Compliant Mechanisms: Principles and Design
Prof. G. K. Ananthasuresh
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 36
Cam-flexure clamp-case-study

Hello, we have discussed a lot about topology optimization last week and this week, and the last lecture we also touched upon shape optimization and size optimization towards the end of last lecture using what are called wide Bezier curves. Now let see what one does when we encounter a practical problem where you want to design a compliant mechanism. So, we are going to discuss a case study later on the course we will see a lot more of these case studies at to see how we can put together all the theories to some good use to solve practical problems. Today’s case study is called a Cam-flexure clamp let us look at what that is and, how we can go about designing?

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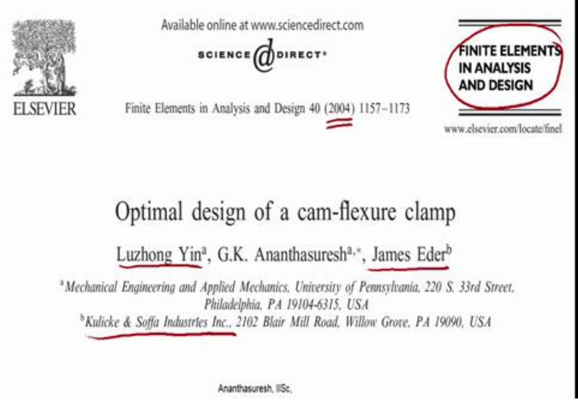

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Finite Elements in Analysis and Design 40 (2004) 1157–1173

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Optimal design of a cam-flexure clamp

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Ananthasuresh, IISc,

So, here I am going to take a paper that is; that was published a very long time ago 2004, in a journal finite elements in analysis and design and to say that it is a practical problem we have a co-author here, Doctor James Eder who was at the time working in Kulicke and Soffa Industries in Pennsylvania in the US while I and my post Luzhong Yin were at university of Pennsylvania and Philadelphia.

So, this problem actually came from the industry, whether; we are able to solve a problem and I am not going to reveal actual details of what the industries problem is, it is always you know tell secrets for them, but because is a co-author we have put only what the company allowed us to disclose in a publication. After the work was done, we wrote this paper where we are able to share with rest of the world how we solved a practical problem; clearly an industry person is involved it is a practical problem. What we will do is look at that paper and try to go through it, so that we understand what the problem was, how we used topology optimization, little bit of shape optimization and then size optimization to solve a problem.

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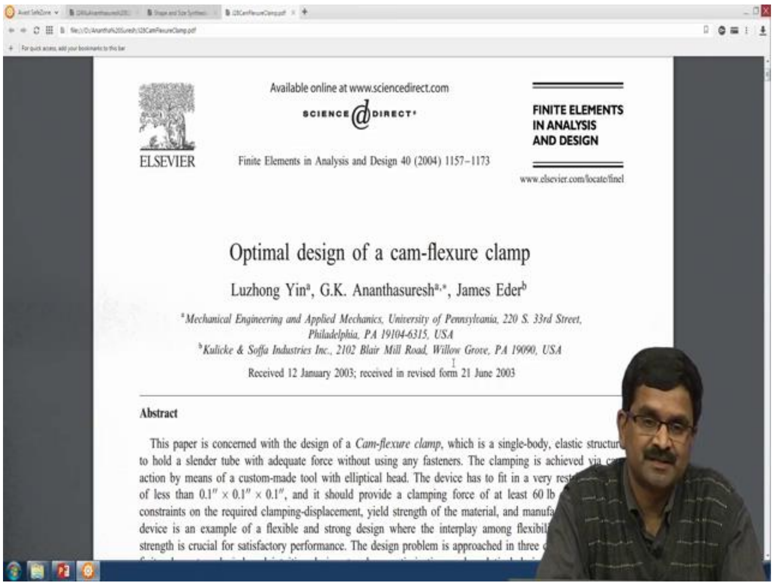

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Abstract

This paper is concerned with the design of a *Cam-flexure clamp*, which is a single-body, elastic structur
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of less than 0.1″ × 0.1″ × 0.1″, and it should provide a clamping force of at least 60 lb
constraints on the required clamping-displacement, yield strength of the material, and manufa
device is an example of a flexible and strong design where the interplay among flexibil
strength is crucial for satisfactory performance. The design problem is approached in three

In fact, interesting part of this case study is that when you encounter a problem you come up with your specifications that is you say I have this much area and this is where the force is going to come this is where I want output displacement and this is how much off stress that I can allow, so far we have not talked too much about stress and there are number of papers that try to deal with stress constrains or strength considerations in compliant mechanisms in the context topology optimization or shape or size we are not talked about all of that, but it is an important things in practical applications because we need to keep the components strong. In fact, at the beginning of the lecture we said the entire philosophy of compliant mechanisms goes towards creating compliant and strong designs.

So, strength is really important enhance stress considerations are paramount in compliant design here we had such a constraint. Now how much maximum stress do we allow in a compliant mechanism? We can make up all your specifications and go to the topology optimization or shape or size optimization and hope that there is a solution to your problem. But how do we know at the beginning that the shapes are the not the shape the design domain that we have set for ourselves actually can give a solution.

So, that was the case here when Doctor James Eder came to us and he had already tried all his methods using finite element analysis and a design expert is that was there and Kulicke and Soffa and they are found that there was no solution or at least they could not find, when they came we started applying our topology optimization solutions taking that we can solve any problem, but then there were issues or then we got around that and I will tell you that story.

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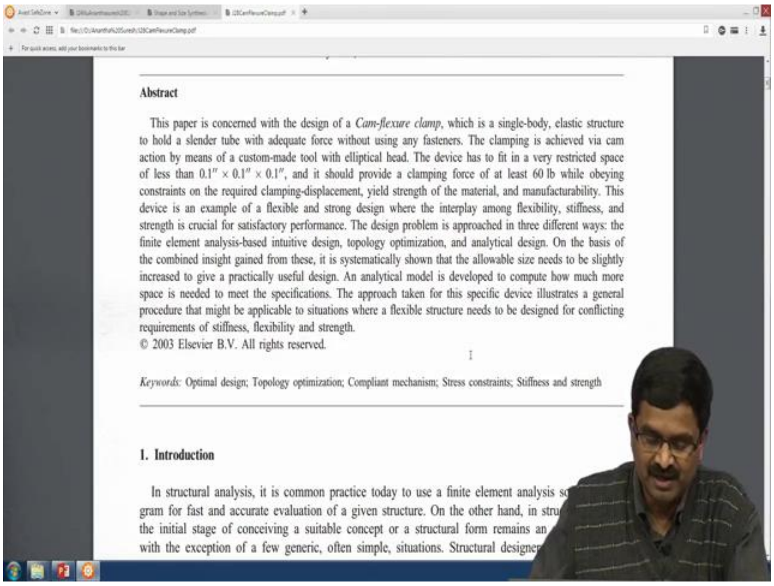

Abstract

This paper is concerned with the design of a *Cam-flexure clamp*, which is a single-body, elastic structure to hold a slender tube with adequate force without using any fasteners. The clamping is achieved via cam action by means of a custom-made tool with elliptical head. The device has to fit in a very restricted space of less than 0.1″ × 0.1″ × 0.1″, and it should provide a clamping force of at least 60 lb while obeying constraints on the required clamping-displacement, yield strength of the material, and manufacturability. This device is an example of a flexible and strong design where the interplay among flexibility, stiffness, and strength is crucial for satisfactory performance. The design problem is approached in three different ways: the finite element analysis-based intuitive design, topology optimization, and analytical design. On the basis of the combined insight gained from these, it is systematically shown that the allowable size needs to be slightly increased to give a practically useful design. An analytical model is developed to compute how much more space is needed to meet the specifications. The approach taken for this specific device illustrates a general procedure that might be applicable to situations where a flexible structure needs to be designed for conflicting requirements of stiffness, flexibility and strength.

Keywords: Optimal design; Topology optimization; Compliant mechanism; Stress constraints; Stiffness and strength

1. Introduction

In structural analysis, it is common practice today to use a finite element analysis
gram for fast and accurate evaluation of a given structure. On the other hand, in
the initial stage of conceiving a suitable concept or a structural form remains an
with the exception of a few generic, often simple, situations. Structural

It is called Cam-flexure clamp it is a name that we made up for it.

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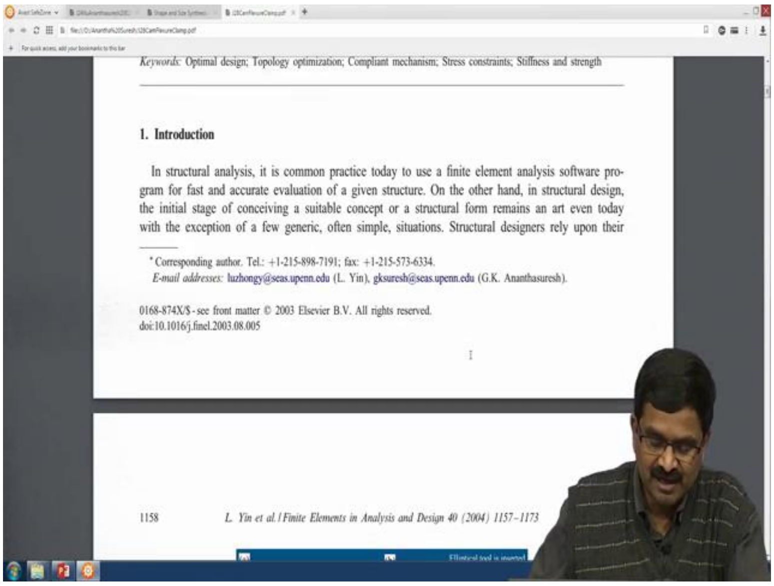

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1. Introduction

In structural analysis, it is common practice today to use a finite element analysis software program for fast and accurate evaluation of a given structure. On the other hand, in structural design, the initial stage of conceiving a suitable concept or a structural form remains an art even today with the exception of a few generic, often simple, situations. Structural designers rely upon their

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Fig. 1. (a,b) 3-D views of the cam flexure clamp.

intuition and experience to obtain a starting design and improve it through repeated analyses. Subsequent systematic shape optimization and optimization of dimensions are not uncommon. However, if the designer begins with an inappropriate structural form (often called the *topology* or *layout* to indicate how many holes there are and how different parts are connected to each other) subsequent optimization might not lead to the best possible solution to the original specifications. In such cases, alternate topologies are considered and the process of refining the design is repeated. It is desirable to minimize or even avoid such trial and error type manual iterative design. Modern methods of topology optimization are helpful in achieving this goal [1]. In this paper, we present a structural design case study to illustrate this point. The device undertaken here is not meant to be merely stiff but also flexible and strong. These types of devices are currently being studied under the name of

And it is actually an interesting device it is quite small actually.

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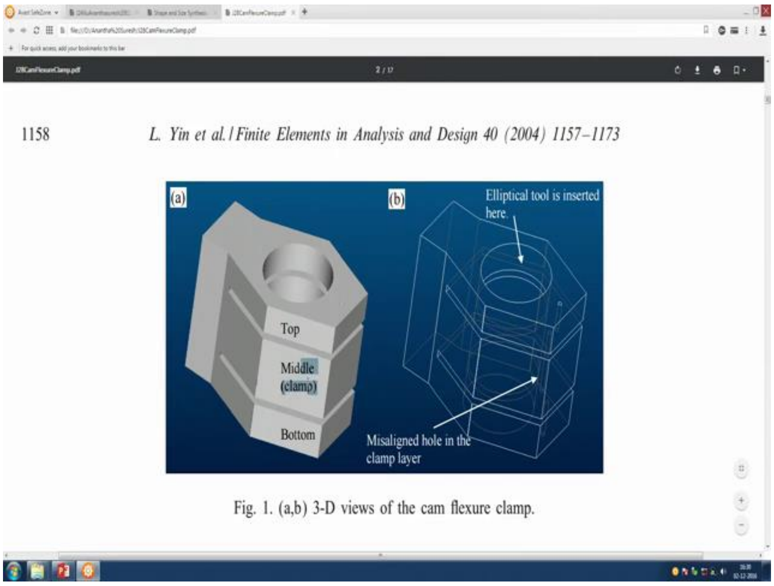

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Fig. 1. (a,b) 3-D views of the cam flexure clamp.

So, if we look at the problem specification that is over here. It looks like a block; it is actually quite small well I have a l l prototype to show this is the optimization design. So, you should not look at the design right now. It is actually a device that this prototype it shows where I am looking at the actual device with a final design.

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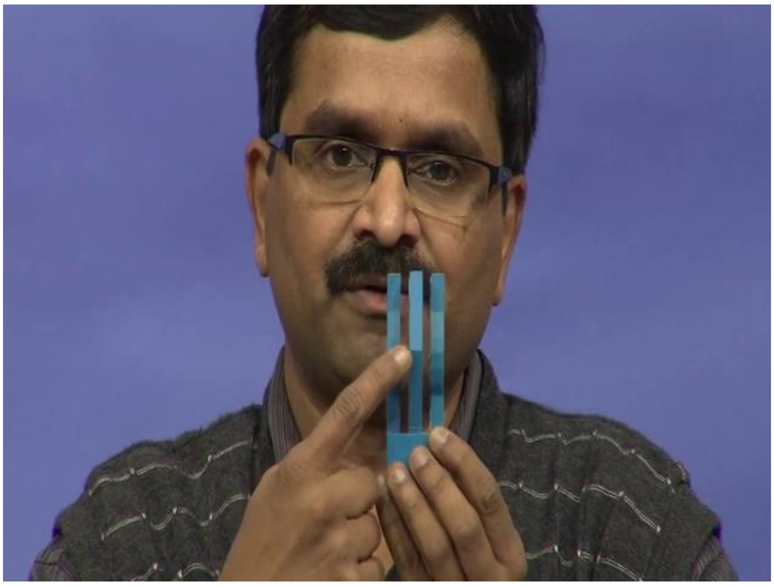

So, what it is, it has 3 layers as you can see. So, we have layer 1, layer 2 and layer 3.

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Cross section if you see top layer and bottom layer are identical, the middle layer there is some design that is the compliant design here and if you look at the holes that are there in the top and bottom they are just normal circular holes where in the middle layer has an elliptical hole.

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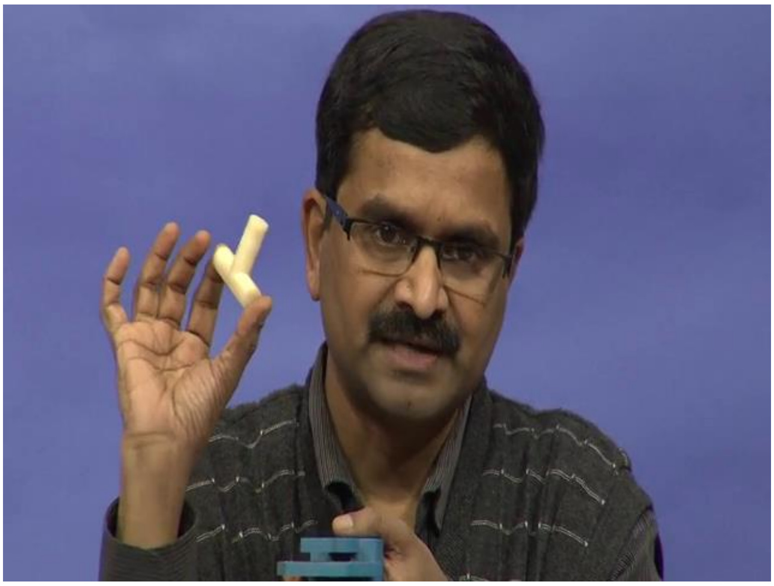

And I have a more like a wrench or a spanner which has actually elliptical cross section here; I can put here and then turn, so that was the idea.

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So, I put in the middle one and turn in the process something happens and there is a comet (Refer Time: 06:14) function that comes in. So, let us go back to the file now. We have the top layer and then bottom layer, we have top layer bottom layer middle one that is the clamp, that clamp layer. So, that clamp layer is where we had to put our compliant mechanism in such a way that when I put this elliptical tool in here and turn the elliptical hole that is there in middle layer and turn it by 90 degrees then that would make this middle portion extend towards the other side that is further, this is the fixed one further from the fixed side.

There are holes, there is a tiny hole here, tiny hole there and those are aligned where as the middle one is misaligned, it is little bit off we need to use the elliptical tool to bring it and align with this, so that I can insert think of this is a wire actually Kulicke and Soffa does a wire bonding of electronic equipment, they make machinery to do wire bonding. So, think of that wire there, that wire when we put in to aligned holes.

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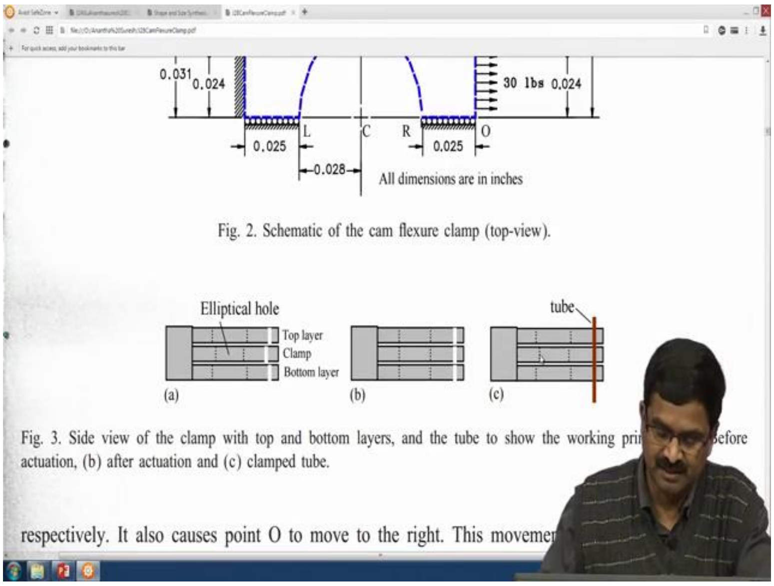

Fig. 2. Schematic of the cam flexure clamp (top-view).

Fig. 3. Side view of the clamp with top and bottom layers, and the tube to show the working pri efore actuation, (b) after actuation and (c) clamped tube.

respectively. It also causes point O to move to the right. This moveme

For that let us actually look at the cross section here, whatever I talked about the top layer bottom layer and the clamp layer. Clamp layer is the one that we need to design this layer it has a misaligned hole in it. This is a misaligned hole and this is the elliptical hole in the center. Right now minor axis been shown rather than major axis when we put that elliptical tool at the top like this and then turn by 90 degrees then this elliptical hole part of it will move this other hole which is misaligned over here.

So, all the holes will be aligned like this, like in the part b here and at that time you insert a tube or a wire a cylindrical tube right, we just insert it now when you insert it what happens is this middle layer which is been stretched towards a right cannot go back, wants to go back because elastic segment wants to go back cannot go back in that process it will very tightly hold this wire. It is all this hole clamp device is took clamp this tube in place and should be very quick this elliptical tool comes it charge by 90 degrees things are aligned tube comes in and then you remove the elliptical tool it is hell now the misaligned holes are aligned and it holds. So, if you think from the compliant mechanism blue point here are the specifications.

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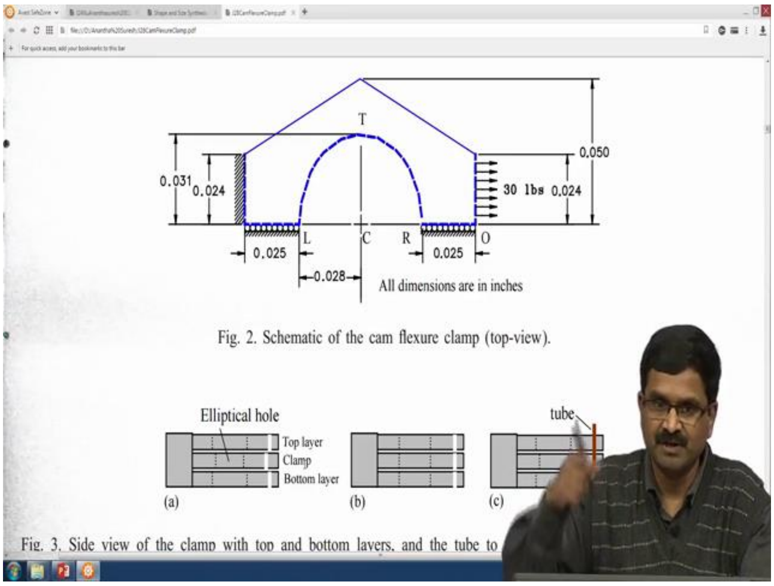

Fig. 2. Schematic of the cam flexure clamp (top-view).

Fig. 3. Side view of the clamp with top and bottom layers, and the tube to

In fact, they had very strict design domain specifications, all the units here are inches where this is united states where this work was done, all are inches. So, the height of this is 0.05 inches as we can see here the height from here is 0.05 inches and they said that you cannot go outside this blue region that is all their company had restriction in the machine say complicate machines in which this is small part, in order to make it metal actually. So, compliant mechanism as we said at the beginning also is not just for plastics it is for any material, if they want to do with metal in particular there were actually thinking of making with titanium because, that has good strength and good a (Refer Time: 09:57) modules. So, it is a small part they can afford to do it in titanium.

There is a hole here elliptical hole that we see the dotted line in which that elliptical tool is put took turn it. It is a complicated actuation is not like a our usual here is a force, there is a displacement type of thing it is the hole ellipse that is like this had to be turned 90 degrees. So, finite elements analysis also quite tricky, now we have to do topology optimization to see what should be the distribution of material in side this, they did not have any volume constraint they said that as long as you do not go outside this region that is 0.05 inches all others are there.

Now, for example, this is 0.025 inches same thing here this is 0.028 this half another half. So, all the things you know all the dimensions are here, if you want to fill this whole thing you are take only symmetric top half here. So, looking at the top view this ellipse will be completed this (Refer Time: 11:00) will be completed and so forth. Now the idea here is that, the misalignment that we see between these two the larger it is the better for them why is it better if it is larger misalignment when apply a force and align it where it tries to go back if you think of the middle clamp layer as a spring we are moving this spring by a large displacement if there is some spring constant k it only uniform. Let us assume that it is a caution spring constant k times that delta how many how much misalignment you have covered, k time delta will be the force with which that tube will be held. When the misaligned holes will you said the tube much like this pen when it was to go back now the tool is removed and it will try to hold the middle layer will hold it very tightly.

So, there is a requirement for maximum output displacement when input is given for this elliptical tool when it turns this elliptical hole you want this point were the hole is that point you wanted to move a lot towards right in this figure, towards right side we want to move a lot so that we will get large force of clamping. That is sounds good, I mean you would have already tried before coming to us there if put of entire material and make it you know move a lot right and you get a lot of clamping force, but then when you take a structure and move it by large displacement the stresses are going to be very high.

So, that is where it was a problem. He had put 120 ksi that is 120 kilopascals as the limit of the stress with all the consideration, they have the titanium material whatever their factor of safety was we said do not exceed the stress of 120 ksi, and a force of clamping they wanted certain force value as well. So, you want to have so much clamping force the same time stress should not exceed particular value and highly restricted design domain you fill that whole thing and try to move it here and there is a little hole here that aligns and he had found that the stress will be so much high when you want certain clamping force.

So, misalignment if you keep it very low your stress will be low, but it would not have enough clamping force. Here is again a compliant mechanism problem where there is output displacement is needed there is a flexibility one in this case misaligning the misaligned hole aligning it, we need to move this hole from here to align with this top and holes in the top and bottom layer that is a displacement or complaints requirement and then the clamping force is stiffness requirement that is what he came up with.

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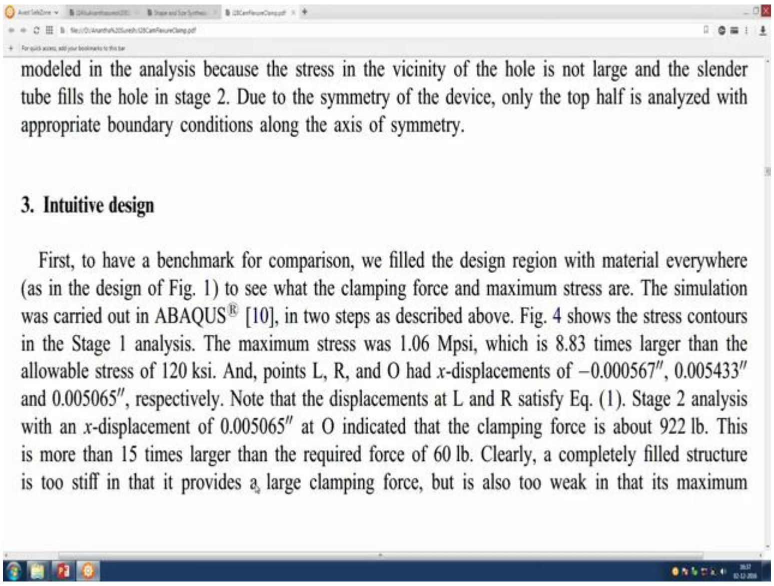

modeled in the analysis because the stress in the vicinity of the hole is not large and the slender tube fills the hole in stage 2. Due to the symmetry of the device, only the top half is analyzed with appropriate boundary conditions along the axis of symmetry.

3. Intuitive design

First, to have a benchmark for comparison, we filled the design region with material everywhere (as in the design of Fig. 1) to see what the clamping force and maximum stress are. The simulation was carried out in ABAQUS® [10], in two steps as described above. Fig. 4 shows the stress contours in the Stage 1 analysis. The maximum stress was 1.06 Mpsi, which is 8.83 times larger than the allowable stress of 120 ksi. And, points L, R, and O had x-displacements of $-0.000567''$, $0.005433''$ and $0.005065''$, respectively. Note that the displacements at L and R satisfy Eq. (1). Stage 2 analysis with an x-displacement of $0.005065''$ at O indicated that the clamping force is about 922 lb. This is more than 15 times larger than the required force of 60 lb. Clearly, a completely filled structure is too stiff in that it provides a large clamping force, but is also too weak in that its maximum

So, when the problem came, we thought that we will put it through our topology optimization and then we thought we will define it later is size optimization. First we went to topology optimization put down and then it was not easy. So, we try to first see how the problem is we looked at it some intuitive design, where we thought that let us put me till everywhere that we had tried, but we have to do it ourselves in (Refer Time: 14:27) tried, but we wanted to do.

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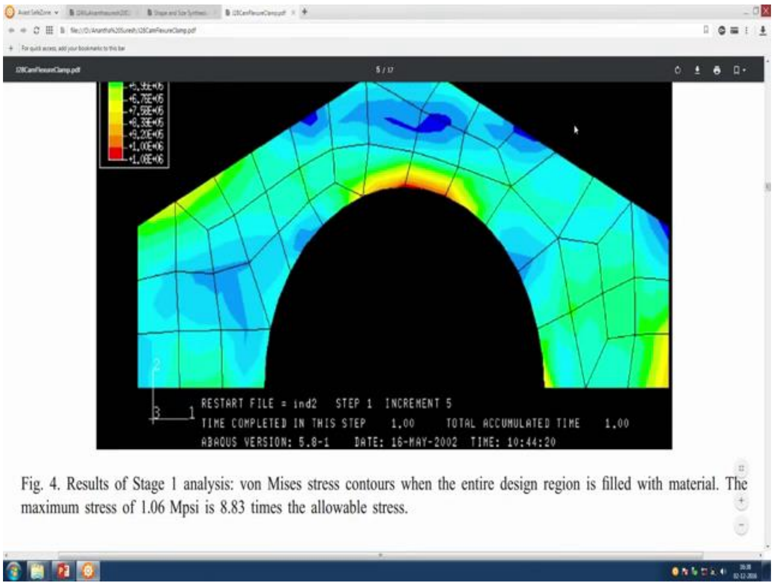

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Fig. 4. Results of Stage 1 analysis: von Mises stress contours when the entire design region is filled with material. The maximum stress of 1.06 Mpsi is 8.83 times the allowable stress.

Again this is an ellipse it is not a circle, because the elliptical tool is a one that turns at to move to the right here we put material everywhere and the stress in that was one mega psi that is 8.83 times the 120 ksi that they have put a limit. So, clearly putting material everywhere it gives 8 times stress. So, what do you do what where do you put a hole. So, we have now there is no volume considered in this problem, this is a small part you fill the whole thing with material and if you try to satisfy the clamping force requirement then you have a lot of stress 8 times that stress almost have a magnitude mode.

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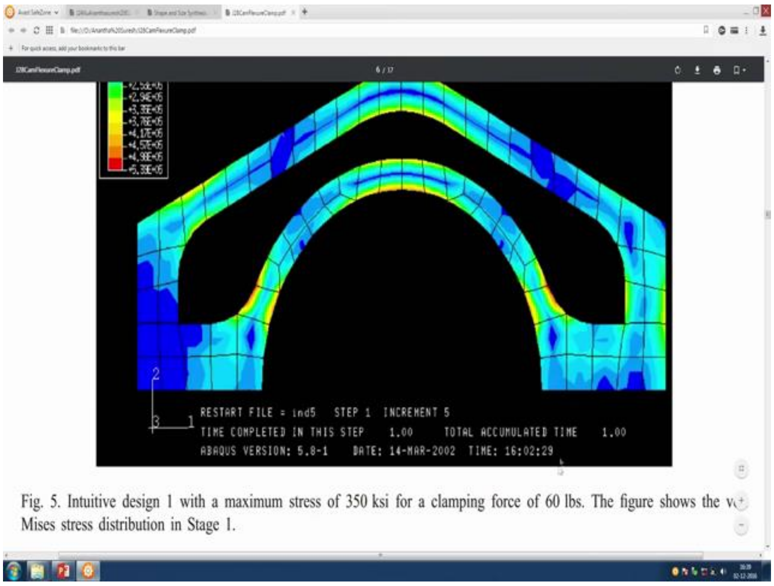

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Fig. 5. Intuitive design 1 with a maximum stress of 350 ksi for a clamping force of 60 lbs. The figure shows the v Mises stress distribution in Stage 1.

So then we thought intuitively, let us you know make a hole not topology optimization just initially this remove some material there is a beam there, there is a beam here curve beam curve beam. Let us see what happens then we were able to be reduce the stress to a 350 ksi, but in the process the clamping force were also reduced because, it became flexible. So, by making flexible we made it strong, because when you flexible we will do it flexible spring, we do not get much force there and a stress somehow was got and down to 350, but still further from 120 ksi that is allowed.

So, intuitive whatever you try it does not work. So, this is one of those cases simple here is the force, there is a displacement type of thing people may be able to solve intuitively, but presented a practical problem one needs a design methods. In fact, some of the some of us who work in compliant mechanism area think that people do not use compliant mechanisms even though they have many many advantages because, there are no accessible design methods. So, now many people have worked you have a lot of design methods that is what we discussing in this course.

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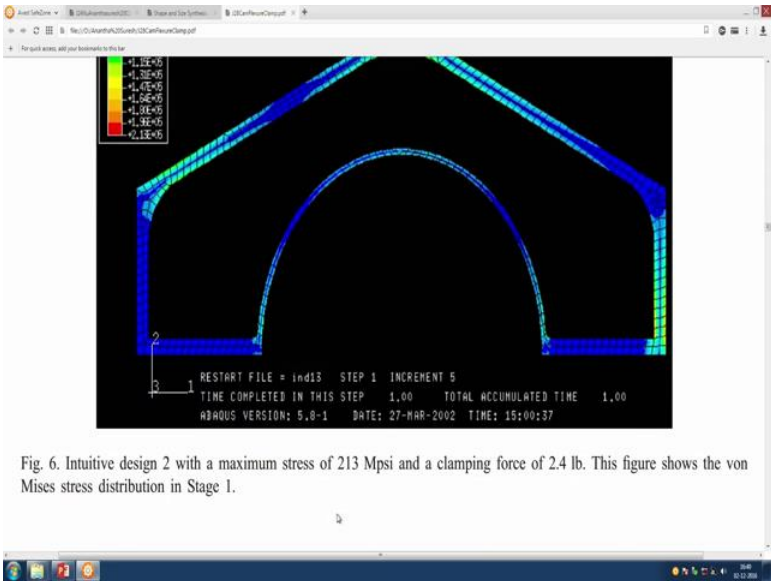

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Fig. 6. Intuitive design 2 with a maximum stress of 213 Mpsi and a clamping force of 2.4 lb. This figure shows the von Mises stress distribution in Stage 1.

So, now we write something else makes it very flexible then stress reduced to 213 still twice as much as 120 our case that we want clamping force drop down to 2.4 pounds. So, it is not able to thing, by arbitrarily trying this and that we do not get any where here which is what James Eder had told us when we began the work what we did it for ourselves or all that we describing in the paper. So one can understand then we went topology design.

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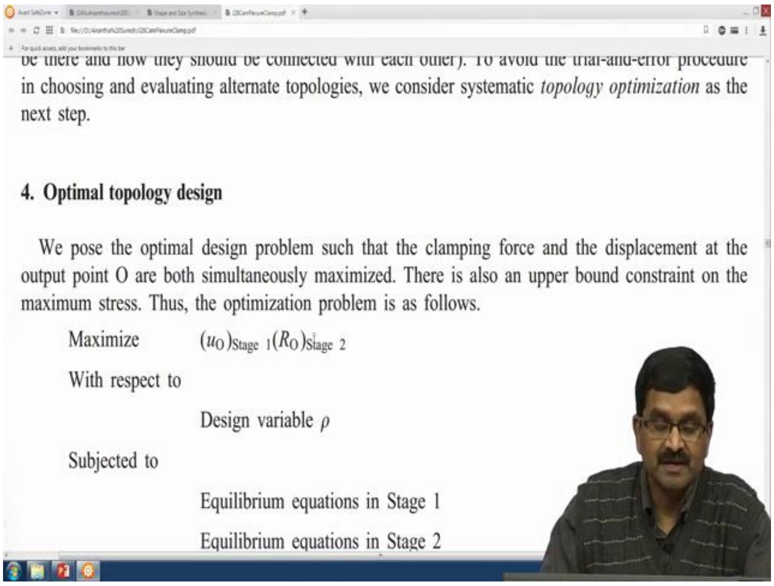

be there and how they should be connected with each other). To avoid the trial-and-error procedure in choosing and evaluating alternate topologies, we consider systematic *topology optimization* as the next step.

4. Optimal topology design

We pose the optimal design problem such that the clamping force and the displacement at the output point O are both simultaneously maximized. There is also an upper bound constraint on the maximum stress. Thus, the optimization problem is as follows.

Maximize $(u_O)_{\text{Stage 1}}(R_O)_{\text{Stage 2}}$

With respect to

Design variable ρ

Subjected to

Equilibrium equations in Stage 1

Equilibrium equations in Stage 2

So, here we are maximizing the R O is the reaction force that is a clamping force, u O is a output displacement of that hole which is misaligned and we have to align it that displacement of that towards a right k is our one objective function, other objective function is to make the clamping force as large as possible. Design variables, the same hold indicator function rho and then equilibrium equations for the load case 1, load case 2 and some constraints on stress.

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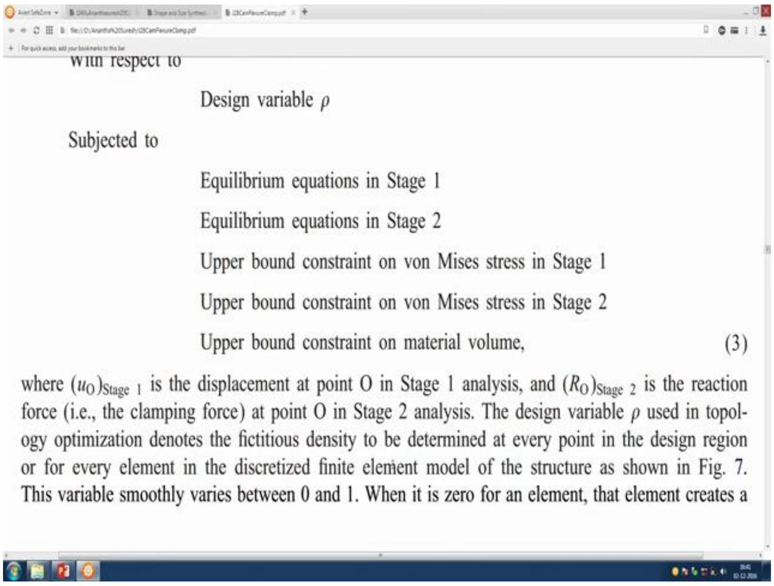

With respect to

Design variable ρ

Subjected to

Equilibrium equations in Stage 1

Equilibrium equations in Stage 2

Upper bound constraint on von Mises stress in Stage 1

Upper bound constraint on von Mises stress in Stage 2

Upper bound constraint on material volume, (3)

where $(u_O)_{\text{Stage 1}}$ is the displacement at point O in Stage 1 analysis, and $(R_O)_{\text{Stage 2}}$ is the reaction force (i.e., the clamping force) at point O in Stage 2 analysis. The design variable ρ used in topology optimization denotes the fictitious density to be determined at every point in the design region or for every element in the discretized finite element model of the structure as shown in Fig. 7. This variable smoothly varies between 0 and 1. When it is zero for an element, that element creates a

For the stage 1 when you are applying the force, secondly, you remove it then it is going to relax a little bit that is going to be another stress in both cases we want to put a stress constraint. They actually put material volume constraint because that is one way we can actually make the algorithm stiff to certain amount of material even though practical problem did not have material volume constraint.

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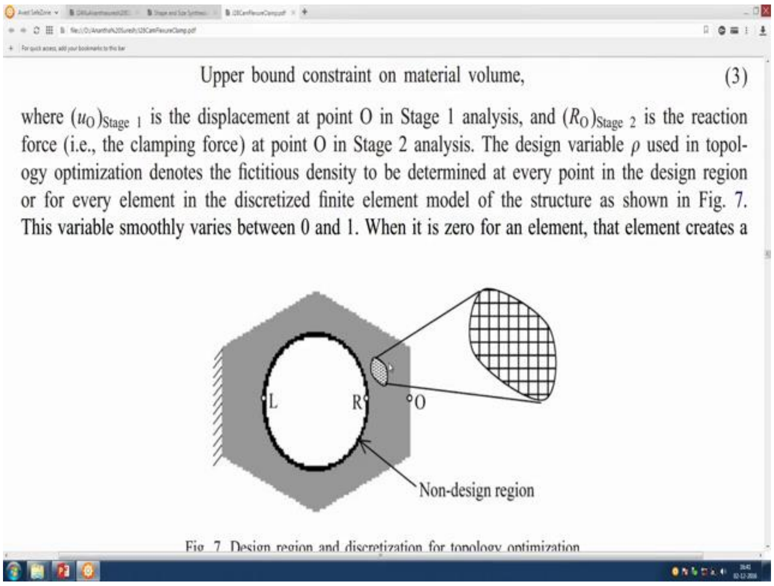

Upper bound constraint on material volume, (3)

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Fig. 7 Design region and discretization for topology optimization

So, this problem when we tried the usual topology optimization there is a hole, are not there we put that and we also put non designed domain in scene actually allows non designed domain as well.

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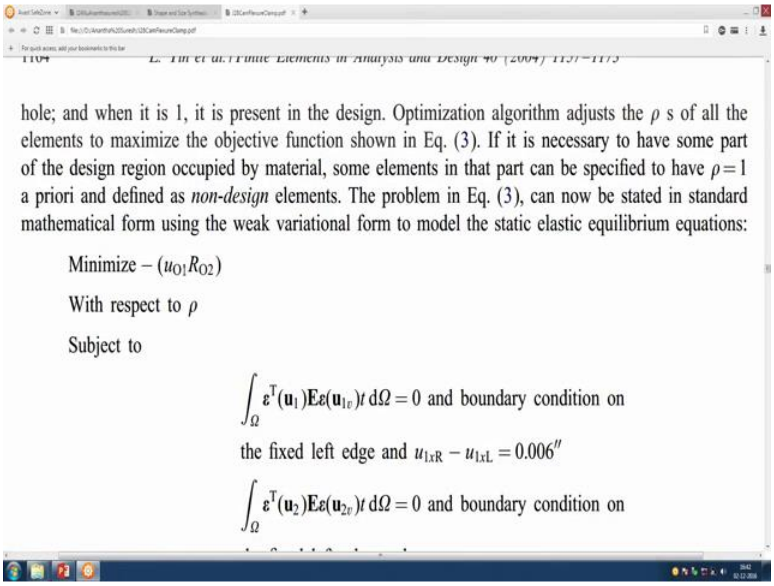

hole; and when it is 1, it is present in the design. Optimization algorithm adjusts the ρ s of all the elements to maximize the objective function shown in Eq. (3). If it is necessary to have some part of the design region occupied by material, some elements in that part can be specified to have $\rho = 1$ a priori and defined as *non-design* elements. The problem in Eq. (3), can now be stated in standard mathematical form using the weak variational form to model the static elastic equilibrium equations:

Minimize $-(u_{O1}R_{O2})$

With respect to ρ

Subject to

$\int_\Omega \boldsymbol{\varepsilon}^T(\mathbf{u}_1)\mathbf{E}\boldsymbol{\varepsilon}(\mathbf{u}_{1v})t\,d\Omega = 0$ and boundary condition on the fixed left edge and $u_{1xR} - u_{1xL} = 0.006''$

$\int_\Omega \boldsymbol{\varepsilon}^T(\mathbf{u}_2)\mathbf{E}\boldsymbol{\varepsilon}(\mathbf{u}_{2v})t\,d\Omega = 0$ and boundary condition on

At this time in scene was there, but we have to write our own code. So, here because the elliptical tool goes there all around this elliptical hole we put some material for sure that to algorithm cannot remove that material, we put that and solve the problem minimize negative or the product of output displacement and reaction force clamping force these are all the constraints.

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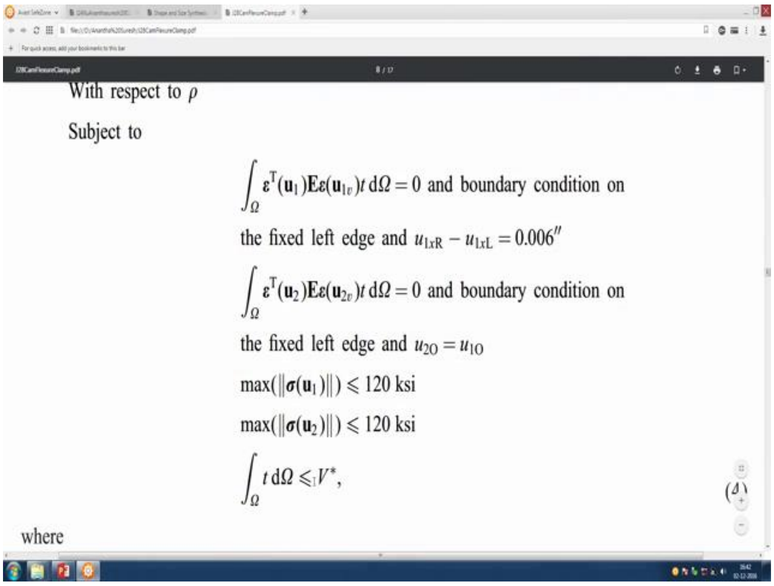

With respect to ρ

Subject to

$\int_\Omega \boldsymbol{\varepsilon}^T(\mathbf{u}_1)\mathbf{E}\boldsymbol{\varepsilon}(\mathbf{u}_{1v})t\,d\Omega = 0$ and boundary condition on the fixed left edge and $u_{1xR} - u_{1xL} = 0.006''$

$\int_\Omega \boldsymbol{\varepsilon}^T(\mathbf{u}_2)\mathbf{E}\boldsymbol{\varepsilon}(\mathbf{u}_{2v})t\,d\Omega = 0$ and boundary condition on the fixed left edge and $u_{2O} = u_{1O}$

$\max(\|\boldsymbol{\sigma}(\mathbf{u}_1)\|) \leqslant 120$ ksi

$\max(\|\boldsymbol{\sigma}(\mathbf{u}_2)\|) \leqslant 120$ ksi

$\int_\Omega t\,d\Omega \leqslant V^*,$

where

And these are the stress constraint, volume constraint, equilibrium constraints all the optimization thing that would have we can take gradients and all that is possible here.

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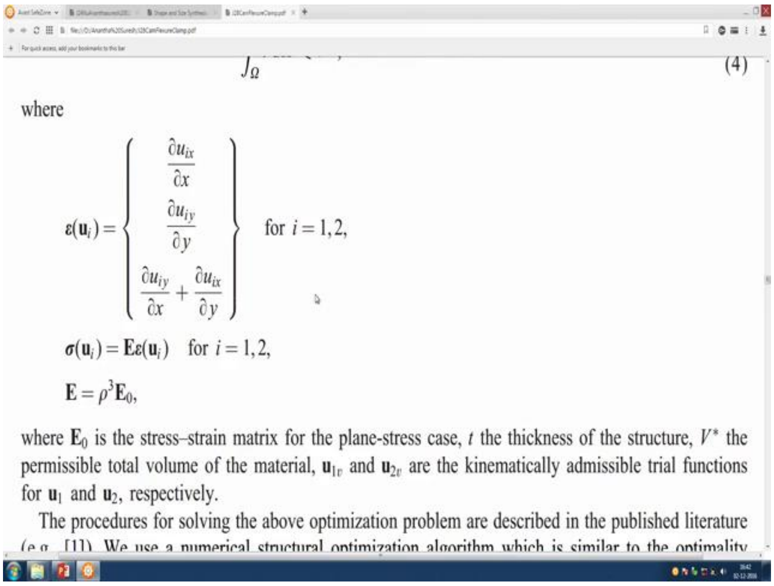

(4)

where

$$\varepsilon(\mathbf{u}_i) = \left\{ \begin{array}{c} \frac{\partial u_{ix}}{\partial x} \\ \frac{\partial u_{iy}}{\partial y} \\ \frac{\partial u_{iy}}{\partial x} + \frac{\partial u_{ix}}{\partial y} \end{array} \right\} \quad \text{for } i = 1,2,$$

$$\boldsymbol{\sigma}(\mathbf{u}_i) = \mathbf{E}\varepsilon(\mathbf{u}_i) \quad \text{for } i = 1,2,$$

$$\mathbf{E} = \rho^3 \mathbf{E}_0,$$

where $\mathbf{E}_0$ is the stress–strain matrix for the plane-stress case, t the thickness of the structure, V^* the permissible total volume of the material, $\mathbf{u}_{1v}$ and $\mathbf{u}_{2v}$ are the kinematically admissible trial functions for $\mathbf{u}_1$ and $\mathbf{u}_2$, respectively.

The procedures for solving the above optimization problem are described in the published literature (e.g., [1]). We use a numerical structural optimization algorithm which is similar to the optimality

Putting penalty parameter of 3 for that rho which we discussed in one of the last lectures used E 0 which are titanium’s properties and we got a solution like this.

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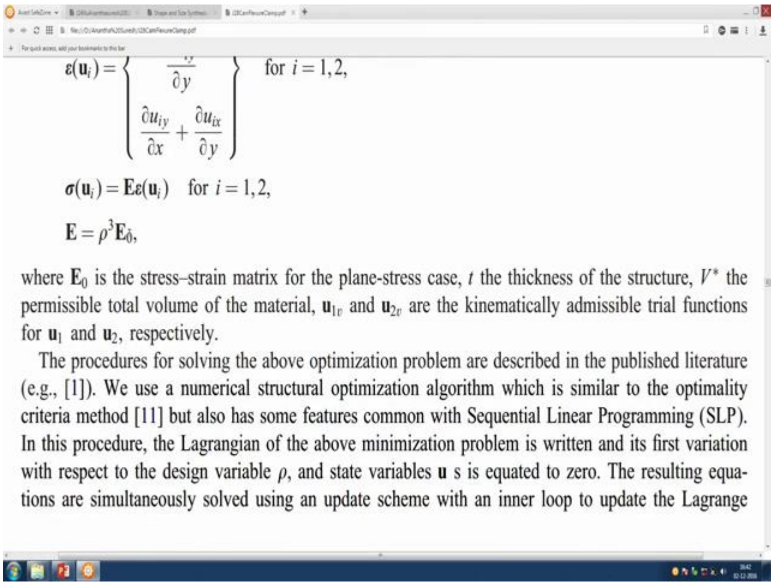

$$\varepsilon(\mathbf{u}_i) = \left\{ \begin{array}{c} \partial y \\ \frac{\partial u_{iy}}{\partial x} + \frac{\partial u_{ix}}{\partial y} \end{array} \right\} \quad \text{for } i = 1,2,$$

$$\boldsymbol{\sigma}(\mathbf{u}_i) = \mathbf{E}\varepsilon(\mathbf{u}_i) \quad \text{for } i = 1,2,$$

$$\mathbf{E} = \rho^3 \mathbf{E}_0,$$

where $\mathbf{E}_0$ is the stress–strain matrix for the plane-stress case, t the thickness of the structure, V^* the permissible total volume of the material, $\mathbf{u}_{1v}$ and $\mathbf{u}_{2v}$ are the kinematically admissible trial functions for $\mathbf{u}_1$ and $\mathbf{u}_2$, respectively.

The procedures for solving the above optimization problem are described in the published literature (e.g., [1]). We use a numerical structural optimization algorithm which is similar to the optimality criteria method [11] but also has some features common with Sequential Linear Programming (SLP). In this procedure, the Lagrangian of the above minimization problem is written and its first variation with respect to the design variable ρ, and state variables $\mathbf{u}$ s is equated to zero. The resulting equations are simultaneously solved using an update scheme with an inner loop to update the Lagrange

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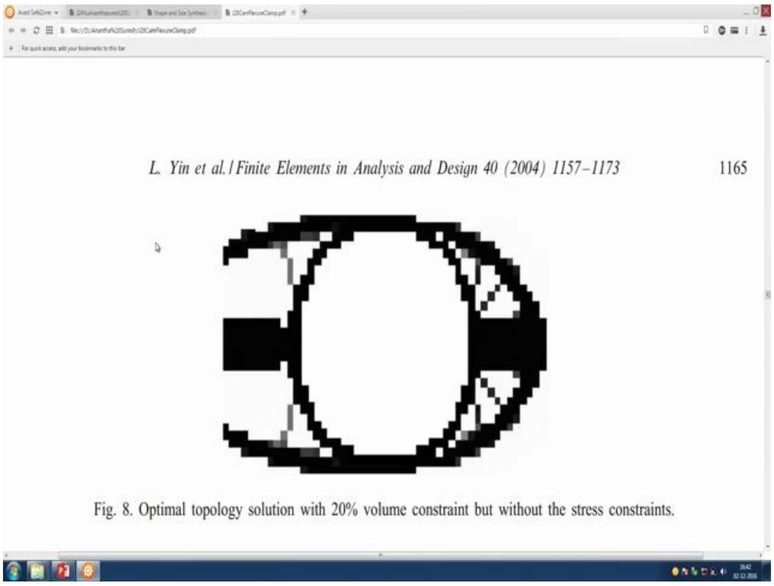

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Fig. 8. Optimal topology solution with 20% volume constraint but without the stress constraints.

Like any other topology problem we get this, again we have this one node hinges point flexures, edge flexures all the usual things and we have this something’s are there. Now we went and looked at the result that we got we did not really get low enough of stress or enough clamping force.

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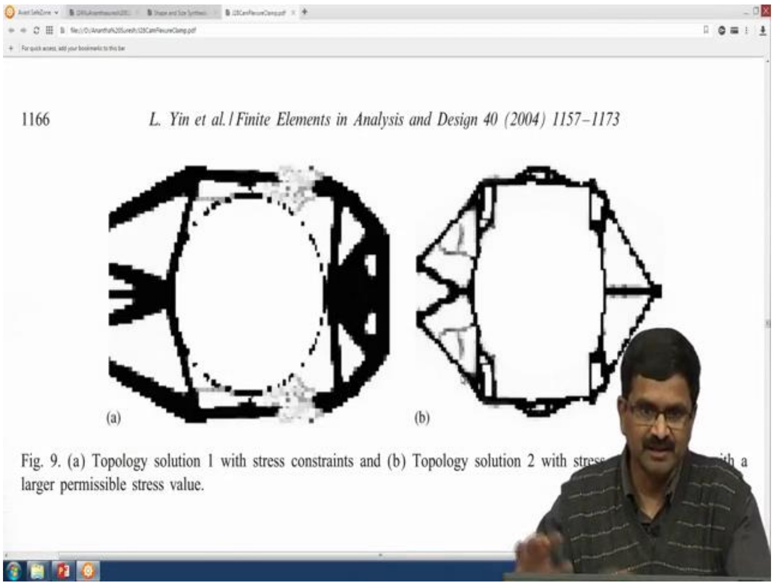

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Fig. 9. (a) Topology solution 1 with stress constraints and (b) Topology solution 2 with stres ith a larger permissible stress value.

Then we had to make some tweets to the thing there in a paper we can look them up and we got something that is not even well designed. For example, we remove this design domain what should be complete circle we thought even if it is partial we can still put elliptical tool then he started giving disjointed in conversed regions with the different type of stress constraint that we had put, right.

(Refer Slide Time: 20:14)

Fig. 10. The von Mises stress distribution in the interpreted design of Fig. 9a.

So, looking at all when we finally, look at that somewhat of a design that kind of close we got where there are different segments and this elliptical hole we have not shown here, but that is there we have to connect it.

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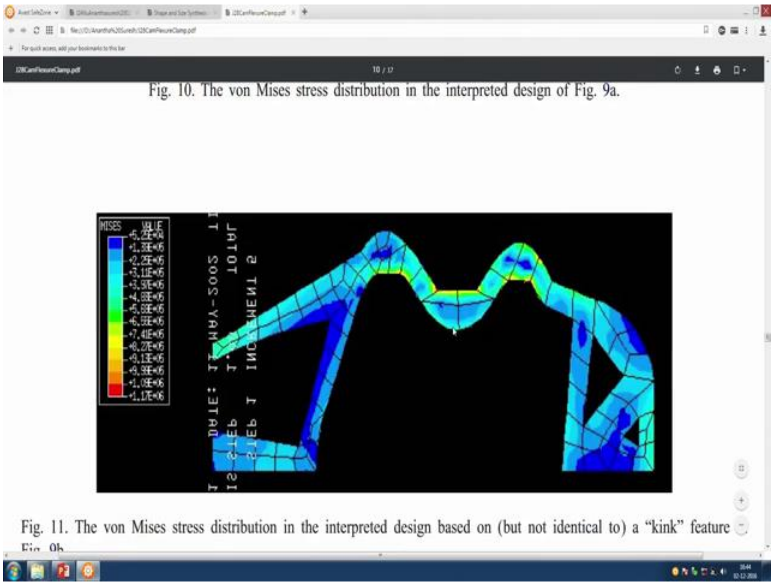

Fig. 10. The von Mises stress distribution in the interpreted design of Fig. 9a.

Fig. 11. The von Mises stress distribution in the interpreted design based on (but not identical to) a "kink" feature Fig. 9b

And another one, so these two are actually inspired by these, they are not actually. So, if you look at this one and this one you do not finding resemblance some whatever closeness here it the algorithm had lot of doubt and we also kind of tried to show that doubt. In fact, when we are looking at all this, all this topology solution that we got by varying parameters, removing stress constraints, adding them and looking at all, there was the idea that what we should really worry about here is what is important - is it bending of the segments or axial deformation.

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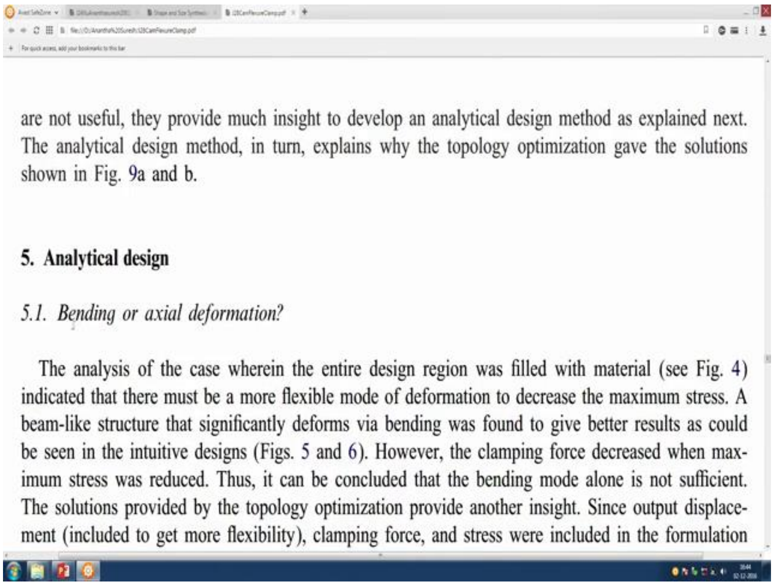

are not useful, they provide much insight to develop an analytical design method as explained next. The analytical design method, in turn, explains why the topology optimization gave the solutions shown in Fig. 9a and b.

5. Analytical design

5.1. Bending or axial deformation?

The analysis of the case wherein the entire design region was filled with material (see Fig. 4) indicated that there must be a more flexible mode of deformation to decrease the maximum stress. A beam-like structure that significantly deforms via bending was found to give better results as could be seen in the intuitive designs (Figs. 5 and 6). However, the clamping force decreased when maximum stress was reduced. Thus, it can be concluded that the bending mode alone is not sufficient. The solutions provided by the topology optimization provide another insight. Since output displacement (included to get more flexibility), clamping force, and stress were included in the formulation

If you think of axial deformation, axial stiffness is quite high for a beam segment right, there we get to serve the clamping force. If it is bending the stress is limited and we can try to satisfy the stress constraint. Finally, we thought after whatever we have learned from topology optimization we did not get close to the 120 ksi and the clamping force of 100 pounds or whatever, instead we went for analytical design where we try to look at for the design that we have if I just take any of that thing that topology optimization gave is (Refer Time: 21:43) that there should be some beams, right.

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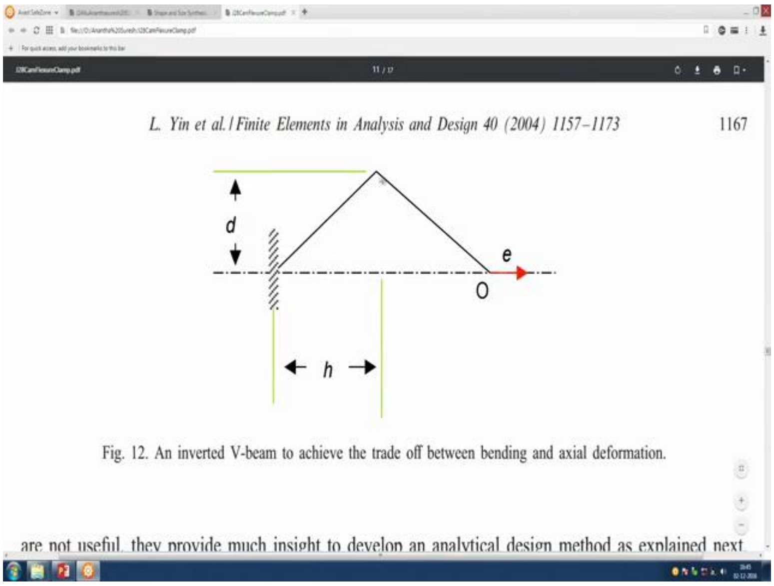
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Fig. 12. An inverted V-beam to achieve the trade off between bending and axial deformation.

are not useful, they provide much insight to develop an analytical design method as explained next.

So, without that the beams may be more like this let us take a V-beam one; there is one segment here another segment. Now let us see what should be the height of this, what should be this span of this edge. Now this h and d and some width of these two beam segments and we try to plot the axial stress, bending stress and a total stress how do they vary?

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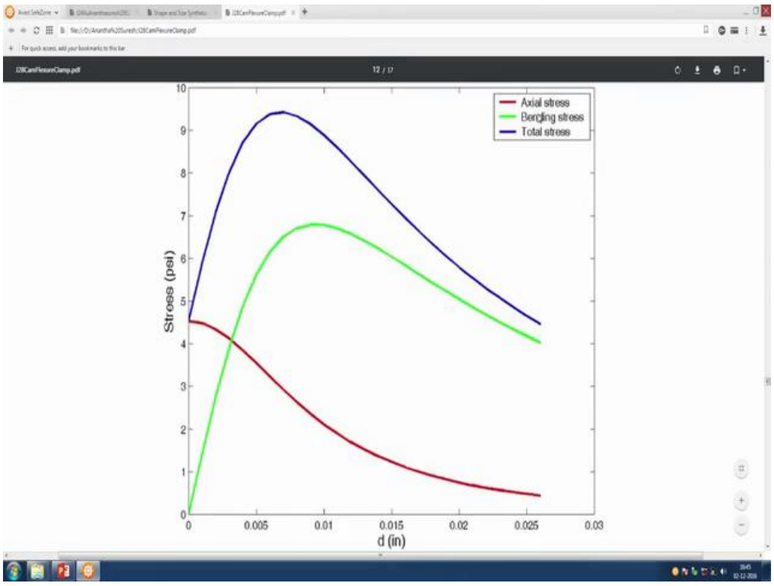

If you see axial stress based on the displacement that we have from beginning to later for this V-beam thing that actually reduces where as bending stress increases and reduces and there is a peak of course, when you add them up there is a total stress blue curve there is a peak.

So, by looking at analytical expressions of the reaction force, by this what I mean is that this V-beam thing we apply a force when this elliptical thing is put and it is moved to the right, how many displacement it goes, what is the stress in that case and when you release it we know how much reaction force will be there. So, we can calculate all that analytically here we put those.

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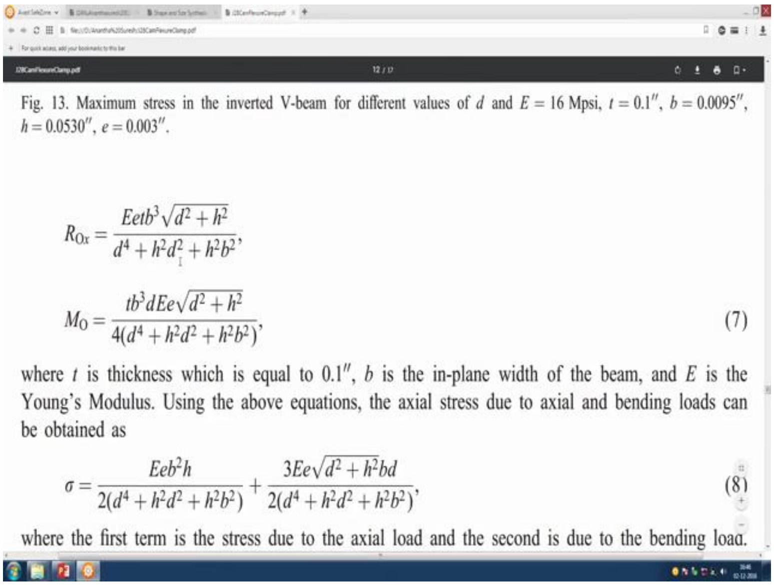

Fig. 13. Maximum stress in the inverted V-beam for different values of d and $E = 16$ Mpsi, $t = 0.1''$, $b = 0.0095''$, $h = 0.0530''$, $e = 0.003''$.

$$R_{Ox} = \frac{Eetb^3\sqrt{d^2+h^2}}{d^4+h^2d^2+h^2b^2},$$

$$M_O = \frac{tb^3dEe\sqrt{d^2+h^2}}{4(d^4+h^2d^2+h^2b^2)}, \qquad (7)$$

where t is thickness which is equal to $0.1''$, b is the in-plane width of the beam, and E is the Young's Modulus. Using the above equations, the axial stress due to axial and bending loads can be obtained as

$$\sigma = \frac{Eeb^2h}{2(d^4+h^2d^2+h^2b^2)} + \frac{3Ee\sqrt{d^2+h^2}bd}{2(d^4+h^2d^2+h^2b^2)}, \qquad (8)$$

where the first term is the stress due to the axial load and the second is due to the bending load.

And try to come up with this two variables d and h or should be the ideal ratio when we did that something interesting happened.

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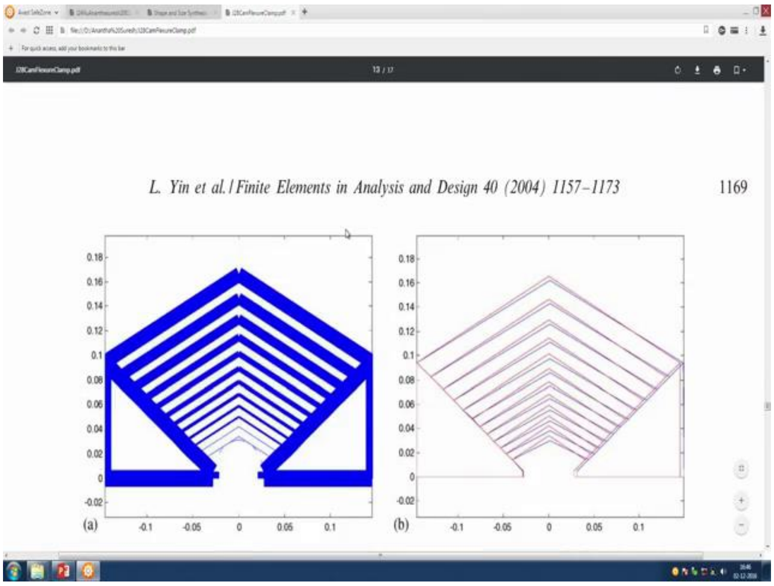

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So, we thought we showed that when we have the elliptical hole here when you take a beam like this from here to here a single beam no matter what width it is it does not (Refer Time: 23:19) the result requirements that they have, right. We thought that how about using the stress each one of them has a particular width here that is the size optimization and the height of this you know compare to the span of this, how high it can be they are all different here even the span is changing. So, there is a notion of shape change here. Topology, we believe that it was suggested by topology optimization in fact it was because, we try to intuitive lot of them did not anywhere, topology is gave up that how it should be connected and then we thought if there is one beam here and one beam here and that V-beam if I take for the stress limit that we have 120 ksi how much reaction force does it get here. We computed that, that was no more force what we wanted then we thought that we should take more and more of them, because then the reaction force decided up each one of them goes to that 120 ksi limit. In a way each V-beam here at have a different span l here and they the d hide that.

So, we got all of them and they all moves like this, this is a rigid portion rigid portion, nice rectangular here and here we put them several of them and then showed that whatever original size that they had we prove to them that it is not possible you have to add a few more and that was a revelation. Because as a designer like at beginning we had started with the space which was all given like this we turned out the beam segments you put how many of beam segments you pack in to this area you are not getting enough force. So, here the force is 30 pounds that they wanted the force originally, the clamping was little more than that. So, whatever beams when you get are not enough so we want to extend the regional little bit when we did that step by step by adding these things one at a time.

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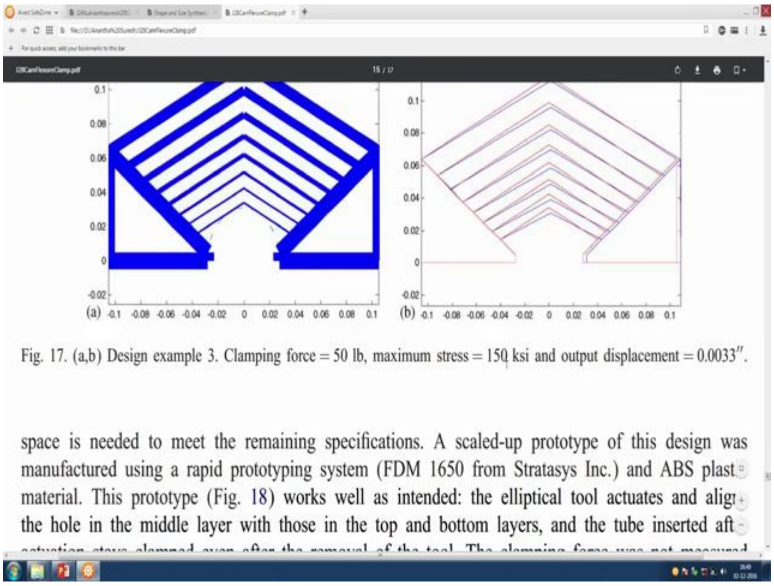

Fig. 17. (a,b) Design example 3. Clamping force = 50 lb, maximum stress = 150 ksi and output displacement = 0.0033″.

space is needed to meet the remaining specifications. A scaled-up prototype of this design was manufactured using a rapid prototyping system (FDM 1650 from Stratasys Inc.) and ABS plast material. This prototype (Fig. 18) works well as intended: the elliptical tool actuates and aligr the hole in the middle layer with those in the top and bottom layers, and the tube inserted aft

So, went to this we are able to achieve it. Here it was maximum 120-150 ksi clamping force was 50 by adding a few more we are able to finally, achieve what we wanted, now you see several beam elements like that.

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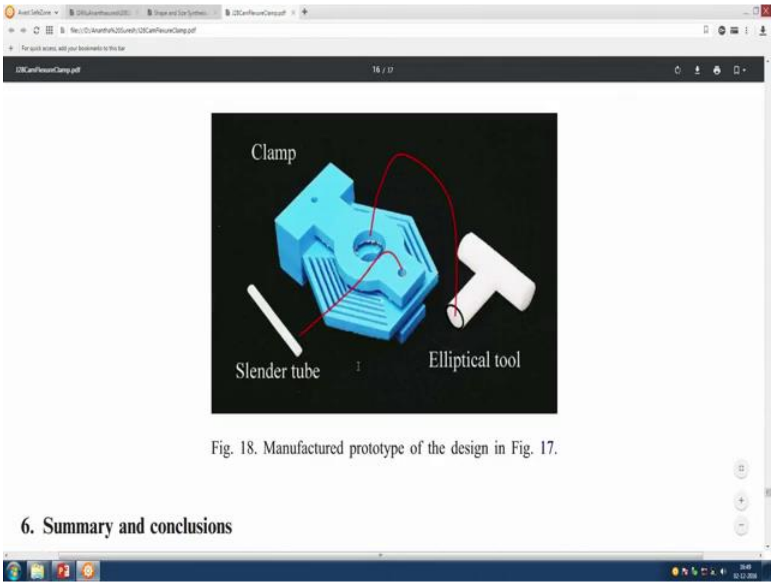

Fig. 18. Manufactured prototype of the design in Fig. 17.

6. Summary and conclusions

And we have to go a little bit more than the original space that they had and finally, it was made this elliptical tool that you have it is a 3D printed modal, but this was to be made an titanium where we have the displacement when I put this elliptical tool here.

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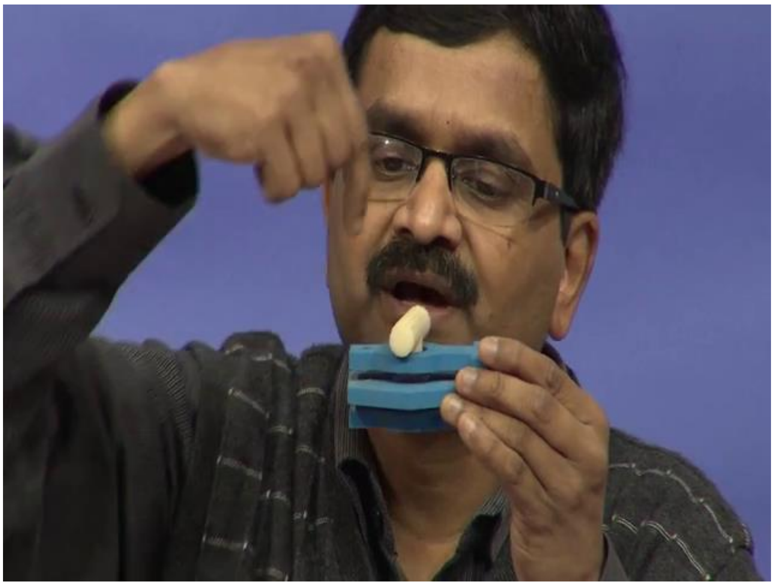

And then turn it when I turn it by applying force it is like this and it turn a 90 degrees that will make the middle portion go a little bit now we can put a wire through this aligned hole.

There are summary conclusions that you can see finally it was able to meet the specifications of the clamping force and within the stress, but with a larger size that is the key. So, when we start the problem we cannot say this is my design domain get a compliant mechanism for that, for what you are asking that space that you have may or may not have a solution. If the algorithm does not give a solution it could be that there is no solution. So, how do we know ahead of time? So far in the last 2 weeks we have discussed this structure optimization inspired methods for compliant mechanisms before that for a week and before that one more week we had used how we can used rigid body linkages and come up with design methods.

So, we have in a way talked about two classes of methods - one are come from one extreme of rigid body linkages, where we use pseudo rigid body model linkage synthesis; the other extreme of stiff structures where we used (Refer Time: 27:20) optimization methods topology shape and size we took one case study then we realize that all the methods that we have are not readily applicable here we have to make some changes as you solve this case study. So, next few weeks we will discussed a few more classes of methods not just these two classes we have a few more methods have emerged in the last decade or 12, 14 years. So, let us look at those in the coming weeks.

In the next week we are going to discuss a method that uses a different model for capturing the kinematics and the elasticity of compliant mechanisms and make it more like a selection space method. So will see and after that we will have few more methods that we can discuss so that we would have a several categories of methods hopefully one of them will be useful for solving a practical problem which is always the ultimate goal for any designed method development research.

Thank you.