Compliant Mechanisms: Principles and Design

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Lecture – 35
Shape optimization

Hello, we have discussed topaz optimization for compliant mechanisms and the advantages, limitations, complications and of course, it does give a Topology all the time with or without this hinges either point flexures, edge flexures but does give a mechanism and we have ways to counter act the point flexures and edge flexures by going for restrained relative rotation are some other base of doing a reducing a erosion and dilation.

But Topology is not ultimate thing for the eventual design of a compliant mechanism, shape also matters. Topology of course matters the most, we keep return in to that in this a course and whenever we talk about compliant mechanism the same breath you talk about Topology atomization because having the connectivity in the right way is very important for a compliant mechanism.

However when you go to practical problems, the shape needs to be tuned, the shape of the beam segments most of the compliant mechanisms do have in them beam segments without that (Refer Time: 01:35) 2 d or 3 d instead of just having beam segments you can also have thin plates and shells, but they are still slander elements which are one dimension wise they are thinner compare to other dimensions.

So, the shape of those things actually needs to be tuned to meet the manufacturability constrains or strength requirements are just available form because unless you have 3 d printing (Refer Time: 02:04) manufacture is taking place now, but you want to use traditional manufacturing methods where the shape is very important of the segments in there. So, let us talk about one publication, one paper where the shape optimization was done for compliant mechanisms.

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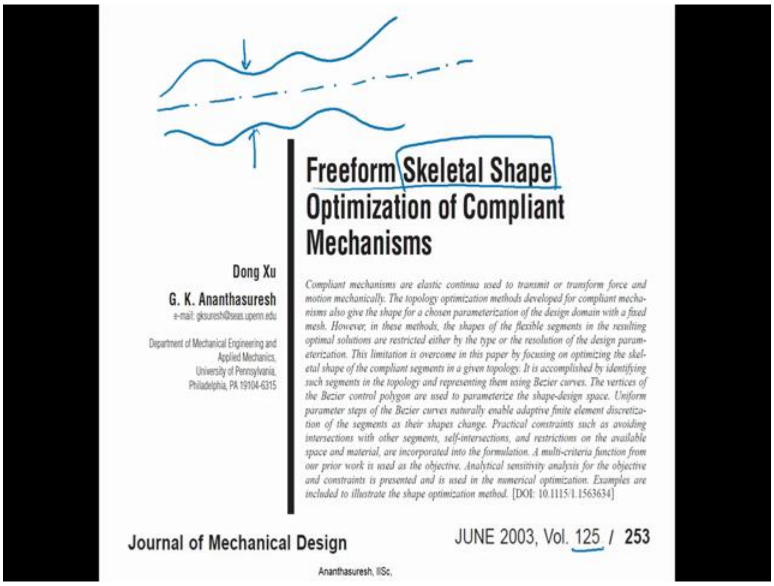

So, let us look at (Refer Time: 02:26) mechanism before that let us actually look at this paper, that talks about what is called Freeform Skeletal Shape, there is a particular word that is used to call Skeletal Shape. By shape one can have lots of different interpretation, one is that if I have a beam element let us say I have beam element, whose neutral plane and neutral access I am showing here, I can have the width profile of that; that is one kind of shape optimization. In fact, that thing people would call a size optimization actually rather than shape optimization because it is a size that width profile is what we are trying to adjust, that is more like size.

So, we are not talking about that type of Shape Optimization, we are talking about something called Skeletal Shape, that is the a compliant mechanism that has beam segments, if you draw them all with this line like this you would actually see a skeletal form of a compliant mechanism and that is what we are trying to optimize here. So, this is in general of mechanism design, volume number 125 and page 253 you can go and read in fact, we are going to discuss this paper in detailed in this lecture.

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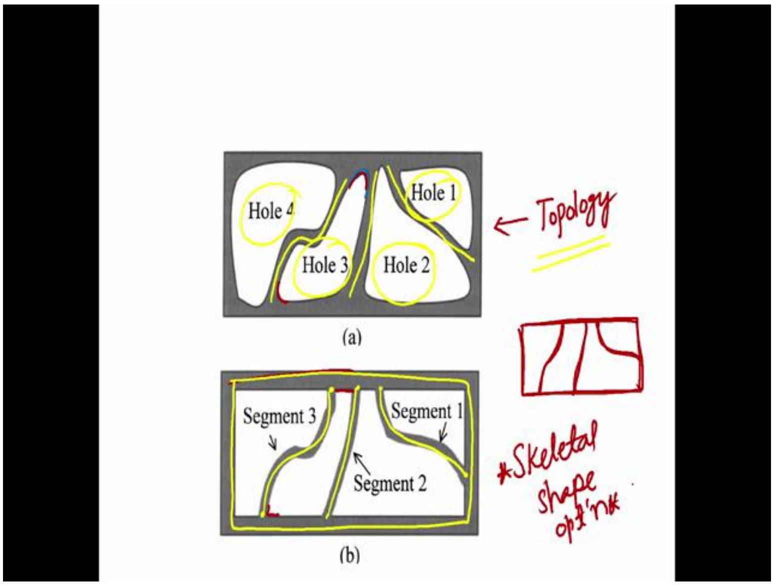

Let us first look at what do we mean by Skeletal Shape optimization, so I am showing two structures here, both of them are identical in a way except a little portion here it is like this, let me use a different color. So, here it is little rounded whereas, this is straight like this and same thing here little straight here corner, here little rounded that is what we have essentially there are holes, there are four holes in this structure this is what we can call it Topology optimization or Topology design.

Other one we say that there is a frame, so let me use now a color that we can easily see, we have a frame out of outer frame, in this outer frame let us say that I have decided to connect this and this because Topology told as that if we connect these two, you basically make the hole four and then if you connect this, you make hole 3 also and then if you connect this to the frame you make hole 1 and hole 2.

So, the Topology can be obtained by putting the holes or identifying this connectivity points and joining them. So, if I join this I create hole 4 that is that one and then by connecting this I create hole 3 and then I connect these 2, I create hole 2 and hole 1. So, now, the shape of the holes also matters, topology optimization the way we call it, it not only gives the Topology, but also gives the size and shape and everything, the shape holes also come as we have seen, but instead we can talk about a skeleton.

So, if I were to draw for this Topology as it is drawn below, if this is the rectangular frame that we have; I say that this is Skelton of this one. Now, this is not the skeletal in the geometric modeling sense, but basically we are looking at beam segments, the frame is also a beam segment inside was the beam segments, we have those if we have optimize those we get in a way a complaint mechanism design or a form by just playing with the shape of these beam segments, if I were the given outer rectangular frame, that is what we mean by Skeletal Shape optimization. Skeletal Shape optimization or compliant mechanism to distinguish between this and what people normally call for beams and bars (Refer Time: 06:52) cross section profile that also can be ensured as shape optimization.

So, what we will do is look at the paper that I have just sited and discuss that. It is free form meaning that you can have any shape that you want and that is the idea. So, once again this is journal Mechanism Design, so we can get this paper and read it.

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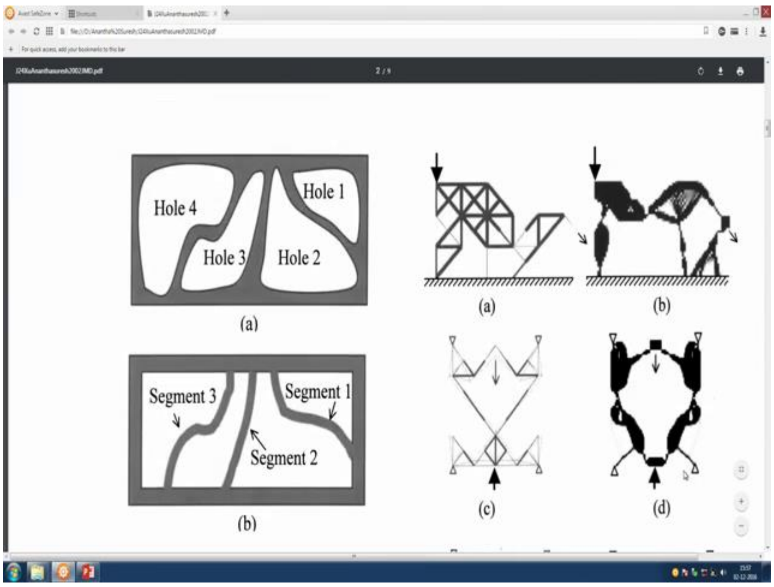

So, what I just discussed is over here, what we have been by Skeletal Shape optimization. Again the motivation for this comes from this point flexure that we have talked about 1 node hinges. Now everywhere we have these joints whether you do with beam elements or continuum elements, we have the same thing. In fact, I would like to point out one other thing here. There is a little rigid potion stiff portion here, which is a check a board pattern.

The check a board pattern is a problem in stiff structure optimization, where you get alternate black and white regions as it occurs in a chess board or a check a board and that is kind of similar to the one node hinge that we have discussed and there again we can come up with the intuitively explanation as to why the check a board occurs? Some people argue this to be in numerical instability into (Refer Time: 08:20) that is also correct, that is a mathematical way of saying intuitively the check a board that it gives is actually stiff and algorithm again exploits a loop hole in finite element frame work and gives you what happens to be numerically stiff structure, but is not something you can easily manufacture, some that we do not want to have.

Whether you take beam elements or continuum elements, the topologies looks the same pretty much and the shape also in a way, but what we want to avoid here are things like this here, we do not want them. So, we do not want things that have this narrow fixtures and that happens all the no matter what problem you solve, again beam elements that is the frame problem this take a c here and the d which we have gotten using continuum elements, they both look the same, so it does not matter which when you take.

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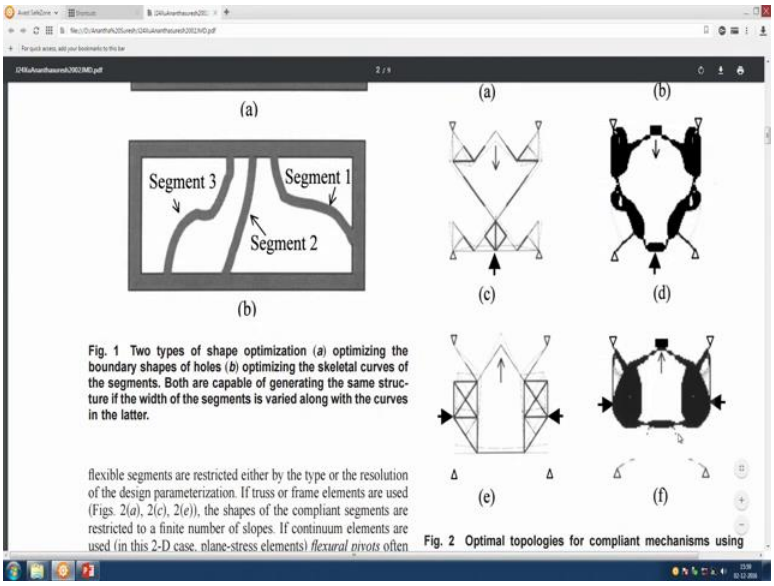

One other example where we have in fact, we have supports provided, to supports here did not want to connect because the force and displacements are over here, it takes this supports whatever beam does that was the continuum does. That is so much identical thing does not matter what type of element he choose we get a same design, but hinges seem to be a problem.

Let us say you take it topologies like this, either this or that for a given problem and then try to identify these segments and try to tweak their shape to improve upon a performance. So, shape optimization comes after you do topaz optimization, you can assume a Topology like you have to do with (Refer Time: 10:06) body model based method or where you start with a rigid body linkage and try to come up with a equivalent compliant mechanism and try to optimize that or synthesize that that is one way or if you assume a Topology then you can do shape optimization as will discuss and try to get a complaint mechanism that satisfies the functional requirements as well as some of the performance requirements.

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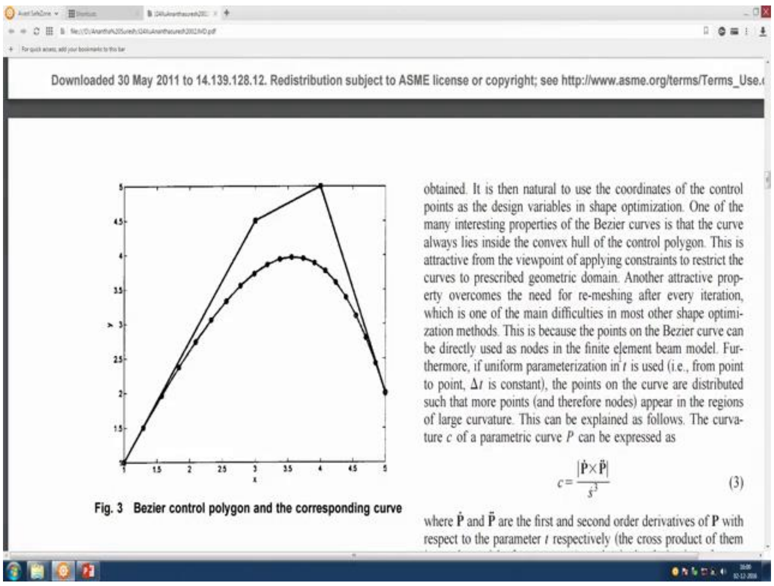

Fig. 3 Bezier control polygon and the corresponding curve

obtained. It is then natural to use the coordinates of the control points as the design variables in shape optimization. One of the many interesting properties of the Bezier curves is that the curve always lies inside the convex hull of the control polygon. This is attractive from the viewpoint of applying constraints to restrict the curves to prescribed geometric domain. Another attractive property overcomes the need for re-meshing after every iteration, which is one of the main difficulties in most other shape optimization methods. This is because the points on the Bezier curve can be directly used as nodes in the finite element beam model. Furthermore, if uniform parameterization in t is used (i.e., from point to point, Δt is constant), the points on the curve are distributed such that more points (and therefore nodes) appear in the regions of large curvature. This can be explained as follows. The curvature c of a parametric curve P can be expressed as

$$c = \frac{|\dot{\mathbf{P}} \times \ddot{\mathbf{P}}|}{\dot{s}^3} \qquad (3)$$

where $\dot{\mathbf{P}}$ and $\ddot{\mathbf{P}}$ are the first and second order derivatives of $\mathbf{P}$ with respect to the parameter t respectively (the cross product of them

So, in this particular thing we are looking at using Bezier curves, the Bezier curves as you know there are some control points there is one here another one there, another one here, another one there. So, control points are the straight line one that we have is actually control polygon and from there we can draw the Bezier curves. It is a free form curve or variables are only the x y coordinates of this control points and everywhere else we get the uniform or smooth curve, which is a cubic in the case of Bezier curve as (Refer Time: 11:20) whatever Bezier curve is a simplest one. So, the entire curve of a beam segment such as the one we discussed in the Skeletal; free from Skeletal Shape optimization, we only need if the n points are fixed which are very well in many problems, then we are dealing with only two control points this and this.

So, with these two control points we have in a 2 d problem, two variables there x y coordinates, another one two variables x y coordinates. So just with four variables, we can do the problem; we can get the shape of this. So, we are using a feature in geometric modeling adopting it for compliant mechanisms.

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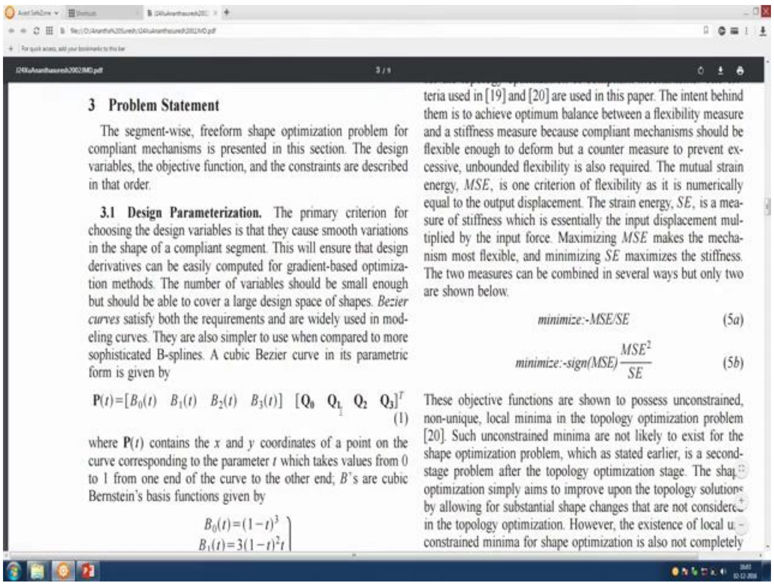

3 Problem Statement

The segment-wise, freeform shape optimization problem for compliant mechanisms is presented in this section. The design variables, the objective function, and the constraints are described in that order.

3.1 Design Parameterization. The primary criterion for choosing the design variables is that they cause smooth variations in the shape of a compliant segment. This will ensure that design derivatives can be easily computed for gradient-based optimization methods. The number of variables should be small enough but should be able to cover a large design space of shapes. *Bezier curves* satisfy both the requirements and are widely used in modeling curves. They are also simpler to use when compared to more sophisticated B-splines. A cubic Bezier curve in its parametric form is given by

$$\mathbf{P}(t)=[B_0(t) \quad B_1(t) \quad B_2(t) \quad B_3(t)] \; [\mathbf{Q}_0 \quad \mathbf{Q}_1 \quad \mathbf{Q}_2 \quad \mathbf{Q}_3]^T \qquad (1)$$

where $\mathbf{P}(t)$ contains the x and y coordinates of a point on the curve corresponding to the parameter t which takes values from 0 to 1 from one end of the curve to the other end; B's are cubic Bernstein's basis functions given by

$$B_0(t)=(1-t)^3$$
$$B_1(t)=3(1-t)^2t$$

teria used in [19] and [20] are used in this paper. The intent behind them is to achieve optimum balance between a flexibility measure and a stiffness measure because compliant mechanisms should be flexible enough to deform but a counter measure to prevent excessive, unbounded flexibility is also required. The mutual strain energy, MSE, is one criterion of flexibility as it is numerically equal to the output displacement. The strain energy, SE, is a measure of stiffness which is essentially the input displacement multiplied by the input force. Maximizing MSE makes the mechanism most flexible, and minimizing SE maximizes the stiffness. The two measures can be combined in several ways but only two are shown below.

$$minimize\text{:-}MSE/SE \qquad (5a)$$

$$minimize\text{:-}sign(MSE)\frac{MSE^2}{SE} \qquad (5b)$$

These objective functions are shown to possess unconstrained, non-unique, local minima in the topology optimization problem [20]. Such unconstrained minima are not likely to exist for the shape optimization problem, which as stated earlier, is a second-stage problem after the topology optimization stage. The shape optimization simply aims to improve upon the topology solutions by allowing for substantial shape changes that are not considered in the topology optimization. However, the existence of local unconstrained minima for shape optimization is also not completely

So, if we do that we can use our usual objective functions that we have already obtained, so let us say I want again go to our MSE by SE formulation, now there is no problem no one node hinges or anything point fluxions here because we already are dealing with shape rather than defining Topology.

So this point fluxions are not going to occur because we are not using those indicator functions and usual Topozation approach; you just design the shape of the thing. Assuming the width of the elements, without this beam segments minus MSE over SE or sign of MSE, MSE square over SE this another small formulation which we have been discussed there are many such formulation in the literature, we did not discuss all of them.

Let us look at this 5 a minus MSE over s e would like to minimize. If there is objective function, we can use the concepts of Bezier curves which are cubic are as it is showed here. So, all the points p of t are given by this hermit polynomials and these are the control points Q 0, Q1,Q 2, Q 3 the x y coordinates making at the cubic, these are the ones that we have the Bezier basis function that we have, so we can get that and define a problem like this.

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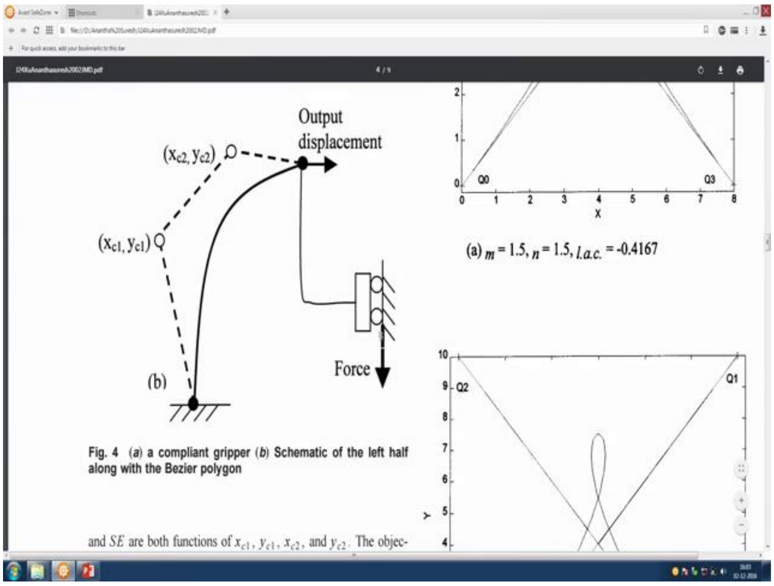

So, let us I want to make a gripper this is a symmetry boundary condition here force is applied when I do that, output displacement should be here let us say gripper problem and I have identified two segments, there is a segment 1 and there is a segment 2.

So, for the segment one this point is fixed, that point is fixed, this point is fixed, for segment 1; we need 2 points that are fixed. So X c 1, Y c 1, X c 2 ,Y c 2 become the four variables for this entire shape and likewise output displacement point is fixed, input force point is fixed, two ends are there shape of this would be the second segments, that will also have 2 control points which are not shown here, so 4 variables 4 plus 4; 8 variables, I have to do shape optimization now.

Since Bezier curves we know all the segments are come, advantages of this Bezier polygon is that when you parameterize this curve, there is a parameter t that is go from 0 to 1, if we take it uniformly you will get this distribution of nodes along the beam, here we can and as well use the beam elements; beam finite elements, wherever the curvature changes a lot our curvature is high more points coming, where it is more or less straight like this portion will get very few nodes. So, automatically if you have this parameter t varied uniformly between 0 and 1, let us say 10 points you take it takes 0, 0.1, 0.2, 0.3 up to 0.9 and point 1.

These points will be distribute in such a way that, where there is more curvature more points will come, where there is less curvature more straight fewer points will come automatically. So, we can do that very easily with this Bezier discretisation, one other problem that can occur is that; this Bezier polygon see if the control polygon this lines intersect you can get actually a loop.

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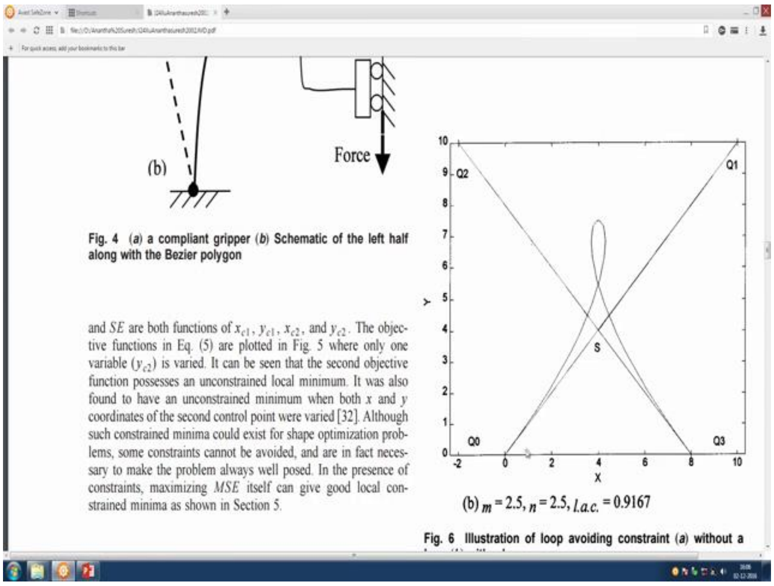

A loop has to be avoided in a compliant mechanism that does not make sense because eventually we are going to make it out of a sheet of material there is a continuum thing. So, having a loop like this would not be referred but there is fortunately a constrained that we can put, what can be called Loop Avoiding Constrained, that again comes from the geometric modeling literature and we can put a length constraint also, we can put a overall how much lengths.

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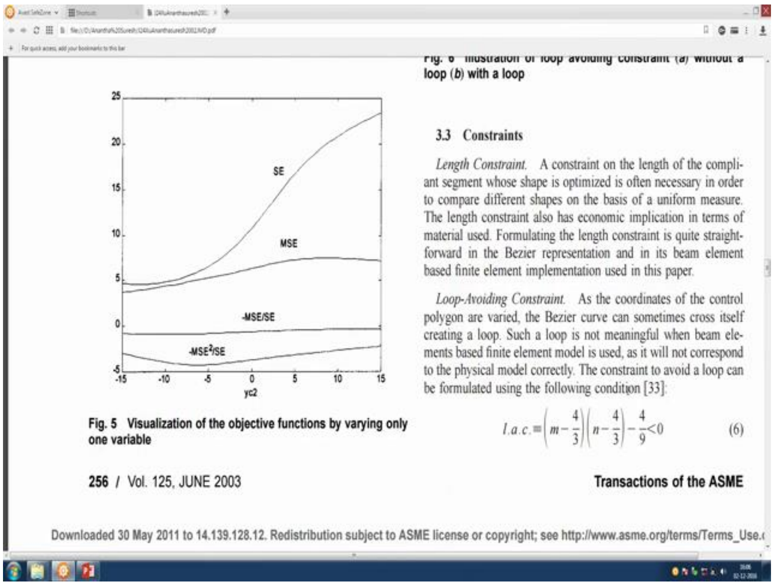

Fig. 6 Illustration of loop avoiding constraint (a) without a loop (b) with a loop

3.3 Constraints

Length Constraint. A constraint on the length of the compliant segment whose shape is optimized is often necessary in order to compare different shapes on the basis of a uniform measure. The length constraint also has economic implication in terms of material used. Formulating the length constraint is quite straightforward in the Bezier representation and in its beam element based finite element implementation used in this paper.

Loop-Avoiding Constraint. As the coordinates of the control polygon are varied, the Bezier curve can sometimes cross itself creating a loop. Such a loop is not meaningful when beam elements based finite element model is used, as it will not correspond to the physical model correctly. The constraint to avoid a loop can be formulated using the following condition [33]:

$$l.a.c. \equiv \left(m-\frac{4}{3}\right)\left(n-\frac{4}{3}\right)-\frac{4}{9}<0 \qquad (6)$$

Fig. 5 Visualization of the objective functions by varying only one variable

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So, we will be go back here, when I get this, how long do you want it? What is upper bound may be a lower bound, lower bound of course is joining from here to here, that is also fine if their combined that straight segment here, straight segment there and that is optimal we can take it. So, we are looking at a gripper such as this one, there is that and there is this.

So, that is that is how it is taken is there a better shape is the question that we are asking, that will give you a better value of the objective function.

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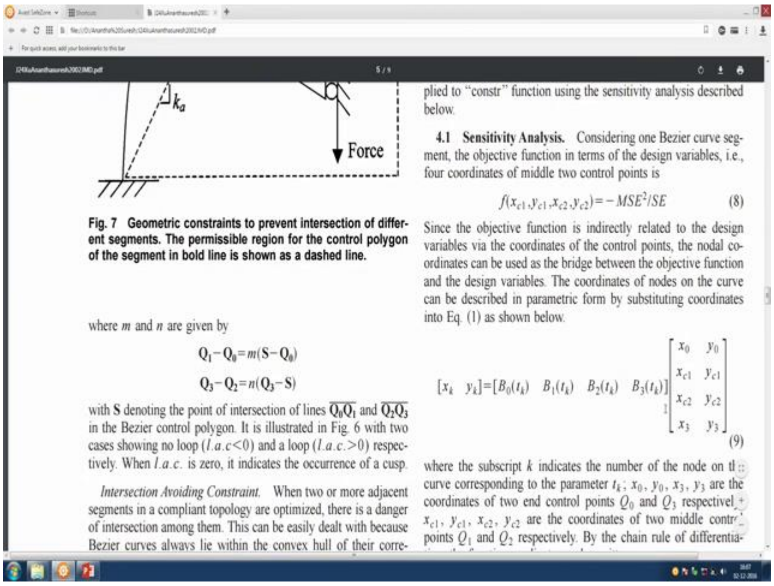

So, we can put this Length Constraint, Loop Avoiding Constraint and also define like a design domain for example, we can take this one for that segment and for this one let us say it is already designed first segment, second segment we say we do not want to interfere with that, we can define a non rectangular design domain for your second curved line and vary that. In fact, both curves you have to do simultaneously, but we have to demarcate the design domain, so that the two curves do not intersect and a curve within itself will not have a loop that is it will not have a double point, it does not intersect itself.

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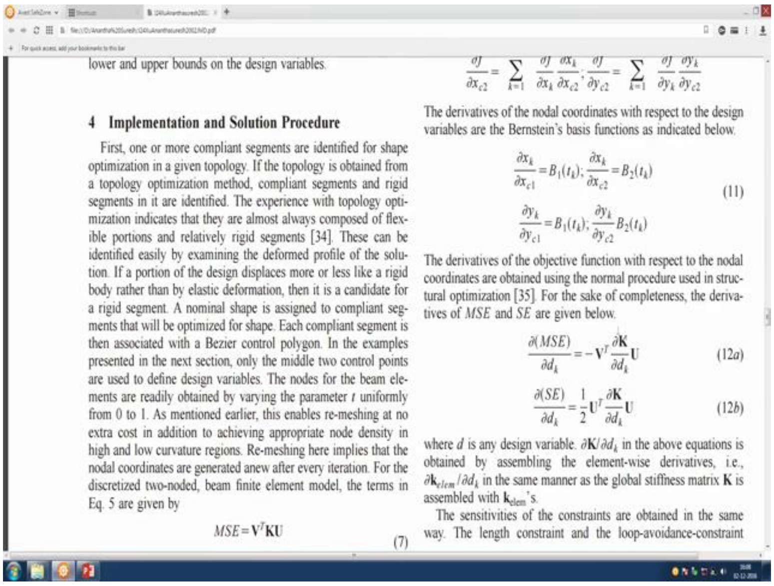

lower and upper bounds on the design variables.

4 Implementation and Solution Procedure

First, one or more compliant segments are identified for shape optimization in a given topology. If the topology is obtained from a topology optimization method, compliant segments and rigid segments in it are identified. The experience with topology optimization indicates that they are almost always composed of flexible portions and relatively rigid segments [34]. These can be identified easily by examining the deformed profile of the solution. If a portion of the design displaces more or less like a rigid body rather than by elastic deformation, then it is a candidate for a rigid segment. A nominal shape is assigned to compliant segments that will be optimized for shape. Each compliant segment is then associated with a Bezier control polygon. In the examples presented in the next section, only the middle two control points are used to define design variables. The nodes for the beam elements are readily obtained by varying the parameter t uniformly from 0 to 1. As mentioned earlier, this enables re-meshing at no extra cost in addition to achieving appropriate node density in high and low curvature regions. Re-meshing here implies that the nodal coordinates are generated anew after every iteration. For the discretized two-noded, beam finite element model, the terms in Eq. 5 are given by

$$MSE = \mathbf{V}^T\mathbf{K}\mathbf{U} \quad (7)$$

$$\frac{\partial J}{\partial x_{c2}} = \sum_{k=1} \frac{\partial J}{\partial x_k}\frac{\partial x_k}{\partial x_{c2}}; \frac{\partial J}{\partial y_{c2}} = \sum_{k=1} \frac{\partial J}{\partial y_k}\frac{\partial y_k}{\partial y_{c2}}$$

The derivatives of the nodal coordinates with respect to the design variables are the Bernstein's basis functions as indicated below.

$$\frac{\partial x_k}{\partial x_{c1}} = B_1(t_k); \frac{\partial x_k}{\partial x_{c2}} = B_2(t_k)$$
$$\frac{\partial y_k}{\partial y_{c1}} = B_1(t_k); \frac{\partial y_k}{\partial y_{c2}} B_2(t_k) \quad (11)$$

The derivatives of the objective function with respect to the nodal coordinates are obtained using the normal procedure used in structural optimization [35]. For the sake of completeness, the derivatives of MSE and SE are given below.

$$\frac{\partial(MSE)}{\partial d_k} = -\mathbf{V}^T\frac{\partial \mathbf{K}}{\partial d_k}\mathbf{U} \quad (12a)$$

$$\frac{\partial(SE)}{\partial d_k} = \frac{1}{2}\mathbf{U}^T\frac{\partial \mathbf{K}}{\partial d_k}\mathbf{U} \quad (12b)$$

where d is any design variable. $\partial\mathbf{K}/\partial d_k$ in the above equations is obtained by assembling the element-wise derivatives, i.e., $\partial\mathbf{k}_{elem}/\partial d_k$ in the same manner as the global stiffness matrix $\mathbf{K}$ is assembled with $\mathbf{k}_{elem}$'s.

The sensitivities of the constraints are obtained in the same way. The length constraint and the loop-avoidance-constraint

You can put those constraints at a shown and then you can do this sensitive analysis because everything is clearly given. So you have a easy way of computing the gradients and all that is shown here, where you take derivative respect to coordinates x c 1, y c 1, x c 2, y c 2, x c 3, y c 3, x c 4, y c 4, we have 4 points control points though are the variables with respect to them you can take the derivative of objective function all the (Refer Time: 18:07) available, that is not at all a problem take derivatives. So, which when we take the derivative of the motivation and energy, that is the output displacement as well as the energy you can do that and once you have them you can solve the problem.

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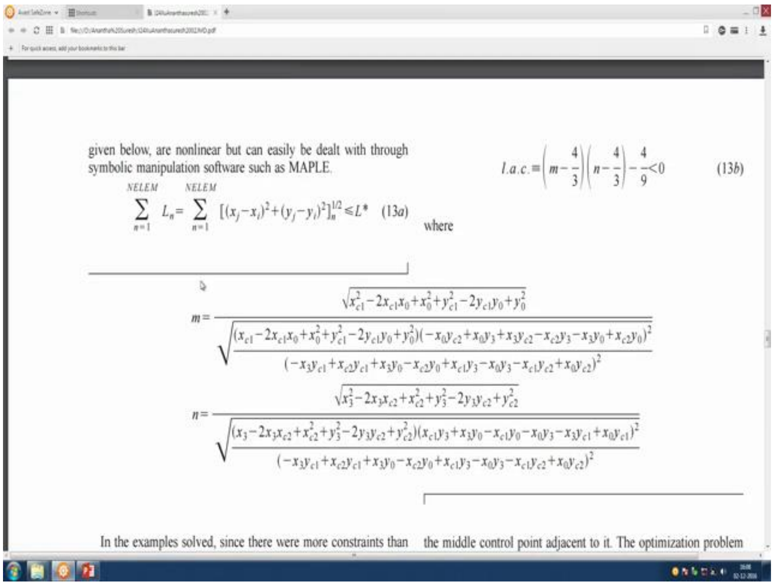

This particular one that m and n were their loop providing constraint, they look rather long, they just long they are not complicated, that loop providing constraint is shown here m given by the long expression, n given by long expression. Basically it is dependent on the coordinates of the control polygon points, that is the conclude providing the constraint and that can be coordinate very easily and when you do this you can actually put let us look at the problem statement.

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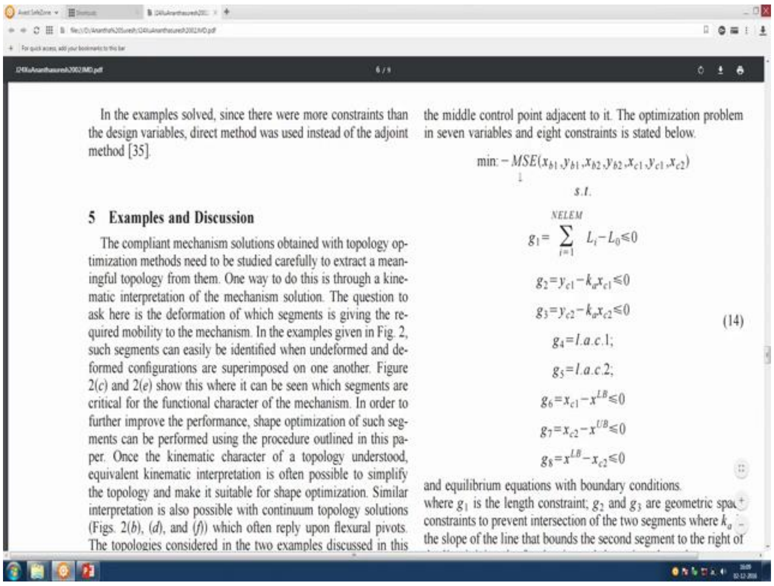
In the examples solved, since there were more constraints than the design variables, direct method was used instead of the adjoint method [35].

5 Examples and Discussion

The compliant mechanism solutions obtained with topology optimization methods need to be studied carefully to extract a meaningful topology from them. One way to do this is through a kinematic interpretation of the mechanism solution. The question to ask here is the deformation of which segments is giving the required mobility to the mechanism. In the examples given in Fig. 2, such segments can easily be identified when undeformed and deformed configurations are superimposed on one another. Figure 2(*c*) and 2(*e*) show this where it can be seen which segments are critical for the functional character of the mechanism. In order to further improve the performance, shape optimization of such segments can be performed using the procedure outlined in this paper. Once the kinematic character of a topology understood, equivalent kinematic interpretation is often possible to simplify the topology and make it suitable for shape optimization. Similar interpretation is also possible with continuum topology solutions (Figs. 2(*b*), (*d*), and (*f*)) which often reply upon flexural pivots. The topologies considered in the two examples discussed in this

the middle control point adjacent to it. The optimization problem in seven variables and eight constraints is stated below.

$$\min: -MSE(x_{b1}, y_{b1}, x_{b2}, y_{b2}, x_{c1}, y_{c1}, x_{c2})$$

$$s.t.$$

$$g_1 = \sum_{i=1}^{NELEM} L_i - L_0 \leq 0$$

$$g_2 = y_{c1} - k_a x_{c1} \leq 0$$

$$g_3 = y_{c2} - k_a x_{c2} \leq 0 \qquad (14)$$

$$g_4 = l.a.c.1;$$

$$g_5 = l.a.c.2;$$

$$g_6 = x_{c1} - x^{LB} \leq 0$$

$$g_7 = x_{c2} - x^{UB} \leq 0$$

$$g_8 = x^{LB} - x_{c2} \leq 0$$

and equilibrium equations with boundary conditions.

where g_1 is the length constraint; g_2 and g_3 are geometric spac[e] constraints to prevent intersection of the two segments where k_a [is] the slope of the line that bounds the second segment to the right of

Here we have mutual state energy and in this particular case, we are not putting constraint energy constraint here; just we want to maximum output placement, but putting constraint in terms of loop providing for the first beam segment, second beam segment and these are the design domain constraint so that the control polygon points lie within our design domain, you can put this and also length constraint for all the arc length of these things you can put a constraint overall you can pose a problem like this and again we have one control polygon for one segment, another control polygon for the other segment and you solve the problem you get optimal shapes like this.

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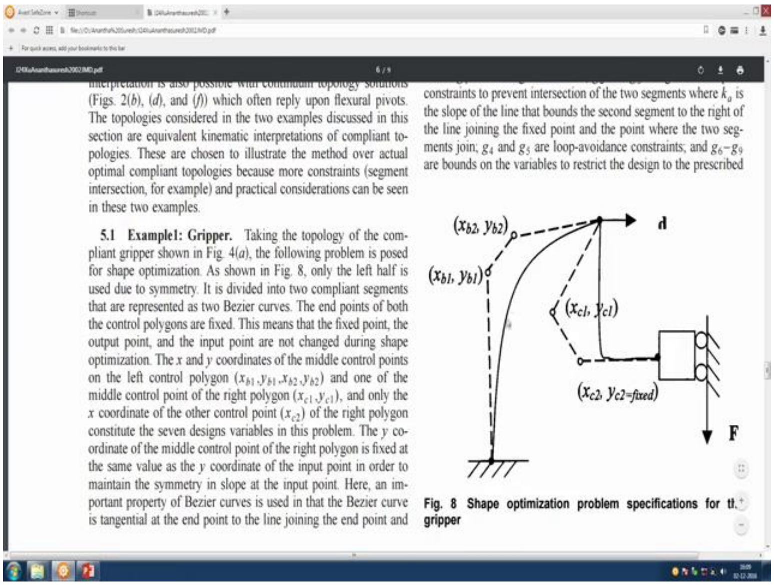

(Figs. 2(*b*), (*d*), and (*f*)) which often reply upon flexural pivots. The topologies considered in the two examples discussed in this section are equivalent kinematic interpretations of compliant topologies. These are chosen to illustrate the method over actual optimal compliant topologies because more constraints (segment intersection, for example) and practical considerations can be seen in these two examples.

5.1 Example1: Gripper. Taking the topology of the compliant gripper shown in Fig. 4(*a*), the following problem is posed for shape optimization. As shown in Fig. 8, only the left half is used due to symmetry. It is divided into two compliant segments that are represented as two Bezier curves. The end points of both the control polygons are fixed. This means that the fixed point, the output point, and the input point are not changed during shape optimization. The *x* and *y* coordinates of the middle control points on the left control polygon ($x_{b1}, y_{b1}, x_{b2}, y_{b2}$) and one of the middle control point of the right polygon (x_{c1}, y_{c1}), and only the *x* coordinate of the other control point (x_{c2}) of the right polygon constitute the seven designs variables in this problem. The *y* coordinate of the middle control point of the right polygon is fixed at the same value as the *y* coordinate of the input point in order to maintain the symmetry in slope at the input point. Here, an important property of Bezier curves is used in that the Bezier curve is tangential at the end point to the line joining the end point and

constraints to prevent intersection of the two segments where k_a is the slope of the line that bounds the second segment to the right of the line joining the fixed point and the point where the two segments join; g_4 and g_5 are loop-avoidance constraints; and $g_6 - g_9$ are bounds on the variables to restrict the design to the prescribed

Fig. 8 Shape optimization problem specifications for th. gripper

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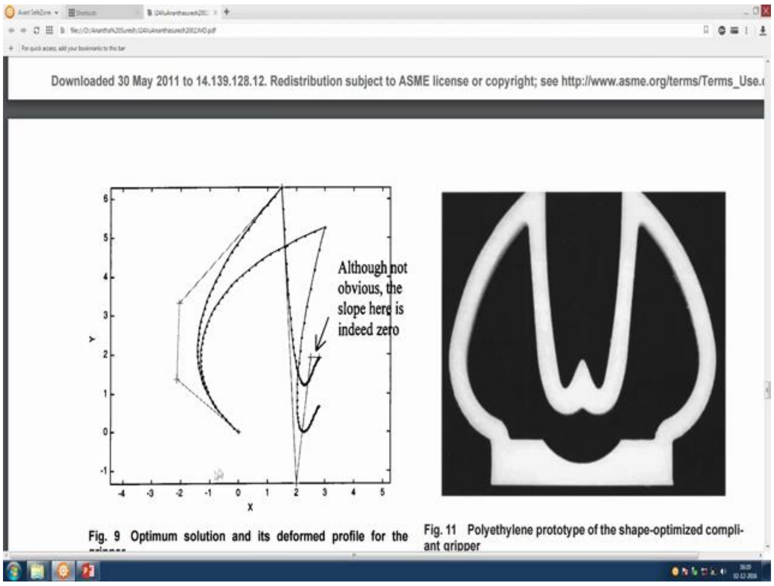

Fig. 9 Optimum solution and its deformed profile for the gripper

Fig. 11 Polyethylene prototype of the shape-optimized compliant gripper

You can see what I told you about when I change the parameter uniformly, so wherever I have rather straight or low curvature regions, these things are further, when I go to the place where there is high curvature, there are more and more point. That is exactly what you want in messing of beams or frames, we want more elements where the shape is changing a lot, fewer elements were shape is not changing that automatically comes here and the optimal shape of these segments is here before and after deformation. So, this is before deformation and this other one is after deformation, so here we have that fabricated with the shape optimized design.

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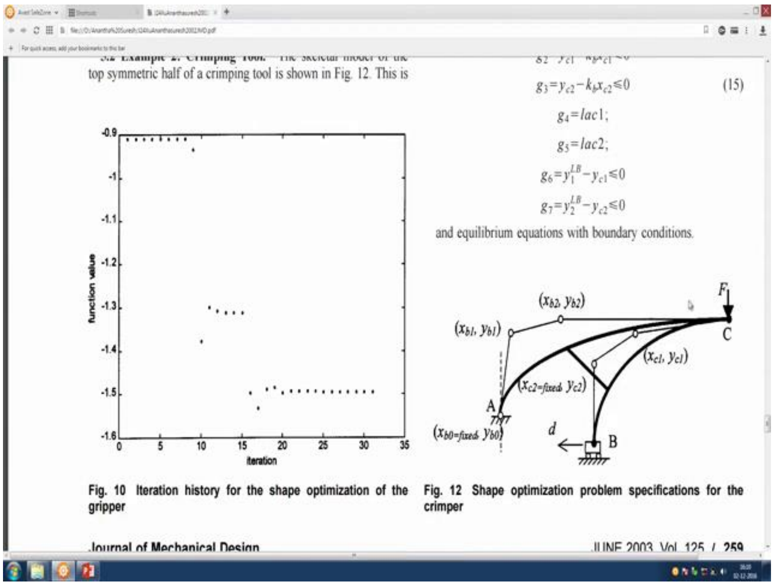

top symmetric half of a crimping tool is shown in Fig. 12. This is

$$g_3 = y_{c2} - k_b x_{c2} \leq 0 \quad (15)$$
$$g_4 = lac1;$$
$$g_5 = lac2;$$
$$g_6 = y_1^{LB} - y_{c1} \leq 0$$
$$g_7 = y_2^{LB} - y_{c2} \leq 0$$

and equilibrium equations with boundary conditions.

Fig. 10 Iteration history for the shape optimization of the gripper

Fig. 12 Shape optimization problem specifications for the crimper

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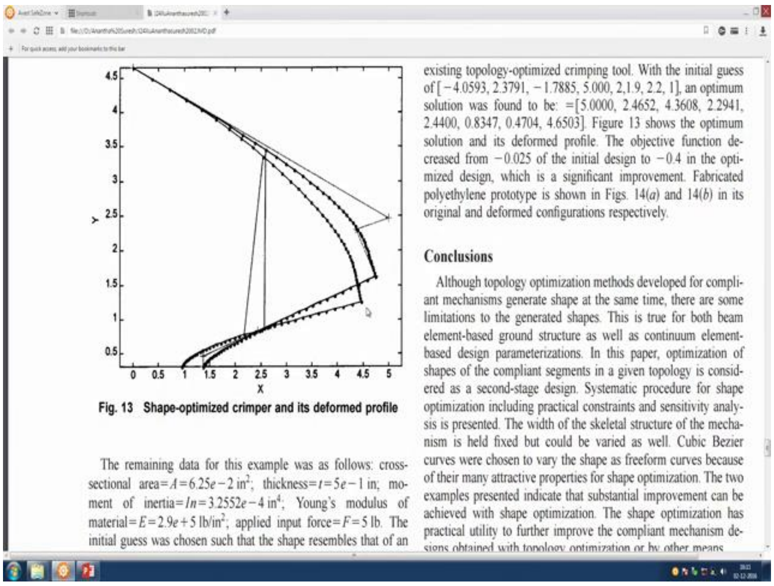

Fig. 13 Shape-optimized crimper and its deformed profile

The remaining data for this example was as follows: cross-sectional area $= A = 6.25e-2\ in^2$; thickness $= t = 5e-1$ in; moment of inertia $= In = 3.2552e-4\ in^4$; Young's modulus of material $= E = 2.9e+5\ lb/in^2$; applied input force $= F = 5$ lb. The initial guess was chosen such that the shape resembles that of an existing topology-optimized crimping tool. With the initial guess of [−4.0593, 2.3791, −1.7885, 5.000, 2,1.9, 2.2, 1], an optimum solution was found to be: =[5.0000, 2.4652, 4.3608, 2.2941, 2.4400, 0.8347, 0.4704, 4.6503]. Figure 13 shows the optimum solution and its deformed profile. The objective function decreased from −0.025 of the initial design to −0.4 in the optimized design, which is a significant improvement. Fabricated polyethylene prototype is shown in Figs. 14(*a*) and 14(*b*) in its original and deformed configurations respectively.

Conclusions

Although topology optimization methods developed for compliant mechanisms generate shape at the same time, there are some limitations to the generated shapes. This is true for both beam element-based ground structure as well as continuum element-based design parameterizations. In this paper, optimization of shapes of the compliant segments in a given topology is considered as a second-stage design. Systematic procedure for shape optimization including practical constraints and sensitivity analysis is presented. The width of the skeletal structure of the mechanism is held fixed but could be varied as well. Cubic Bezier curves were chosen to vary the shape as freeform curves because of their many attractive properties for shape optimization. The two examples presented indicate that substantial improvement can be achieved with shape optimization. The shape optimization has practical utility to further improve the compliant mechanism designs obtained with topology optimization or by other means.

So another example that is shown here again two segments, control polygon for that control polygon for this and here is the optimized one before and after deformation along with the control polygons that are given and this is the one that is fabricated proto type.

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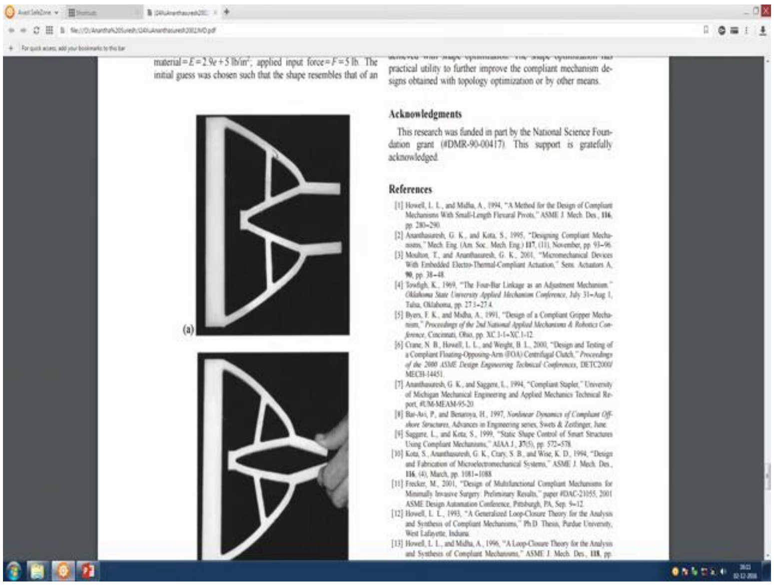

In fact, we have seen this earlier this proto type, it is a very good mechanical advantage for this. It is very flexible, it is distributed complain design in a way and it has a very good mechanical advantage as well and you cannot say that one point is deforming more than the others and it has uniformed width everywhere, that is what we have assumed in this Skeletal Shape optimization.

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So, there is another word that is actually a quite interesting where instead of having uniform width everywhere and changing only shape, one can do this simultaneously both shape of the skeletal segments as well as the width of the thing and that is called the concept of Wide Curves.

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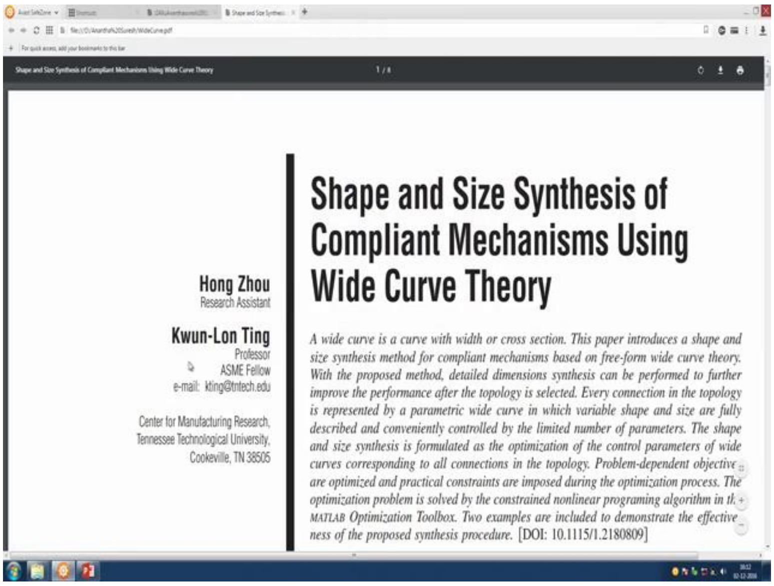

So, this is Hong Zhou who is now a professor himself and professor Kwun-Lon Ting who are a Tennessee technological university and this paper is published in journal of mechanical design in 2006 about 10 years ago, what they did was shape and size synthesis, so they use this Bezier curves for shape like we discussed here, but also they change a size meaning they would change the cross section profile at the same time, but in the previous paper that we discuss the cross section was the same, but here they change that also, they call that wide curve is basically a extension of Bezier curves, where in addition to the shape of the curve, you also have the width or cross section dimension also varied.

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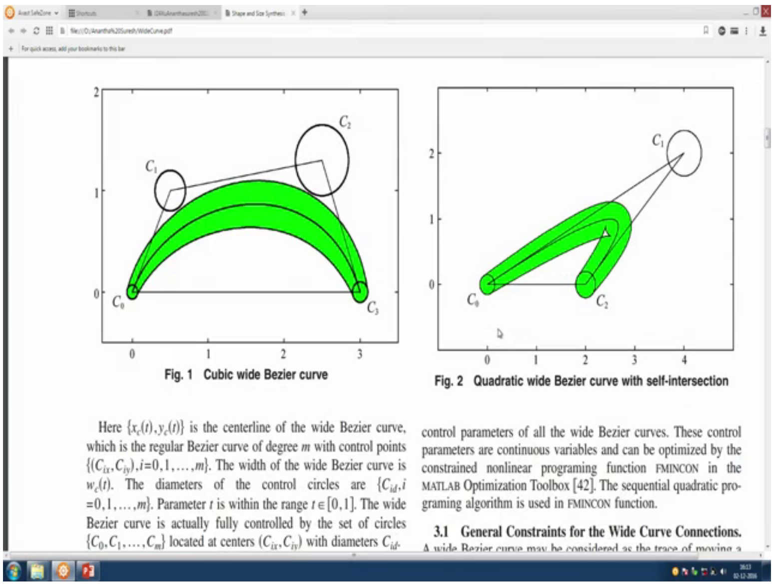

So, they use something like this, so here you have a Bezier control polygon but you also have a radios function there, a cross section function may be small here, larger, largest and then decreasing again. So, it increases; goes to the maximum and then decreases are something that is shown there. So, you can have this wide curve theory used so that you not only vary the shape of the skeletal segments of a complain mechanism, but also vary the cross section shape.

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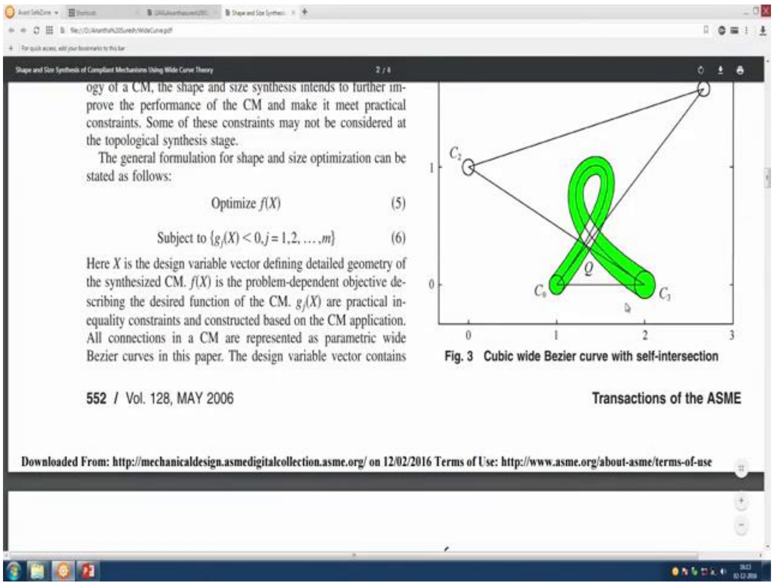
Shape and Size Synthesis of Compliant Mechanisms Using Wide Curve Theory

ogy of a CM, the shape and size synthesis intends to further improve the performance of the CM and make it meet practical constraints. Some of these constraints may not be considered at the topological synthesis stage.

The general formulation for shape and size optimization can be stated as follows:

$$\text{Optimize } f(X) \quad (5)$$

$$\text{Subject to } \{g_j(X) < 0, j = 1, 2, \ldots, m\} \quad (6)$$

Here X is the design variable vector defining detailed geometry of the synthesized CM. $f(X)$ is the problem-dependent objective describing the desired function of the CM. $g_j(X)$ are practical inequality constraints and constructed based on the CM application. All connections in a CM are represented as parametric wide Bezier curves in this paper. The design variable vector contains

Fig. 3 Cubic wide Bezier curve with self-intersection

They also wide a loop a winding constraint they put that in so that you do not end up with things like that, it basically translates whatever done in the previous paper for the wide curve theory and again they also do sensitive analysis to get the gradients and try to get the design.

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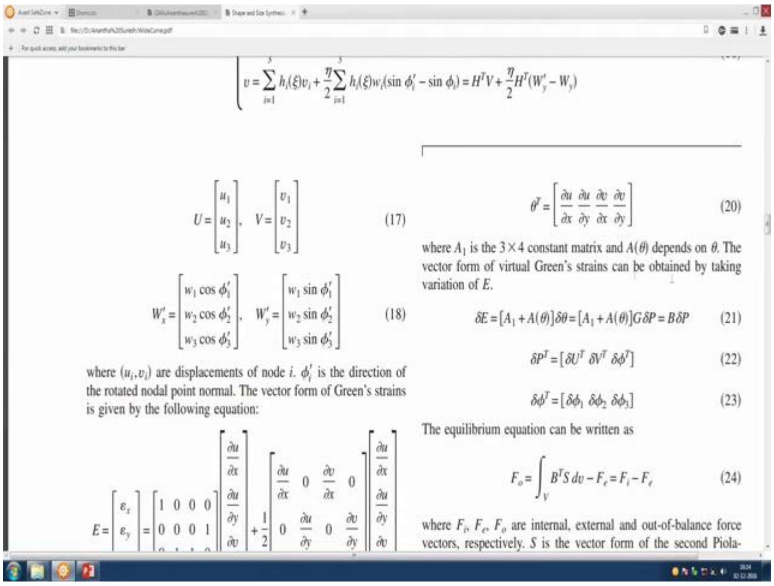
$$v = \sum_{i=1}^{3} h_i(\xi) v_i + \frac{\eta}{2} \sum_{i=1}^{3} h_i(\xi) w_i (\sin\phi_i' - \sin\phi_i) = H^T V + \frac{\eta}{2} H^T (W_y' - W_y)$$

$$U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (17)$$

$$W_x' = \begin{bmatrix} w_1 \cos\phi_1' \\ w_2 \cos\phi_2' \\ w_3 \cos\phi_3' \end{bmatrix}, \quad W_y' = \begin{bmatrix} w_1 \sin\phi_1' \\ w_2 \sin\phi_2' \\ w_3 \sin\phi_3' \end{bmatrix} \quad (18)$$

where (u_i, v_i) are displacements of node i. ϕ_i' is the direction of the rotated nodal point normal. The vector form of Green's strains is given by the following equation:

$$E = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \ldots \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \ldots \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} \\ \ldots \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \frac{\partial u}{\partial x} & 0 & \frac{\partial v}{\partial x} & 0 \\ 0 & \frac{\partial u}{\partial y} & 0 & \frac{\partial v}{\partial y} \\ \ldots \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} \\ \ldots \end{bmatrix}$$

$$\theta^T = \left[\frac{\partial u}{\partial x} \; \frac{\partial u}{\partial y} \; \frac{\partial v}{\partial x} \; \frac{\partial v}{\partial y} \right] \quad (20)$$

where A_1 is the 3×4 constant matrix and $A(\theta)$ depends on θ. The vector form of virtual Green's strains can be obtained by taking variation of E.

$$\delta E = [A_1 + A(\theta)] \delta\theta = [A_1 + A(\theta)] G \delta P = B \delta P \quad (21)$$

$$\delta P^T = [\delta U^T \; \delta V^T \; \delta\phi^T] \quad (22)$$

$$\delta\phi^T = [\delta\phi_1 \; \delta\phi_2 \; \delta\phi_3] \quad (23)$$

The equilibrium equation can be written as

$$F_o = \int_V B^T S \, dv - F_e = F_i - F_e \quad (24)$$

where F_i, F_e, F_o are internal, external and out-of-balance force vectors, respectively. S is the vector form of the second Piola-

So, let us look at couple of things examples here, we get things like this.

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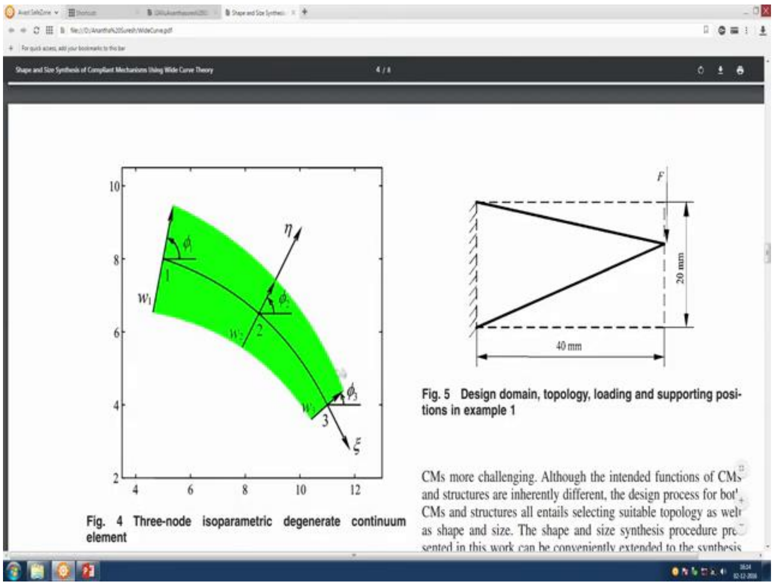

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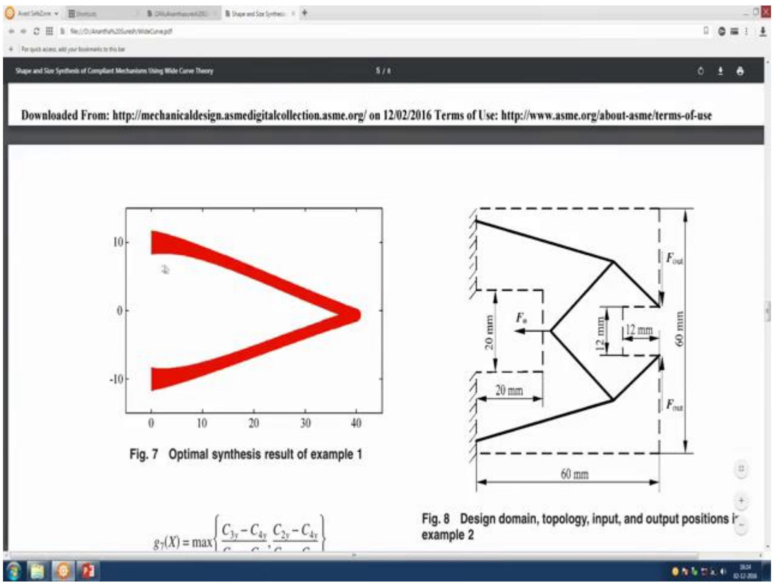

Now, you see there is a nice distributed couple and mechanism because of shape optimization not Topology and you have a cross section vary rather smoothly, is another one, he had to take this skeleton first that this Topology you assume and then you do the shape optimization.

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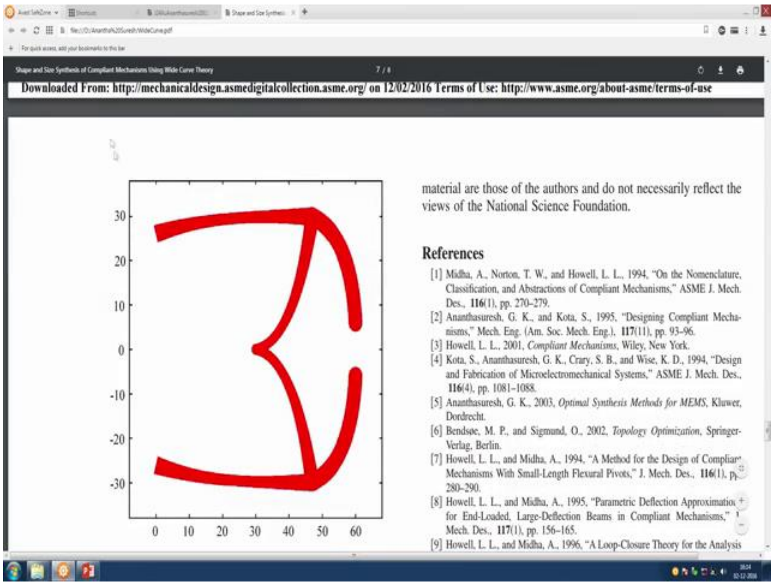

So, you end up with designs like this, you have taken that original Topology; now we can get the nice shape, which has variable crossing in profile also, but this is the second step. In optimization we have this hierarchy for the geometric form, what is Topology and then we have the shape and then we have the size. So, today in this lecture we discussed shape optimization using Bezier curve, one can do explains and many other ways as well and also discuss the second paper by Zhou and ting, where they weighed the size also, so shape and size that is the wide curve theory paper that you can look up.

So, we have discussed enough about topaz optimization and the shape optimization and size optimization, of course there is a lot more literature on this. But now let us pass and see how we can use in a practical application and what are the limitations and what else could be done, so will take up a case study in the next lecture, so that we see how we can use topaz optimization and shape and size optimization to solve a practical problem which is the final aim for any research that raise develop design methods.

Thank you.