

Compliant Mechanisms: Principles and Design
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Lecture – 32

5 different formulations for compliant mechanism and design and some benchmark problems

Hello, in the last lecture we used yinsyn program, which is implemented in fortran, but now works in the environment of matlab. So, you can give a text input file for any problem that you have in mind, for single input and single output compliant mechanism design, other the program does allow for multiple inputs and multiple outputs as well, but that will be you can average sense, but you can play with that.

Now we are going to discuss alternate ways of formulating the problem and a need for it and when you do it, what do you gain what you lose. So, we will discuss a comparative study of various formulations that are reported in the literature. And then see why people have try to come up with many different formulations to solve this compliance and stiffness problem.

let us look at that alternate formulations today for compliance and stiffness design that is what you want to have because we already discussed that, compliant mechanisms need to have flexibility, that is a compliance parts at the same time they should also be stiff, otherwise as we saw in the case of process and beams we can get imaginary areas or square roots sign will have some negative values, that shows that there is something wrong when you holds only a compliant requirements that is design for deflection requirement without stiffness. As we had seen the early papers in this area had already pointed out, but it takes some time for other people to realize that that was the case and how to fix it that was happened in the last 2 decades also.

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MSE/SE Ratio formulation

Min $V^T \rho$
 Subject to
 $\Lambda: u^T K U - \Delta \leq 0$
 $\Gamma: \frac{1}{2} U^T K U - SE^* \leq 0$
 $\lambda: K U = F$
 $\mu: K u = f$

Min $-\frac{u^T K U}{\frac{1}{2} U^T K U}$
~~Min $\frac{1}{2} U^T K U$~~
 Subject to *Linear*
 $\Lambda: V^T \rho - V^* \leq 0$
 $\lambda: K U = F$
 $\mu: K u = f$

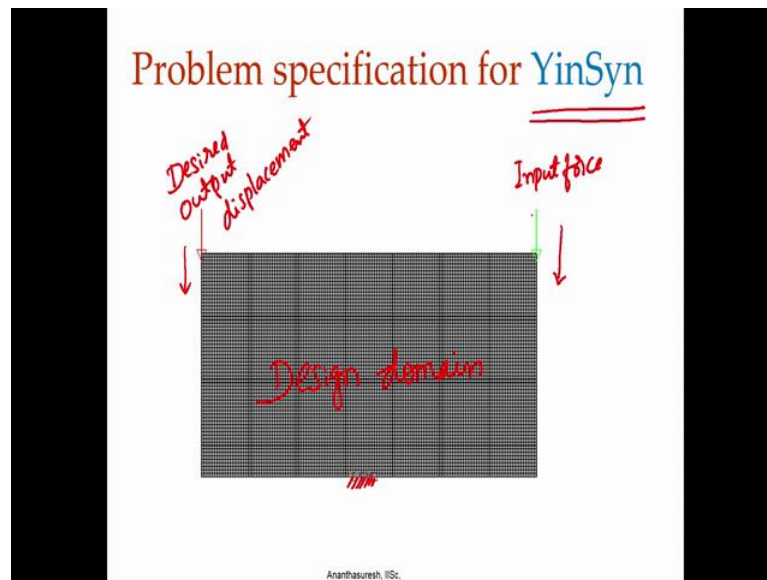
Min $-\frac{u^T K U}{\frac{1}{2} U^T K U}$
 Subject to
 $\lambda: K U = F$
 $\mu: K u = f$

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Now, let us continue with what you had used call this MSE by SE ratio formulation. So, MSE as we know now is simply numerically equal to the deflection or displacement of the output, that is the mutual strain energy when these energy because of unit virtual load it is actually unit force times the displacement and SE strain energy that is the measure of stiffness of a structure, here it is also stiffness of the compliant mechanism. So, MSE by SE is what we try to maximize so we that wrote we could write minimize volume of the material subjected to the reflection constraint as well as the strain energy constraint and the governing equations, later we said well why not combine these two and put in to objective because dealing with a linear constraint that is the volume constraint is easier than dealing with this non-linear constraint. These are non-linear constraint whereas, this one is a linear constraint in terms of the design variables here indicator functions rho.

Later we also said that even if you do not put volume constraint you can still solve the problem, although the yinsyn program that we demonstrate in the last lecture uses this formulation there is a volume constraint. Because by putting a volume constraint you can actually see, how the topology changes topology does not change qualitatively as we already saw, but in terms of distribution of materials and more and more well defined image if I am call it, will emerge from topology optimization. So, these are ratios formulation, this is the ratio formulation and s is this? Without the volume constraint these are with the volume constraint.

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So, we tried a number of examples are rather one example with lot of variance where my students and the program both try to solve. So, we are able to get intuitive solutions as you saw in the last lecture, but sometime this specification is non intuitive in this particular case, the fixed portion is a tiny bit over here in the middle and this is where the input force is applied and here is where our desired output displacement is sought . So, these a problem look at it and whatever percentage of material that you can use can be 10 20, 30 50 whatever that can go in to this design domain. These are our design domain there you can distribute material and try to get this specification out of it, when you push down over here input force then it should also come down.

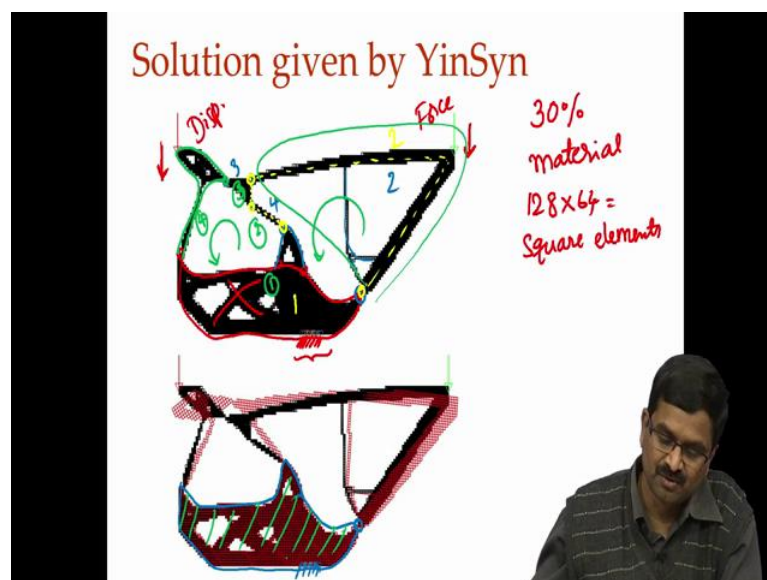
This is somewhat of a non intuitive specification because normally if you fix this is like a fulcrum in the in the where we have fixed, if you pushed a lever in this direction you would expect this point to go up, that is a intuitive behavior.

But here we are asking it to move down. So, there our intuition might fail or it might take a little longer for a human designer to come with a solution, what one finds with this topaz optimization algorithm is that especially this in yinsyn implementation that we demonstrate in the last lecture, almost always I would say even always gives a solution. If your input is proper meaning that it does not make the program crash in the sense that if you say there is a whole and you put an input force or output displacement there it would not.

And also when you give discretization, when you give very small at that fix with in that one element then also it will have problem. Other than those inconsistencies it always give a solution, you say good tool try out our intuition also it instructs us that is the thing I will illustrate with this example, when you see the solution given by the program then you would realize that oh that is the way to solve the problem because, we are asking it to be fix only at one midpoint at the middle almost at point, this not a really point I cannot give this, but a narrow reason rather one support when something you know comes out of this let say, how does it solve and by the way in the last lecture we said that is it easy to implement.

Well the theories simple enough which we discussed in the last few lectures, when it comes to implementation there are some tweaks that one has to in program any numerical analysis that is (Refer Time: 07:31), but other than that it is very simple to that so simple that these days one can write an app that runs in a smart phone, one professor at denies technical university has even done that and in a smart phone you can have an app and it may solve the problem. Stiff structures it works very well compliment mechanism sometime there will be some issues but overall it is not a problem when it is small displacement analysis base synthesis which is what yinsyn also is.

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Let us see what solution the algorithm came up with. So, again this is where it is fixing and we have displacement downwards will see the deformed one when we apply the

force like this, again this is force and this is displacement. This was with 30 percent volume constraint in that rectangular region that we have we did not put any holes we could have, with a very fine discretization I think it was 128 by 64 square elements that means that, these two whatever that number is that many variable are there in this problem and it gave a solution.

If you look at it, how do you interpret it what has algorithm done? Algorithm has done something clever I would say, the clever in the sense that we are giving only a small portion where it can be fixed. But that is not enough to visualize a mechanism compliant mechanism that will give us this desired thing when apply the force here it should move down like this over that point. You multiple the points, multiple points are there you can actually solve the problem.

So, what algorithm has done is, it has created it is own fixed surface. So, we allowed it to fix only here, but it created a surface how did you do? By actually incorporating a stiff structure there, in fact if you were to run topology optimization for stiff structures where it is to be fixed here and some loads are there at this point and at this point and at this point at this point the stiff structure would look like this.

In fact, there are other some characteristics that it has that (Refer Time: 10:04) structure that will talk about little later. So, it has all this you know if you actually give it, it will come up with these stiff structure and then puts it all the material most of the material in this portion to make it stiff. That gives us an idea, when you have a very little portion where you can fix we use that and create a stiff structure, so that other points do not move much.

As we will see if I were to get the deformed profile, you can see that this portion over here let me change the color so you can see if you see this portion it is not. In fact, I can put this whole thing so here also I can add that right. So, this is not deforming much, rest of the portion is deforming because red is the deformed and a black is the undeformed like the original image of the solution.

So, it has created a stiff structure with little support there that is; what stiff optimizations do. So, when since we have compliance and stiffness formulation here the algorithm knows how to create stiff structures and then it will come up with this, if you also look at it little closely what we got here? There is a something like a hinge here this is (Refer

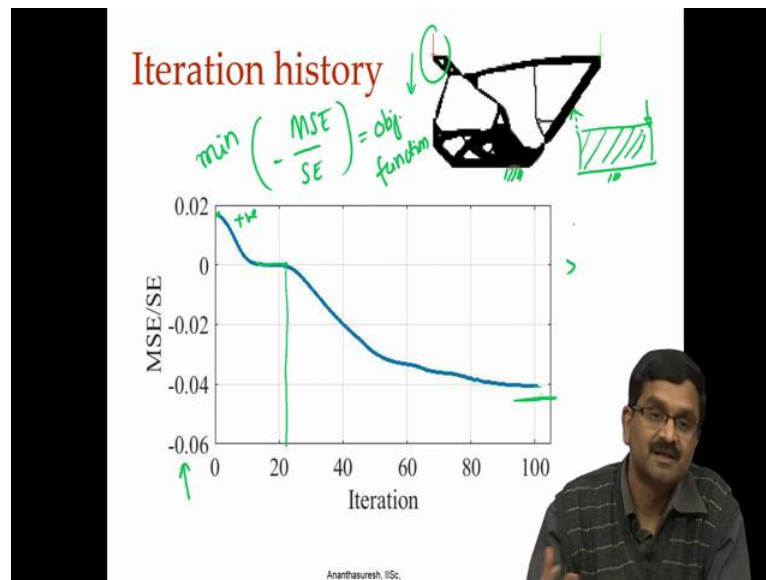
Time: 11:26) whether is a hinge here and this is just a intimated portion we can kind of ignore that for now, there is a hinge here there is a hinge here and this whole things that is look little slender segment; effectively will have may be a hinge, other words we have a kind of a bent beam here so let me change colors, I can mark with yellow.

So, I can say that there is something like a bent beam, which sort of looks rigid relative to the other things and there is one slender thing there and we have one hinge here, one hinge here and there is a another rigid body over there, there is a hinge effectively there is a hinge right. So, if we see this it is like a four bar linkage.

So, we have the fixed body here, this is 1 and this whole thing is 2, I have yellow colors we can see it let me change the color. So, this thing is 2 and this is 3 and this is the 4th. And then that is 1 four bar linkage, other one if we see I have 4, 3 and then there is a hinge here there is hinge there, once again let me do with a different color now another four bar, again 1 here the same one has before one and then we have this as 2 and this thing there is a hinge here there is 3 and there is 4 this whole thing. This whole thing is one thing. So, it has put two four bar linkages in tandem. So, 1 2 3 4 over here is a 4 bar, there is another four bar here with two 4 bars you can actually do whatever you want one going up down going this way that way that is intuition, but then the real thing that the algorithm has told us is that, when you have a small support go for a stiff structure that actually gives you more support on which to build on mechanisms..

So, this way the algorithm is clever. Where is that cleverness coming from? It is merely coming from the algorithm, the gradients that we have put in all we have is a necessarily condition and using that to update the indicator function go through on the structure that is all these come. You are welcome try yinsyn all possible ways and see that every time you give a specification solution come you can actually interpret it and understand what the result is.

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Now, interesting thing is in this problem if you look at the iteration history. So, we have or objective function MSE by SE that we want to maximize, so we put a negative sign. This is what we are minimizing should you should decrease look at when we start out where it is. These numbers actually do not mean much there are some constants and so forth. So, 0.02 0.0 does not mean much, but the fact is that it is actually positive, here the axis is here that is iteration number initially MSE by SE this objective function this is our objective function starts over positive meaning that the same problem which is fixed here, where apply for any given designs our initial guess will be uniform 30 percent if I give, the whole thing will be 30 percent if I am fixing here when I apply a point will load here it would try to move up, meaning that it is going opposite to what we want.

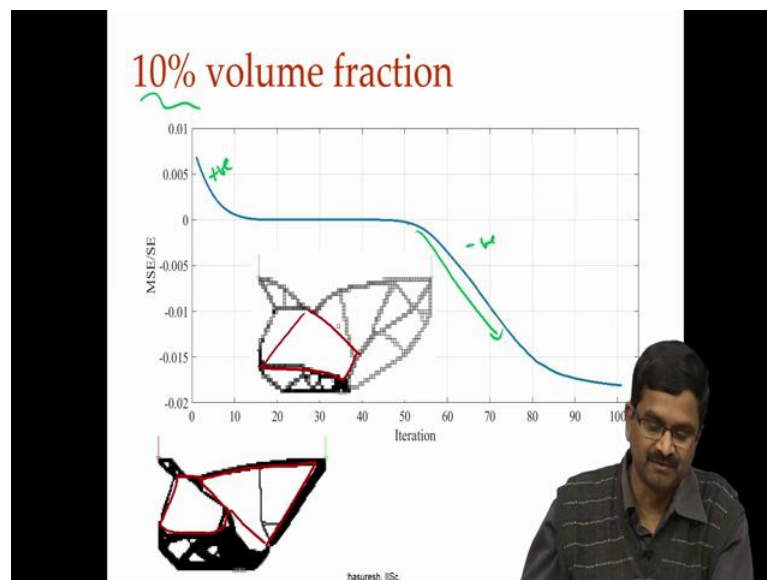
So, it is positive because energy is always positive, MSE can be negative. That is why we said we have to add MSE and SE to constraints in the problem because that product of capital P small p capital M small m in the case of stress and beams respectively, we said that the sign can be for MSE positive or negative, it depends on which your output is going that is what we are trying to get. It is positive and it comes down and see that it actually to turn to negative it takes about you know 22 iterations also. So, that is where up that iteration are needed for it to turn to negative.

Initially it is positive and it actually has a kind of a (Refer Time: 16:13) region here because it is trying to see what change in the design would make this point go down?

That is the desired direction. So, that is an interesting thing the algorithm does in fact, for highly non intuitive specifications algorithm starts with a positive sign and then goes negative and negative and that decreases steadily and then little bit slows down and finally, kind of converges here it is like already it had gone this what happens.

So, algorithm is also struggling for it is intuition to get to the right direction this is highly instructive for human designers when you use this, because as you can see this thing does not take much time you can write an app in a smart phone. It must be easy enough, if you can run a finite element analysis that is all it takes in order to run this image iteration.

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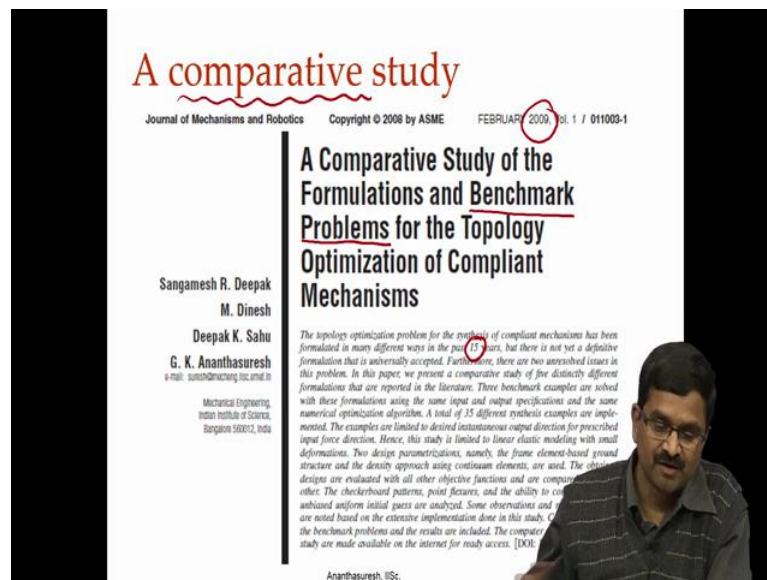
Now instead of 30 percent if I give 10 percent volume fraction, that design is topology is (Refer Time: 17:16) the same again it went for a stiff thing here does not have as many supports as we had for did not have enough material to make this other connection that the previous one had if I little bit (Refer Time: 17:29) it could have done, but rest of this stiff structure that is that is here we had in the previous one if we go back we wanted this you know big rigid thing.

It try to do it within that material, but pretty much this also when you look at the deform profile, you would see that it also would go down and again it is starts positive and then goes negative it actually in this case to turn negative it had go almost 50 iterations. Once it there it quite stiff it just that initial trigger is needed for it to going in the right direction

and then it is steadily decreases and then it is taper solve, we stopping in 100 because already tapering out.

So, it is able to give solution and now when you see you get this again this thing is 1 there is a hinge here and there is a hinge here, there is hinge here and this whole thing effectively if you think in terms of pseudo rigid body model, that can be a hinge also what you have essentially is a four bar linkage again? It is actually interesting that again draw a skeleton 4 bar, there is a four bar here and then there is a stiff structure because allowing it to fix only here it made a stiff structure, that is a stiff portion and this is the compliant portion. Most often we can interpret the solution in terms of linkages, again if you compare that the thing that we got with 10 percent and 30 percent you would see that there is topological similarity as already said this particular thing is missing over here because it did not have enough material instead of two 4 bars that we interpreted for this it became one 4 bar. So, here we said there are two 4 bars right, here it became one. So, from here to here the way we had sketched in the case it is a linkage, but it is a compliant mechanism we can actually make a continuum version of this and like I said 3 d printed you know right away.

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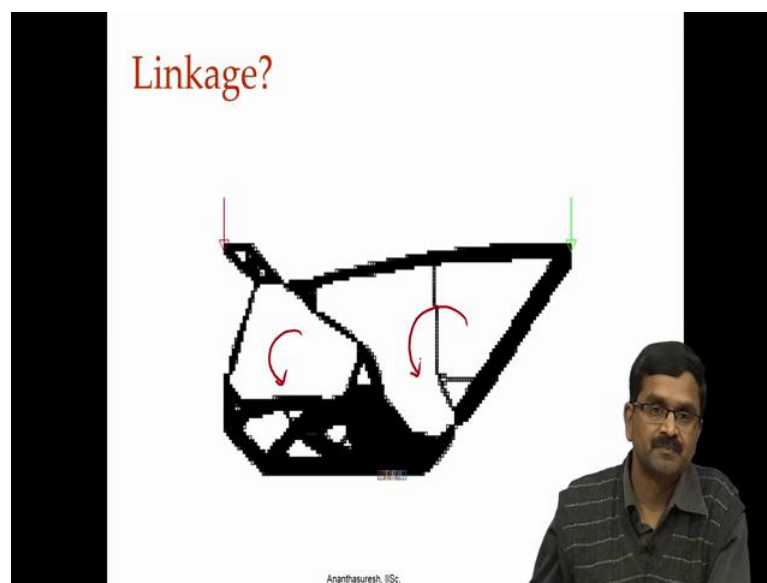


Having said that it seems to work in all things that you give, but yet in the literature lot of formulations have been tried, a number of papers in a last I will say 25 years lot of papers have appeared now it has decreased little bit, but still happens. So, in 2009 we did

a comparative study of many of these formulations as you can see it is a comparative study of formulations and some benchmark problems in the literature when too many formulations occur for a new entrant it will be confusing now where people are trying a lot of different formulations for compliant mechanisms. For stiff structures, most often people use something called mean compliance it is basically strain energy, work done by external forces or strain energy and that works and everybody uses, but in the case of compliant mechanisms there is no one way to put it a universally accepted formulation.

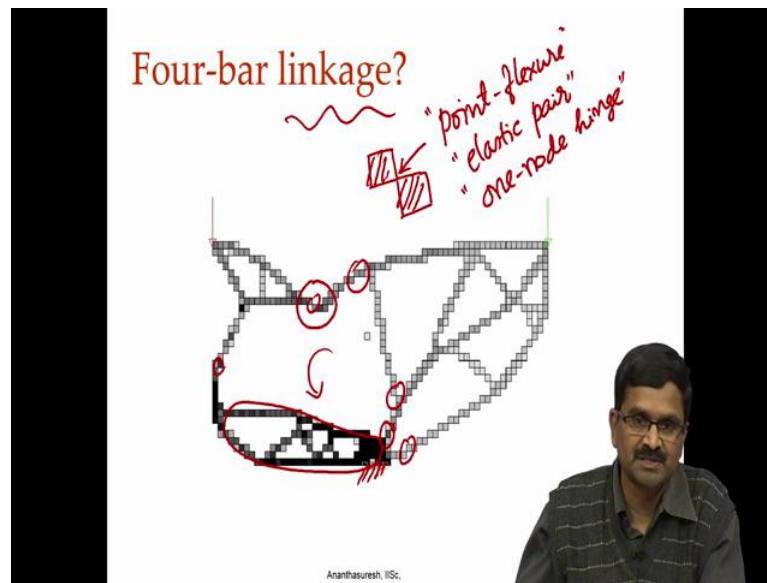
So, we thought let us look at all of these things and that time it was 15 years now it has been a little longer. So, you can look at this paper and will be actually looking at this paper to see what we see when we compare all these different formulations, all of them fall under this compliance and stiffness formulation.

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As we saw we are getting linkages, so already saw there are there is a four bar here and there is four bar here, 2 loops or when we look at the 10 percent one we got one four bar here that is one four bar that actually four bar linkage.

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So, how we developed a method to design linkages by going through this structure optimization route, we have to design compliant mechanisms but then it came up with a stiff structure here but fix things and a compliant mechanism; that is why we should look at compliant mechanisms as a middle ground between stiffest structures or stiff structures and rigid body linkages, it is not different it takes ideas from both. But now a method that is developed for structural optimization is able to give linkages.

If you think about pseudo rigid body model based compliant mechanism design, we have to first assume whether a four bar or 6 bar 8 bar some linkage and then try to copy the body lengths and joint locations that comes to the angles and torsion spring constraints for the joints and then use pseudo rigid body model to go to compliant mechanism.

Instead here it gives a conceptual solution and sometimes if you use (Refer Time: 22:37) discretization for this thing you actually get a manufacturable image.

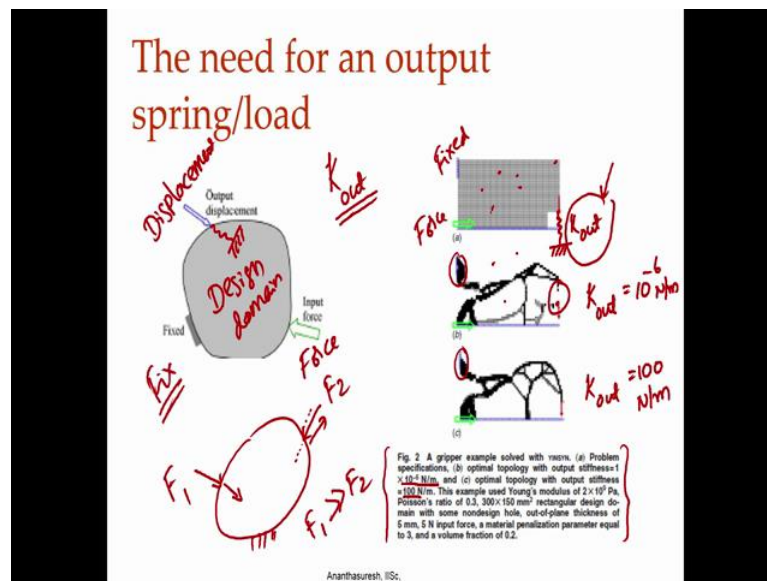
That is but then we are actually looking at more like a linkage and this joints that come you see everywhere the diagonally connecting points and that especially that over here let me erase a little bit to show you if you see here that has one element like this, another element like this, that one square another square what does that mean? That is what we can call point flexure that is the what we called elastic pairs, one can call it elastic pair like kinematic joint it is an elastic joint some people call it one node hinge. It is basically

hinge; the algorithm is going for this hinge lot of different names for it elastic pairs one node hinge point flexure whatever.

It is able to come up with those by itself to give you topologies are connectivity for linkages. So, why is it coming will discuss it in another lecture, but it is actually rather disappointing because, we are trying to get compliment mechanisms and is going for this flexures everywhere you see their flexure, there is flexure there is a flexure probably here it is going towards linkages rather than giving as a compliant mechanism.

So, that is why people think may be this formulation is not good enough let us try something else, that is why a number of papers have appeared to see what are the other ways of doing it.

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So, our problem is this we have a design domain and it can be in any arbitrary shape does not matter we have input force and we have desired displacement and we can fix it wherever we want, whatever boundary conditions does not matter.

Now if you remember from the last lecture, we said the output should always have a spring or an output load in the yinsyn program we had this output spring constant k_{out} what if you do not have it. So, let us see an example here; we are asking the force over here the green arrow output displacement as (Refer Time: 25:03) much let the gripper

examples we did and it is fixed here and we always imagine that here there is a an output spring and that is what we specify in yinsyn also, if we do not have it what happens.

So, here it is taken these example (Refer Time: 25:19) paper where we going to look at when the output stiffness is 10^6 Newton per meter, in these example that k_{out} is very small 10^6 Newton power meter meaning that it is not there at all, then you see this was our output point it is not even connecting to it.

So, the reason you are connecting is that this point that point this point whatever we have here none of the points have springs, how is our output point different? All we are saying is that we are calculated displacement there and try to maximize it, but other points are also similar for the algorithms you cannot, because if there is input force it will always connect? It is connecting here because it needs to take support because we are fixing it there, but for an output point, we do not put a spring or an output load there is no difference, you put an output load that becomes like an input except that it is in the opposite direction.

So, when you have an object elastic object let us say I fix it here, apply load here and load here there is no guarantee that if this force let says this is F_1 and F_2 , if F_1 is much greater than F_2 when apply F_1 and F_2 simultaneously this point need not move in this direction it can move the other way.

So, when you have an output load it actually becomes like a stiff structure problem because you just have 2 loads and a structure. So, instead of here if F_1 is much greater it will definitely move there, but here it may actually move that way, like that is not a compliant mechanism may if you want it to go down. So, when we want to do output load then that becomes like stiff structure problem, we go for spring and vary this principles.

So, in this case K_{out} if it is very small we did not even make a connection that is a problem. So, we always keep K sufficiently large here second one it is 100 Newton per meter. For the number constructed in this problems all the details are in the paper, so you can look that up. So, you sometime it does not even connect that say a defect in the problem.

One is that there are springs where that this one node hinges or point flexure, other is we need to have the spring. The spring is unrealistic because if it is if you have an object for this mechanism deal with, you can give a proper k out based on the problem specification, but if you do not have such examples are also there because there is no output load there is no output spring, you should just move in the direction. In which case if you put k out and solve the problem then actually unrealistic.

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Five "different" compliance-stiffness formulations

1. $\text{Max}_p \frac{\text{MSE}}{\text{SE}} = \frac{U_{\text{out}}}{0.5 F_{\text{in}} U_{\text{in}}} = \frac{U_{\text{out}}}{0.5 F_{\text{in}} U_{\text{in}}}$ (Ratio)

2. $\text{Max}_p \frac{F_{\text{out}}}{F_{\text{in}}} = \frac{k_{\text{out}} U_{\text{out}}}{F_{\text{in}}}$ (Sigmund)
subject to $U_{\text{in}} \leq U_{\text{in}}^*$ $F_{\text{in}} U_{\text{in}} \leq \frac{SE^*}{2}$

3. $\text{Max}_p \frac{F_{\text{out}} (U_{\text{out}2} + 0.5 U_{\text{out}1})}{0.5 (F_{\text{in}1} + F_{\text{in}2}) U_{\text{in}}}$ (Hedgecock & Kota)
 $GA = \frac{U_{\text{out}}}{U_{\text{in}}}$

4. $\text{Max}_p e^{-\frac{(GA - GA_0)^2}{k_{11} k_{22}}}$ (Rahmatallah & Swan)

5. $\text{Max}_p U_{\text{out}1}$
subject to $U_{\text{in}2} - U_{\text{in}1} \leq 0$

Efficiency
M. Wang
 $\left\{ \begin{matrix} U_{\text{out}1} \\ U_{\text{out}2} \\ U_{\text{in}1} \\ U_{\text{in}2} \end{matrix} \right\} = \left\{ \begin{matrix} F_{\text{in}1} \\ F_{\text{in}2} \end{matrix} \right\}$

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So, that is one problem, when people came up with all these different formulations there was a reason you know because the original formulation of this ratio formulation as we have discussed so far. So, MSE over SE maximizing that was thought to be not good enough. So, a lot of formulations we are chosen four more formulations.

Here is one this was reported by ole Sigmund, professor ole Sigmund at Denmark technical university DTU, where he put it maximize F out by F in, again indicator of function is (Refer Time: 28:38) just like here, if you say F out is inherently putting K out also he has to specify that and the solution is sensitive to that.

If we make it very low you do not get a proper design, you make it very high become like a structure K out, F out is K out times U out, U out by F in, F in if I were to that is given and then if you put U in that becomes like strain energy. So, if F in is already specify in the problem which in this case effectively what he is trying to do is maximize

output displacement because K out is specified F in specified and you (Refer Time: 29:19) in a deflection on the input displacement.

When you input deflection with the constraint that is same as if F in times U in half of it is strain energy; that we are putting a strain energy constraint just like in the previous formulation. So, these are both actually identical, the beginning lot of papers were (Refer Time: 29:42) but they are similar, there is not much difference at all here that is second formulation, but any way without we will put it and then see because here is a constraints on the displacement here both are together in as one ratio and volume constraint, here also one can put a volume in the second formulation.

There is the third formulation this is called Efficiency formulation let me write in a different color this called efficiency formulation, this was proposed by Hetrick and Kota at university of Michigan. So, they called it Efficiency formulation this is a little efficiency formulation, they defined an efficiency in kind of I would not say complicated, but something that these discussion what they do is you see the subscripts 1 and 2, they take 2 load cases.

Let us say I have an object and I have a input point and output point, input output in the load case 1 they actually fix input point and apply if this is that desired displacement you apply a load in the opposite direction and measure the input reaction force because you are fix there you will get some F in F in one comes like that and then F out how much hour is there put that F out here.

The second load case you free this, now you fix this input point and then apply some U in one here we have this U in you apply some displacement here and then measure the output displace actually they would not fix it here they measure the output displacement here, in the first case they fix this one and apply a opposite load they retain the load in the opposite direction and then try to free it and apply a displacement there.

So, they had some reasoning as to if we read the paper you will understand and finally, what they have showed in the numerator is the output energy the denominator input energy, so they says it energy efficiency formulation. We can see when you do that of course, we cannot argue that you know these 2 and these 2 are the same right away because they slightly involved.

And then there is a fourth formulation which uses raise to minus geometric advantage GA is this is due to Michel Wang who is now at Hongkong university at that time he was he put forth this formulations where geometric advantage is U_{out} by U_{in} , that is there is a desired one sometimes you want 7 10 whatever output displacement and input displacement, what you obtained he used exponential, so that it will always be positive GA sometime can be negative if U_{out} is not in the direction that you want it can be negative.

So took care of that he actually exponential to always positive and then you used two stiffness which also if geometric advantage is taking care of maximizing this U_{out} by U_{in} and actually trying to get target one; So you have 3 5 10 whatever amplification you want, that is embedded in these exponential function you also has stiffness here. This K_{11} , K_{22} for him was this.

So, if you have a whole compliment mechanism he would reduce that to let say you put U_{in} here and then U_{out} and then you have F_{in} F_{out} , F_{out} output may not be there, but you also has to put a spring that is a different matter. So, here you have 2 by 2 this is K_{11} and this is K_{22} and this will be K_{12} and K_{21} , he ignores this cross things he takes this K_{11} and K_{22} and acts that they.

So, there is some input side stiffness, output side stiffness will come back to this formulation later. So, it also a stiffness and flexibility formulation rather interesting and he showed that, when he has geometric advantage if you keep it a low value, then he gets a nice design, but if we ask for let us say 30 50 100, then all the problem that we see that is it is not connecting to the output point and it is has lot of hinges all those things appear.

There are one more formulation this is by Rahmatullah and Swan, they were at university of (Refer Time: 34:51) at that time. So, they came up with another one maximizing output displacement and then subject to the constraint, it may look like it is a normal strain energy thing, but actually slightly a difference see again you see U_{out} 1 and input 2 like in this formulation, they also take two different load cases.

In load case they put an output load and then solve the problem, in the other second load case they put only input force and put a spring, there is an output spring in both of those depending on the value output spring, they thought that they could eliminate this point

flexures of one node hinges, but that also depends on the spring that spring caution that you have.

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Design domain, geometry, and material properties

$t = 5 \text{ mm}$

$F_{in} = 5 \text{ N}$

$k_{out} = 1 \times 10^4 \text{ N/m}$

300 mm

150 mm

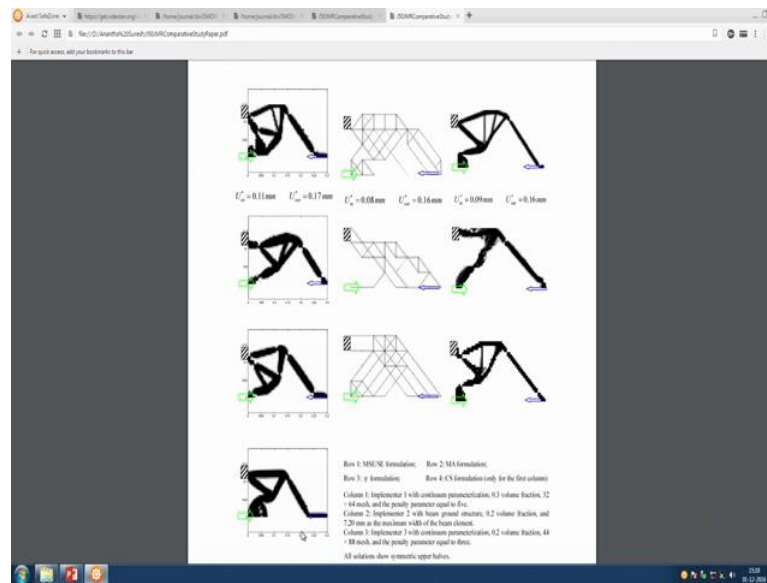
$Y = 2E9 \text{ Pa}$
 $\nu = 0.3$

Axis of symmetry

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Even though there are looks like these two are quite similar, but all others are slightly different seemingly. So, we thought will solve all of these things with the same problem. So, we took a design domain to the thickness t , F_{in} , K_{out} and young's modulus and poisons ratio with some no symmetry (Refer Time: 35:56) axis symmetry, changed input output points, but all 5 formulation were implemented for 3 different problems to see what came out. So, let just go to that and try to see what we get.

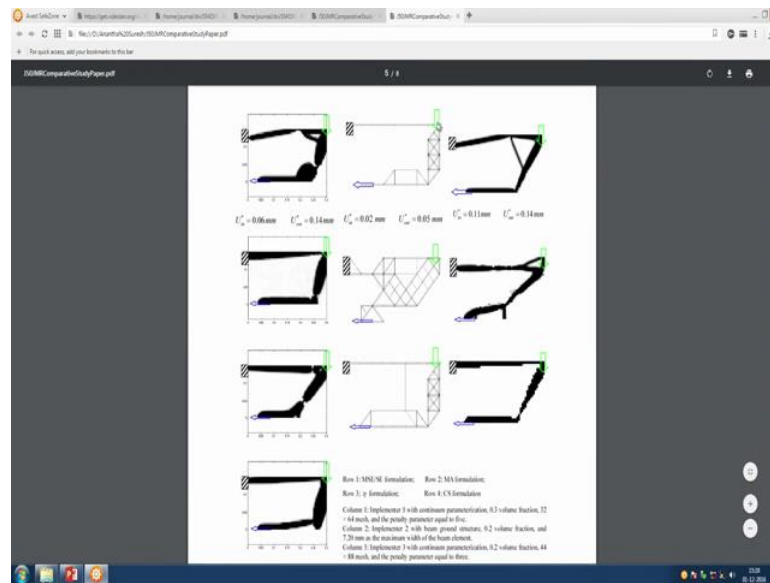
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So, let us go to this paper, so where we implemented all these 5 different formulations, all the details are here look at the results, let me show one example and these examples were actually solved both with beam elements as well as continued elements while by 3 different students in the class (Refer Time: 36:42) this was implemented by first column prepared by one student, second column other student third column by third student may be 2 different implement they have their own tweaks.

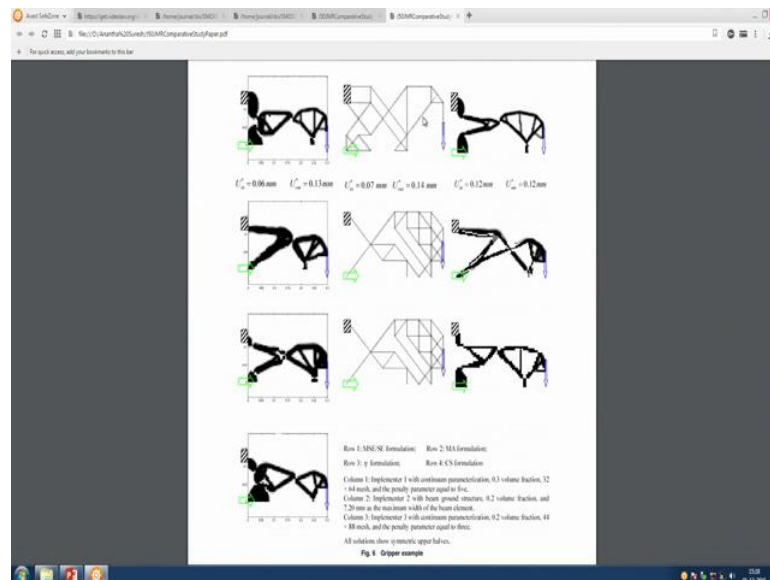
So, we can see the similarity whether you use beam elements or continuum elements middle and other left side and right side, topology is unmistakable it looks almost a same topology and the hinges are very prominent they are there everywhere and the first one row one was MSE by SE formulation, row 2 this mechanical advantage formulation F out by F in and the third one is ah the efficiency formulation it did not seem to matter there are only very slightly deform one another and the 4th row is Rahmatullah and swan formulation.

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So, any examples we took it was the case the things were not different, we took another example where input force is the green arrow, output force is this is blue arrow and again the beam formulation continuum formulation look the same, difference are topologically connectivity wise they are not different and even quantitatively also they were the same.

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So, what we did was we took another example, if this is more like a gripper example again you can see the beam and continuum look the same, it does not matter what element parameterization you take, the topology seems to be the same when you look at

it. So, then we thought how do you compare these numbers may be little small let me make them little bigger.

(Refer Slide Time: 38:17)

Table 1 The optimized objective function values of the examples shown in Figs. 4-6. See the text for explanation of the data. The italicized entries indicate apparent anomalies because we expect that the diagonal entries to be the largest for the corresponding columns. It is reasoned that these anomalies occur if the data given to a formulation violate its nonlinear constraint. It may also be due to a local minimum.

	Implementer 1				Implementer 2				Implementer 3			
	MSE	MA	η	CS	MSE	MA	η	CS	MSE	MA	η	CS
MSE	0.63	0.34	0.72	7.8×10^{10}	0.7443	0.3842	0.5106	0.7440	0.1725	0.5406		
SE	0.73	0.40	0.85	8.4×10^{10}	0.0847	2.1203	0.3224	MA	0.2535	0.2081	0.3371	
MA	0.59	0.56	0.91	1.1×10^{11}	η	0.1618	0.9869	0.6813	η	0.6699	0.2337	0.6487
η	0.53	0.05	0.16	2.9×10^{10}								
CS												
MSE	0.97	0.27	0.82	1.6×10^{11}	1.2116	0.1210	0.5375	MSE	0.5269	0.1947	0.8084	
SE	0.95	0.30	0.85	3.3×10^{11}	0.3538	1.8689	0.7098	MA	0.3448	0.4493	0.8002	
MA	0.39	0.60	0.92	6.7×10^{10}	η	0.6518	0.1219	0.5477	η	0.5175	0.2133	0.8879
η	0.97	0.10	0.39	5.1×10^{11}								
CS												
MSE	0.80	0.26	0.72	5.9×10^{10}	0.7580	0.3054	0.4598	MSE	0.4274	0.1584	0.5336	
SE	0.78	0.25	0.69	8.1×10^{10}	MA	0.0453	3.0080	0.6458	MA	0.4191	0.2119	0.7134
MA	0.27	0.10	0.88	2.2×10^{10}	η	0.3112	0.8777	0.6712	η	0.4617	0.1971	0.7529
η	0.72	0.09	0.52	1.2×10^{11}								
CS												

So, what we did was each formulation that we took implementer 1 2 3, different formulations when you use ratio formulation also calculate mechanical advantage, efficiency on this characteristics stiffness of Michal Wang and this Rahmatullah and swan formulate where the different spring constraints.

So, we calculate other matrix also to see if you do with MSE over SE ratio, does it do well in terms of mechanical advantage also and efficiency and other things. We calculate all of that we did not really see a pattern here, so if you were to focus on MSE over SE as the primary objective that of course, will be better than the other 3 or 4 objective function that they use. As similarly if I use mechanical formulation what a solution you get not fare well in terms of MSE over SE.

So, algorithms are doing their job for the formulation you ask but when you look at the topologies they were not very different. So, it was concluded that no matter what formulation you take there is actually no not much difference.

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MA 0.78 0.25 0.69 8.1×10^6 MA 0.0453 3.080 0.6458 MA 0.4191 0.2119 0.7134
 η 0.37 1.00 0.88 2.2×10^6 η 0.3112 0.8777 0.6712 η 0.4617 0.1971 0.7529
 CS 0.72 0.09 0.32 1.2×10^6

Fig. 7 Topology of the inverter obtained with $K_{in}=1 \times 10^4$ N/m and $K_{out}=1 \times 10^4$ N/m

Fig. 8 Topology of the inverter obtained with $K_{in}=1 \times 10^5$ N/m and $K_{out}=1 \times 10^5$ N/m

Fig. 9 Topology of the inverter obtained with $K_{in}=K_{out}=1 \times 10^7$ N/m

Table 2 Comparison of the five formulations. Up-arrow indicates an advantage and down-arrow a disadvantage.

Criterion	MSESE	MA	η	CS	IO spring
The need for output springs or load	Yes	Yes	Yes	No (!) but needs	Yes

And the real problem was that there are these hinges that are encircled here with in red color. So, this also if you look at this ah Rahmatullah and swan formulation when K out is 10^4 Newton per meter, the hinges appear. When it is 10^6 again the hinges appear, when it is 10^7 it looks like the hinges are going away, but it is actually giving a stiff structure, it may look like Compliant mechanism, but its displacement will be very small for the same force.

The topologies are same it is moving away from hinges to no hinges. So, there when I look at the images you may think that oh hinge problem has gone away, but it has actually not. It is giving a stiff structure because the loads are here and you put a output spring if you put that is like a stiff structure problem you are saying with design domain and put a spring and a your time to overall try to get some stiffness out of it.

But at least people thought that hinge problem can be removed by putting artificial spring outside, that spring is really artificial in a real comparison it may not be there, it would not give you the same displacement when you do it, but you get some topology and we try to actually compare various formulations here to see what are the advantages.

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obtained by minimizing the mean compliance. So, why is mean compliance preferred over the others? This is because it is simple to implement and gives a physically meaningful optimality criterion.

1.9 Topology of the inverter obtained with $K_{in}=K_{out}=10^7$ N/m

Table 2 Comparison of the five formulations. Up-arrow indicates an advantage and down-arrow a disadvantage.

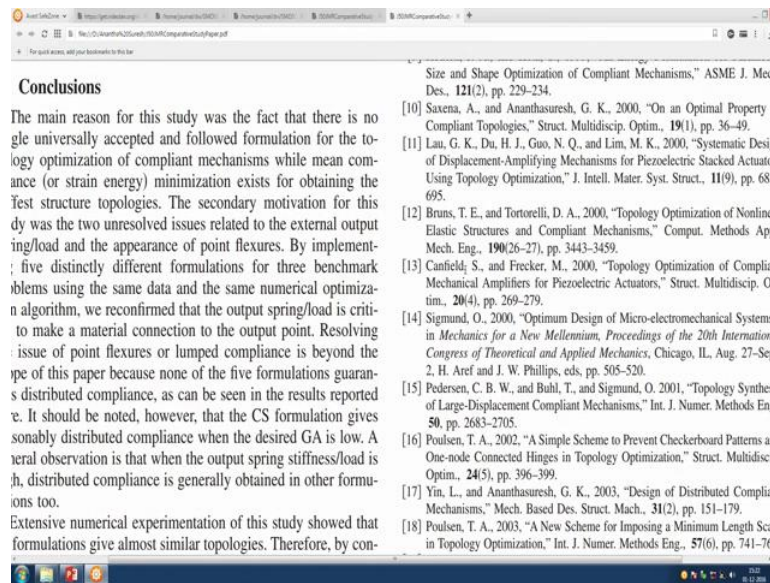
Criterion	MSE/SE	MA	η	CS	I/O springs
The need for output springs or load	Yes↓	Yes↓	Yes↓	No (!) but needs $GA_d(\cdot)$.	Yes↓
Computation in the sensitivity analysis	Low↑	Low↑	High↓	High↓	Low↑
Nonlinear constraints	No↑	Yes↓	No↑	No↑	Yes↓
Converging from an unbiased initial guess	Yes↑	Yes↑	Not always↓	Yes↑	Yes↑
Gives point flexures	Yes↓	Yes↓	Yes↓	Yes↓	Yes↓

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If we look at this the need for output spring is there in all, but this Michal Wang formulation of geometric, but then he has to (Refer Time: 40:59) net formulation you have to get desired output displacement input displacement.

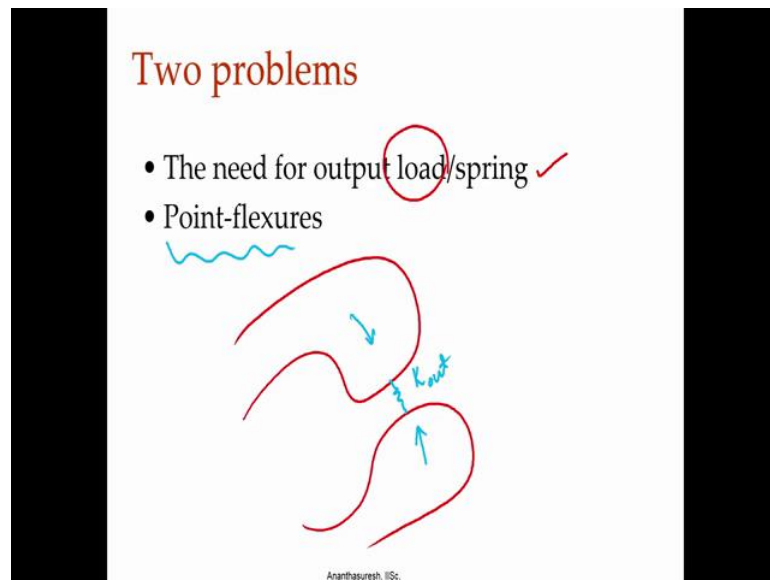
Computation sensitive analysis MSE over SE and MA are very low, efficiency formulations is high because you have to solve 2 load cases and so is Rahmatullah and swan formulation and these character stiffness formulation also and non-linear constraints there are no constrains in MSE over SE mechanical advantage formulation is was there, efficiency formulation did not have there is an advantage like that you can get. Converging to unbiased initial guess, most of them do except we found that efficiency formulation sometimes get trouble, point flexures are there in are in all of them that is the big thing you know all the formulation view this point flexure. So, we will spend in the next lecture how to resolve that? How do you avoid? How do you get truly compliant mechanism or distributed compliant mechanism?

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Look at the conclusion in this paper; we conclude that all the formulations pretty much have the same difficulty that is there is a need for, I will select the pen now, there is a need for output load or spring as already said when you put output load it becomes primarily a stiff structure problem.

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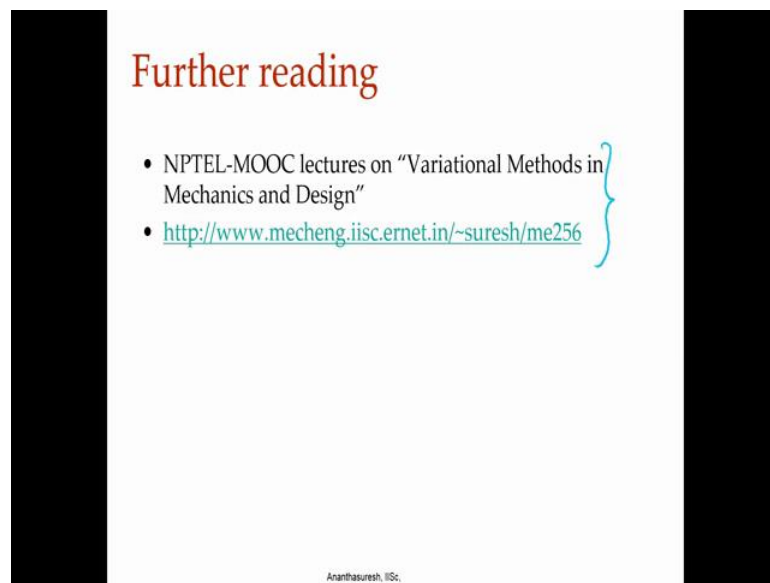


When you put a spring for that particular object you can say elastic object, this let us say I am doing a gripper problem, let us say gripper comes out like this and there is an object there see there is an object; I can think of that object as instead of an object, I can model

it as a spring that is K out, if that is what you want I will be the (Refer Time: 42:42) job. So, you are garbing that like that.

But then that may be artificially in some problems, where there is no output load output spring how do you do? Bigger than this is point flexure problem it is giving linkages, it is giving joints and we need to fix that; that is what will discuss in the next lecture as to how to avoid.

(Refer Slide Time: 43:04)



The slide features a central text area flanked by two vertical black bars. The text is as follows:

Further reading

- NPTEL-MOOC lectures on "Variational Methods in Mechanics and Design"
- <http://www.mecheng.iisc.ernet.in/~suresh/me256>

A blue curly bracket on the right side of the slide groups the two bullet points. At the bottom center, the text "Ananthasuresh, IISc." is visible.

And further reading is of course, the paper that I showed and if you want to know about implementation details and you have to understand (Refer Time: 43:13) theory for that there are other sources for reading up.

Thank you.