos} n_H} u_{inH}, \frac{n_H F_{outH} k_{cos}}{k_{extH} + k_{cos}} < F_{inL} < 2n_H V_H$	$\frac{a}{n}$,H,H,H,H
6	$n_H > \frac{F_{inL}(k_{cos} + k_{extH})}{k_{cos} F_{outH}}, n_H > \frac{F_{inL}}{2(F_{outH} + k_{extH} u_{outL})}, u_{inL} < \frac{V_H + k_{cos} u_{outL}}{k_{cos} n_H} < u_{inH}$	$\frac{a}{n}$,L,H,H,H

So, they are all condition when you analyze the KKT conditions, there are some intenerate variable that it define like v h and so forth. So, when this condition satisfied you have particular solution that is possible. Sometimes you go to this is U in U out, n, F out and K external. In this case whether they will be it a high high it is some very intermediate and then high high sometimes it is low and so forth. You check this condition there are all analytically obtain; you can get these various values and generate the boundary of the curve.

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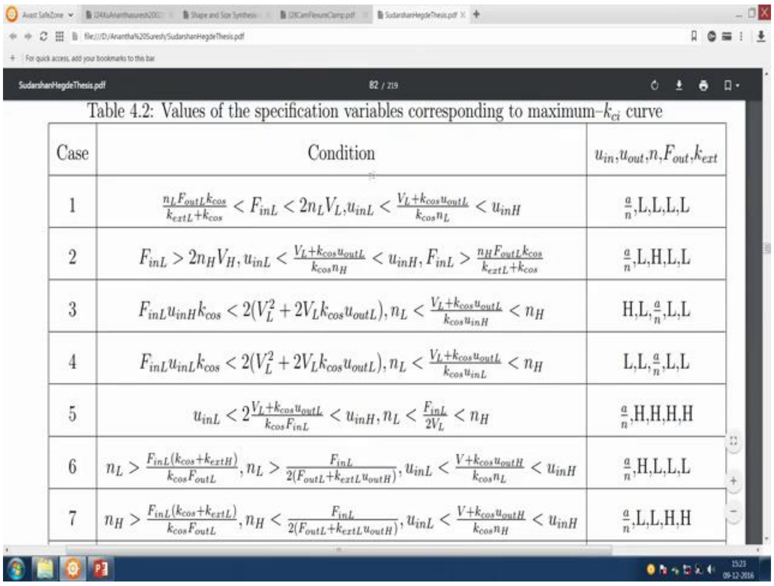
Table 4.2: Values of the specification variables corresponding to maximum-k_{ci} curve

Case	Condition	$u_{in}, u_{out}, n, F_{out}, k_{ext}$
1	$\frac{n_L F_{outL} k_{cos}}{k_{extL}+k_{cos}} < F_{inL} < 2n_L V_L, u_{inL} < \frac{V_L + k_{cos} u_{outL}}{k_{cos} n_L} < u_{inH}$	$\frac{a}{n}$,L,L,L,L
2	$F_{inL} > 2n_H V_H, u_{inL} < \frac{V_L + k_{cos} u_{outL}}{k_{cos} n_H} < u_{inH}, F_{inL} > \frac{n_H F_{outL} k_{cos}}{k_{extL}+k_{cos}}$	$\frac{a}{n}$,L,H,L,L
3	$F_{inL} u_{inH} k_{cos} < 2(V_L^2 + 2V_L k_{cos} u_{outL}), n_L < \frac{V_L + k_{cos} u_{outL}}{k_{cos} u_{inH}} < n_H$	H,L,$\frac{a}{n}$,L,L
4	$F_{inL} u_{inL} k_{cos} < 2(V_L^2 + 2V_L k_{cos} u_{outL}), n_L < \frac{V_L + k_{cos} u_{outL}}{k_{cos} u_{inL}} < n_H$	L,L,$\frac{a}{n}$,L,L
5	$u_{inL} < 2\frac{V_L + k_{cos} u_{outL}}{k_{cos} F_{inL}} < u_{inH}, n_L < \frac{F_{inL}}{2V_L} < n_H$	$\frac{a}{n}$,H,H,H,H
6	$n_L > \frac{F_{inL}(k_{cos}+k_{extH})}{k_{cos} F_{outL}}, n_L > \frac{F_{inL}}{2(F_{outL}+k_{extL} u_{outH})}, u_{inL} < \frac{V + k_{cos} u_{outH}}{k_{cos} n_L} < u_{inH}$	$\frac{a}{n}$,H,L,L,L
7	$n_H > \frac{F_{inL}(k_{cos}+k_{extL})}{k_{cos} F_{outL}}, n_H < \frac{F_{inL}}{2(F_{outL}+k_{extL} u_{outH})}, u_{inL} < \frac{V + k_{cos} u_{outH}}{k_{cos} n_H} < u_{inH}$	$\frac{a}{n}$,L,L,H,H

Similarly, another problem we generate all these conditions, they are all simple I am not saying that they are actuarial, but they are not complicated in the sense that if you know optimization theory, we are understand this part.

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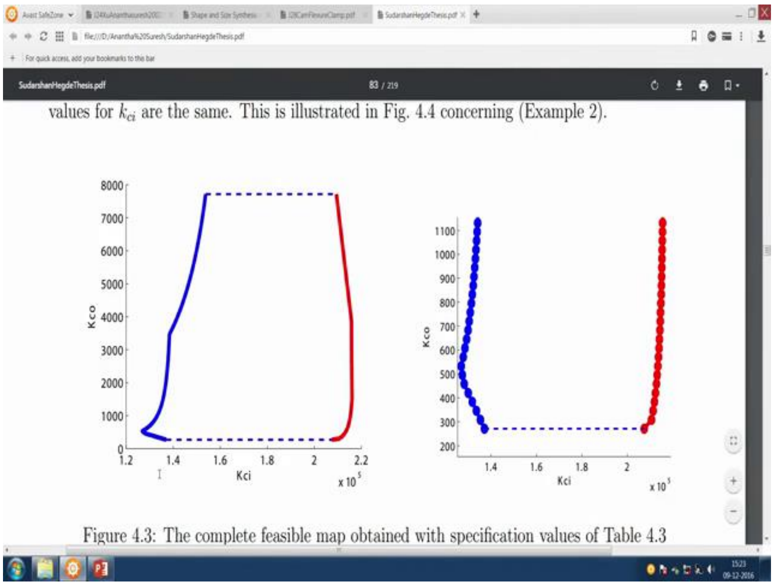

And with all of these we construct the boundaries here, we can construct a boundary for K co given value and K ci we can get this, here we do not necessarily think that they need to be just joined. Then you have to do K ci various things and then see what comes out here if at all it is there, but we know that there is nothing below this, that is why this even dash line a shown because if I take K co that is a value that is lower than this, it all ready tells you that we did not get a feasible n we did not get K ci min, this side K ci max the other side.

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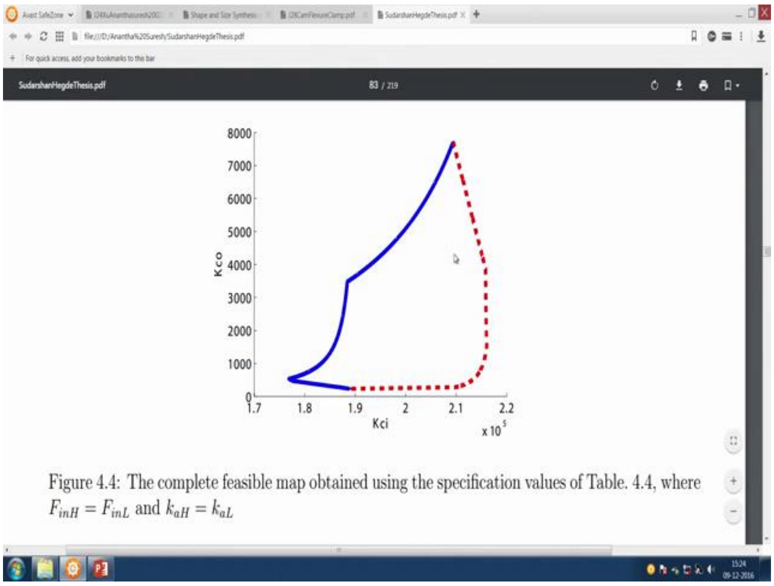

Another thing to remember, the moment we finish the map like this, any point I take inside can have several values of n, per K ci and K co there is only one value like a this a slice, but there are several n values are there.

So, need to generate that whole n, that is the three dimensional volume that has a range, that n has to be continuous is what is also proved in this chapter. It is not like, it is goes like this and goes up that actually continuous not only the feasibility volume is a singly connected domain, the n value at any point is also continuous and that is also proved in this chapter for this continuity of n. So, this construction of the slice and that point is also discussed and proved.

So, this chapter four is actually the key chapter of this is, let us talk it all is user eccentric and user will be using it like more like a graph user interface manner, but this construction of this is the key here.

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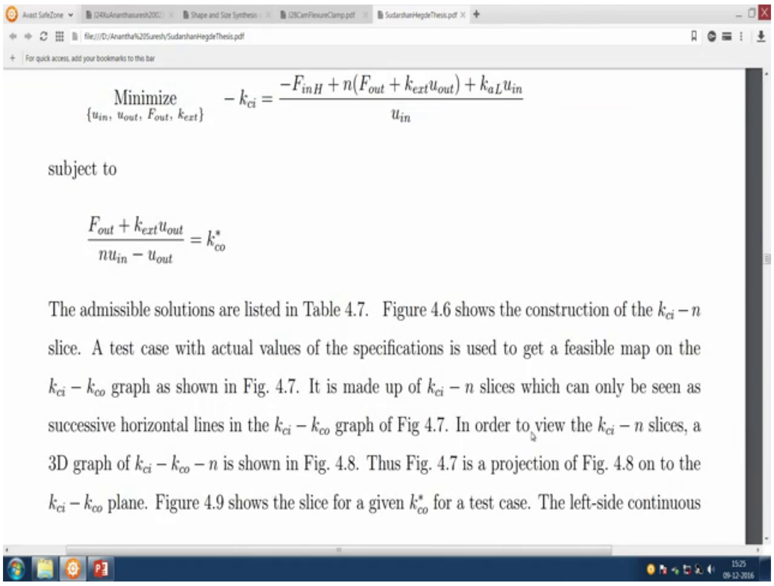

The continuity of n the right out about between n low value and high value at any point it is a continuous value and that is also proved by some analytical arguments. So, again I am skipping all those details, but telling you that what exist, is all what you see here is the one that argues why it is continuous, so anyway. Now I would like to switch back to our I think here, to say that with all of these what we have discuss now is how to compute the feasibility map, chapter four in that PhD thesis tells you how to compute.

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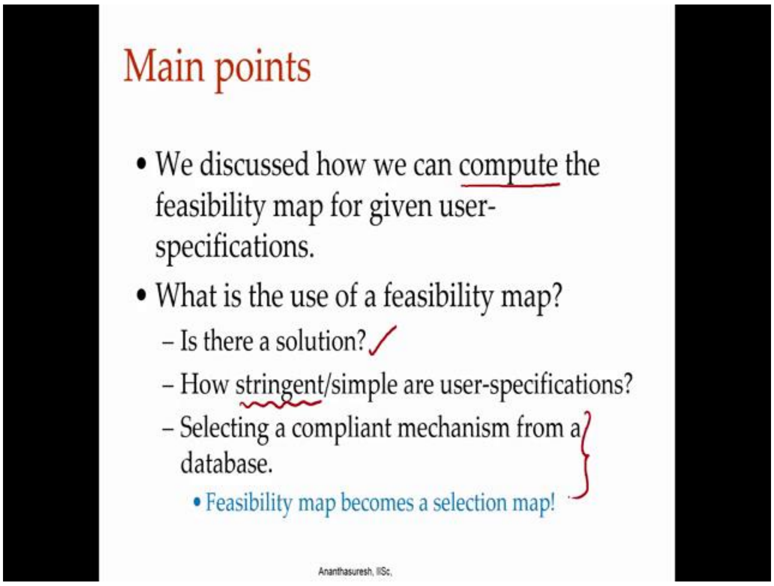

But what you really need is that as a user to enter these specifications, we will get the feasibility map, and then what do we find from that? First point if there is a solution or not, if there is a solution that means, that feasibility map is not null you see some blog or projected one you see some area.

If that area is too small, then you would understand a user, but what you are asking is too stringent, that is such a small space of K ci K co n actually exist. It saying that if you want to choose a material for a design that you have an hand saying that, my let us say Young's module talk about stiffness, Young's modulus has to you between 74 and 75 gigapascal that is all, that small range.

So, if you are lucky then will be a alumina for you, but if you have some other value for which there is no material or that material is not available in the form that you want, then you would know that what you are asking is not possible or very stringent you have to go for a special material. So, it tells you the feasible space is very small your specification is stringent.

Similarly, if the feasible space is very large, then you know that they could be many many complaint mechanisms that we do your job. Because what you are asking is common place. So, your K ci, K co and n are all over the place wide range, that one can expect that they will be mechanisms to meet your specifications.

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Now, let us say that you have done that, you understand that there is the solution and there is a stringent or simple specific at that you have, what we will discuss next is how to use the feasibility map to actually select a complaint mechanism that is the next thing. And further reading for this apart from the PhD thesis are these 2 papers which we have also noted this first paper there and this second paper, that you can read more about what will be discussing in this week's lectures.

Thank you.