Compliant Mechanisms: Principles and Design
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Lecture – 26
Early works on design for compliance

Hello, in the last lecture we discussed how the approach to synthesis of complaint mechanisms will be benefited if we take the structures view point that is we use the techniques that are developed in structural optimization and apply them for complaint mechanisms synthesis. So, when we do that the first question that we need to ask is how complaint mechanism design problem is different from the stiff structure design problem? That means that, we have to ask question we need design for compliance or flexibility, what differences do we see from structural optimization proper?

So, let us look at that problem here today design for complaints and we will look at some of the early works in this field, normally early works would have a lot of insight into the problem because somebody would have posed it and solved it. So, we look at this design for deflection or compliance of flexibility today.

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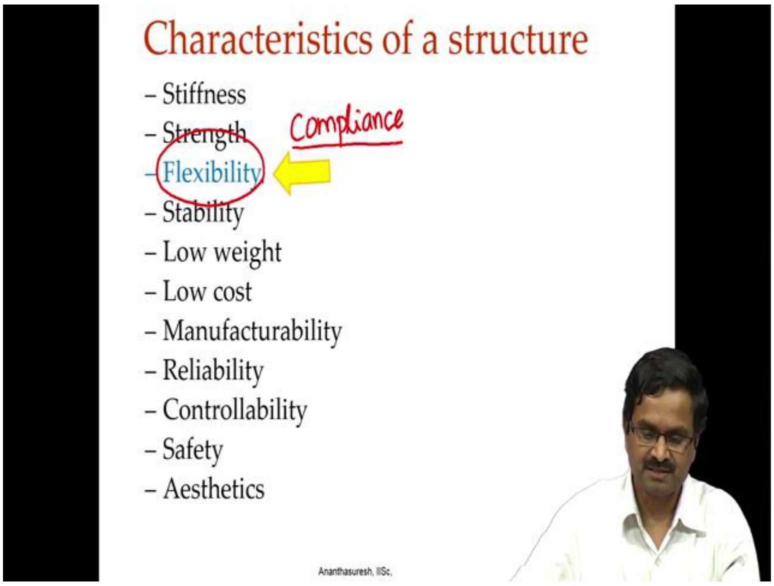

We had put these characteristics that are expected of a structure when we try to design it stiffness, strength, flexibility that is our focus today. Flexibility is what we want are in our contest, we can also call it Compliance. Structural optimization techniques have been developed for stiffness, strength, stability, weight cost all that are listed in here and more over, but when it comes to designing for compliance or flexibility or deflection in general there is limited literature in the literature of structural optimization.

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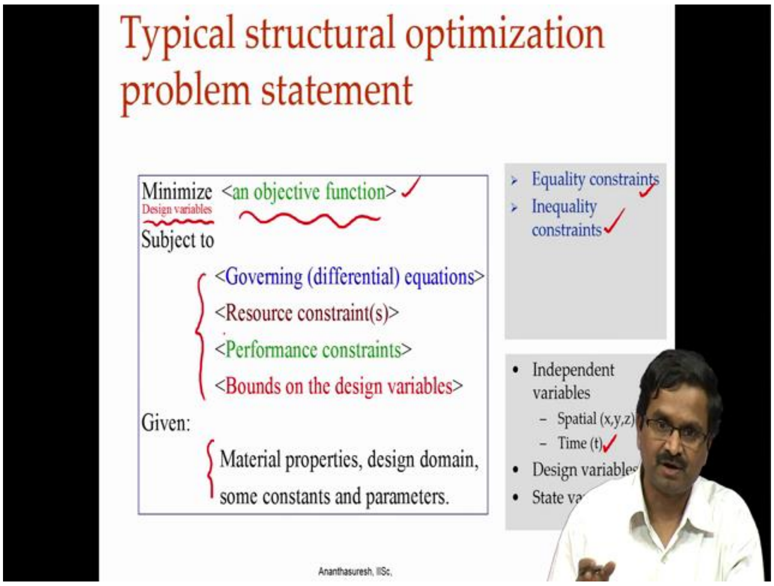

Before we discuss that let us ask the question, when we pose the problem in this manner is a Typical structural optimization problem statement, where there will be an objective function, there will be governing equations, resource constraints, performance constraints and bounds and design variables and define desire variables a critical step which like optimization, basically formulating the problem that is what is the objective function and what are the constraint? What are the design variables? These are all important. The constraints can be quality constraints, inequality constraints all of those, it can be in 3 dimension x y z or only x or only x y 1D, 2D, 3D.

If we take (Refer Time: 02:48) there will be time also will be a variable, these are all the state variables and we also have deign variables all of those now when we have and of course, we would assume material property design domain, when we pose this problem for stiff structure design and for complaint mechanism design, they both look the same. What will be different is the objective function will be different. In the case of stiff structures, we try to minimize the strain energy or work done by external forces or displacements there all the things that we minimize in order to get the stiffest structure because stiffest structure is not suppose to undergo substantial deformation

In contrast when you take compliant mechanisms what do you do? Simply maximize reflection because that is what you want. You want a structure or a mechanism that deforms a lot, so you want to maximize the deflection or what do you do? How do the constraints look like when you pose it for complain mechanism verses stiff structure?

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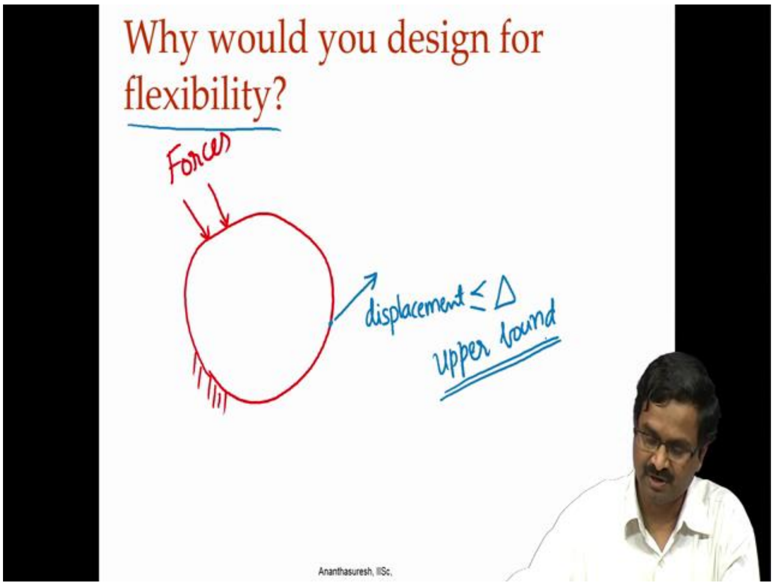

One thing that will be very different is, when you design for complaints or flexibility or deflection, first question to ask is why you would want to do that? So, whenever you look at the literature, you have to look at what has been done, also when you look at the publication or books we have to see what is the motivation for some of those design methods that are developed for that matter, any subject matter? If we ask in the past before complaint mechanisms were studied, why were people interested in designing for flexibility? By flexibility what I mean is that, if I have a an obituary domain, let say it is fix somewhere, we apply some forces this is our forces and we say that we want to design the interior of the structures such that, when apply forces there a point has to move in some direction. So, this is your displacement of a point it can be anywhere on the body we are applying forces in one place and then asking for displacements somewhere else because that is what complaint mechanisms do, you apply force somewhere else you get displacement

But in the past there was no effort to design complaint mechanism until recently, that recently itself is 2 decades old, but I am talking about really a way passed, why were people interested? There are some papers where people try to design for flexibility why were they interested? The interest it appears to be that, they wanted to limit the displacement.

So, displacement should be less than some delta it is like an upper bound, they were not really interested in flexibility in the structure because structures were suppose to be stiff that is what it designed for, but sometimes you cannot really avoid. Especially if we try to make structures optimum, then you would use the given amount of material in the most efficient manner as least material as possible, then you cannot avoid deformation. So, they try to put an upper bound on the reflection and that seems to be the motivation for these papers to look for flexibility as a requirement, either in the objective function or in a constraint.

But there are some very nice papers from which we can learn to do complaint mechanism design. Now there is a lot of interest in complaint mechanism design, but when I started my work in 1999, there was very limited literature. So, when I first found some are the very important papers, it was really exciting to see that people had thought about accounting for flexibility in structural design.

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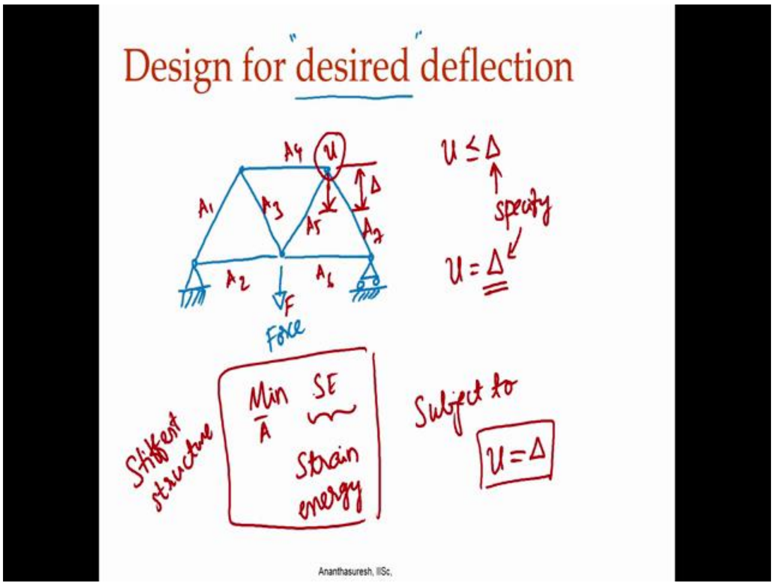

So, this problem of Design for desired deflection, desired deflection is the key here. Just like in complaint mechanisms we want deflection. So, at the time also people wanted to

Design for desired deflection. It could be just a truss for example, let us take a truss like this, let say this is fixed here and this is on a roller support and we have these.

Let say apply a force here, this is the force, and then I ask for let say deflection somewhere else. So, I want this point to move by certain value, but not more than that, that we were design for desire deflection problem, you say the displacement here if I call that let say u, you say u should be less than equal to some delta that you specified in some cases you may want u to be equal to delta also, that is really desired deflection.

You want this deflection when this force is applied, then you ask the question what should be the area of cross sections of all these elements a 1,a 2, a 3, a 4, a 5, a 6 and a 7. So, we have 7 variables which are cross section areas of all these bars in a truss, then you say given this force F you want this point to precisely have a deflection that U specify, that will be desired for desired deflection problem. How is this different from taking the same truss under the given force again you want to make your cross section areas of the 7 truss elements, so that overall truss does not deflect much, that is you want to get the stiffest truss. So, that you can pose you can say I want a stiffest truss, then you would say minimize strain energy. So, SE here stands for strain energy under the load that is applied here if we do that you get the stiffest structure.

Of course you should have a volume constraint, your given limited amount of volume to make this trusses, you minimize the strain energy with respect to these a vector meaning A 1, A 2 A 3 up to A 7; 7 areas are there we can do that. But instead if we also have this deflection constraint then you will say subject to a particular degree of freedom if I may call U say should be equal to delta.

That is among all the trusses which give you U equal to delta, you want to pick the one that has maximum strain energy for a given volume that also you fix. So, you can pose a problem like that.

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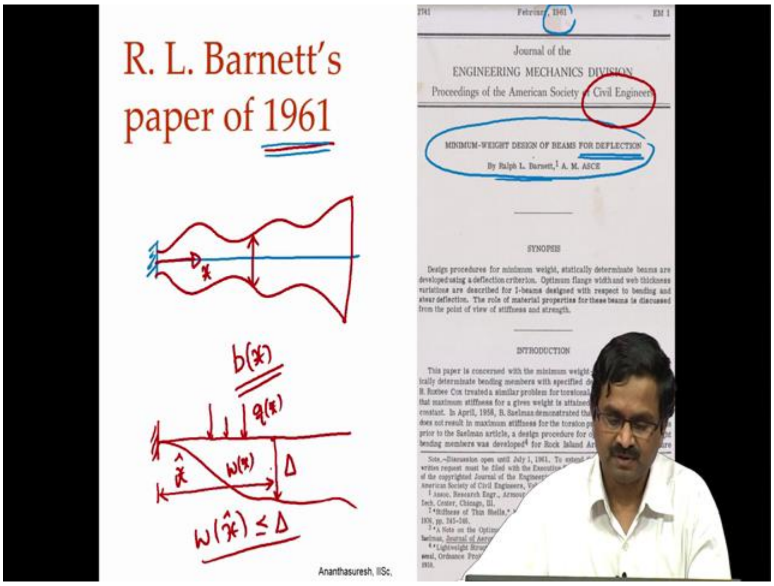

When you do that what changes, what becomes a problem; that is what was first discussed in 1961, I consider this a landmark paper because there is nothing before this. So, that that makes it the really pioneering paper by this person name Ralph L Barnett, look at that date 1961 February. 1961 very old paper and the title it is little small, but I will read out for you it is “Minimum Weight Design of Beams for Deflection” for deflection is the key here. So, Barnett had considered problems of designing beams.

So, if I take a beam and there are cases where if it is starting determinate boundary condition it is easier, if it is starting indeterminate it is little harder, it consider all of those this is a very long paper and he has consider this is a first paper. So, let us say I have the beam like that whatever boundary condition it can be a cantilever or a (Refer Time: 11:19) whatever. So, what he tries to do was how to get the cross section profile of the beam.

So, something likes this. So, if I call this let say the width b of x where, this is our x here, this function is what he try to determine. So, that under some load which is the transverse load, this at a particular point if I what to do the side view of this, if I do the side view of this beam let say this is fixed over here, there is some loading given arbitrary loading does not matter you can call it b of x, you say when it deforms in some fashion.

So, at a particular point, he imposed a constrained that if I call this deformation w of x, hw would said w at some x hat that is let say this point from here to here, if we call that x hat that imposed a constraint that should be less than equal to delta or sometime this equal to delta, that is what he meant by design for deflection again if you look at his association, this paper was published in the proceedings of the American society of Civil Engineers.

So, Civil Engineers normally want their structures not to deform. So, what was Barnett motivation for this, again the motivation as I said is to limit the deflection at particular points there could be. So, when you do optimal design everywhere this scatter beam bend a lot, now we want to prevent that and make it bending in a way, that deflection at a particular point does not exceed in a upper bound that is specified.

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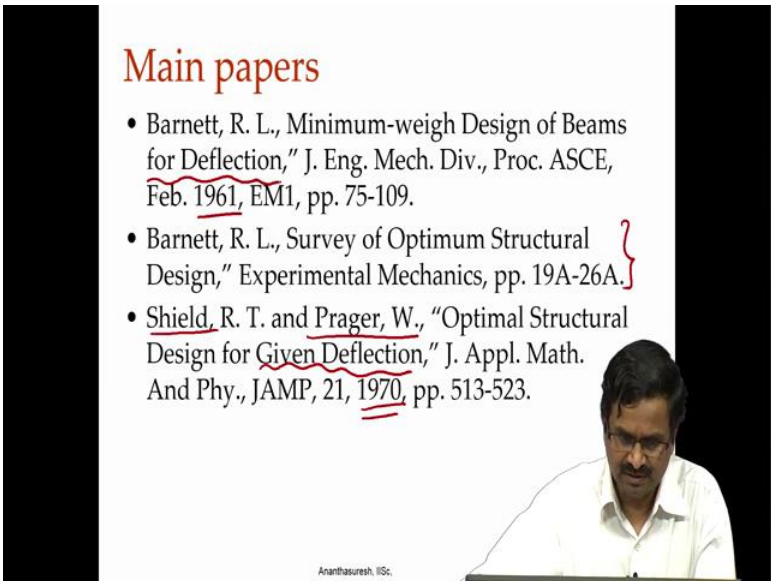

So, after that 1961 there were a few other papers very important papers Barnett himself had another paper. in 19 soon after that it was this is 1961 this was 1965, and the 1970 shield and Prager, William Prager is a big name in mechanics and he and Richard shield had a paper, where again if we seen in all of those given deflection is the key word here; designed for deflection.

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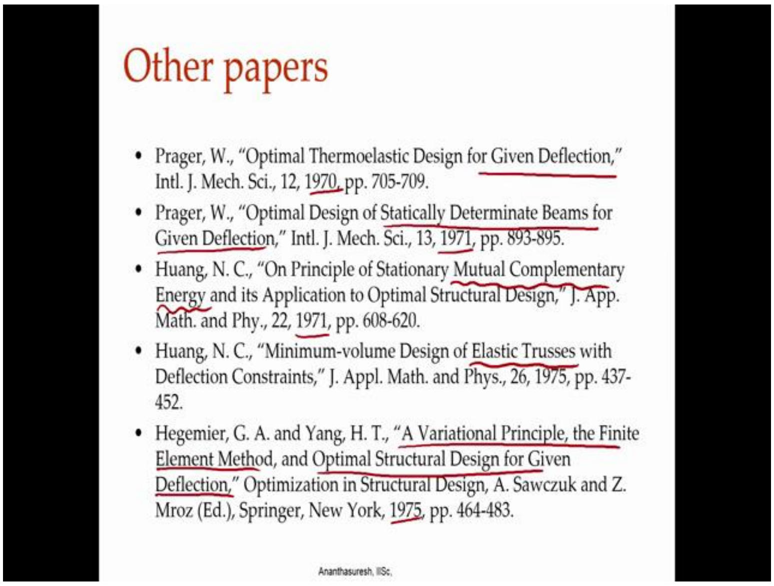

Following this for a while for about 5 6 years, there were number of papers I am giving them all because you do not find them that easily everywhere in the literature, they are all there but somebody has to tell you that they are there. So, here again for given deflection Thermo elastic design 1970 by Prager and then statically determinate beams again for given deflection 1971 and then another paper 1970 Haung who looked at and talked about what is called "Mutual Complementary Energy" we will talk about mutual strain energy which was defined by Shield and Prager.

In the other paper that I showed and trusses beams trusses all that and even a general one a variational principle the finite element method and optimal structural design, for given deflection that is the key word today given deflection that is in 1975.

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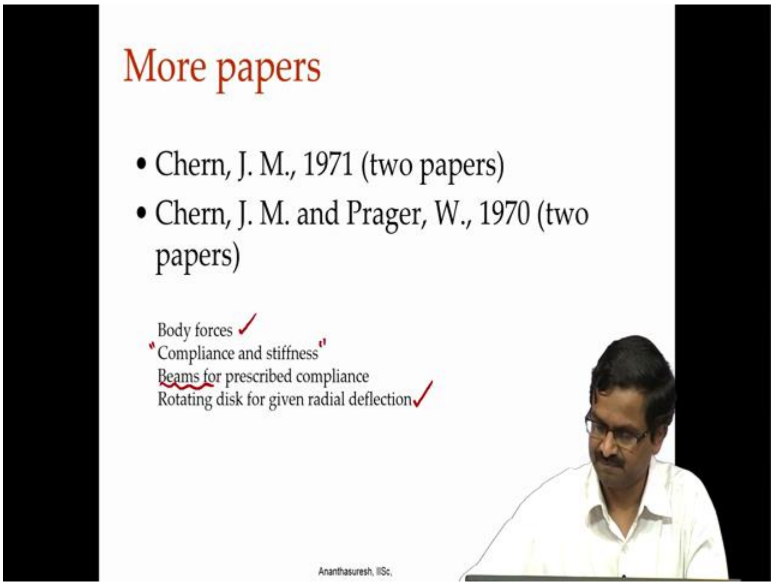

And there were a few more papers there were 4 papers by Chern. And Chern and Prager, where they consider body forces for a disk that is rotating with a constraint on the radial deflection.

So, you imagine a turbine blade goes at very high speeds, that will have body forces and there you should not touch the housing. So, radial deflection there was a constraint and they had looked at Compliance and stiffness. Compliance is a synonym for flexibility. So, where that consider many time they consider beams and trusses to solve problems that is what we will also do.

These are the some of the very important papers in this field and then after 1975, there was a gap and people did not look at it when complaint mechanism field started we had to look at some of this old papers and learn for from them to see how the problem of designing complaint mechanism is different from designing stiff structures. So, let us take a problem where we can start with a truss it itself.

So, let us look at the truss problem. So, we can take any truss we can take one more. So, we will fix for now will take statically determinate boundary condition these are truss.

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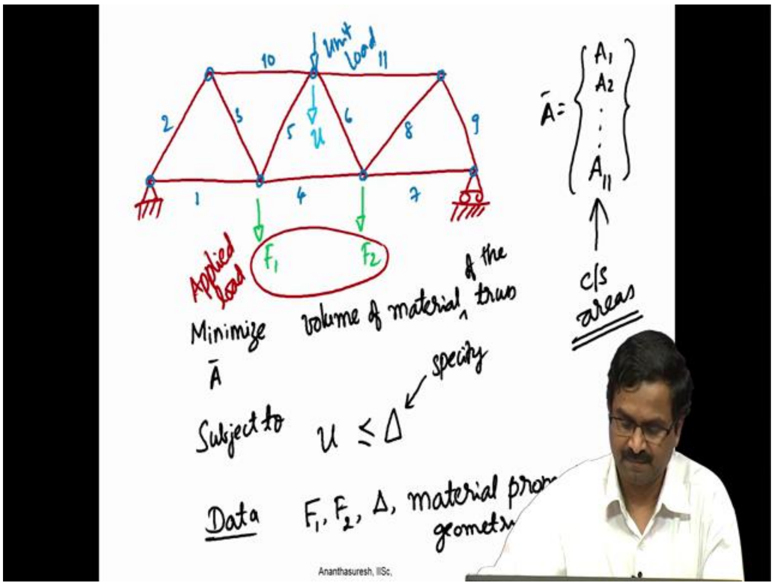

So, there will be this hinges this everywhere (Refer Time: 16:30), we have many elements. So, we have 1 2 3 4 5 6 7 8 9 10 11. Now let say that there is force acting somewhere a typically number of forces does not really matter let say there is force F1, force F2 any which way, then we say we want this point to have some deflection. So, we want this point to have some deflection, let us call that U, we want to optimize it.

So, the problem that we would pose is as follows, will say that minimize let us say we want to minimize the volume of material, let us consider simple problem volume of material of the truss subject to this cross section variables, when I put A like this, what I mean is that we do not know or these are the design variables A1, A 2 in this case we have A 11 these are the where 11 variables are there you want to minimize volume of material subject to a constraint that under these forces of course, a constraint that the deflection at whatever point we have chosen, should be less than or equal to delta that we specify.

So, delta is what we specify. What is the data in this problem? The data would be are the forces F1, F2 and this delta and some material properties and geometry we need that, we need (Refer Time: 18:39) modulus and the geometry, the lengths of the truss and so forth. But what we do not know are the cross sections, cross section areas that is what we will use a design variables. If we do this, if we take this problem and pose it mathematically, so let us do that whatever I have written.

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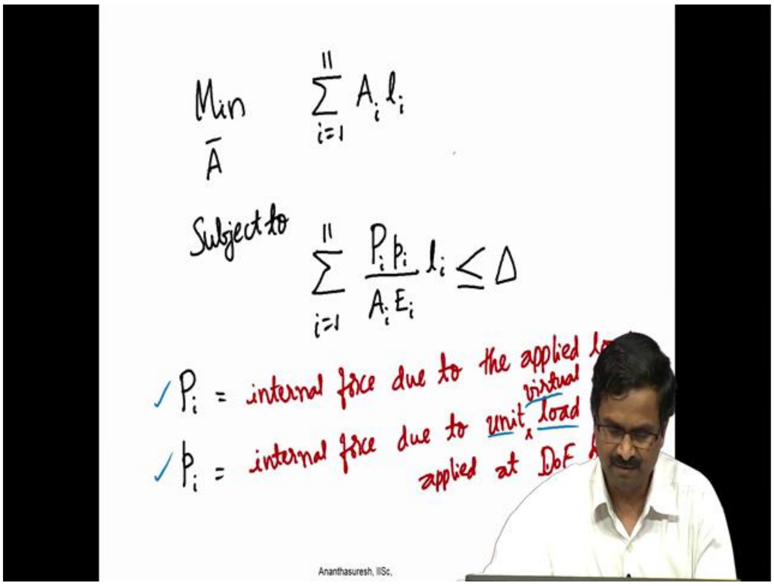

Now, I will do it mathematically minimize with respect to that factors a and volume I can write it as sigma I equal to 1 to 11 here area of cross section I am say length of that cross element, lengths are given geometry is known to us subject to we have to write that U that we had, there is a simple way which I do not have time in this course to go through, but you can go through the other courses, that are there variation method structural optimization and a few others that the NPTEL course as well as a course i teach at i a s c where you can write this as capital F i small f i divided by actually that we use different symbols not f here, I will use capital P i small p i by A i and Ei then there is l I, i equal to 1 to 11, this gives you that u and that we say less than or equal to that delta that is the problem.

What are these p capital P i and small p i, capital P i terms out to be the internal force, this is the internal force in the truss element we have 11 of them. Internal force due to the applied load and this one is internal force due to a unit load which is actually a unit virtual load, it is not real load that f that we have F1 F2 were real load unit load, applied at it is virtual, applied at degree of freedom of interest. D O F stands for degree of freedom of interest, what I mean by this if I go back to this truss these are the applied loads.

So, these what is given to you apply them and every truss member will have some internal force, they are all captured as capital p s there will be capital P 1. Capital P 2 up to capital P 11. Now what do you mean by virtual load? Virtual load basically wherever u is there I apply unit load here in the direction this is the unit load. It is virtual you just imagine that is it not real, you apply that, then again in absence of applied force you applied loads you remove and apply this virtual load. Then again you can compute the internal forces in each truss member that is denoted as small p i. So, we have capital P i and small P i due to applied load unit virtual load at degree of preventers wherever u you want that is what you do.

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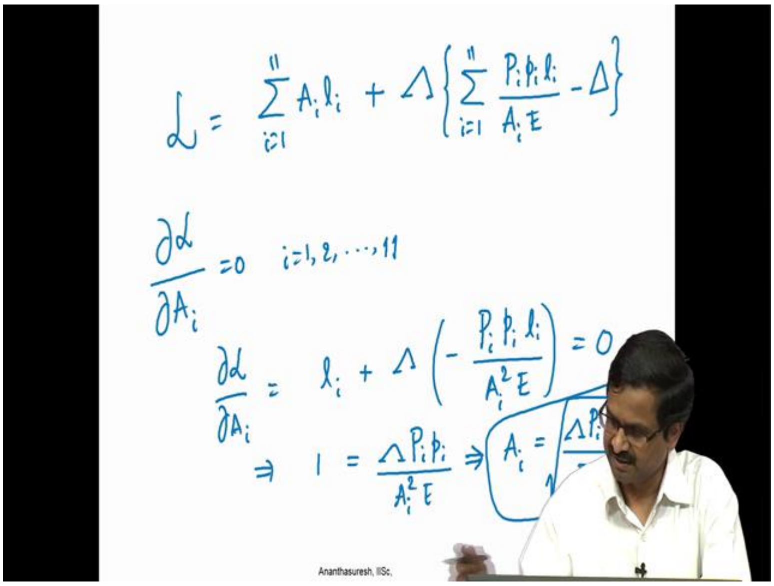

Now, we have this problem this can be easily solved, what do we have we can write the Lagrangian for this those of you do not know optimization should brushed up. So, we write our Lagrangian, where we say we have sigma A i l i and then plus we have this lambda which is Lagrange multiplier, times the constraint which is again summation i equals to i goes to 1 to 11, this I goes to 1 to 11 and we write the deflection constraint capital P i small p i l i by A i E i are just E, hence modulus minus delta basically that less than or equal to bring out this side Lagrange multiplier So, these Lagrangian.

So, for optimality what we have is dou l by dou A i equal to 0 where I goes from 1 to 11 we have 11 variables, we have 11 equations because each of them i equal to 1 to 11 we do get an equation we can solve, we can actually do that. What is dou Lagrangian by dou A i? The first one will simply give you l i because all of them A l i if you say with respect to a i only, that will be there and here also the same thing lambda derivative of this only where a that i that you take will be there, that will be minus capital P i small p i l i by A i square, because one over A i is there A. So, this is equal to zero that is the equation, you can actually solve for this now. Because from here you do get if we solve it l i actually gets cancelled. So, what I get will be 1. So, this implies (Refer Time: 24:42) 1 equal to one equal to lambda in to capital P i small p i by A i square t. So now, I can get A i to be square root of this Lagrange multiply capital P i small p i by E that is the solution.

So, all of the equal to i equal to 1 2 3 up to 11, we get an expression in close form. So, which is lambda square root of P i P i by e i. So, let me just write that there.

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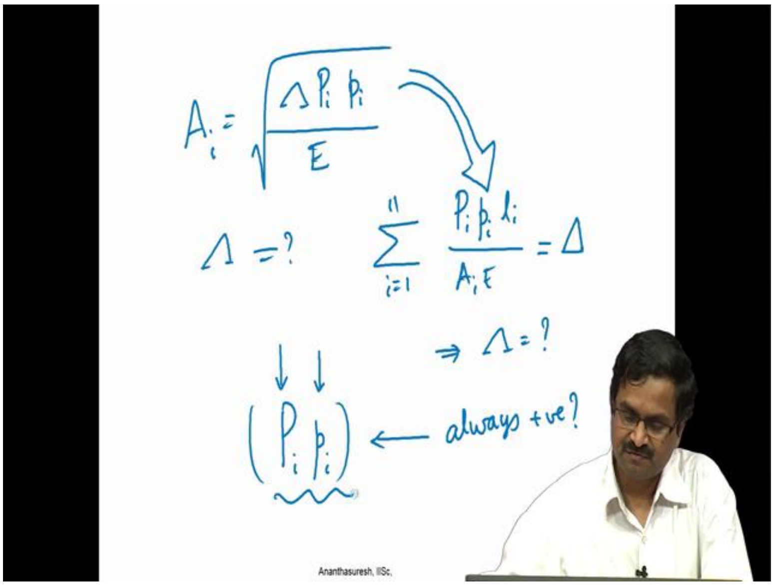

So, I get A i equal to square root of lambda capital P i small p i by what would we have there, I think it was just E. It is just E, we have the solution. So, who gives us lambda because we have the constraint that the. So, how do we get this we have the constraint which is active which is i equal to 1 to 11, we had P i P i l i by A i e is equal to the delta that is a deflection constraint based on that if you substitute all the as in to this here, you can compute this thing.

So, everything is done. So, what is a big deal, but this is a big deal that Barnett observed in this paper, if you look at the A I, you have in the square root capital P i small p I, what is a guarantee that the product of capital P i small p I, again recall this thing is the internal force due to the applied load, this is the internal force in the I th member due to the unit load that you applied.

So, what is the guarantee that this is always positive? In fact, it is not that is what he observed that sometimes it can be negative sometimes it can be zero of course, when something goes from positive to negative it has to pass through zero, when that becomes zero and becomes negative we have a problem and that is a difference.

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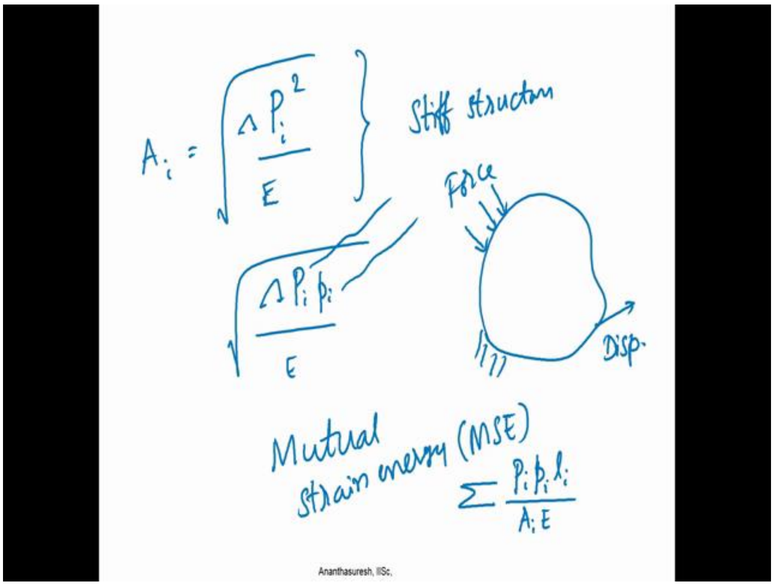

So, if you where to look at compliant mechanisms problem, in the case of stiff structure problem, the same thing would appear it appear as P i square by whatever we had that was E. So, lambda E with a square root is what your area of cross section would have been we considered stiff structures. The movement you put complaint mechanism, we get the same thing to be lambda capital P i small p i by E I, capital P i again applied load and this is the unit virtual load. So, we have 2 points that we had worry about again we go back to the abstraction, if I apply force some place, I want deflection in some other place.

So, here is where I want displacement applying force here. So, there is a mutual relation between the applied load and the displacement, where you apply the unit virtual load. In fact, the quantity that we wrote is called “Mutual strain energy”, Mutual strain energy we will just call it MSE, will be written in to this what I am call Mutual Strain energy? Where we had sigma capital P i small p i in to l i by A i and E is called the Mutual strain energy because there are 2 forces mutually they are interacting, they are applying the force there is a displacement. These the key and that can become negative and you will get interesting possibilities in this and that is what we will be discussing in the next lecture we are looking at trusses first and then also the beams.

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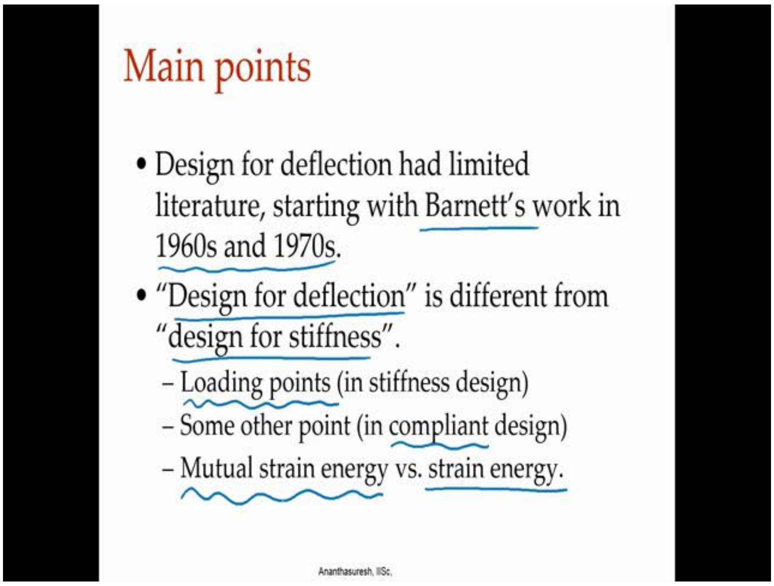

So, what we have discussed today is that design for Deflection had very limited literature, starting with Barnett's landmark paper and Prager and his associates starting this the decade 1970 to 1975, 15 years and there is a big (Refer Time: 29:32) for some two decades and then we starting looking at complaint mechanisms in this case Design for stiffness is different from design for deflection.

We mention today that loading points are of interest in stiffness design, whereas in complaint design we want some other point, where the loading may or may not be acting and that leads to the concept of mutual strain energy rather than strain energy as you will discuss. (Refer Time: 30:04) situations where things can lead to this under square root sign we have some that can go negative, which never happen in the stiffness design.

And (Refer Time: 30:14) to problems are also new opportunities, as you will see in the next lecture on trusses which can be related to mechanisms, because in a truss if you remove certain elements it actually become like a linkage and likewise we can consider frames with beam elements. There again their interesting complaint mechanism that will emerge, if you look at in the perspective of design for deflection.

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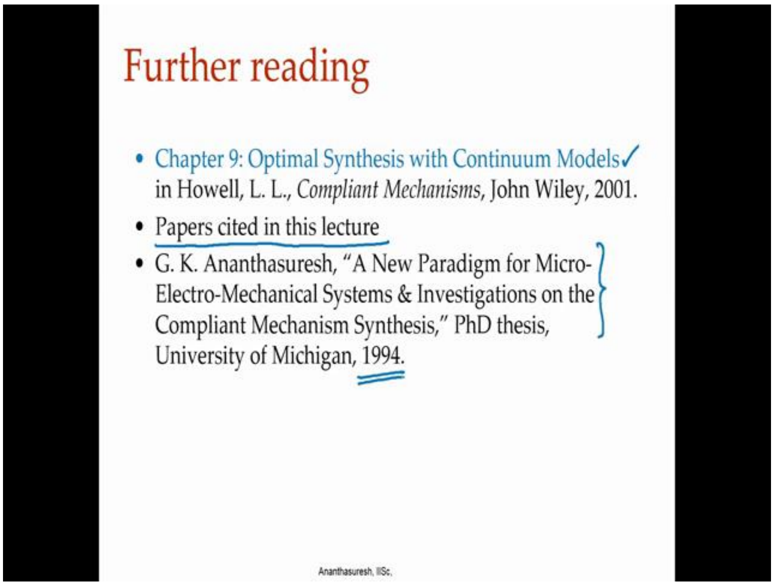

So further reading, again you can look at this chapter which is limited, but very important for those of you want to learn is to look at papers that are cited in this lecture. There are many papers from Barnett, it is very very important to read those papers understand how they did it? How they defined the mutual strain energy concept and all of that.

Another thing is the PhD thesis that I had written in way back in 1994, where is a big discussion on Design for deflection, which led to the complaint mechanism synthesis from the structural optimization view point. And we will discuss that from trusses to beams and then continue in the next 3 lectures.

Thank you.