Compliant Mechanisms: Principles and Design
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Lecture – 34
How to achieve distributed compliance

Hello, again the last lecture, lecture number 33; we ended with a question saying that how does one achieve distributed compliance and for that how do we quantify distributed compliance so that we can incorporated in to topology optimization. There have been couple of method; actually couple of method; just two methods are essentially that are worth mentioning and how those methods are able to achieve this distributed compliance is what we will discuss now.

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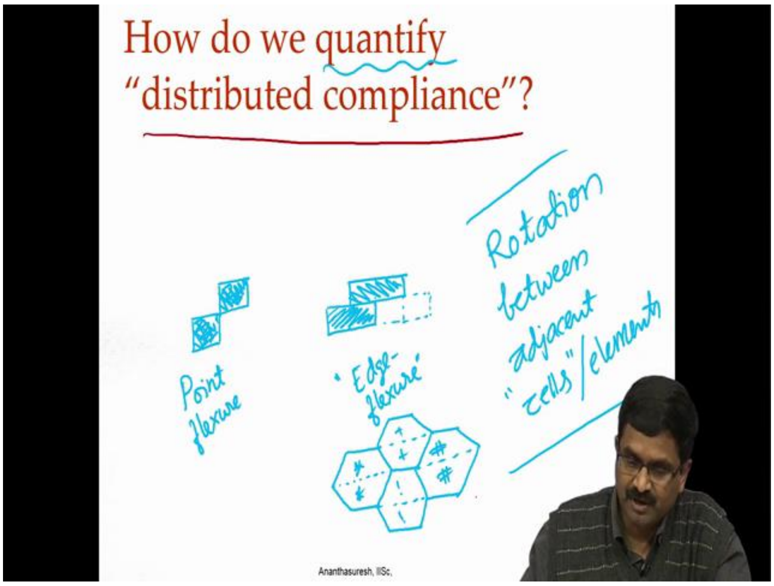

So, we will start with where we left off in the last lecture, how do we quantify distributed compliance. One way is to use this relative rotation, so what do I mean? so whenever these point flexures are edge flexures you know something like this edge flexure that two elements. You can call them you know elements you have staggered like this actually in finite element frame work; we will take these elements everywhere. So, these are all there, but for the purpose of design, you keep this densities of those two the same and also those two the same and as we just said in the last lecture then instead of getting this point flexure; we get what we can call narrow edge flexure slightly better than this, but not a whole lot.

The other way as we have all ready discussed in the last lecture, if you have hexagonal parameterization of your element and you divide this into 4 quad laterals for the purpose of finite element analysis, but otherwise now it is basically normal finite element frame work, but these two elements will have a same density and let us say I will put indicate you know like that are plus these to be will have the same thing is some other symbol.

We basically have half the number of a design variables if there are let us say n finite elements which are all trapezium like quadrilaterals, there half of the regular hexogen you have half of them to be design variable there we get edge flexure, that is not what we want but what we seen both cases either edge flexure of point flexure is that, there will be a lot of rotation between adjacent elements, that will be the characteristic that is how the a hinges work lot of rotation between adjacent we can call them now cells, because these are call hex cells right are elements finite elements that we have.

So, which we are way you do, we will end up having lot of rotation and that what causes this is rotation, lot of rotation between adjacent cells or elements. So, if you want distribute compliance meaning if you want the entire structure of the compliant mechanism to undergo elastic deformation we should not allow too much of this rotation between the adjacent a cell or elements.

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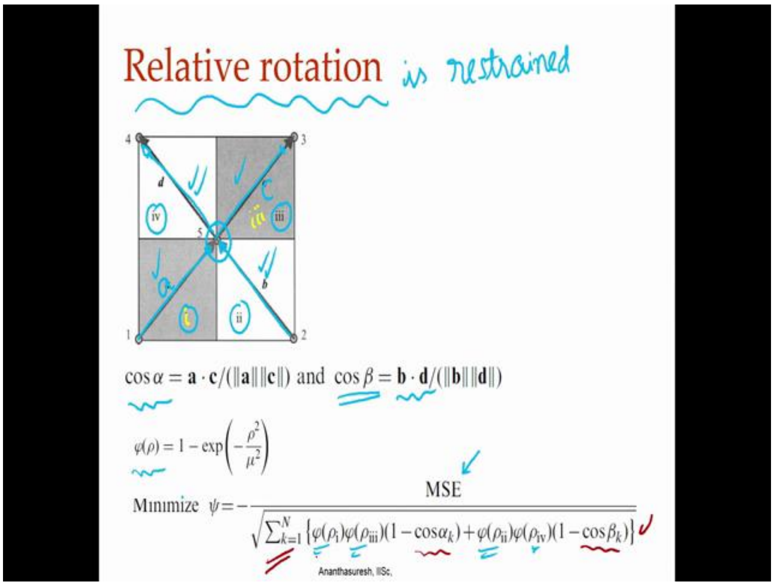

So, we can look at a formulation which is sub what complicated, so what we do is, whenever a situation that is shown here happens that is there is material here and here and there is no material in the diagonal elements then what you say is you try to measure the rotation and that point. How do you do that? you had you define this vector a here and vector c here, so this is there are two vectors that has shown between those two you compute the angle of rotation alpha or cosine of that for because cosine is simply dot product was easy, so a dot c.

Similarly if it is other way around that is material is here and here and these two are empty that is I this is 1, 2, 3 and 4. If 1 and 3 are empty and 2 and 4 are full than you can be find the rotation cosine beta by either dot product of this vector b here and then d there. If you do that, then we can pose on objective function such as what is shown here again we go for this MSE which is flexibility or compliants and this is more towards strength in a way, because we are trying to minimize the rotation and hence hopefully the distortion energy did was minimize the strength because distortion energy figures in determine the strength of the component right. So, it is a flexibility and strength formulation what we are discussed in the last lecture, but now we are looking at a formulation where this relative rotation is restrained, that is we do not allow between adjacent elements, we do not allow excessive rotation, so the algorithm is trying to exploit by going for this 1 node hinges.

Now, we close the door for the algorithm and then see what happens, how do we do that again why do we need all of this actually this is; in this case a phi rho is something like a filter that is we put it only at the points, where this type situation occurs that is two things are diagonally connecting, so we have this filter functions there whether it is between 1 and 3 or 2 and 4, so let me show this 1 and so this is 1 and this is 3 and 2 and 4 can already see very well.

So, we put that between 1 and 3 we have this cosine alpha, between 2 and 4 we have cosine beta we put that in the denominator on be some over all the elements, so that we filter out or we pick out computationally all the possible things even when these corresponding rhos that is indicator functions whether they are 0 or 1, we try to do that over the entire thing, capture that put in the denominator; that means, that when we are minimize in negative meaning maximizing where essentially make denominator become as small as possible that is we are trying to minimize or restrain this will relative rotation.

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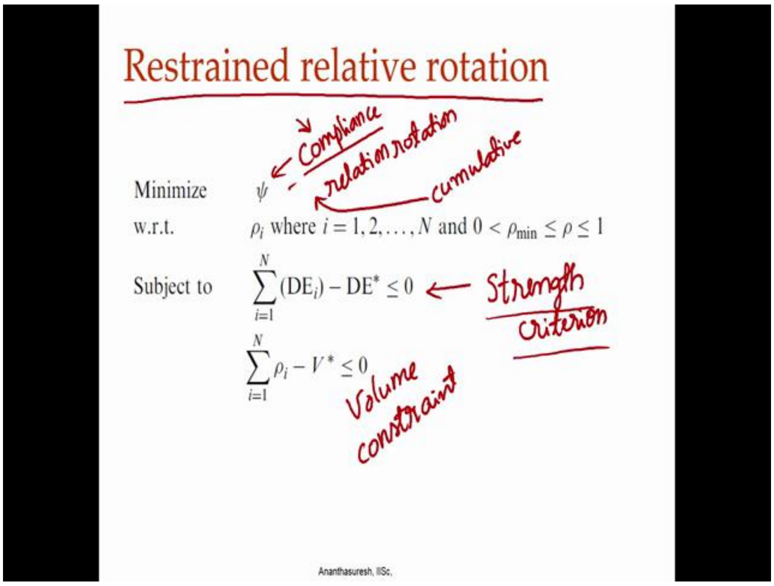

If you do that you are not allowing this hinges to form that is what should happen, let us see that works or not. So, the problem now is one of restrained relative rotation it is little complicated, because we have to get this complicated function psi which was there in the last slide, this psi. You want to make this psi as large as possible, because MSE and this both; you want to make it large, so we are minimizing in negative of that right; so we have that psi with the negative of what we have.

So, rho similar indicator functions, but we can also put our distraction energy constraint this is the in a way a strength constraint, because the distraction energy from which we calculate what we call von mises stress, if you recall your discussions in failure analysis courses in mechanically design, we actually start in order derive the concept of von mises stress, we start from distraction energy where we take the (Refer Time: 09:16) energy subtract format the isotropic component and live in the deviatory component.

So, that is basically strength a criterion, so now we are moving towards a formulation for getting distributed compliance, we have here already compliance and now we have strength criterion has a constraint and we have in the denominator of that the psi compliance you want to maximize that there the negative minimize and then here we have relative rotation cumulated will to rotation of all the elements, the way we just discussed in the previous slide; it should add another word here calling cumulative relative rotation, let us go back to the previous slight and see what we mean, see this denominator here this part is the cumulative relative rotation.

So, we are trying to minimize that cumulative relative rotation or everywhere, so that everywhere it rotates only a little bit, bends a little bit. If you want a large displacement, which is where asking, where we have this compliance requirement here, we want large displacement, but you do not want any two elements to rotate a lot that is the idea of distribute compliance; that means, that two elements having a joint are point flexure is not a load we want something to have the whole thing elastred deform and we also put the strength criterion so that in doing so, any one point should not be a stress too much also and then we have the volume constraint.

So, this formulation actually works quite well and its logic is quite strong, but implementation is little bit difficult because of the complexity of this. So, we can actually compute that is not a problem, but imagine taking sensitivities that is for your gradients then things become quite complicated, because your cosine of angle, angle in the dot product to vector and these vectors will have the nodal coordinate and then we have this filters which have some exponential functions there other forms, but whatever you do it becomes quite an ordeal so, but in terms of logic it does ensure distributed compliance.

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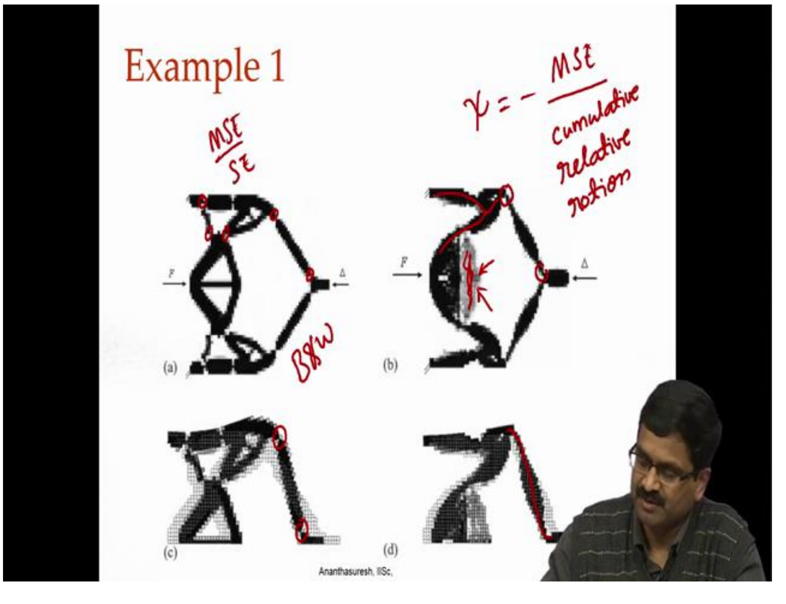

So, here let us look at an example, this one is the usual way that is the usual MSE over SE formulation and this is our new formulation where psi we have negative of MSE divided by I can call it cumulative that long a summative expression, cumulative relative rotation in the entire image of the topology doing the course of the optimization.

We can see that we have joints lots and lots of joints right whereas, here are the joints not there they are there actually I am showing in example, to show that somewhere else they become more you know uniformly distributed if I look at the top have, but other places it does have, but if we zoom in a little bit adjust parameters, now we can see that at least it has become edge flexure rather than point flexure that happens here. So, by constraining or restraining relative rotation where able to go towards a distributed compliance of course, there are some in converged region also here it is quite close to black and white right, but here there are somethings basically the complexity of the formulation causes problems in implementation.

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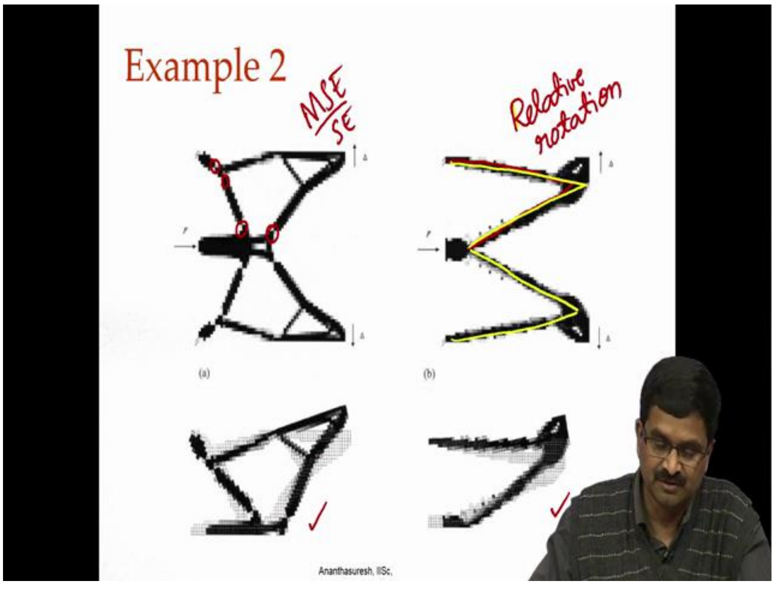

So, that is a problem here let us look at another example, so here again we see the joints here and there is a joint whereas, here this has done quite well because what we see here it just 2 beams that came from continue formulation. So, if we look at this we have a nice 1 beam their another beam their such has the one then one would sketch as just 2 beam elements that what is we have is the symmetric problems you have to (Refer Time: 13:57) elements this came from this relative rotation formulation that we have, cumulative relative rotation that when you restrict or restrain you know get this is usual MSE over SE formulation, so it is able to give us something useful.

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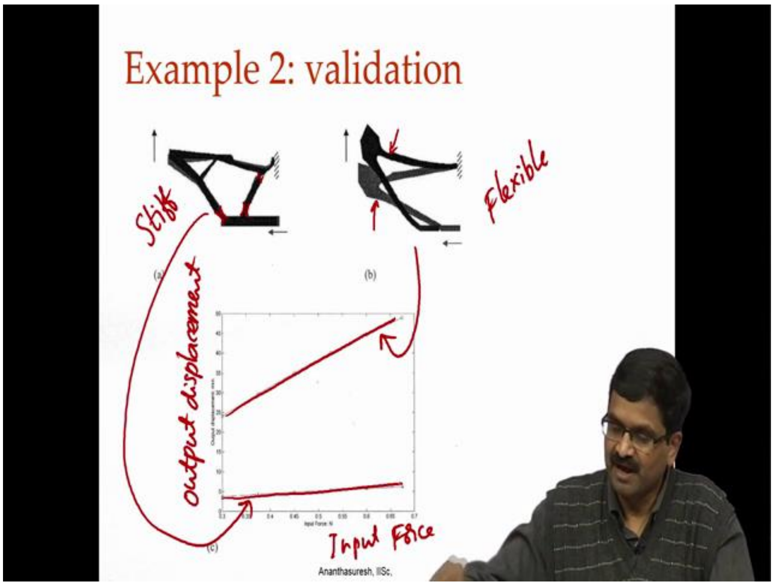

Now let us validate it, so in the sense that if you take the topology that we have, this is the usual one, this is the restrained relative rotation result these two let us actually you know clean it up. So, how to speak the just math lab program software has you know routine where you can actually smooth it out by just using one of the readymade functions.

So, let us smooth it out like this where we have this flexures, these are all the flexures and other one anyway is actually both original one that is this one the deformed one are shown in this figure, and why are we not seeing deformed one here in the discrete case for the same force, it turns out that it is actually quite stiff and this is flexible that is actually validation, what you see here this is this curve here we have output displacement, everything is the same there two synthy example is similar data and we got two designs use in the usual MSE over SE and with an without this relative rotation think input force here.

So, you see this one; this curve small displacement it is linear, this shows a little bit of nonlinear, but any way that is not the point here you get a lot more displacement for this mechanism as opposed to this.

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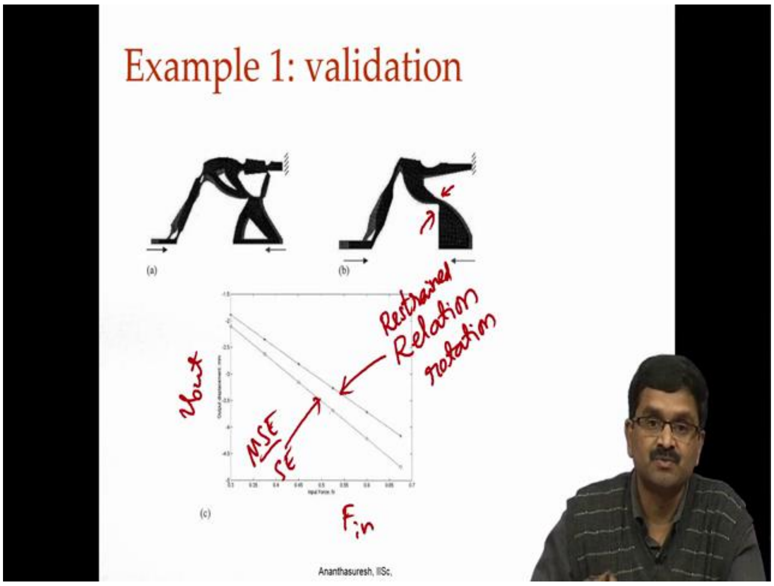

To prove to you I even have proto types that are made using this and this is true for the other example where there was actually hinges occurred in the distributed thing also, that is why this is the restrained relative rotation and I should always put restrained relative rotation formulation and this is our usual MSE over SE, again this is the output displacement this is input force, here it is only slightly better because this also has the hinge as we all ready saw. The example one that we showed this also had a bad hinge, we could not completely avoid it, may be the tuning on the parameter was required in the example, but we can see both extremes. So, in this particular case that this is very stiff, we can see by looking at a proto type that I have.

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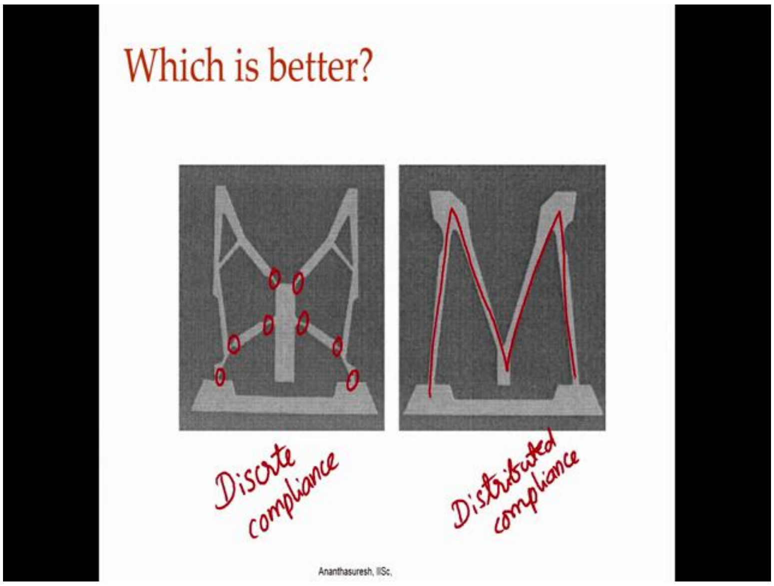

So, here if I look at this, this is our discrete both were given by topology optimization this is discrete compliance and this is distributed compliance. We can when you look at them you see is this where the joints are right and of course symmetric, so we have joints here, here, here; whereas here it is smooth beam like this. These were; is same almost I want of material in a topology images and we have made these where the purpose of make in progress time, which is better clearly as we will see, the right side one we have this. So, when I applied force this is the input force here, it is supposed to go and pinch something, so that is the output here. So, it pinches in fact; this is the quite good mechanical advantage to when I when you put your finger in a mechanism like this and apply force, you actually feel lot of force and it is quite flexible, now the looks like I cannot say 1 point is experience in lot of large deformation compared to other points.

More or less everything is deforming of course, this handles are intestinally made so that we can put your finger over here and do that otherwise they are not really important, but otherwise the rest of the structure at least I would say 80, 90 percent of the structure is actual elastic deforming. As opposed to, let us take the proto type of the other mechanism, which is what we have to orient you, I should put it like this right. So, are upside down right so this is of it is so by the right side one, so when apply force it is going right it is also seems to be flexible, but it is actually a lot more force is required to actually apply this and more over if I put here my finger and try to feel the force it actually does not give us much force and in fact, if I am not careful here what it does is actually go out of plain, because its flexibility in the out of plane direction is more than emplane it would actually go.

So, and of course if we closely look at it these are the once that outgoing to undergo plash deformation very quickly and they might even fail there. So, the MSE over SE formulation, when it gives hinges it is very good for designing rigid body linkages rather than compliant mechanism, compliant mechanism for distribute compliance we have to go for this relative rotation restrained, but it is a little involved. So, we look at compliant mechanism topologies as something that gives us conceptual design for as to sketch and tell to avoid the hinge based designs whenever it is possible, these what people have done and there is an alternative way, instead of this complicated way of computing vectors in that products and get in the relative rotation quantify, but if you think about our usual strain shears strain.

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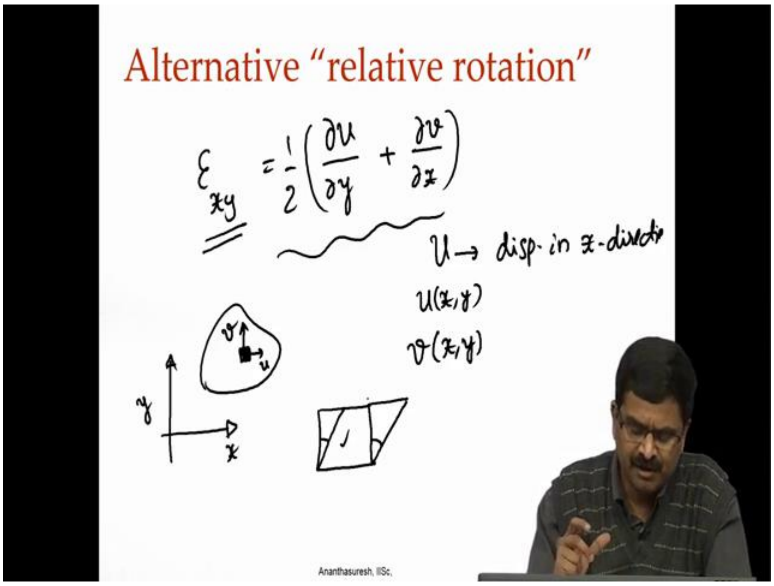

So, shears strain if I say epsilon x y into 2d that will be, dou u by dou y plus dou v by dou x with a half, where u is the displacement of a point in the x direction that is if I take a elastic body let us I have a x and y axis our elastic body, if I take a point the displacement of that is u function everywhere. So, u is actually a function of x y everywhere and likewise how much it moves the y direction v, I can call it also a function of x y in 2d then at an; if I take element there around that differential element then the shears strain in it epsilon x y is given by half dou u by dou y plus dou v by dou x and that is actually nothing, but when I say shear, if a take an element the shear is how it would actually go like this that is what we call the shears, so this angle we call up talk of shears strain.

So, if you take this for a every element and basically integrated over the entire domain because this is the a measure of relative rotation in a way how much the element is the distorting, you can take this how much a shears is there with this element at the next one we take the difference and quantified rather than go if the vectors this can also be done and it gives similarly results meaning that it does avoid hinges not always, but butchering you can make it work, but it still quite of a difficult formulation to implement, but that is there and there is no other idea that directly tackle his problem, people have tried a lot of methods to avoid this 1 node hinges, but this relative rotation contain relative rotation restraining it seems to be the best way.

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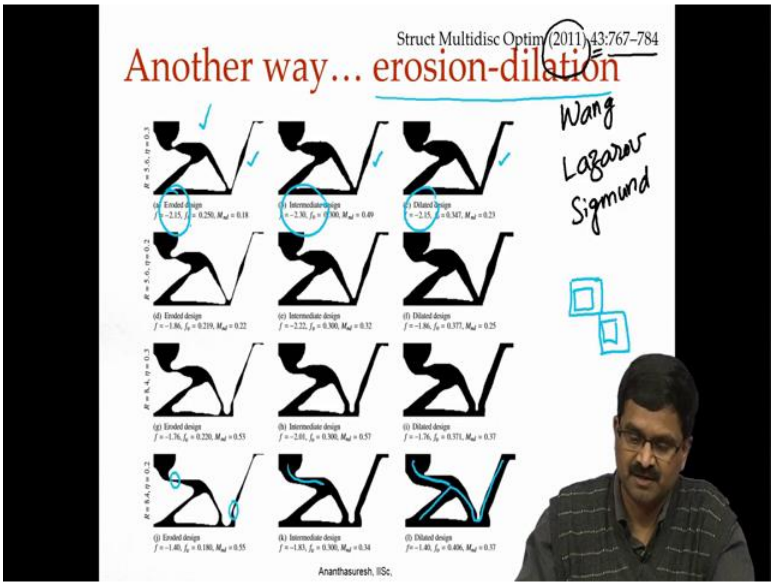

Recently about again (Refer Time: 23:11) about five years ago in 2011, I think the authors are Wang and Lazarou and Sigmund, their paper appeared in structure material optimization volume 44 in 2011, page numbers are given here. So, what they did was is; like a different way, they used what they call erosion and dilation method. What they do is let us say in the course of topology optimization, you have all the indicator functions rhos everywhere, what you do is every element that you have you eroded did a little bit meaning that you make it thinner, that is if I have let us say in element like this and element like this right. So, you make this element a little bit smaller little bit smaller rather they bit are whatever you can see compare to this, this is little bit weaken as if it as does not eaten for some time right, it is eroded whereas this actually has become fatter it fed more. So, they use this erosion dilation which is a concept again in image processing where you can take in image and like change in the contrast of the image, you can make things go narrow or fatter.

So, they used that and for any given intimated design, they construct a model that is eroded and the constructed model that is dilated, expanded and in both cases in all three cases in say both cases, all three cases; we compute your performance measure whatever it is MSE SE or a five different things we talked about, any that you want your compute your performance measure take the worst one and try to improve in that optimization, optimization always you start initial guess your performance measure, you try to improve it. Here they take a they candidate one and try to eroded and dilate and all three you compare; take the worst one and try to improve of quantity, that way you get a design that kind of looks distributed.

You can see here, so these all distributed, but very weak you know these are like almost like point flexures right, whereas here is slightly fatter right and of course, this is the fattest design. So, they try to see in a given application you try to get something that is intermediate between, so these the intermediate one it will erodent dilated so that particular one if it is eroded, it will become very weak are your performance criterion something is stress constrained are whatever is a compromised, it will not go towards eroded design, it will not go dilated design whether make it very stiff you do not get maximum output displacement there do this.

So, these all in implementation wise not very straight forward it can be done like all papers in the publish papers, you have a method that is tried out and tested and validated, but for us to for everybody else to use its a lot of work, but that is how the state of the r t is in terms of this distributed compliant mechanisms.

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Again to conclude all that we have taken today is taken from this paper the talk about design of distributed complaint mechanism in this journal, but what we need to understand is that avoiding excessive relative rotation is the key to distributed complaint mechanisms. We should not let things bend too much, so let us look at actually what exactly constitutes excellent distributed complaint mechanisms. So, let us look at that point before we end this lecture.

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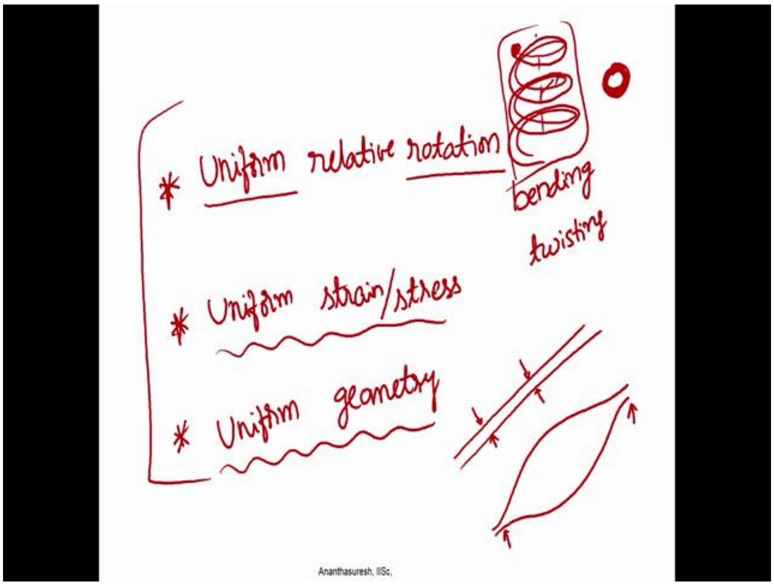

So, what exactly would be ideal distributed complaint mechanism, so ideal distribute complaint mechanism is one, which has uniform relative rotation that is the bending I am calling rotation to mean bending or twisting a complaint mechanism in 2d works by bending in 3d if we have beams. There will be bending and twisting I am not so worried about stretching because stretching like tension it is actually in fact, complaint mechanisms that are based on strings in tension are quit welcome, because there will be quite strong and there will be nice to deal with.

We are worried about bending, so bending or twisting is focused on to a few regions what we called discrete regions that is actually bad, because that is where thinks will break we want to avoid that, so you want to have uniform relative rotation. Since this is not enough for difficult to implement with any formulation all that is been done in two papers that I described today, we also want to have uniform strain; at the same time it would mean that will have uniform stress also this is one characteristics, this other character uniform strain or stress.

If we have that actually very good, but a compliant mechanism unlike structures cannot leave with only uniform stress because it has to have functionality. So, there is disparity in terms of some regions being more stress compared to other regions, so the first one is uniform relative rotation, second is as much as possible we should have uniform strain and stress throughout compliant mechanism, but there actually third one to characterize this distributed complaint mechanisms, which is uniform geometry actually, what do I mean by that most of the complaint mechanisms are slender beam segments that is what they have in them and those (Refer Time: 29:39) segmentation should be uniform in their size profile, that is you want to have things that are like this; not like this, this actually amongs to again the hinges we try to avoid. We want things that are uniform wherever you take; you want to have uniform geometry.

So that bending and twisting will be more uniform, you need to have uniform relative rotation that is our aim and ideal aim is been uniform stress and strain, but those things can be achieved if you have actually uniform geometry, is there such a design, the best thing that comes to my mind are comes to anybodys mind is actually our helical compression spring, think of helical compression spring or the extension spring helical spring. The geometries uniform the cross section; I am talking about the spring that is there in all ball point pens and so forth right.

So, we have let us say circular cross action; the same circular cross section is there everywhere. So, that has uniform geometry and does they we have uniform stress have strain, in fact that is true wherever you cut as you know from analyze of the beam spring, the same torsion is there at every joint, because the force times the radius if the helix is normal rectangular helix, we have the same torsion torcks everywhere. So, you have same strain and stress of course, not at the cross section may be cross section as a whole with the cross action if we make it a helix operation spring with a annular cross section everywhere also it will have the same stress and strain.

What about relative rotation, in fact that is also true because in the case of helical compression spring or extension spring when you are compressing are extending, what is happening is everything is actually twisting a little bit the same amount, because every section has a same torcks; everything is rotating little bit and no wonder we see this helical compression spring everywhere, not only in engineer components, but also in proteins and everywhere and creepers everywhere, these the one of the best designs that one can have when it comes to elastic are compliant design, because they have uniform geometry, uniform stress and strain and uniform relative rotation everything is contributing towards the final goal of what we want

That will be the ideal distributed compliant mechanism and this is where to achieve what we have listed here a lot more work is needed and that is an open problem in compliant mechanism synthesis. So, the last lectures at this lecture tell us why this 1 node hinges occurred and it is actually a problem in any of the formulation that we have discussed five different formulations. So, we have discussed two difference formulations; one in detail, the other one by 1 lager over segment, which is give a reference you can look that up how they used image processing erosion and dilation concepts to get something that is distributed compliant, but the idea is that we need to have relative rotation restrained, if we do that we do get distributed complaint mechanisms.

Thank you.