Compliant Mechanisms: Principles and Design
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Lecture – 37
SL model for compliant mechanisms

Hello, we are into the 7th week of this course. We have discussed to classes of design methods one based on rigid body linkage synthesise other based on structural optimization. We have discussed pros and cons of both the methods and these two methods have been extensively worked on by many researches in the last I would say in two and half decades.

Now, there are a few more methods that have been developed in the last few years, and they are apparently developed to a plug the difficulty that are in the previous two methods that is based on rigid body linkages and that based on slut optimization. This week we will discuss one such method which is more of a selection method rather than synthesis method, when you have specifications for those you can select a compliant mechanism from a data base, because of the work that has been done on compliant mechanisms in the last two and half decades has given history to a lot of compliant mechanisms rather we can have a data base compliant mechanism. Now out of which it is possible that most of the applications there can be new applications or new problems one can choose one of those, how does one choose, how does one validate that a particular mechanism suitable for a specification or not is what we will discuss this week.

So, we will begin today, with a new model it is a spring lever model it could also would I have been called pseudo rigid body model, basically this also models a compliant mechanism with something like a very simple linkage, simple lever and with a spring to account for the elasticity of the compliant mechanism. So, we are going to look at this spring lever model and then discuss in the next lectures, how this is going to be useful for selecting or rather designing a compliant mechanism.

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So, before we start let us look at the two methods that we have discussed. First is the rigid body linkage approach, here we have to assume a mechanic type whether it is 4 bar, 5 bar, 6 bar, or how the connectivity of various rigid body is and the joints are and then at which joints we want to put a torsion spring to account for the elasticity or rather what we call rigid body model. When you get a compliant mechanism out of it, it will be similar to flexure hinge based design because, that is essentially how we have started. Although it is possible sometimes a hinge and a rigid body can be replaced with a cantilever beam or a fix, fix beam or fixed guided beam and so forth. So, it will be even more than flexure hinge based design, but it will remain close in spirit or in topology to a linkage. So, this is best suited for partially compliant mechanism that also good for quantity it will design as we have discussed you can say these are the points have want in a function to be generated or a part to generated you can write down the equations and get a solution for quantitative specifications so that is the strength of this approach.

And the other approach is the topology, shape and size optimization approach, this is purely what is develop for special optimization and using it for compliant mechanisms, and next we have seen it is a great tool for conceptual design even when you do not know how the compliant should look like just from these specification of the problem you can get an idea of how the elastic members or segments should be connected in order provide the functionality that one wants, and to some extent it is also useful for quantitative design so that we can say the other performance constraints mechanical advantage or manufacturability or stress all those considerations can also be incorporated, but optimization problem becomes more and more complicated. After getting the conceptual design or compliant topology one can go for shape and size optimisation as we have seen in the last two lectures on this topic, and we had one on shape optimization specifically and a case study that also about size optimization, but the clue is usually given by topology optimization.

Now, neither of these can tell us apriori that the specification that the user has can be met or not, we have write the equations in the first case and solve them and then if you do not get the solution you may conclude that these specifications do not have a solutions. Similarly topology or (Refer Time: 05:39) optimization approach also we have to first solve the problem and then see if there is a solution or not, but apriori before you begin to do at they cannot tell you whether there is a solution or not or as we saw in the case study and in the last lecture when there is no solution how can we say what should be modified in order to have a solution. In that case study we saw that if you are to increase the space or design domain in which the mechanism has to fed then there is solution otherwise, there is no solution we should be able to prove that usually possible in analytical methods, but we will discuss a method today where it is possible even with a numerical method to tell if there is a solution or not, that is where the scope of this week's lecturers and design method that we will discuss.

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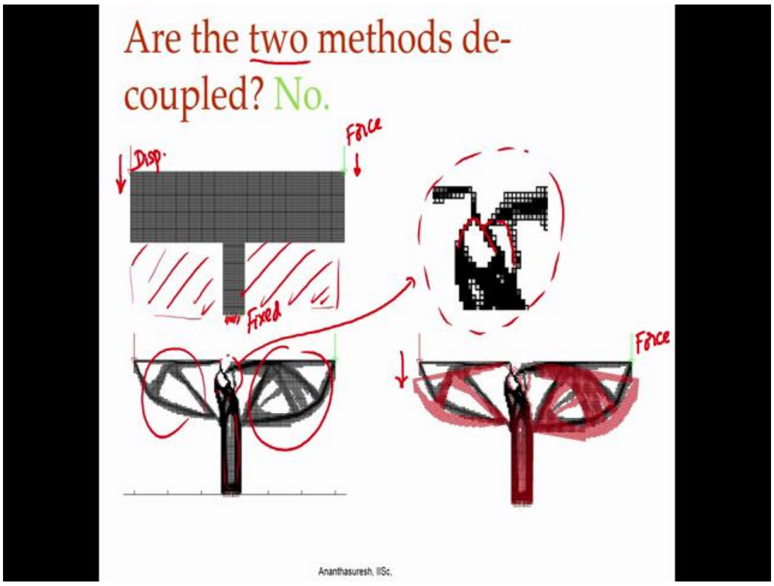

Now, we had seen this the topology optimization we had considered that if you fix only here, and apply the force over here and want displacement over there is the resolution, because this is going down and it that goes down that is the force and our displacement this points should also be with over the direction, if we normal level if you take if you fix it here that is what we have done here, we saw that it did give a intuitive resolution. Then (Refer Time: 07:14) suggested that how about putting a whole here that is design domain give it in this T shape other than including this additional portion, which we had include earlier then it came with nice mechanism when we included this place also it did come with the nice mechanism.

But now even if you put that it is still came up recognize mechanism, which as we can see here it is in dead going down when we apply force on the side and see it has taken all the material that you have given made it a very rigid one here, who supports can be taken under this portion is also rigid, this portion is also rigid and it has come up with a nice mechanism, which is shown here right. It has segment that segment that little segment and that segment so it is somehow is able to come up with a topology and that is a strength of the topology method always gives resolution, but this method when I say these two methods the structural optimization based method, and the rigid body linkage based method or the decoupled. In fact, they are not this figure we had seen in the case study lecture.

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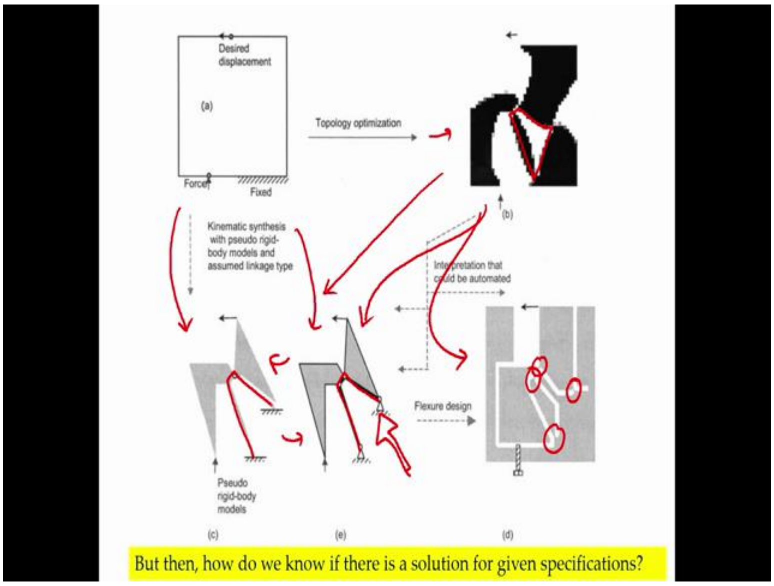

When you have topology optimization you get a topology and from there you can go to rigid body linkage and a compliant mechanism that is fully compliant, but only thing is we have this flexures right, those flexures are there which is like this rigid body linkage. So, you could also have come from this side assuming a complaint that is partially compliant and then way a rigid body model we get arrived at here that is we go there and then go here, once you know how these topology is from here also we can get that idea and make that, that is what we have done this is the 4 bar linkage that we have over there are over here. So, these two methods are not really de-coupled one can combine them and get a solution.

Now, again the question how do we know if there is a solution for given specifications that remains in question both the methods.

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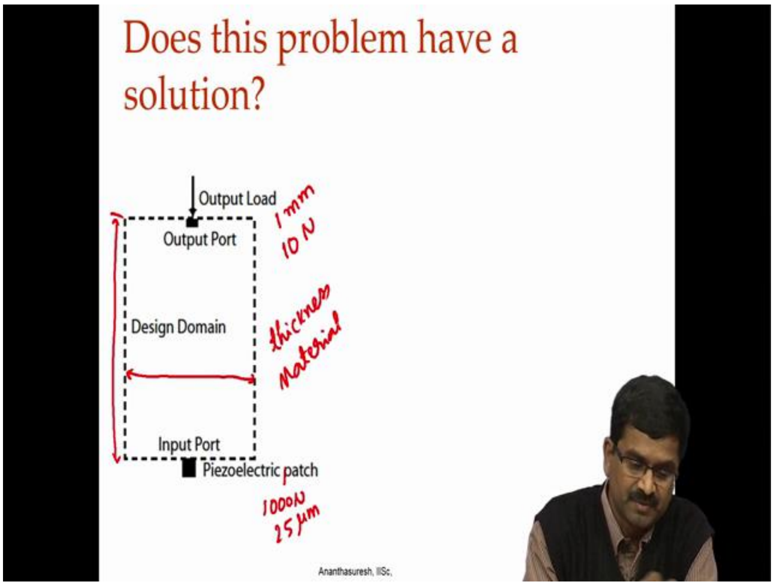

So, now let us ask this question, let us say that there is a problem of some piezoelectric patch is there it can apply some force over certain displacement, let us say that we have some you know 1000 newtons, over 25 microns at the input. Where is the output we want one millimetre right that is 25 into 40000 microns or 1 millimetre we want, against much smaller load let say, I say there is only 10 newton load right, and then we give some dimensions, so this is so much and this is so much and some thickness you know everything we take are possibility material also we can choose. Given this can we say that is there a solution for this problem before, we do it just because an algorithm is does not convert the equations do not a low solution it is not possible for to conclude that this problem does not have a solution. That is in rigid body linkage approach if the 4 bar that we have chosen does not have a solution when you try to solve the equations that does not mean that some other configuration, some other topology would not lead to, same thing which are optimization just because algorithm does not give a solution does not mean there is no solution. So, that question we cannot answer.

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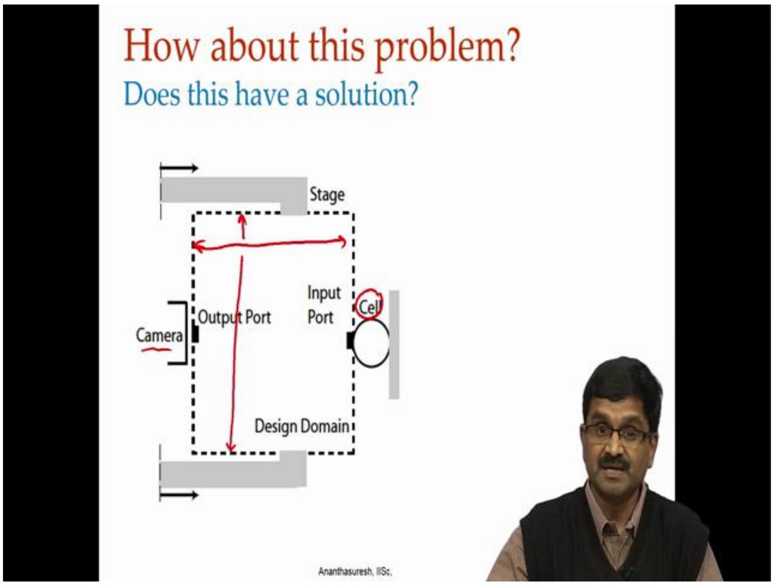

Similarly, if I take another one which is to manipulate a biological cell as it is written here the force will be extremely small here and this place also small and through a camera we want to observe the displacement here, and it should be large enough and there is no force here because, it just moving their external output force here is 0. So, such a problem if we have to be done at micro scale then for given size that you want is the resolution right so that size and this size is if we have is a resolution. So, these methods cannot answer that.

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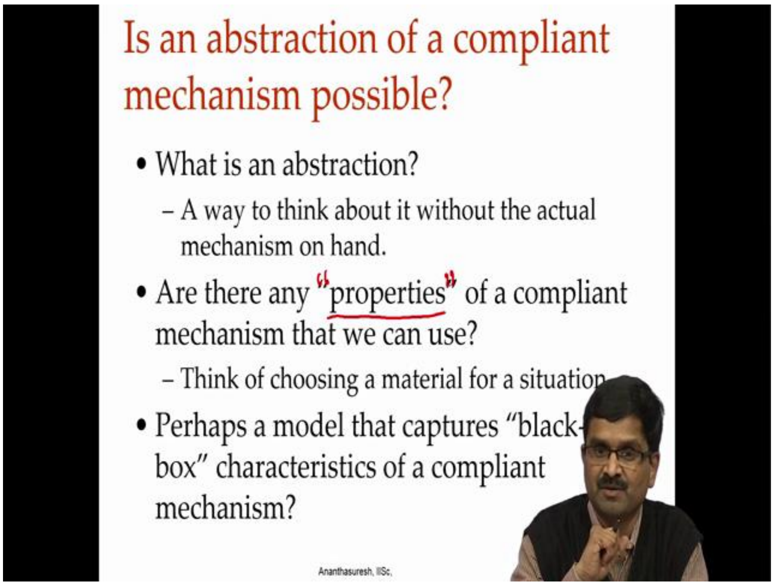

So, what we want is kind of an abstraction of a compliant mechanism that will enable us to think about, what is possible, what is not possible, without having a mechanism on hand or in mind we just have specifications and from there we want to discuss whether there is a possibility of mechanism being there or being designed or not. So, what we want is abstraction. So, we should be able talk about it without talking about the actual mechanism that is what we call something abstract.

So, towards this we ask are there any properties of a compliant mechanism, why we call it a property will become clear later this week in the lectures what by that what remain is that if there are different compliant mechanisms, and let say three of them can do a job the same task that one has in terms of specification. If all three of them can do what is different about those three compare to some others which cannot satisfy these specifications of the user right. There must be some properties let us say there more like material properties, if we say material property youngs modulus of aluminium is 70 giga pascals, stilling (Refer Time: 13:09) pascal some polymers have one giapascal, those are properties. When we try to design a mechanism which we did into use we think of let say for a lasting design (Refer Time: 13:23) modulus then we have to see which material is suitable the range that we have in mind for youngs modulus will decide the material.

In similar way, we can get ranges of some properties that is why we put in codes here, the properties if we have then we can also say this mechanism is better for the application and so forth. So, it is going to be more like a model, again a model like a pseudo rigid body model is going to be a black box model that is we do not want to know what is inside the box it is just that when we try to look at the mechanism at the input or output, we should be able to feel what the mechanism is that is kind of an abstraction that we want.

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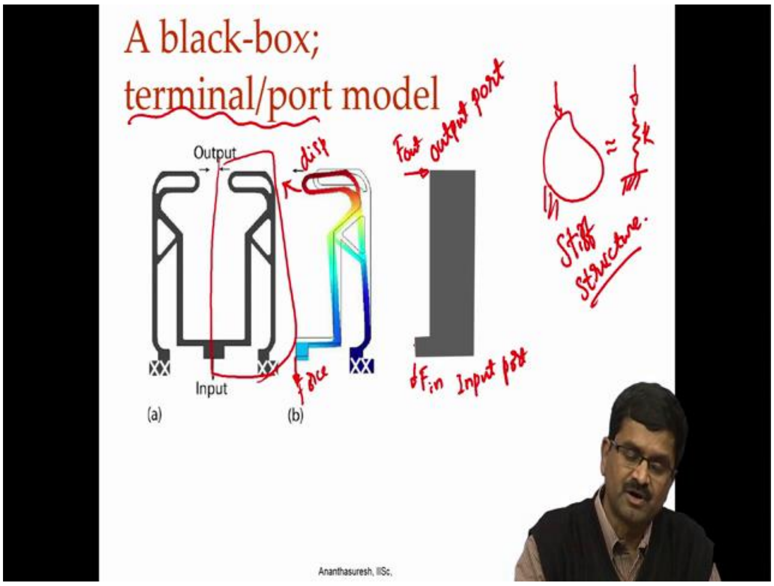

So, this black box if I have a compliant mechanism then if I taking half of it right so if I apply force, here there is a displacement in that direction, force is in that direction, displacement is in that direction right.

So, now this is a black box so if I want to understand whether it can give what I want as a user all I would do is when apply force on the black box I feel some displacement. Similarly, if apply a loading because suppose to move that way move to the right, so apply a force in that way this is we can call it input force, this output force I should see that there is some displacement there is some displacement here and so forth right. So, we do not want to know what is inside, and just have a terminal or port model meaning that this we would be input port and that would be output port. More like what electrical engineers use for resister capacitor (Refer Time: 15:27) things like that transistor they have terminals and what happens said a lot of detail, but what they want is that macro scale behaviour of that, that is what we want. Like normal a structure that is there with some forces then we can replace that with that force and basically is spring right that is abstraction for a stiff structure, this is for a stiff structure where we just have one point and then four supply and then we can have a lamped model like this. But in compliant mechanism we have two points at least because there is input there is output. So, for that we need a different type of model, which is input output port model.

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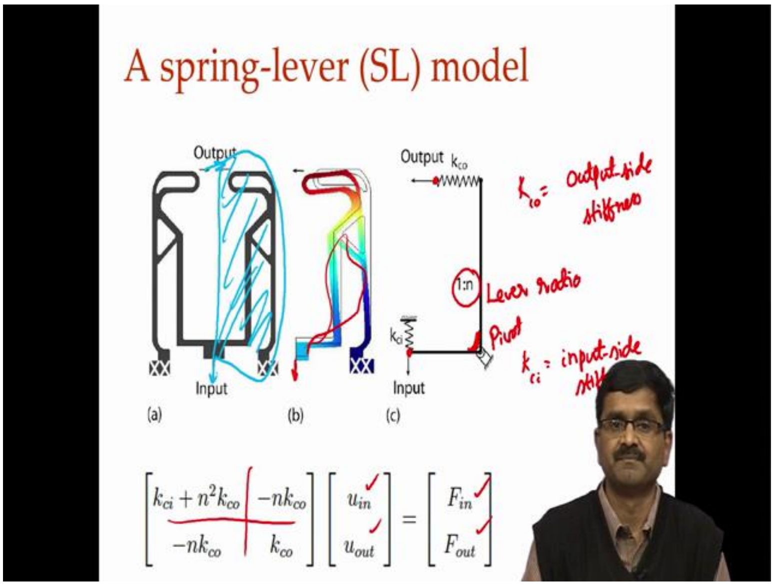

And that is exactly what figure shows here there is a spring, which basically captures what all the mechanisms does on this side when we feel that over there how does it feel is k ci. So, here k ci is input side stiffness similarly, we have k co, that k co is output side stiffness, if you want to play with it on the output side it look like that input point is here output point is there, and then there is lever ratio because most of the compliant mechanisms input output displacement are the same; this n that lever ratio can be a positive value or negative value depending on the directions of input displacement and output displacements. So, have this spring lever model, which is essentially lever because it is a pivot and its one body you can apply forces and try to see how it works and that should imitate the real compliant mechanism. So, if you do that if you look at it mathematically then you have u in, u out 2 degrees of freedom, 2 points their directions surface the 2 degrees of freedom there is F 1 and there is F 2, and we have a 2 by 2 metrics here, that is in terms of this k ci, k co and n 3 parameters we have this. So, this is the spring lever model we do not want to know what is inside everything we cover up, and some something like this if I say we cover up.

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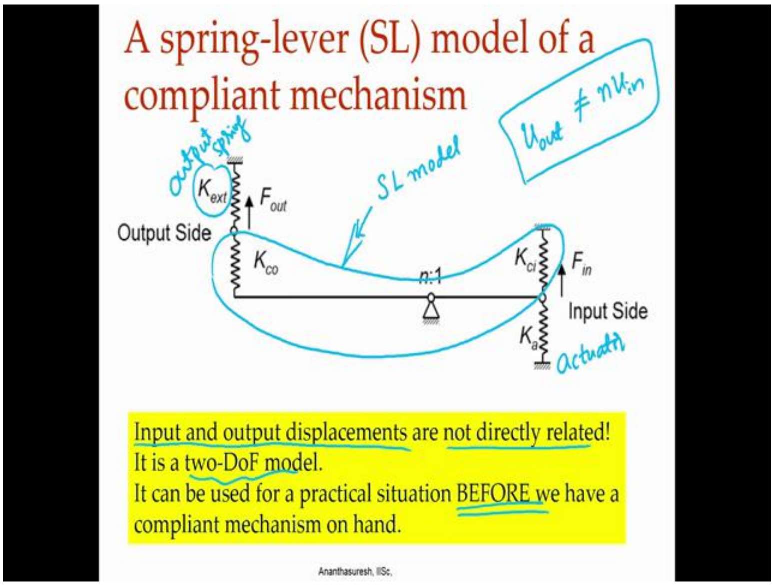

So, from here onwards we covered this whole thing, and not know what is inside here and, but we should be able to feel that there and feel that here that is the abstraction we drawn in board this spring lever and s l model.

So, if you notice this is the input - upper down that you can decide or if this is input moves by let us say u in here this is lever ratios n. So, if this moves up this point is going to move down that is going to be n times u n, but that is not was output is somewhere here. So, let us indicate that to be the output u out. Over all were we this point go there will be n u n, which is trying to bring get down u out is take into up if we somewhere in between. So, we have 2 degrees of freedom for this model.

Now, if you notice we have that model once we have it we can connect this model to an actuator and to in output load are output spring that is in a elastic object if we use model for the box piece, then that are also have stiffness that can also be put in. So, what we can do is we can do this at this model just like in a real device this is our SL model this is the spring lever model, and we can connect to an externals spring or sorry, this is the other one this is actuate stiffness likely this one is our spring lever model.

So, this is a actuator side because input side is where actuator will be this is the external, which is the output spring, external spring we can connect it and we can also see that input output displacements are not directly related meaning we are not saying that u out is equal to n times u in where you saying that it is actually not equal to, u out is a in depend degree of freedom from the inputs side. And it can be used for practical situations before we have a compliant mechanism that is the idea, and say 2 degree of model simple it will also make it complicated by making at a multi degree of freedom mechanism.

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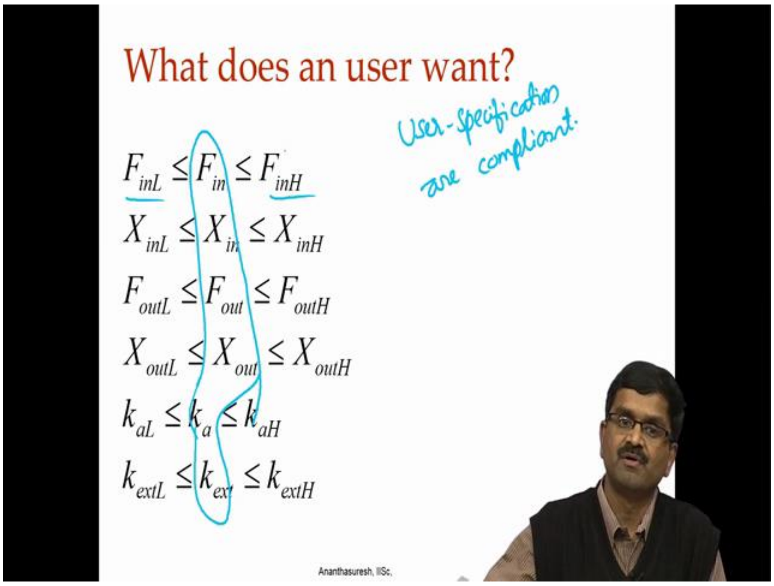

What does the use of want? The user usually specifies, or I have some actuator that can provide. So, much force the actuate can move by, so much displacement and a output I have I want this much displacement it has to move against some load there is an external spring and there is an actuator stiffness every actuator will have its own stiffness. So, you can specify that, that the usable specify user may not specify that I have a force this way it has to go that way for which we have discuss the topology method, user may say it has to go along an curve like function generation, which we have discussed using linkage based approach.

Now, user will specify not a single number, but and upper bound and a lower bound high and low values in every parameter that is here, all parameters we are trying to let the user specify upper and lower bonds. There is a reason is that users specifications are also flexible compliant in the sense that users can change their preferences their numbers based on, what is possible? What is not possible, like we do when we go to buy something in the prices to high we will be willing to reduce some features or prices low will add some more features and buy something we like.

Similarly, here user can specify upper and lower bonds and then based on whether solution is there or not and they can be changed again, these about knowing the fusibility of these specifications before you have mechanism on hand.

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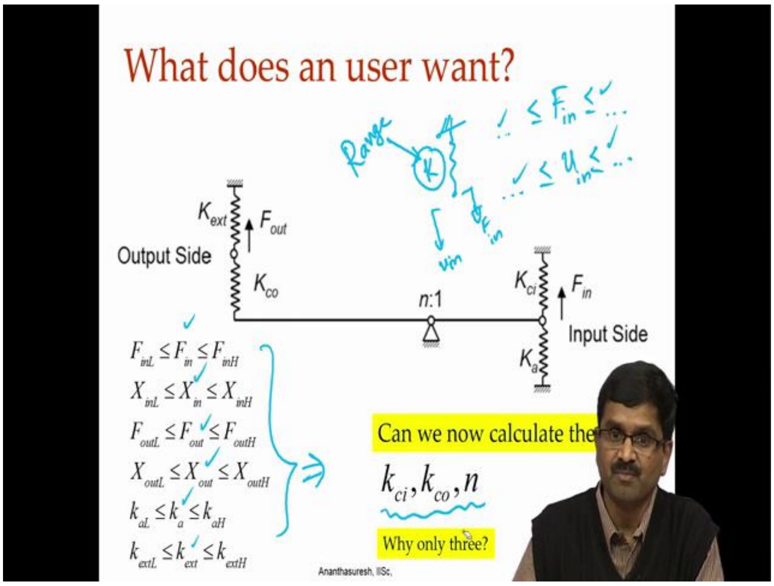

So, we can put this SL model and put this in equalities there are 6, because we have 1, 2, 3, 4, 5, 6, quantities and they have upper and lower bound, so that basically 12 in equalities. Now, given these inequalities that is users specifies the question that we have ask is can we calculate the ranges of k ci, k co and n; it is like if I have just spring, it applied some force and I want some displacement here, let is call at u in, F in. Then if somebody gives me upper and lower bonds for F in and u in that is will have F in some number here some number here and then u in, some number here and some number here. We can come up with range for this k, by giving this numbers these four numbers lower limits upper limit where actually we can make a sense of what should be the range of stiffness of the structure we want.

Similarly, here we can get range of values for k ci, k co and n from these specifications. We will discuss how could do it mathematically, but understand for now that it is possible can we calculate, yes we can calculate k ci, k co and n why only three if you ask there can be more.

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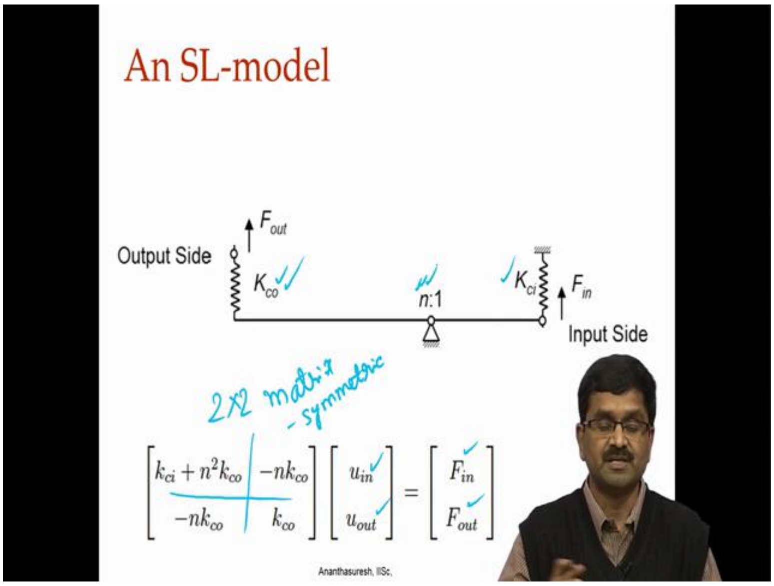

If we think about that and SL model when you take, when we write this input output relationship that is u in F in, u out F out will have a 2 by 2 matrix here. This 2 by 2 matrix is symmetric, 2 by 2 matrix will have 3 parameters that is point we have k ci, k co and this n so as per 3 parameters here.

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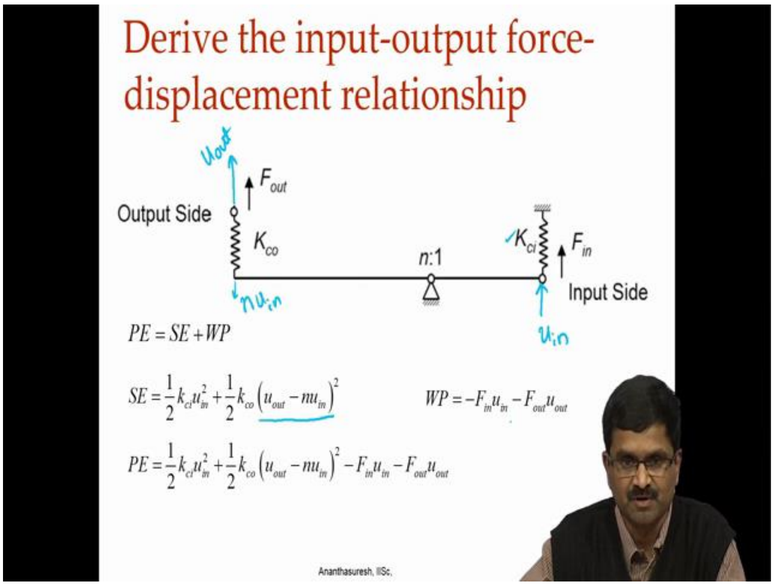

Now, let us say we want to derive the input output relationship for this spring lever model to go with the simple problem with potential energy strain energy plus work potential, strain energy we can write we have the spring here k ci, k ci times u in let us say that is u in and that is u out. So, k ci time u in square a strain energy plus k co times u out minus n u in because if this is u n this is going to go down because the lever ratio n u in. So, if it is moving out its coming down. So, we have to take that into account and write it here.

Here we assume that n is positive, here in the sense that it is inverting also I am subtracting otherwise you could put plus also here, because the spring will stretch so much when this is moving up this going down we have to add, but here we are taking that n is positive rather than being negative, negative inverts it. This (Refer Time: 26:21) inverting, but otherwise it could be positive. Do not worry about that it will be different and addition n is pass to the negative, we have that and then we can write work potential minus F in times u in minus F out times u out and rewrite the potential energy strain energy plus work potential.

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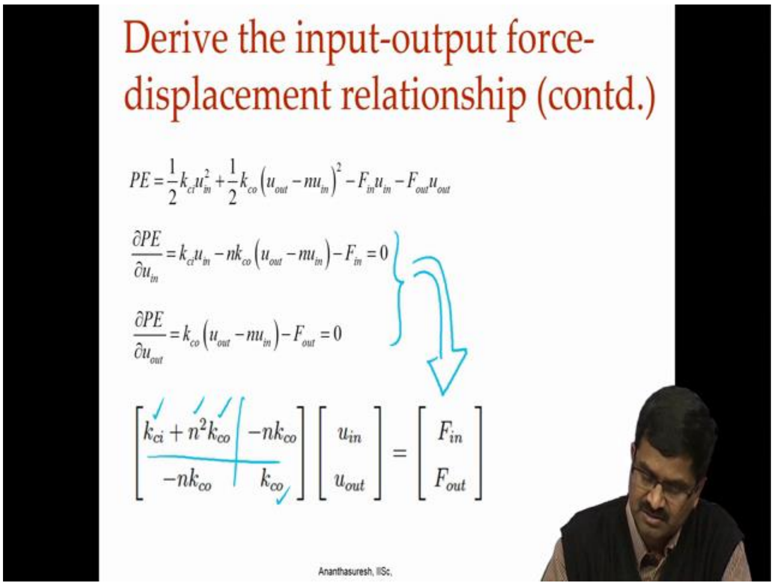

And for equilibrium equations you have to take the partial delivery of potential energy with respect to u in or static equilibrium also respect to u out equated to 0, we have two force displacement relationships and we can get what we have right, u in and u out we get this 2 by 2 matrix. So, are the inputs side it feel like k ci, but it here also feel k co, but n square times is more like transformers if you think output side resistance we will feel like n square number of turns there. So, similar here or gears for that matter will have this stiffness felt output side will also we felt at the input side, but n square times and k co go outside and coupling 1 is minus n in k co. So, we can get that basically these equations liveries to this input output relationship.

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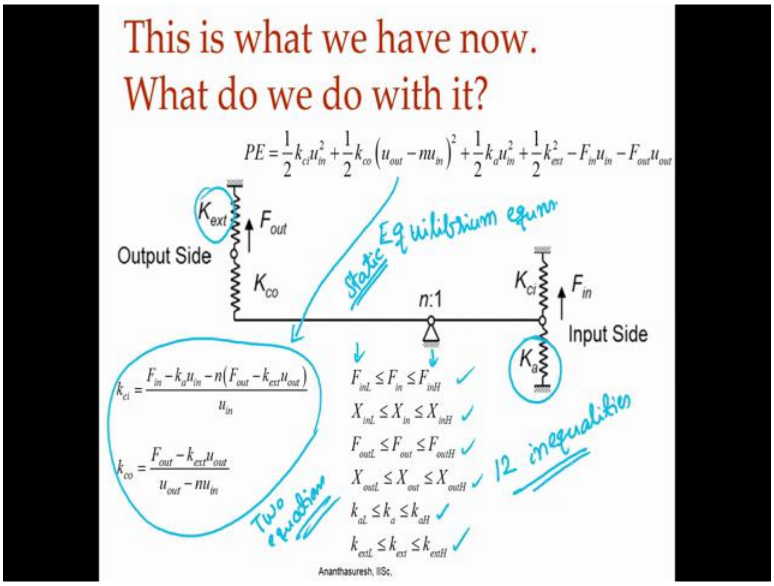

Now, if we also put in the k external and k actuator (Refer Time: 27:49) actuator will have stiffness then you can have the user enter those also along with these four that we said. Now from here if I write equilibrium equations I get these what we just did by writing potential energy it is derivative with respect to u in and u out will give two equations they are this static equilibrium equations that is important to know we are going to about dynamic next lecture, studied usually we have two. Now can you solve these inequalities and these two equalities to arrive at range of values for k ci, k co n that satisfy the user input, users is going to putting these numbers lower limits upper limits and the equations can you do that right.

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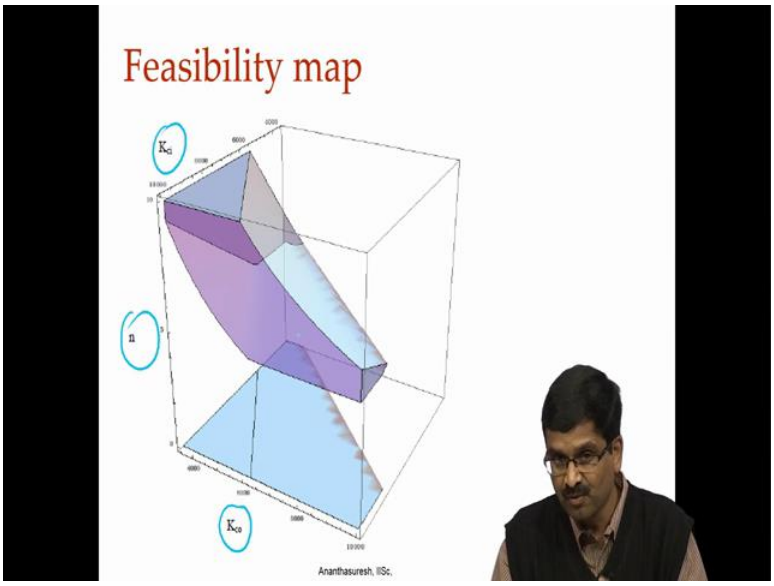

Turns over that you can do and that is what we call it feasibility map that is we do not know a mechanism ate that is the idea; before we have mechanism on hand we would like to know whether this specification due to some feasible values here it is feasible. So, if we are inside here the n the 3 were showing it 2 dimensional plot here, were is k ci here there is k co and the n is indicated n is some value if you click there will be a value that is why there is a gray scale map in reality, what will have is a volume the feasibility volume here with k ci 1 axis k co another axis n as a axis.

So, were inside your specified are satisfied. So, we have the inequalities, 12 inequalities and 2 equalities, equilibrium equation these are the 12 inequalities and we have 2 equations, here 2 equations let us solve this normally we are used to solving equations, but we also can do inequality solutions will discuss how it is done in one of the future lectures understand that we can get a feasible volume like that and the projected space for k ci k co it can be feasible 2d map it is a map. So, we are inside the map then your solution your specifications are possible, when we are outside there not possible at least u know you get a finite volume you know that your specifications are actually possible.

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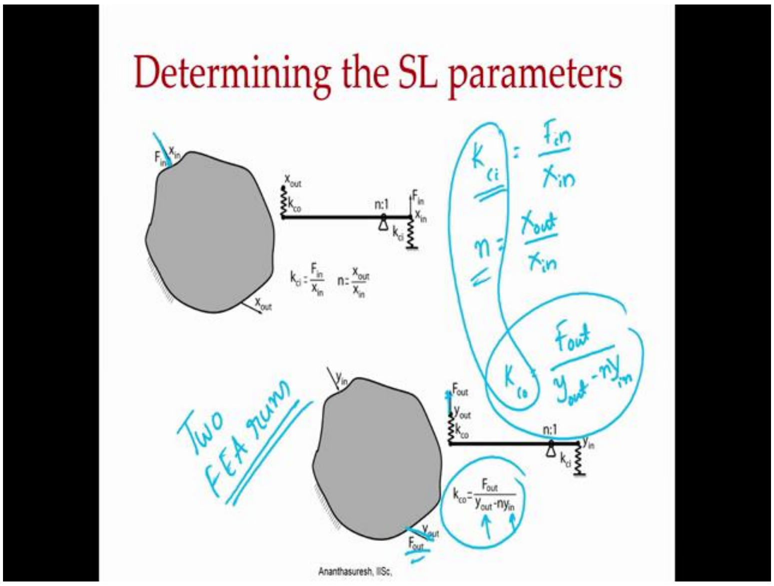

So, now how do you determine this parameters given a compliant mechanism, what it do is they are finite element analysis you apply input force when you measure the input displacement in this case x i n and then x out output displacements, then you can get k ci as whatever input force you apply it and divided by input displacement that you have measure and n also x out by x in at we have to apply this force and measure x in and x out they are and F in of course, you apply. So, you get these 3 parameters in order to calculate the k co you have to apply F out now. So, we have to apply F out in the direction, whichever we have chosen in this case is up like that you apply that then you measure y in and y out and get k co as given here, k co is going to be whatever F out you apply divided by y out minus n y in.

So, we can get that also we have to do 2 finite element analysis t runs a needed here, but we can do it or if you want to do in experiment you can do that and get them. So, when you get these 3 numbers k ci and n k co they become properties, because two compliant is you can have this same numbers. So, if they are like Youngs modulus on ratio thermal conductivity they are like properties of compliant mechanism, we see why we are calling them properties in a later lecture their characteristics of compliant mechanisms.

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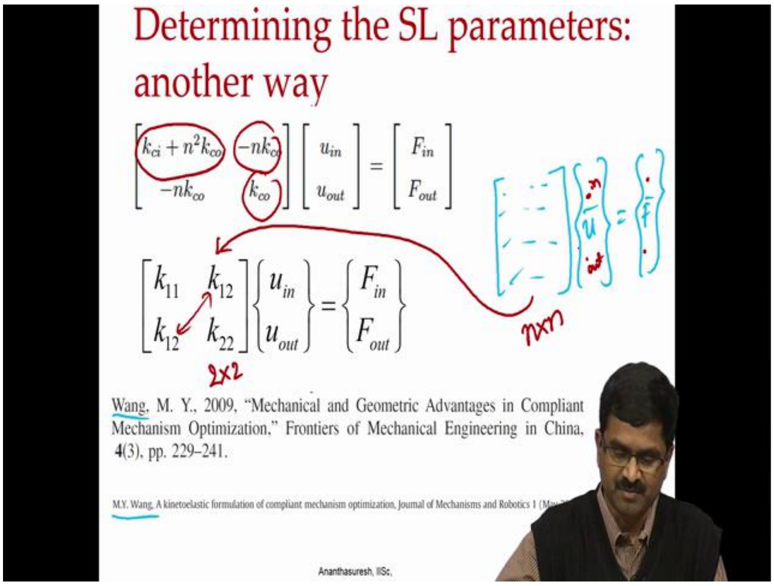

There is another way professor Michael Wang in two of his papers came up with the way he would work is if you have the big stiffness matrix of a compliant mechanism or (Refer Time: 32:30) structure, there is displacements and there is forces. And we have all these things this going to be one variable somewhere here, which is the input there be one that is output somewhere and there be corresponding F in and F out what he argues is that and its actually quite also straight forward that you can reduce all that to this. In other words you can solve for the remaining degrees of freedom other than the input displace out displacement then that matrix, big matrix.

If you have n degrees of matrix of freedom n by n matrix you can reduced 2 by 2 matrix where we will say k 11, k 12 that is symmetric and k 22 and we can say k 22 is nothing but k co and k 12 is nothing but minus n times k co and k 11 is k ci plus n square k co. We can do this way, you have the big one you manipulate all of these you can get sometimes this is more efficient other tie there is a efficient in any case with two finite analysis you can easily get this 3 parameters.

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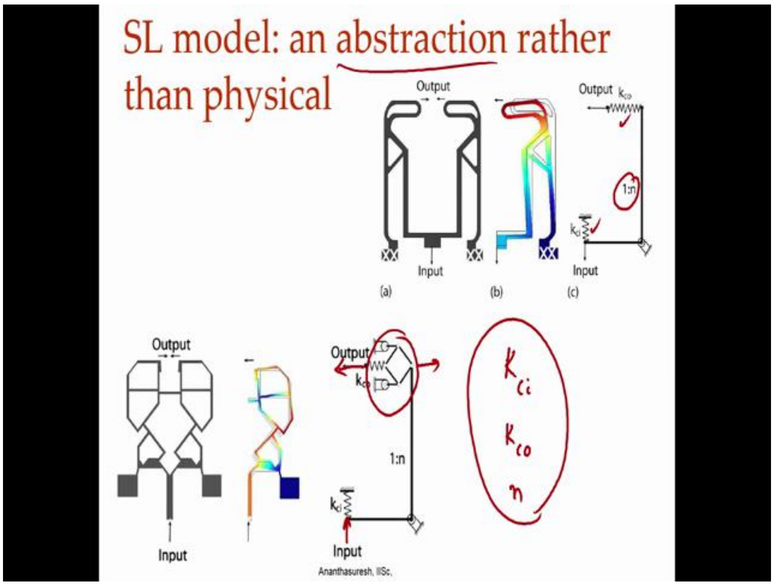

Once we have those we can talk about this subtraction, subtraction meaning that we have a real mechanism and we put a spring here and level here with a pivot we are thinking, but it is not a physical thing it is merely in abstraction, because if I have a mechanism like this where I have input that way now when I put a pivot here output has to go in that direction. But now if you do this it is going to go the other way right we apply this is going to go this way, but output is going that way we can come up the some physical mechanism for it to invert the displacement to go to other side as it happens here. But it is not really needed what we think of this is that they are all three properties of compliant mechanisms that are important of static applications this more of an abstraction.

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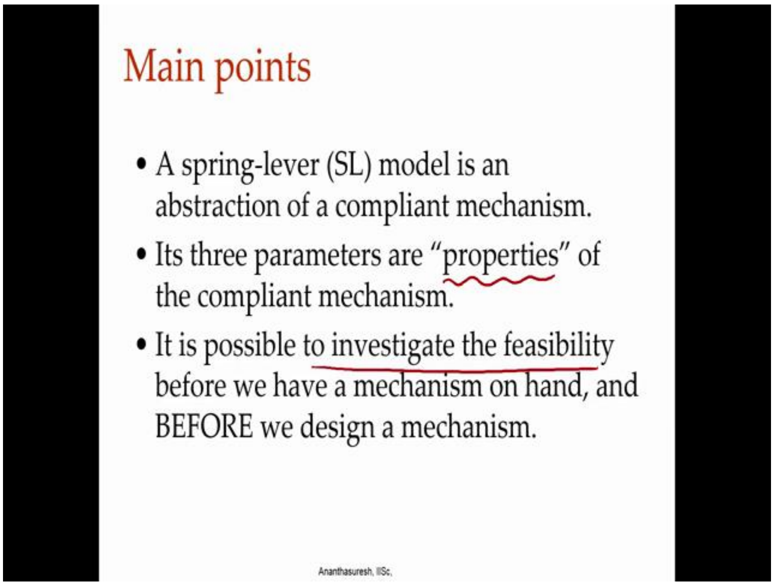

Now, to conclude this lecture we have a discussed what we call a spring lever model, which is an abstraction not a physical model and like pseudo rigid body model it is an abstraction in general, but sometimes it cannot be a physical as we are seen an example, and its 3 parameters k ci, k co, n and can we call properties and it is possible to investigate the feasibility as we just showed that we can construct the feasibility map by solving the inequalities and the equations. So, it is possible feasibility what we take advantage of. So, that we will know whether something is possible or not before we have the mechanism on hand what we have discuss today can be found in these two papers.

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There is first paper, and second paper this is a mechanism and machine theory and 2012 and this is 2010 a paper from journal of mechanism design. It was based on Sudarshan Hedges PhD thesis at the Indian institute of science, you will look up this papers will be coming back to the content of these two papers in the remaining 5 lectures of this week.

Thank you.