Compliant Mechanisms: Principles and Design

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Lecture – 33
Distributed compliance

Hello, now we are going look at to our discussion of synthesis of compliant mechanism is using topology optimization. We discussed in the last lecture five different formulations and why all of them actually behave in the same way even though there are different formulations literature, but there is in really much difference in the way that the behave there only certain differences in a sense what we are after is a formulation that can give a compliant under the same time a stiff design that will meet the functional specification that we have. See it is not enough to have only compliants we should have compliant and stiffness, as I emphasized at the very beginning of this part of the lectures that this was figured out a long time ago in the case of beams and the other things where this concept of mutual strain energy was put fourth.

Now, today we are going to discuses two problems that we identified in the last lecture, which is the appearance of these point flexures are what we are called one note hinges. So, we have a method now that came from settle optimization applied to compliant mechanisms, where we are getting basically something that imitate rigid body linkages because all topology that we get have this joints in them. So, we are essentially designing a rigid body linkage which is not a bad thing, but our original goal of getting a compliant mechanism that does not have joints is not served. So, let us analyze that today why the joints appear and how to avoid them, so that takes us to what we are going to call distributed compliance. So, let us look at what distributed compliance is and how to get it, so our focus today is distributed compliance are a tag line is point flexures and how to avoid them in optimal compliant topologies.

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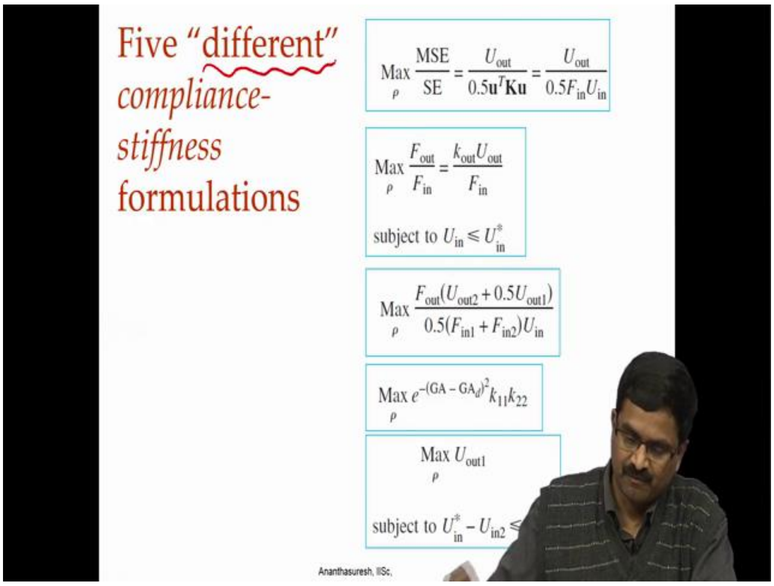

So, let us recall that there are these different so called are seemingly different formulations, five of them we chosen and we showed that they give pretty much similar results in terms of designs on all of them suffer from this problem of giving point flexures or one node hinges or elastic pairs, there are is a narrow portions where most of the rotation takes place that is a clue to what we are going talk about and basically they are like rigid body linkages right.

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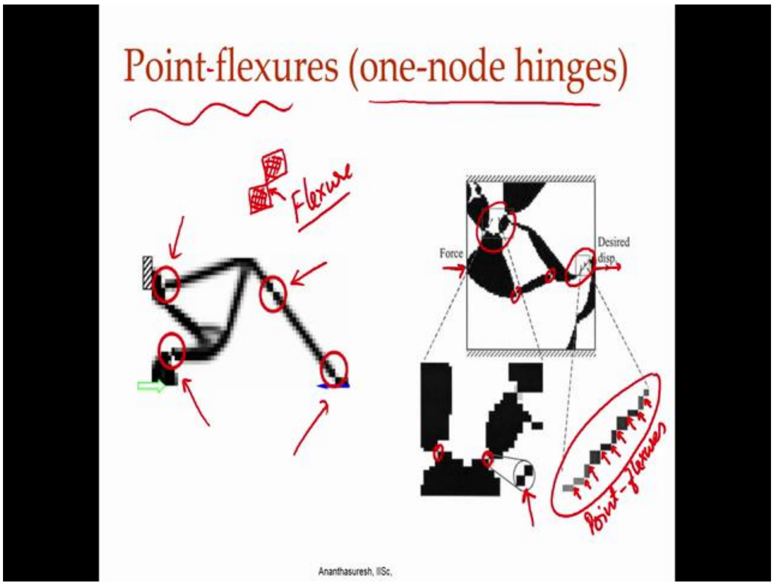

So, first let us see one more time recalling what happens, so where we get a topology like this all the things that we have encircled in red are basically this point flexures are one node hinges, how do the hinges occur if we notice any one of them we have an element and another element which are both black not is in black color, but basically material there and this acts like a flexure it is at a sin single point, that is why call point flexure with the (Refer Time: 03:57) may be point flexure are they translate to elastic pairs, and we can take another example little bit more complicated where it is fixed the top and bottom edge force is applied over here, and we want displacement in this direction the algorithm for this specification give a design like this, if we zoom in this portion which is shown here we see really if we zoom in we get two to these solid elements connected digenetic corner and that corner is our point flexure are where it is urging us to put a hinge, put a hinge, put a hinge, put a hinge, that is what we are getting or if we take a portion which seems to be somewhat distributed like that if we enlarge it, which is this one again and we have this point flexures everywhere.

So, the algorithm seem to be dependent on all these point flexures in order to give maximum displacement, because we are maximizing which was strain energy which is nothing, but displacement and output point way to get this is to go for this point flexures, where there is rotation taking place and the bending takes place and its able to give the this displacement, if that is what we want then there is another way to get easier way you start with the linkage.

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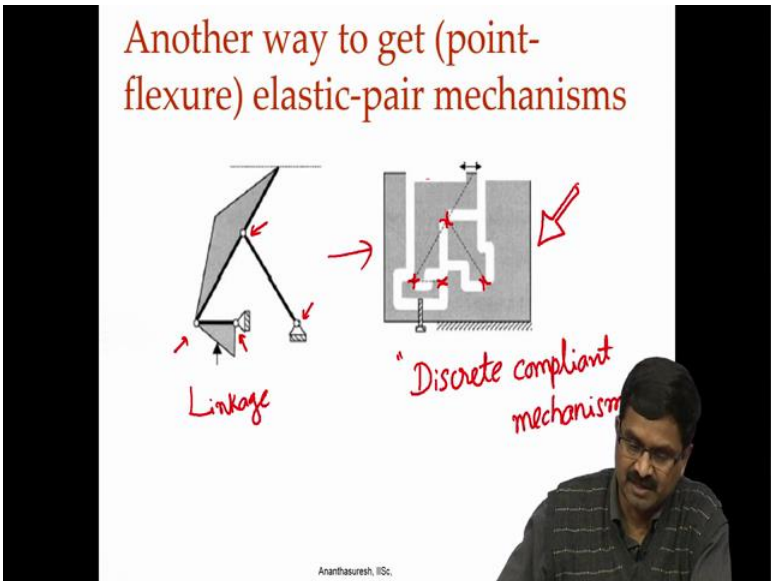

So, here we have a linkage and we take that linkage and wherever the joints are it is a 4 bar linkage. So, we have four joints right so, if wants four joints that is over lade here with dash line in this thing here. So, we have the 4 bar linkage right so, there is one joint, another joint, third joint, fourth joint this is the same body. So, it is a straight line motion mechanism, but we have that, so if we really want to get that you can take a sheet of material and run your milling tool along all of these in this passion and remove all that material, what will be left out will be your mechanisms with this point flexure.

So, here we have point flexure what we call elastic pair at the begging of this course and so this elastic pairs we can have on we have the topology or of course, in the case of the linkages we have something called pipe synthesis number, synthesis that tells you what type of joint you should have and how many you should have with all that knowledge you can design a linkage and go to recovalent compliant mechanism, but such a compliant mechanism let us call that a discrete compliant mechanism, because it has basically discrete rigid bodies connected with elastic pairs even if it is a one piece one meaning that like this particular thing here, where will be able to cut it out of a single sheet of material, but its complaints is limited to a few discrete locations there are discrete number for a rigid bodies and there are discrete elastic pairs joining them.

So, it is basically a rigid body linkage, but it is just using elastic pairs why are they bad where there not good enough. They are not good enough because the range of motion quite limited if you uses elastic pairs there excellent when the range of motion is limited if fact they are widely used in procession mechanisms, where the range of motion is not as important as avoiding backlash and friction related and were related problems, but here in compliant mechanisms that require large range of motion to complete with rigid body linkages in terms of the things that they do we cannot depend on only this discrete compliance, where we have a few elastic pairs connecting rigid bodies who want something that is more distributed compliance. So, what do we mean by distributed compliance?

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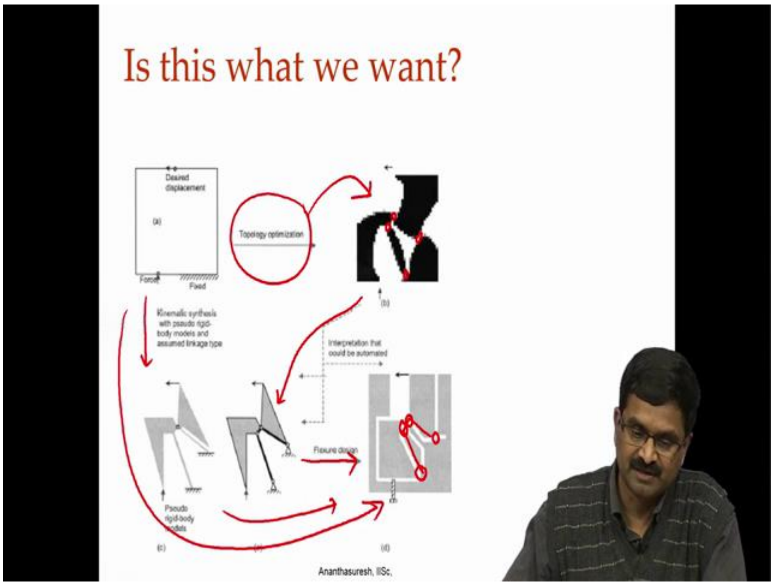

So, again if we look at what we have if solved a topology optimization problem let us say we get something like this, we immediately see that there are joints there 4 joints four bar linkage very often it happens. So, you can go there and then from there you can get to be flexure design again by taking a sheet of material and cutting out all this. So, we get this elastic pairs over there, over here, and over here, and over here.

Four of them just corresponding to what we got, but we could have gotten directly from here, like synthesis that we discussed for suraj body module based method, where we can gets something like this we can get here so, there are 2 ways to get here. So, why bother using structure optimization all we are going to get is something that we can actually happily do from linkages this that is the question that bothered the people working in this area required sometime and everybody was after are at least everybody one wanted these distributed compliance mechanisms.

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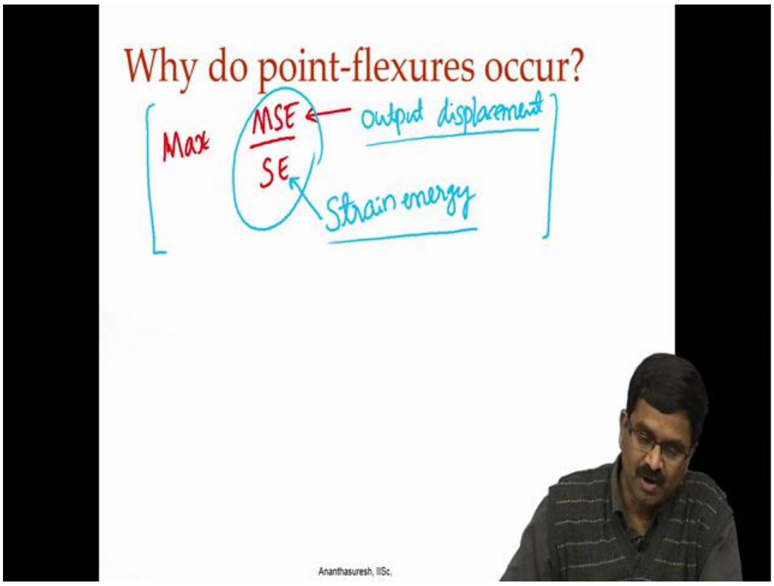

In order, to understand how to get distribute compliance mechanisms, first we will to understand why this point flexures actually occur; it is actually quiet straight forward if thing about it, what are we doing we are trying to maximize, MSE over SE in whatever formulation that we want right, subject to volume constraint or not that is a different question this is our objective function. So, if tries to make in order to maximize this or we say minimize negative of that its a same or equivalent will you want maximized mutuals energy which is nothing, but output displacement then we emphasizing that it a lot this is nothing, but output displacement you want the algorithm to maximize output displacement and what is a dominator, which is SE that is strain energy the ratio is what you want maximize.

So, ratio you can maximize if you maximize numerator and minimize dominator. So, on the one hand we want maximize the output displacement, on the other hand or at the same time you want minimize the energy so that the ratio will be large. Now think about it this can be achieved very easily are almost a perfectly with a rigid body linkage, or a rigid body linkage for given force will be lot of displacement with basically maximizing displacement and strain energy in them will be 0, because there is no elastic pair let say rigid body linkage with kinematic pairs or kinematic joints there is absolutely no elastic energy, there is no strain energy there is actually 0. So, when you want you to maximize you can actually make it infinity, even when output displacement is finite based on the mobility of rigid body linkage that we have taken the denominator is closed to 0, it actually can go to the infinity the objective function MES or SE. So, that is exactly what the optimize algorithms develop per complaint mechanisms topaz that algorithms are able to achieve they try to come up with is hinges.

So, that strain energy stored in them is quite small is not 0, because they still be little bit of strain energy or it is very small suitable to maximize the ratio in the process, because strain energies very small at these hinges you get enough this displacement here able to also get maximum all very large out displacement. So, it is as simple as that why do, they occur the reason is that we are trying to ask what rigid body linkages very clearly eminently satisfied. So, what we were formulating so far is not for compliant mechanisms, but actually for rigid body linkages where this satisfies so very well they will have 0 strain energy under lot of displacement or finite displacement rather large displacement. So, that is why point flexures occurs its actually interesting that algorithms figure this out, how do that figure it out, how are the able get to the joints it is all in the gradients of course, we in there able to give what we are asking. We are asking it to maximize the ratio they are able to do it by getting this joints and how are they actually doing it.

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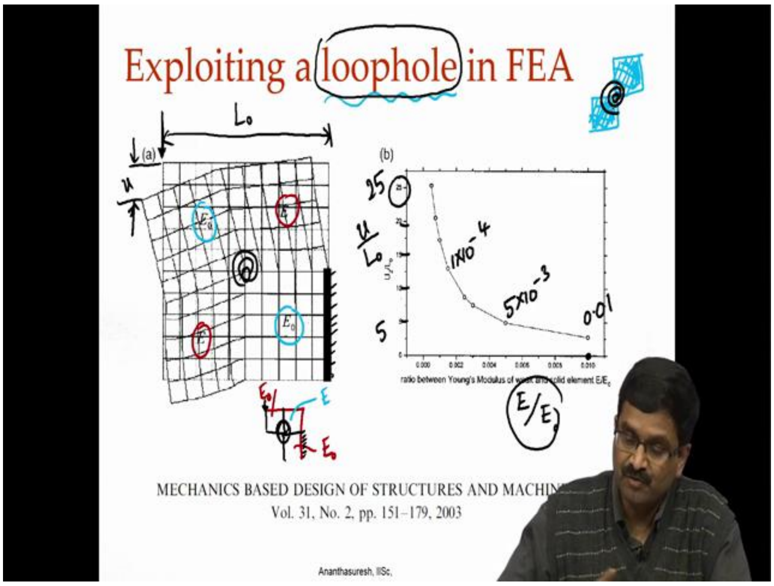

So, one way of saying is that these algorithms in the iterative process we are doing finite element, this finite analysis has a loophole when it comes to this point flexures that is if I have one element which has complete material and another element also complete material, when these two are join at a point you might think that finite element analysis might have some problem actually does not have any problem it runs happily, but rather something that is not desirable, it is a loophole in finite element analysis of continuum structures, if you what to do this that is when something happened like two elements are joined at the corner, you do not getting the error the algorithm gets some result and from finite element analysis it moves on and another reason is that, if we were to take a problem where we have taken if you notice this is e 0 and this other thing also e 0 and there is a anther one let me use a different color there e and e meaning what we are try to do simulation here is for a f force self problem rather large we had fixing it forced self problem, where the material is completely filled and so is here region it is not a small cell rather large problem whatever one is interested in and keep these at ends model is e and e 0 and e 0's meaning some material other things they are kept at in very low are varying I am say that is called this e, let us call this e is to portion and we are fixing it here in this problem, here fixing this edge as it is shown their this is fixed an applying the force over here rather, we are apply the forces over there. So, now, if I indicate the displacement here as u and plot that u by L 0, L 0 let us say is this length. In fact, that is what it is L 0 and the displacement of this point in this direction let us call that u; u by L 0 if our took plot against e by e knot.

So, the numbers may be small here, but starts from this one is 0 point 0 1 that is it is 1 percent of the material. So, very little material is kept here applying force how does it go so u by L 0 we have and then this one is 4 point actually this is 5 into 10 bar minus 3 the ratio e by e 0 is 5 into 10 power minus 3, and this one seems to be 1 into 10 power minus 4 that ratio e by e 0, but you can see at every point you are not getting infinite displacement where getting finite displacement u by l 0 at this point as the largest value here is 25, here it is5, 10, 15, 20.

So, what we see here even though we have we are going to n power minus 4 next one is 10 power minus is something now even then even though the ratio e by e 0 so small we are not getting infinite displacement for u meaning that something like this you think force I am going to get a lot of rotation their and algorithm is going to crash, because it is large displacement and e conditioning and so forth and nothing happens finite element analysis somehow happily allows and that is why we call it in loophole. So, that we get finite stiffness is large stiffness it is small stiffness, because displacement a lot more compare to small change in the force here, but it has be find finite stiffness and it does not crash and algorithm continues.

So, we have a hinge here and hinge now has a torsion springs so we cannot actually think that this hinge is not simply a hinge between two things here; we actually have a torsion spring. So, that is how this is so as like in elastic pair so here we have a hinge and a torsional spring between the two things, and that is exactly what finite element analysis he is being exploited here in order to give us a design that has hinges again we goes back and saying that our objective function is actually not entirely appropriate if you want to avoid hinge based complaint mechanisms are discrete compliant mechanism. So, what we want is actually what we called distributed compliant mechanisms.

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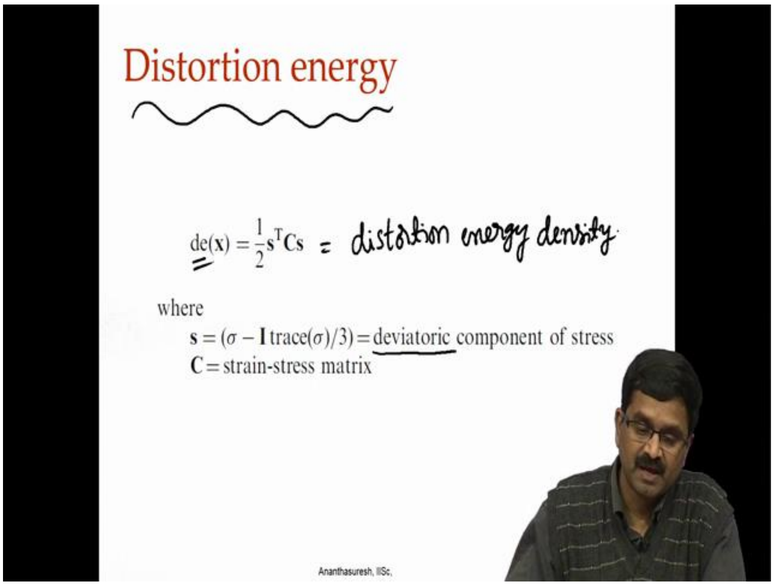

So, how do we get that, here is an idea because we define the mutual stamina energy rather it was defined in early works starting from bar net and Sheldon Prager, and then later we used them to develop compliance and stiffness formulations for complaint mechanisms. Similarly now you have to think about distributed complaint mechanisms lest think of an energy, and energy we know from of failure analysis that we are learned something called Distortion energy. Meaning this is the energy that distortion material mostly due to share and that is what causes the failure in materials in that the materials. So, let us look distortion energy where we subtract the isotropic component living out the deviatoric component of stress that is s here stress strain matrix c and that we can call as the distortion strain energy density we are using small d here name that distortion energy per unit volume.

So, we have the distortion energy if you have this distortion energy idea is that when things are excessively stressed and they are going to be share, bending and all that and if we minimize the distortion energy can we get because, strain energy we already have put we are trying to minimize strain energy and that is going towards rigid body link cages, what about putting distortion energy that is we do not allow the elements to distort in terms of their shape, but they can stretch if they want to that is what we are subtract in this component, because this what causes the actual failure in that (Refer Time: 21:38) materials right.

So, we do not make complaint base of little materials although we could and we do use silicon for example, little material per to limit our range of motion yard design should appropriate not for it to just break with little material precede that till materials then distortion energy becomes important.

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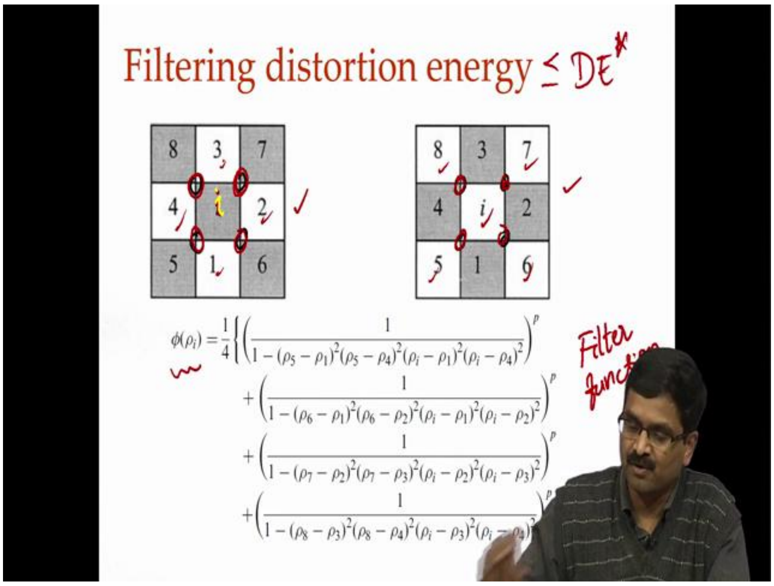

And this distortion energy will be there in the continuum when you are doing this, but we know into in (Refer Time: 22:06) some elements will disappearing, some elements will be present how do you know where to apply this we do not want to put this distortion energy constraint, that is you should not be more than certain value, we not put everywhere we want to put only those places, which are the potential hinge locations there are two base there can actor for an element I, and if that we have taken here element I, that element i; we can have this one node hinges are conferences occurring in two different ways. So, when these are empty, these are empty, are these are empty and this is also empty, i this is also empty.

So, we have a filter function so, what we have here this fi is actually a filter function meaning that in your topology image there are going to be all of them at varying levels of grey between black and white and you have to scan that not visually, but mathematically if use this expression what will tell you is, this will have a large value whenever either this situation occurs or this situation occurs that is all these are candidates right, this point flexures whenever they happen this function will rise a flat meaning that it will assume a high value, that is how this function is constructed you have to pass that and look at it then you will understand the numbers are greater and this is i the neibours 1, 2, 3, 4, 5, 6, 7, 8, in both cases how those value should be because, let us say we take this between i and seventh element right.

So, when the element in question is there and then we have the seventh one also there, so we want to penalize this particular joint and there distortion energy should not be very much only for those things we should apply this distort energy constrain that this should be less than or equal to some value that equal to upper bound, so that things do not bend a pole knot.

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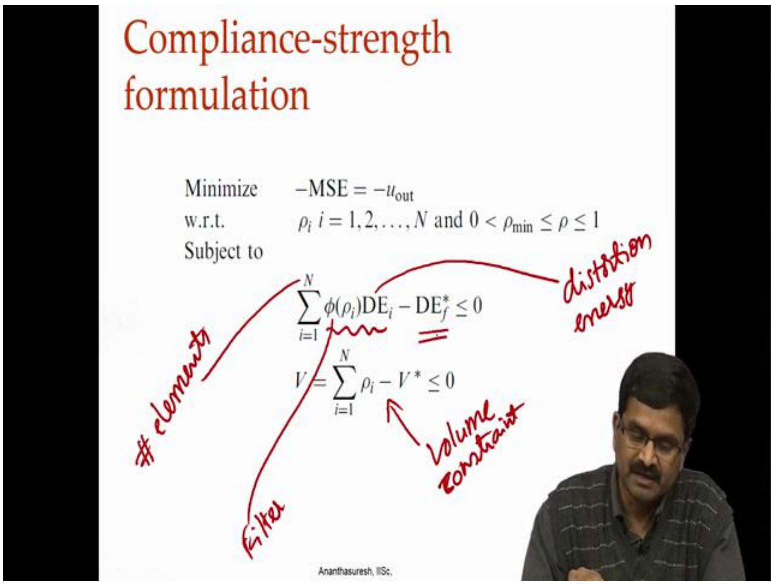

So, if you do that we can go back to our objective function of maximizing the output displacement or minimizing negative of that subject to this filtered distortion energy constraint, we go from 1 2 n where n is number of elements, this is number of elements and this is a distortion energy of that element that we have discussed, distortion energy of the i-th element and then we have this filter that we just discussed and then we have the volume constraints as usual.

So, when you have all of this we hope that we do at a solution, because now we are not targeting strain energy because that is strain energy will 0 and then we go to towards are very small we go towards rigid body linkages, but now we are trying to see that we are avoiding the distortion are limit in the distortion.

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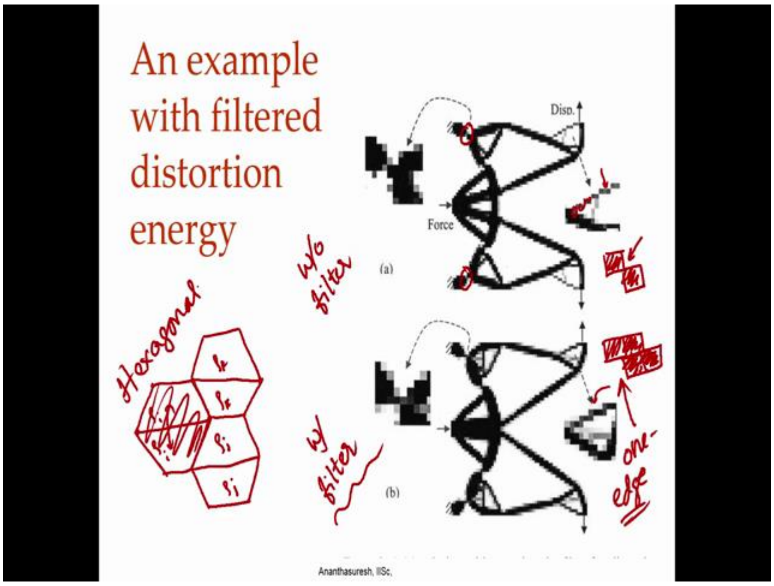

So, that things do not bend excessively at a few points of course, these are integrated form with one value put in locally is what we want to get at this that is why this particular formulation still cannot completely avoid the joints, if you see there are two things this is with actually this is without filter and this is with filter. In both cases distortion energy was put as a constraint and we do have this hinges now we they do not away there is just slight difference, so where there is this type of you know thing is here one node hinges, now they become little bit better.

So, originally in this formulation without filter when we do it, it was like this here what it does is, it does this. So, that is material is here and here this how it is here when you actually start penalizing the points by putting this constraint then it barely avoids by going. In fact, because of this some others in particular professor Anupam Saxena IIT Kanpur, has extensively worked on what is calls x elements. So, that is you discredited your structure not with square elements, but do with hexagonal elements and each hexagon you divide into 2 quarter laterals, so each of these is now a finite element quarter lateral.

Now, what it does this both of them will have this and this will have the same rho. So, let us call it a rho i, rho i and similarly this and this will be rho j, rho j and then rho k rho k; that means, that together they exist are the disappear then the contact within an existing element always were an edge and not a point and that anyway we are getting here an edge contact, if you put this with filter distortion energy constraint it does avoid things like 1 node they become 1 edge a little improvement, but not a great improvement a same thing with these hexagonal element. So, what you get here is instead of point fluxes we get edge flexes we still have an arrow. So, like this people have tried a number of methods, but algorithm wants to go towards this joint, jointed design and that is something that we are not going to avoid, but there is a way to do this.

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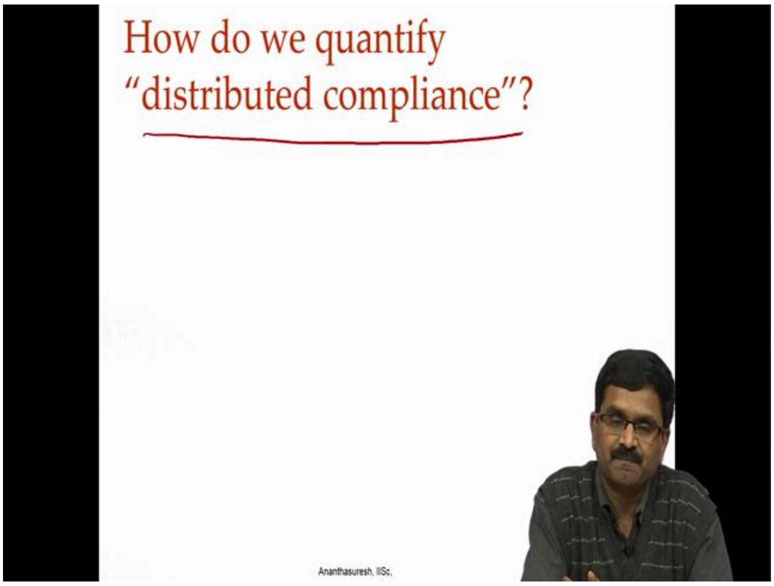

And that is dependent on how we quantify this distributed compliance, how do we quantify what is distributed compliance; what do we need to do? we need to a compliance mechanism can you tell that this is a discrete compliance mechanism probably it is easy, because we see that there are a few portions where the strain energy is concentrated in a discrete way that is where most of the last deformation takes place we call it a discrete compliance mechanism, from what do you call a distributed compliance, is this the one where elastic deformation is uniformly deformation, yes inwards that what it means we have to have the elastic deformation happening throughout the structure, but how does it happening is it possible at all to come up with a criterion quantitative to measure that says that this mechanism is this much distributed complaint, right as supposed to discrete if you put some spectrum one end is discrete other end is distributed fully, uniformly everywhere is that really possible to have a complaint mechanism where the stress is the same throughout, where a structure it is possible you can do that that is what that a uniform or fully stressed design is that possible to do in a complaint mechanism, because mechanism the strength or stress is not in the criterion, criterion is the actually functionality has to move have give a motion in a particular way, if that is there can you get uniformly distributed stressed and that is the question that is to be answered. So, will end this lecture now and continue this point in the next lecture.

Thank you.