Compliant Mechanisms: Principles and Design
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Lecture – 25
Structural optimization approach

Hello, this week and the next week we are going to talk about an alternative design technique for compliant mechanisms. So, for we have been talking about how to use kinematic based or Kineto elastic mechanics based design using pseudo rigid body models, but now we turn to an alternative technique and in the coming weeks we will be exploring a few more techniques for designing compliant mechanisms. Today we are going to talk about how to use structural optimization to design compliant mechanisms.

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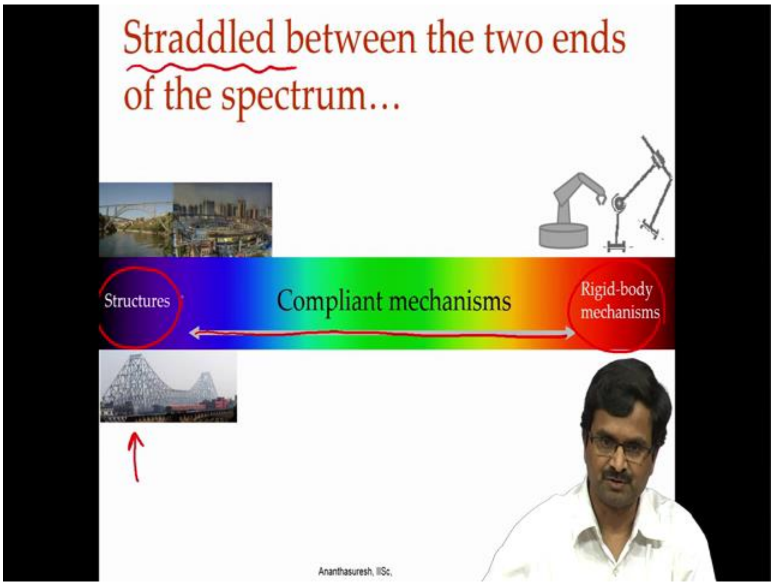

This approach is a different perspective on compliant mechanisms and that will be become very clear. If we look at the entire spectrum of things that we have around us there is one end we have structures, the other end we have rigid body mechanism. This is one end of the spectrum where we see bridges, buildings and all the things that do not elastically deform much; they of course, deform but they are suppose to be static structures right. The other ends we have things that are made with rigid bodies with joints connect in the rigid body so, that we obtain the motion robots, mechanisms and so forth.

In between are the once that are straddled are the compliant mechanisms. The entire spectrum that we have here, this entire thing is compliant mechanisms on one end they touch the structures, the other end they touch the rigid body mechanisms. It makes sense to borrow the knowledge that exists in this traditional field of structures as well as rigid body mechanisms. We have done in the case of a design or synthesis; how we can use the knowledge of rigid body mechanisms in order to synthesis compliant mechanisms there the pseudo rigid body model was helpful.

Now, let us turn to structures and then see if we can use that knowledge to design compliant mechanisms. In many things if we recall the mobility analysis also we had borrowed some from the structural field, some from rigid body mechanisms field. We do the same thing for design as well.

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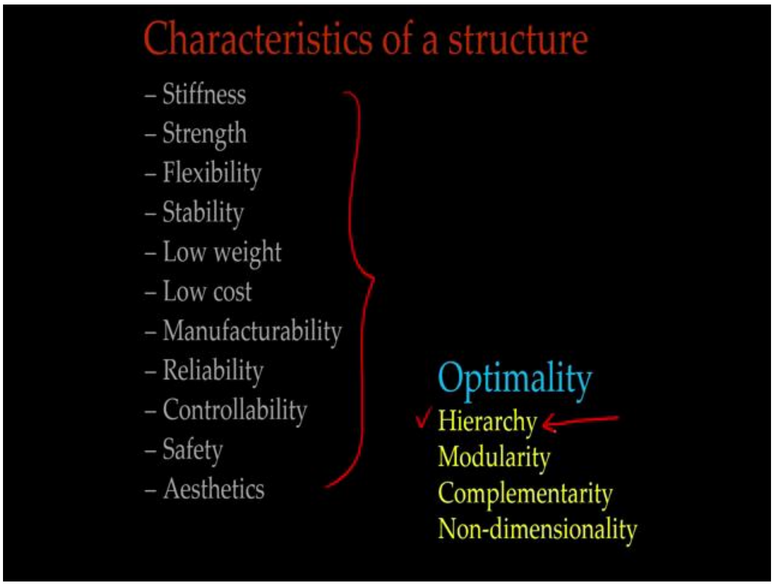

Now moving on to what a structure does. A structure is designed for stiffness, strength, we also have flexibility here sometimes structures are designed for little bit of flexibility in the case of compliant mechanisms, we need a lot of flexibility which is what we will be discussing in the next few lectures, how to design structures to be compliant or flexible and then stability is also an important characteristic of a structure mechanism any system for that matter. Then it should be of low weight, as low as possible and low cost, again as low as possible there it should easily manufacturable it should be reliable, controllable, it should be safe and also aesthetics, all these are the character that one would like to see in a structure, when we designed that is what we design for.

Beyond that we also want optimality in a structure that is the structure in order to achieve all these objectives in an optimal way it needs to be optimal and when we take this structural design view point optimality naturally comes in. With rigid body linkage synthesis also optimality is possible where we take (Refer Time: 04:11) points and move them around and get the best one that is possible, but here the optimality comes in a much more effective manner and apart from this characteristics that we have listed here there are few more that optimal structures have and that is hierarchy modularity complementarily and so forth. Today will discuss only the hierarchy other things will discussed in the context of compliant mechanisms in the coming weeks. What do we mean by hierarchy?

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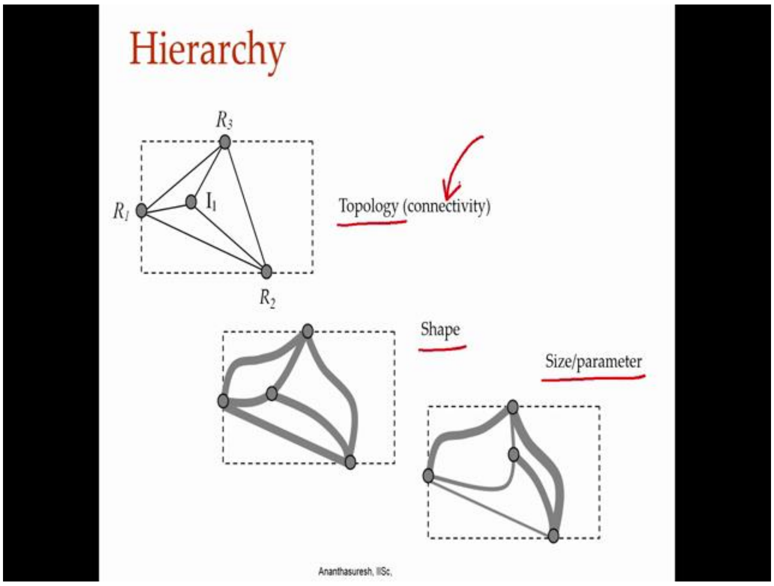

Hierarchy is important for optimality and if we look at what you mean by hierarchy, there is a view for any structure at the level of what we call topology and that becomes important for compliant mechanisms as well and then there is shape on level below and then there is size or parameters the level below that. Optimality in the contest of compliant mechanisms R structures such as trusses, frames and other things that we see in bridges and buildings and so forth. It has a lot to do with connectivity, how are different portions of interest connected to one another. Then the shape of the force what the word says shape of the segments that connect in the size, again what is the size? It can be variable it can be thin, thick, wide, narrow all of that. This is the hierarchy in the view of structural form that one would like to have in a structure.

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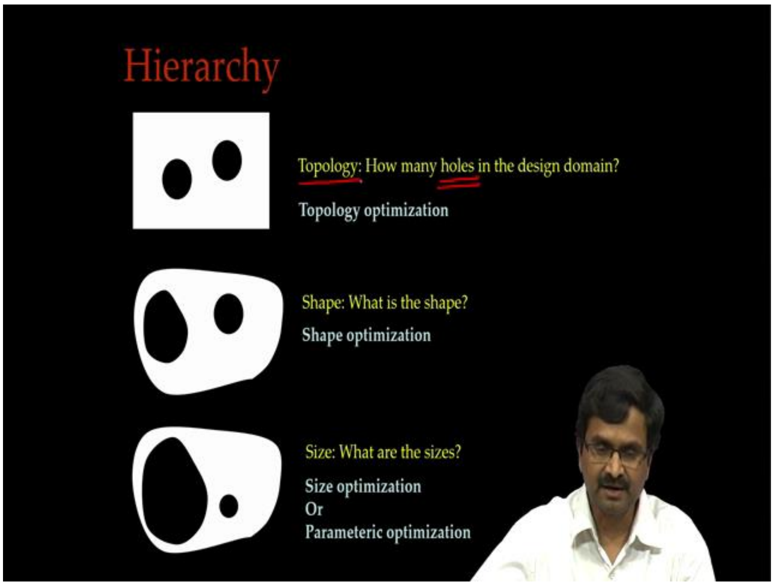

Another way of looking at hierarchy is in terms of the wholes and their whole shape; because the wholes are the one that characterize what we call topology. We will be talking little bit about this topology optimization that is developed for compliant mechanisms. Topology in the mathematical sense indicates the number of wholes that are there in a structure and once you have wholes, what is the shape? What is the size? Naturally comes in we get shape optimization as well as size optimization.

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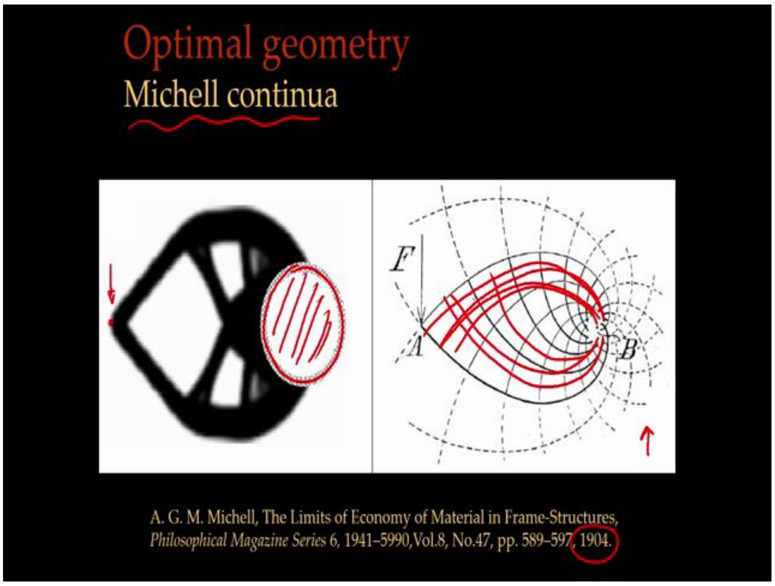

These are the hierarchy, but hierarchy the time reference to now is much more than that. This is a famous design due to A. G. M. Michell a long term ago this is 1904, more than 100 years ago they are called Michell Continua. If you have as a structure where there is a let us say a force acting here at this point, then if you want to connect it to something like a you know circular support of some kind then this is what is fixed here; the optimal structure that today’s structure optimization methods give is what is shown here and electrically if you were to solve which is what Michell did this figure is taken from his paper, then you see a lot of this grill age patterns. In fact, if you have you can draw a lot more of these in between everyone of them, we can draw a lot more of this if you do all this you get a grill age pattern here some go like this some go like this.

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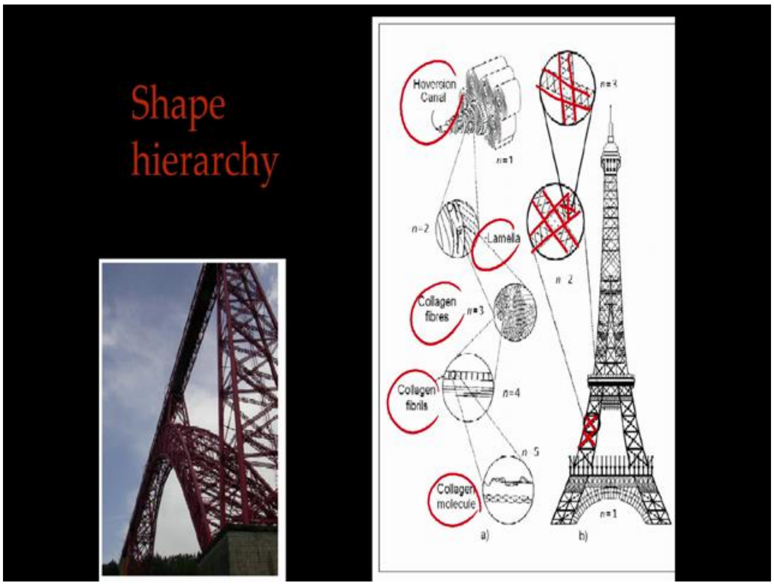

There is hierarchy here in the sense that there is an overall shape and then underneath that you get this smaller element that look exactly the same. The point will become clearer if we see some other famous structures many of them, but built by the Gustave Eiffel the Eiffel tower. Eiffel tower has this cross beam that we see and within the cross beams if you look at there is another level of cross beams as we see and then within that if we zoom in there is one more level of cross beams if you zoom in they look like that. Here we have hierarchy in the structural design and that also exist in the bones starting from Collagen molecules, they become Collagen fibrils and then fibres and Lamella and then this Haversion Canal that make the inner part of the bone. There is again hierarchy there. The shape hierarchy we can call it topology hierarchy we call it, but there is a structure at different levels and that seems to be optimal.

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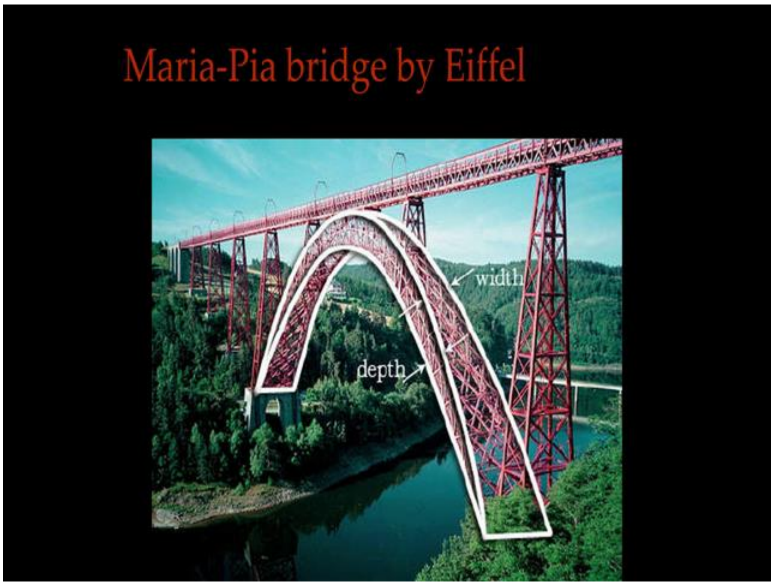

There is also when you have the topology next thing is the shape here you see, the shape of a bridge that was designed and built by Eiffel had a depth which was varying from end to end and so, was the width in a different way that the shape optimization and of course, the cross bars and cross bar, even cross bar is also evident here of this hierarchy that we can talk about.

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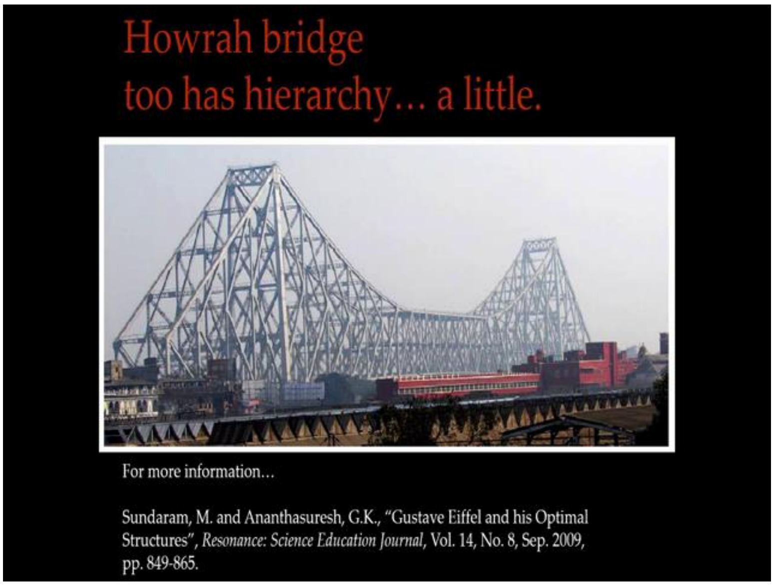

It is there in many bridges and structures that are around us this is a Howrah bridge in Kolkata that also has this hierarchy to some extent.

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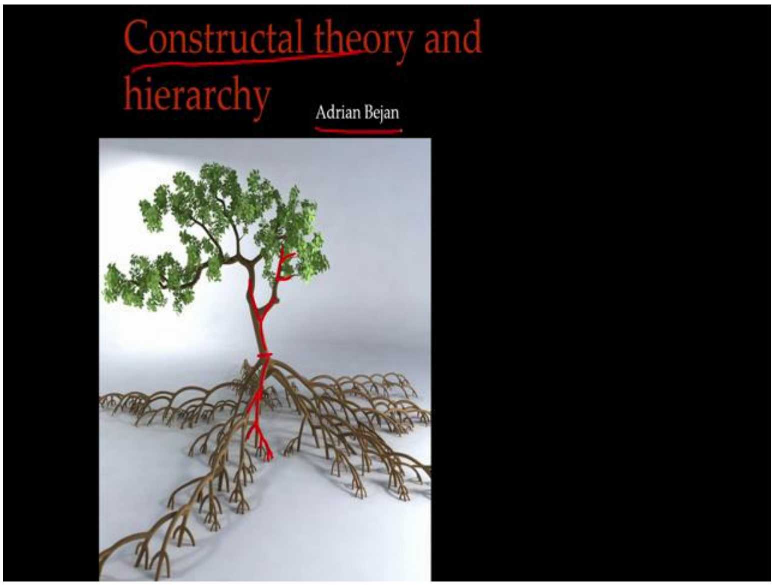

This is also there in nature if we look at a tree and it is root system there is a hierarchy. There is a big trunk that divides and sub divides and so forth. That is what we call a tree structure and there is a root structure also dividing and sub dividing into smaller and smaller pieces right. There is a hierarchy here; there is a big one and then going to the small level. Both above and below for trees and this is something that is called constructal theory by professor Adrian Bejan is called the constructal theory by professor Adrian Bejan at Duke university. There are lot of theories about how we can get optimal structures.

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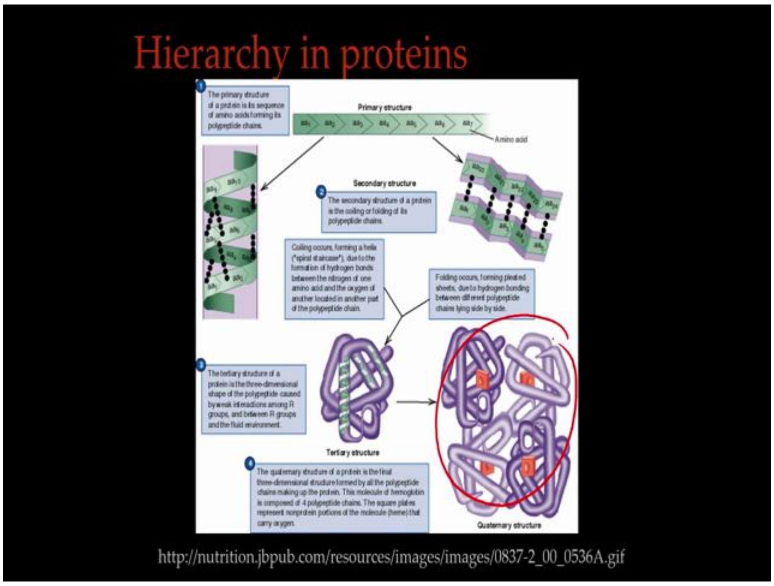

It is there in proteins, proteins are the molecular level building blocks for all leaving organisms and these are also hierarchical; they are just straight line like a chain and they have secondary structures, tertiary structures and then quaternary structures like it is shown here they are fold into that there is hierarchy in proteins as well.

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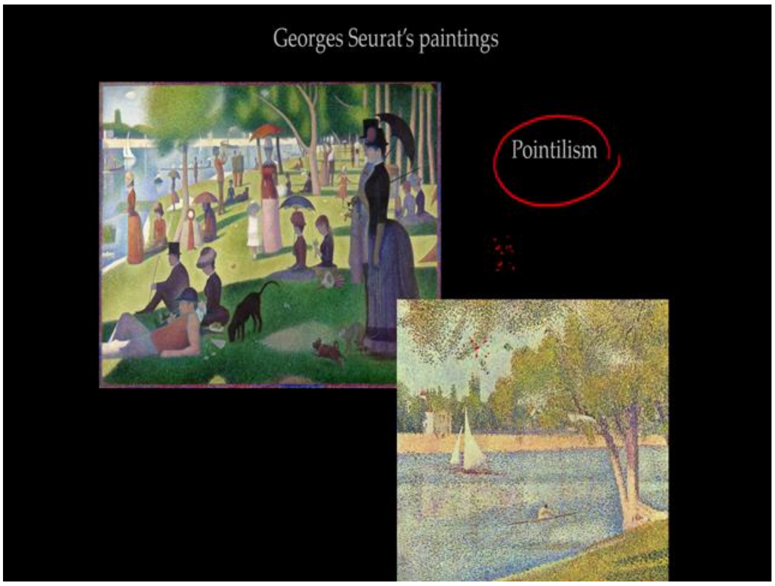

It is a very rich concept will extend that people have compared these two some other famous paintings done by a painter name Georges Seurat, where we had this concept called Pointillism and we will see why I am bringing out the painter into a structural designed lecture, if you see this paintings all they have at some dots. Dots have different colours as you see they are just dots without dots there is a painting emerging just with a few dots that concept can be actually extended. In fact, its call Pointillism you put this and there is some description you can pause and read.

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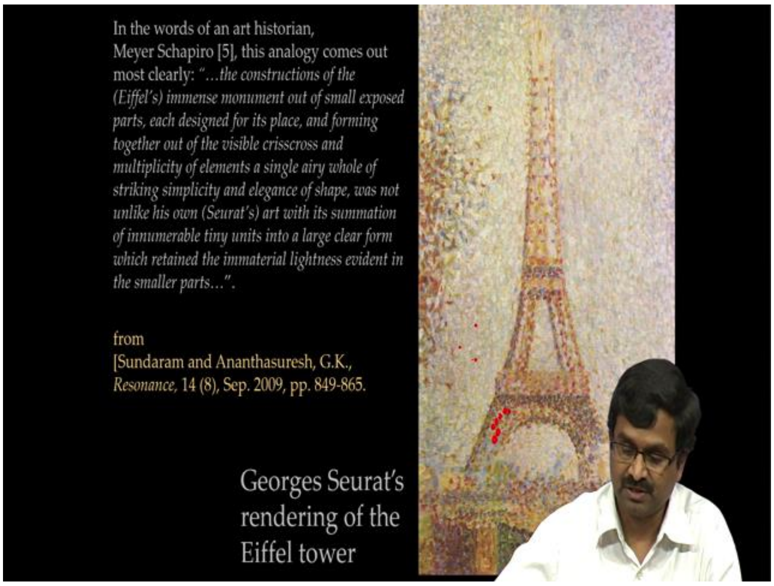

Where in fact, he had painted Eiffel tower itself because he found some who found that in Eiffel's design structural designs, there was this notion of this little dots being there that make at hierarchy, little dots make a bigger one bigger one and is much bigger structure over all it becomes structure.

This is a way of looking at a topology of a structure to say that at every point do you want to put the material or not or what material you would put and so forth. Since exactly how today we design structures these very rich field in its self.

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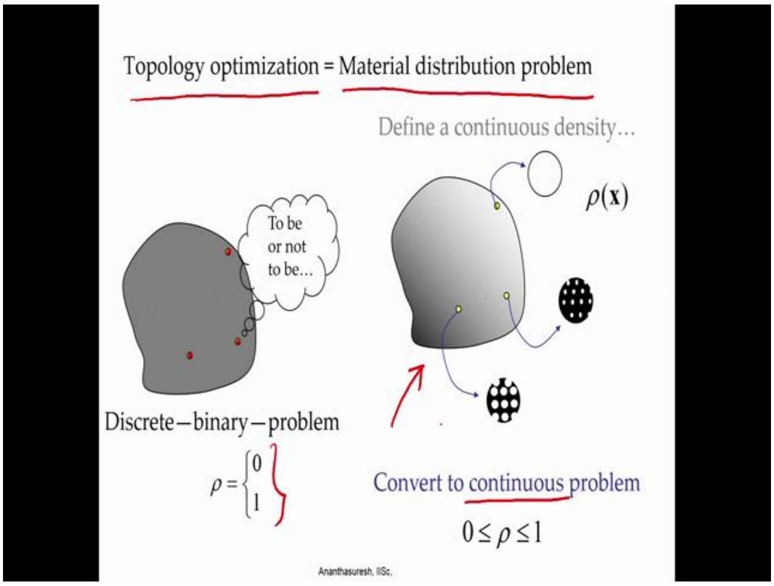

In a few lectures we want to be able to cover everything, there is another NPTEL lecture that one can take to talk about optimization and underline theory and so forth. Basically what it says is that Pointillism, you take different points and you say at these points should I put material there are not. Topology optimization structural design becomes something equivalent to distributing material in optimal manner. We have binary problem 0 indicating absence of indicating a point 1 the material been present.

We can convert at a continuous problem to get some grace scale image as it is shown over here and one can do design of compliant mechanisms as well as one can do structures these are view like I said at the beginning that we borrow from the structures. Structures have been designed for a long time and optimization structures is much older than the field of compliant mechanisms not necessary older than rigid body kinematics field, but old enough, mature enough there are lot of interesting techniques that one take because compliant mechanisms are really straddle between structures and rigid body linkages.

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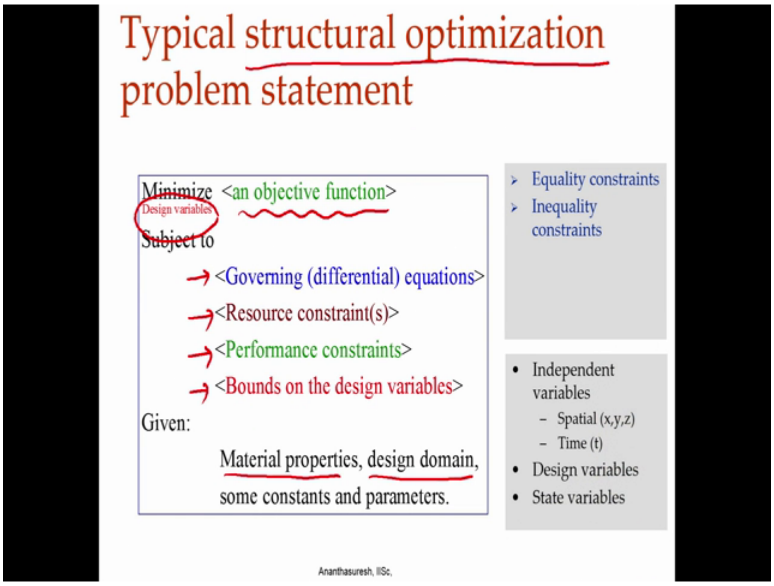

If we take this view, then we can pose an optimization problem when you take structural view point structural optimization naturally comes in where we can have an objective function, we had listed objective functions for a structure similar thing we can do for compliant mechanism as well as we well do later. Subject to some constraint governing constraints, resource constraints, such as the cost material available and so forth, and performance constraints something that you want a lot of displacement or stress being limited by certain value and also bounds and design variables given material properties the design domain in which the design has to fit and some parameters and constants.

We can do this by defining design variables that define the structural form the structural form a compasses the topology the shape and the size. One can now use this frame work of structural optimization for designing compliant mechanisms as well.

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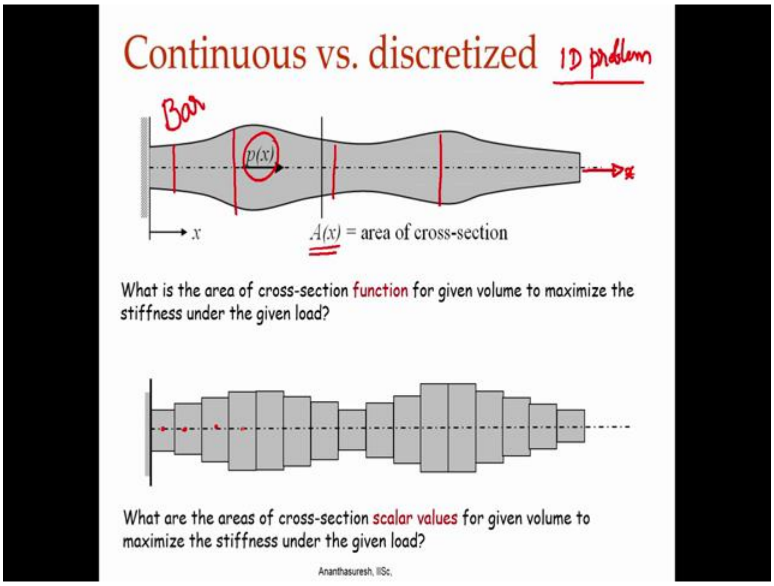

Here there is this view of continuous versus discretized, if I take an axillary reformable bar 1 dimensional. So, this is 1 dimensional problem which is simple if you want to design a bar which is this we want to design this bar for given load p of x; where there is a design we want to find the area of profile that is changing along the x. This is the x direction it a changing from point to point it is little here, large here and then becomes small and then becomes large, some particular fashion.

We can think of this as a rigid one if I discretized you know with different elements here we can also discretized in (Refer Time: 15:07) optimization problem.

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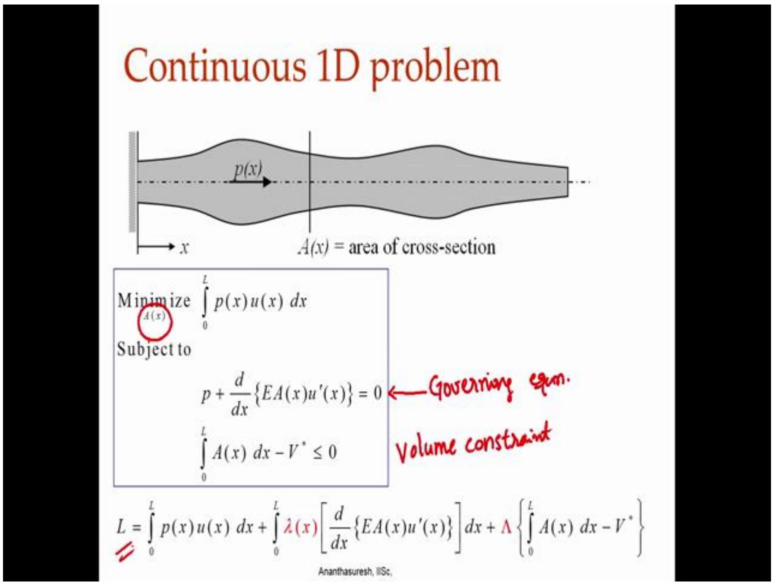

For this purpose we need the frame work of calculus of variation, where the unknown is the continuous function A of x and we can pose a problem subject to some constraints this is in this case the governing equation that gives the static equilibrium equation here and then there is volume constraint this is our a resource constraint. If you are given limited amount of material how do you use that optimally this is a few point that is very germane to structural optimization and we can do that we can use lagrangian approach those of you do not know the apportion theory, can pick up on that will be talking little bit about it later on.

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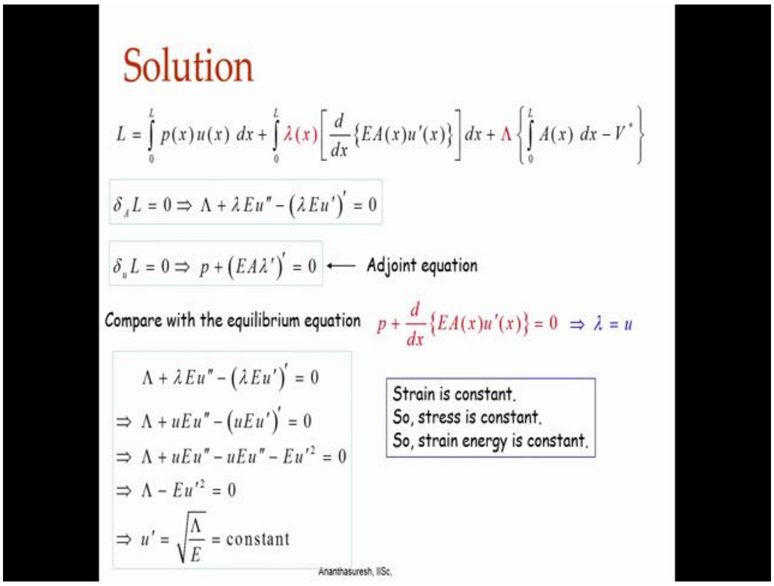

We can do this and solve the problem an electrically and where we can interpret optimal designs as some of the equation show there, show here. We can get a design in this particular case a thing that looks like this area of cross section is optimal one.

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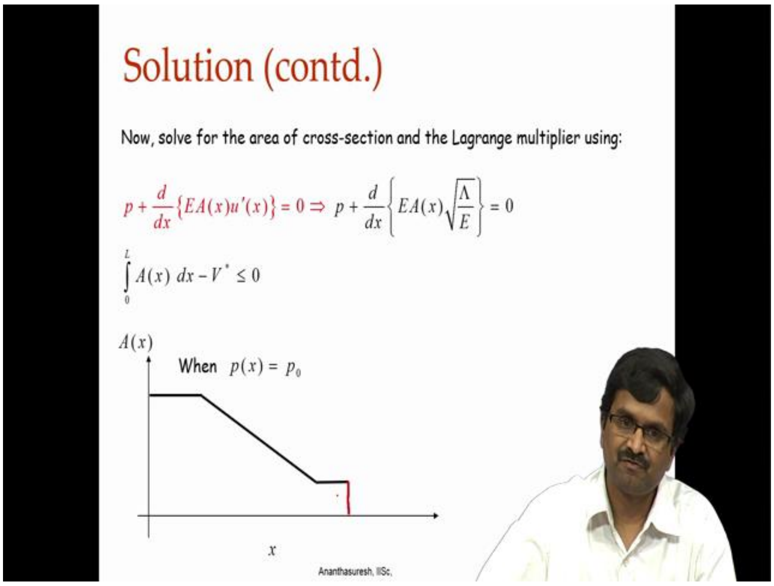

We can pose this problem and solve whatever has been done for structures one can also do for compliant mechanisms.

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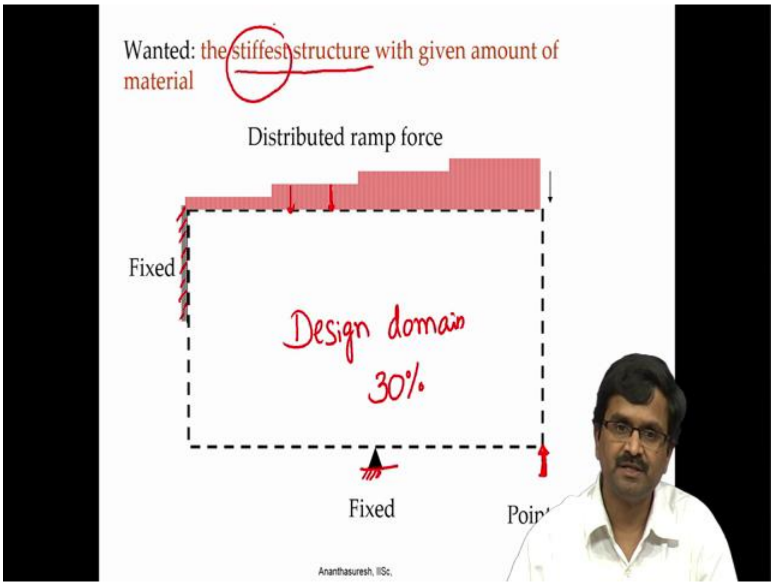

Now, let us take an example to see that if I want to get the stiffest structure for a given amount of material that can effect. Let us say we have this design domain this is whatever is enclosed by this dash line is a design domain there is a distributed load here all that is shown is distributed load. There is a fixed support over here and there is fixed support over here and we also have a point force over here right.

Now if I give you let us say 30 percent of material to be put in here, where would you put to get the stiffest possible structure or 30 percent of material. Today topology optimization technique can give a solution for that without realign on the intuition of a human designer. That is also another thing in the case of kinematic base design of compliant mechanisms, we have to first choose a type of linkage whether I want to use 4 bar, 6 bar, 8 bar or 3 bar whatever. We need to first choose it and then make a modal using pseudo rigid body model and then design right.

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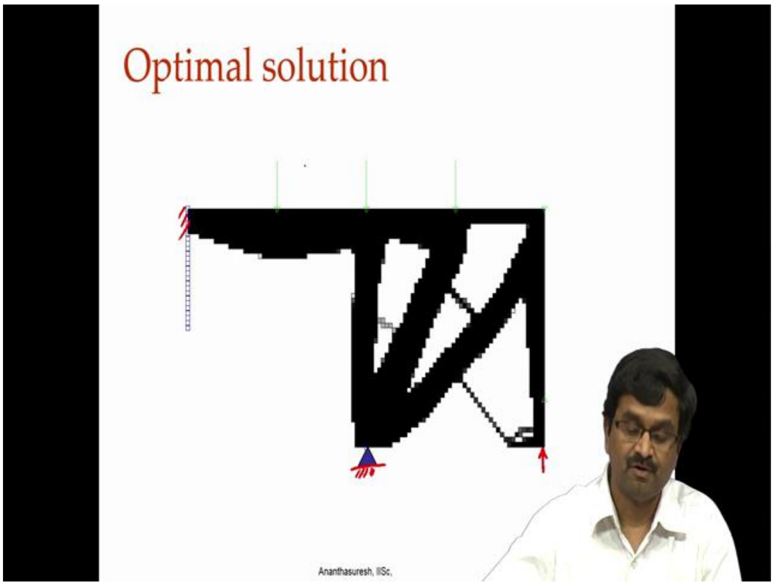

But here do not need to assume anything; we will get the design automatically from the algorithm it looks like this. It says that you have fixed here and there is fixed here right with the load that is over here and the distributed load over there with structure right away.

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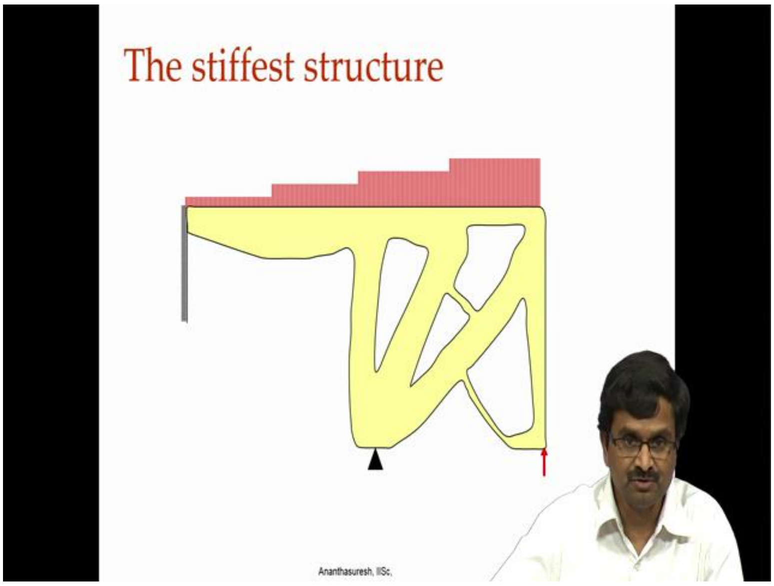

From which we can do some processing to get a structure that look like this, we had done you got a solution for us right, similarly can we apply this technique to compliant mechanisms is what we want to talk about.

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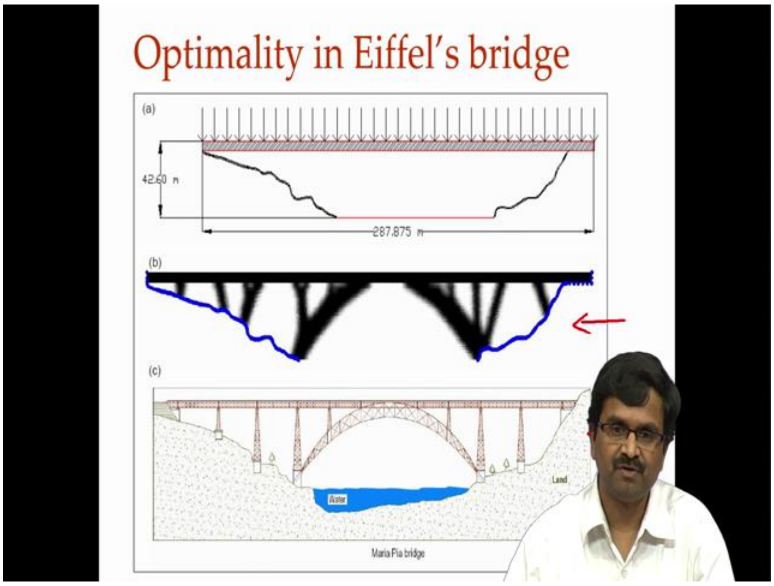

Similarly, if we look at the bridge one of the things that Eiffel's have built. If we take this same (Refer Time: 18:30) and use about measure you get a solution like this which is not too far from what Eiffel had done. That is what we find today, if you pose the problem properly we get the best possible design; that is the optimum design. It is a very attractive propagation to use this technique that is developed for structural optimization and apply it for compliant mechanism.

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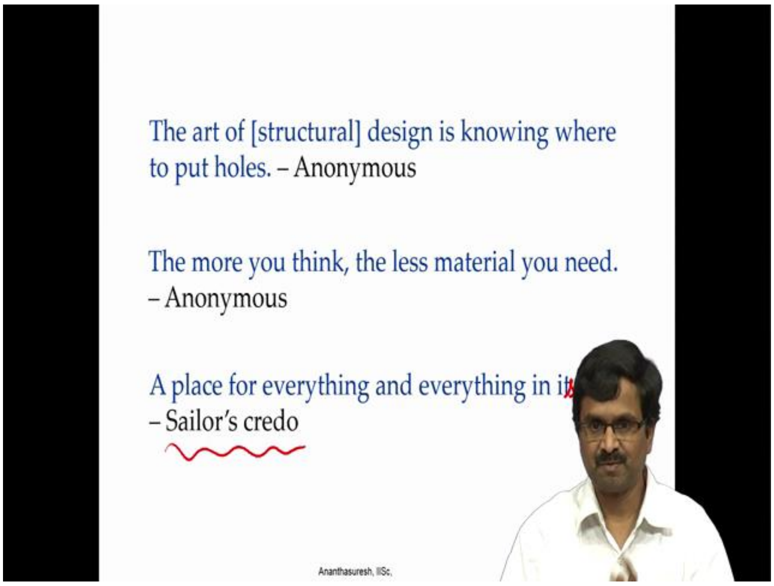

Before we start that a few things will be of interest to note these are the interesting course about the structural design. Now we are looking at compliant also as structures only thing is that structures that deformed to do something useful; let us say is an anonymous the art of design or structural design is knowing where to put holes. We want to use as little bit as possible meaning we need to make a lot of force the art of design is knowing where to put holes and another way of saying is that the more you think the less material you need and that is actually true. You do not want to put too much material in fact, it will you know the encounter product if you do that. There is also something called a Sailor's credo which says that, a place for everything and everything in it is place. That is if you choose some material it should be placed exactly where it is the best thing for the objective that try to achieve, that is exactly what the structure of technique do.

If you have some material they placed exactly where you want much like this Pointillism in the painting, you want put this little dots of material in the right way so, that overall image emerges. Let us keep this in mind as we think about applying structural optimization method for the design of compliant mechanisms.

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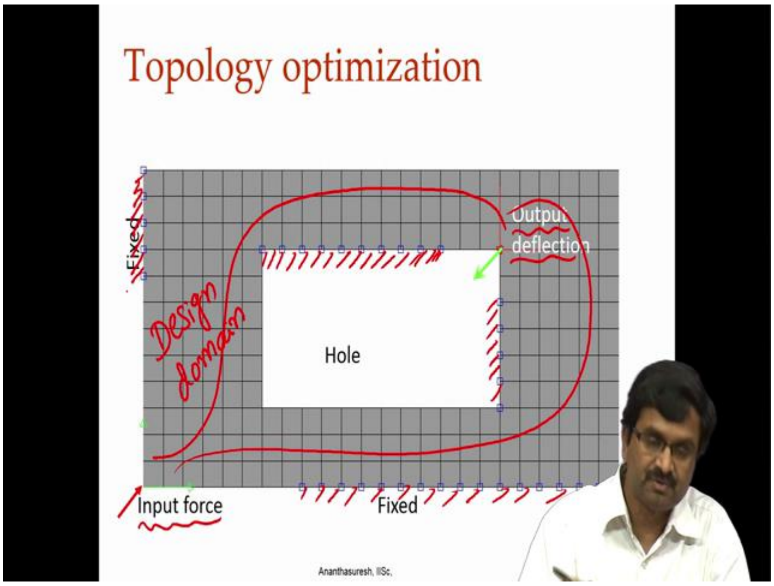

Let us take an example first before we discuss anything, that I have a domain like this where whatever is a grey portion this is our design domain. Here is where we can put our material right and it is fixed somewhere here, that is where you are allow to fix and somewhere here also you are allow to fix and over here and over here. There is a whole in the domain to being with let us say there is some obstruction we can put your mechanism there. What you are asking is when you apply the input force in that direction over here we want the output deflection in that direction.

Let us take that as the compliant mechanism problem we want to get a mechanism a compliant mechanism without any joints such that when you apply force here that should move in that direction it will basically some more circumvent this whole around and connect to this and that is what we will want. You can sit down and thing and come up with your mechanism solution intuitively, you may be able to right. You do that, but you are only allowed to fix where it is shown right you may get that.

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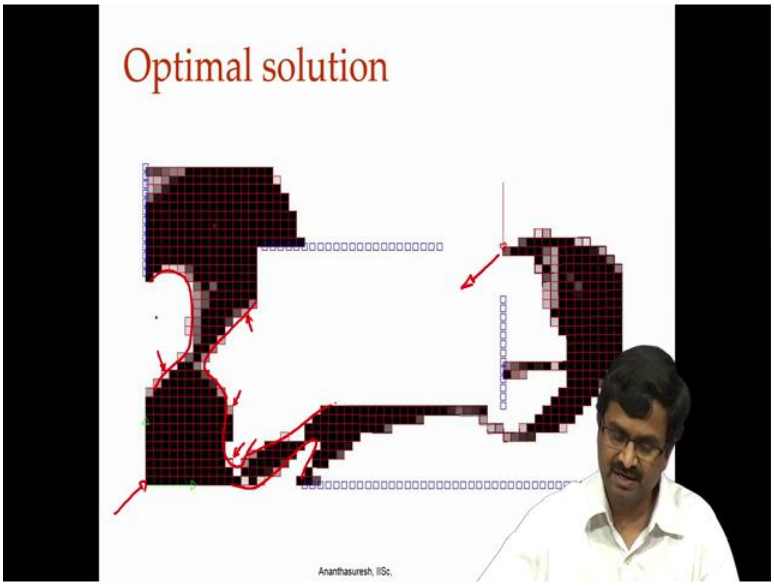

But now with structural optimization you get it in a straight forward manner, you just give that some amount of volume of material it will automatically distribute the material the greyness here some of this things are gray, that is because we have this row between 0 and 1; 0 means that a material is not there, 1 means that material is there in between they are few the boarder, boarder line cases.

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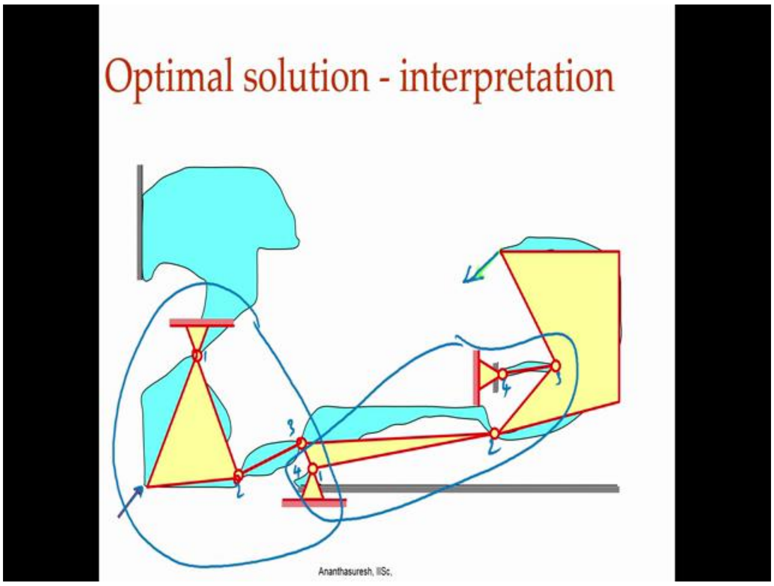

Now, in this case if apply a force here this actually will come down like this and then the hole is interact right. We will get a solution like that that is the power of this structure optimization or apology optimization. Indeed from there you have to join everything from that structure one more time from there if you joint that smoothening of for the things here if you do that every where, if I smoothen we get something that looks like this. This structure if we call it you apply a force here it will defiantly come in that direction, because that is how it is posed and solved. Now if you pass and look at it for little while we will see something interesting which is that wherever there are thin portions?

This are the thin portions here wherever those are there if you put joints rest of it has rigid one; we actually see two four bar linkages you know this is one four bar linkage, let me change the colour. This portion is 1 four bar linkage then there are four joints 1, 2, 3 and 4 rigid ones and then from here if you see now from this one here another four bar linkage again this is 1 now 2, 3 and 4. The way turns then this force is applied in this direction here the output displacement goes that could we have imagine this possibly some of you may be able to do it by just looking at the specification given, but now we can get these things what we call types synthesis rigid body leakages without using an exciting solution we can get a new solution, that looks like that to begin with and from that interpret and then we can interpret it further with a linkage solution.

Here is an interesting way to do design linkages that is notion of optimality, but this is also this notion of very little reliance on the existing knowledge or previous prior knowledge and intuition of the designers. Designers always good, but it aids the designers when they cannot come up with the conceptual solutions conceptual design.

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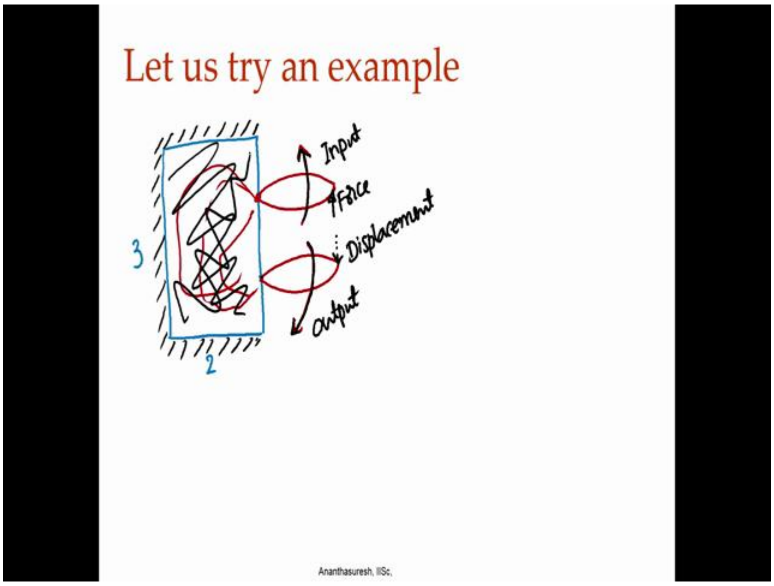

Let us try another example, let us say that we have a designed domain, let us say this is 2 units, this is 3 units and we say that there are couple of rigid, in fact this is from in insect problem some of the insects people how found that when you move let us say dead insects when you rotate about this point, if you were to rotate it like that other wing may go this way or in the same direction it depends right. They are somehow mechanically coupled inside, now how the coupling is we do not know some where they are coupled one wing rotates like this other wing should rotate like this.

Let us say we take this problem we also stipulate that, this thing is you can fix it anywhere on this 3 boundaries, now imagine a compliant mechanism that does not have joints so, that when I rotate this one about this point; this is we can call it input; the output should go like this. If we are watching this it will be nice if you pass here and sketch your own design at a compliant mechanism whether no more joints it should appear that it is rotating about this rather we can apply a force there. Then apply a force there this point should displace here if this is the force upwards this other one should displace the downward direction and this is your design domain. Whatever we have here is our design domain where you can fit your design. Can you come up with that right?

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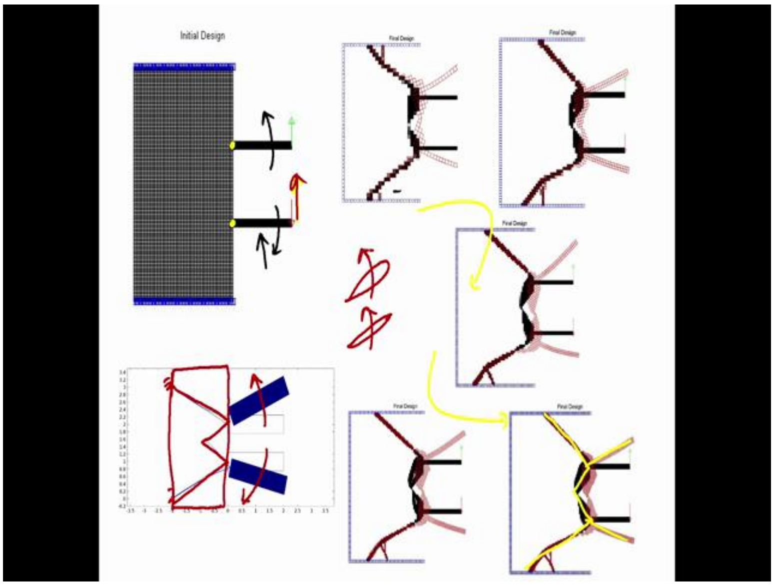

Let us look at that, this is the problem now if you have not passed again, now is a time to pass and sketch your own design, when you apply the force in that direction and we want to get the displacement in the downward direction. So, that this thing appear to be kind of rotating like that even this rotate like this both have to pervert about those points. The points are these points; let me sketch again for those points.

If you give it to optimization it actually gives the solution like this as we refined we get more and more clearer designs which some of you may have gotten with your own intuition. In fact, I had given it to one of my students you have to (Refer Time: 27:42) this course saffron he came up with this designs. This is where you are fixing these your box to begin with, these are box that he had taken when this is rotated like this, and this will rotate like this a simple design that looks like this.

What is optimization given interestingly optimization also gave a design, that is very much like what a human designer has come up with you know this is various refinement that happens with finer and finer mash that you define design buts exactly what we get right. It is possible to intuitively design, but in algorithm that can work with anything suddenly, if I say instead of going up that it should also go up rather than going down then how do you do it again we have to sit down and design. If these goes up these also has to go up rather than going down, that is when this wing rotates like this the bottom wing also should rotate like this if you ask again you have to sit down and design.

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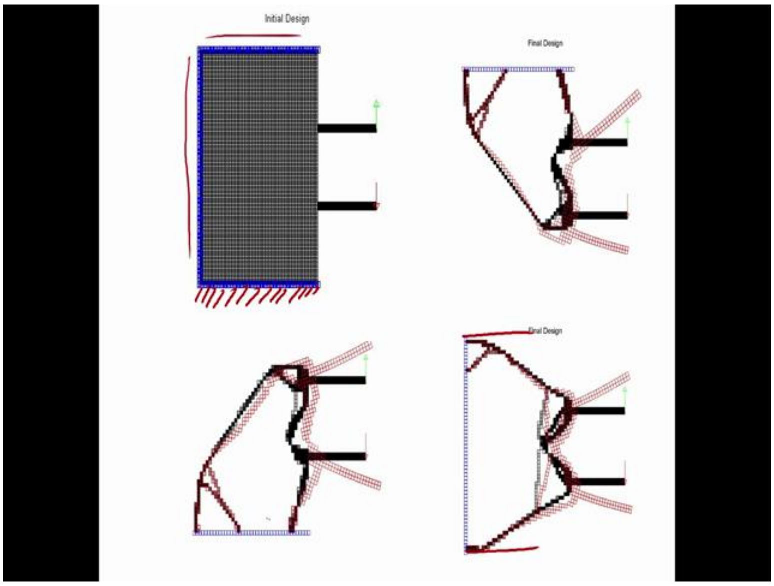

Let us take another example the same problem, but now I say that only here you can fix. Let us say there is restriction in the mechanism you can only fix it there and not over here or over here. Then what design? When you allow to fix everywhere we got a designs even though it did not fix here and here we get a design right, but if you are allow to fix only here what kind of a design do we get and it comes out to be if it is only top that will be that bottom this is here right. It is possible that anything we ask we get a solution which is structural optimization based approach.

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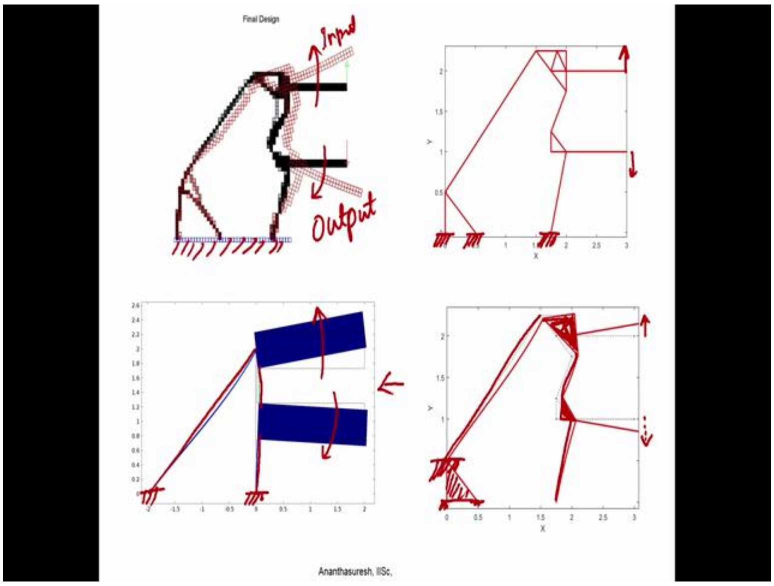

Let us see this particular one is a design, when we fix at the bottom it cannot this when I am rotating like that this is rotating like this. This is the input and this is the output right. Now we sketch in our beam code that will while ago where we had use them right; it is fixed at this points now when I apply force here this point will go down as I showed here when I move that up this comes down. What did a human designer come up with? Something looks like this, it is fixed here because only bottom at he was allowed to be fixed and when we do this is coming right.

There are multiple solutions after algorithm gave something like this again this is more like you know if I say fixed here-here this whole thing that is tangle that is fixed essentially, it is as if that is fixed over here there is a bar and that is what here also there is and then there is algorithm came up with this other one right; which is straight here, that is linked line same thing in between there is a beam here also it became up with a beam and except that it made it a little rigid here and this whole thing is little rigid.

It as well as would have gotten right there. In some sense their intuition an amenable designs. That is what are algorithm gives is something that human designer can do, but much more quickly and we can do a lot more as we will see.

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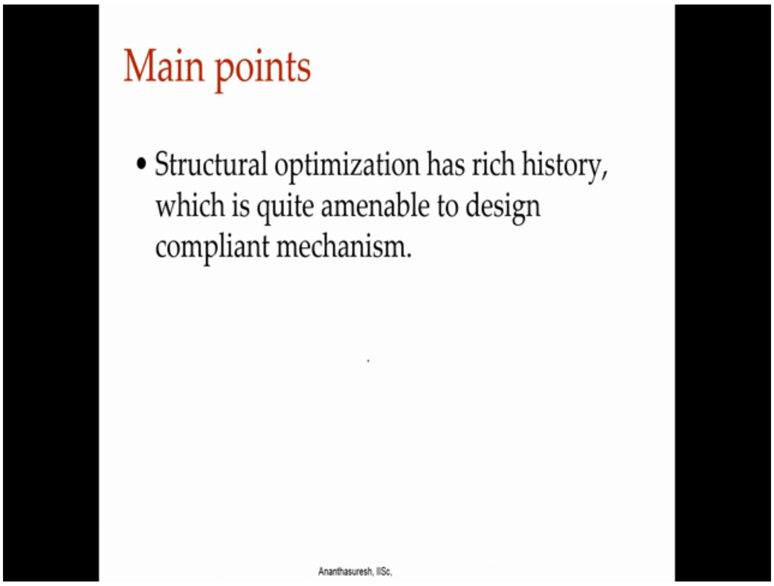

What we have discussed today is structural optimization which has a rich and matured history for designing stiff structures is also amenable to design compliant mechanisms. We can design a compliant mechanism as easily as we design a stiff structure and there are lot of benefits to it which we saw today as something that is very easy to design. You just say these my objective function may constrain and there you get a solution for various amounts of material that one can give to the algorithm. These are interesting approach that is what will discuss in the remaining lectures of this week as well as next week.

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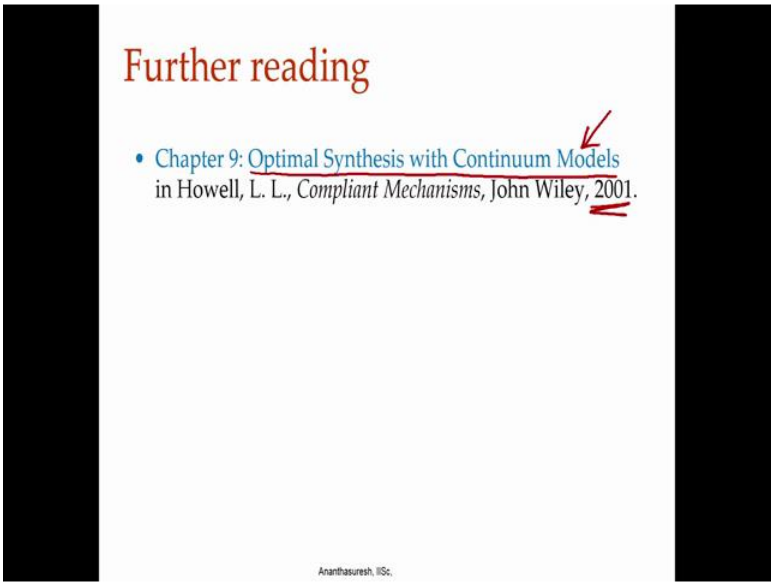

Further reading for it there is a chapter in this book of Larry Howell, where there is one chapter in chapter 9, optimal synthesis with Continuum Models which will be touching upon this was 2001 when this chapter was written by myself and Professor Mary Fraker, but now a lot more has happened some of which we will study in the next few lectures.

Thank you.