Compliant Mechanisms: Principles and Design
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Lecture – 21
Solving equations of PRB modeling and comparing with finite element analysis

Student: let us start sir.

Hello, in the last lecture we had looked at an example of constructing a pseudo rigid body model and simulating that for a compliant mechanism and comparing that with finite element analysis we also into the detail procedure in the last before lecture. Today let us look at two more examples to see how we can use PRB analysis for simulation of compliant mechanisms and comparing with finite element wherever it is possible.

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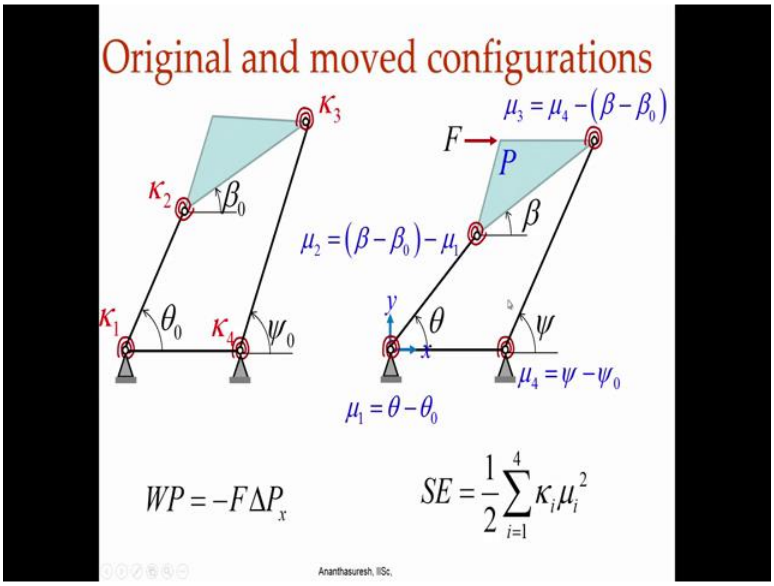

So, let us look at the two examples before we go to the examples let us just recall that finally, any compliant mechanism that we have where one or more of the members are elastic we can come up with a PRB model where we have the 4 bar linkage where there are these springs torsion springs at each joint and we denote their spring constants kappa 1 kappa 2 kappa 3 kappa 4, we have discussed how to arrive at them based on the nature of the member that connects the two hinges there and we have a point where you can apply the force or we can apply the torques both are possible and then we look at the rotation at each of the hinges which we are denoting by mu 1 mu 2 mu 3 and mu 4 based on that we can write the strain energy and then we can also write the work potential and get the potential energy for this system which we minimize for the condition part that is d PE by d theta equal to 0 theta is your input variable here.

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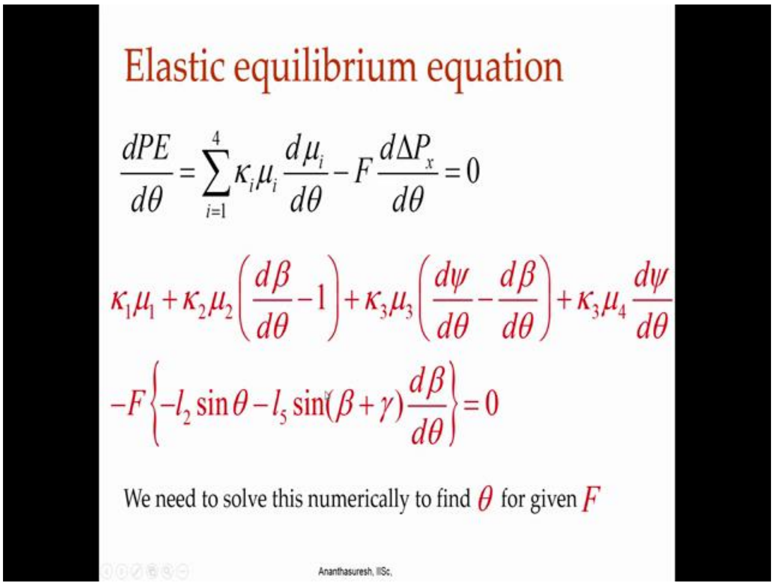

The other two angles beta and psi are automatically decided because, it is a closed loop linkage single degree of freedom if you differentiate equate to 0 you get what we call elastic equilibrium equation or force displacement F is here, this is the force and displacement here is the theta that we want to know once you know theta you can get the kinematic coefficients here d psi by d theta d beta by d theta and beta and this is like a constant here you can get a force distance relationship and we can find for a given theta you can find F or for given F you can find theta. What we discussed in last lecture was that finding F for given theta is straight forward you have an expression F is unknown the rest of the terms are all known because we are assuming theta. So, you can compute F easily, but if you are given F and you want to find theta you have to use immediately iterative analysis or incremental iterative as we discussed in the last lecture, that is numerical solution.

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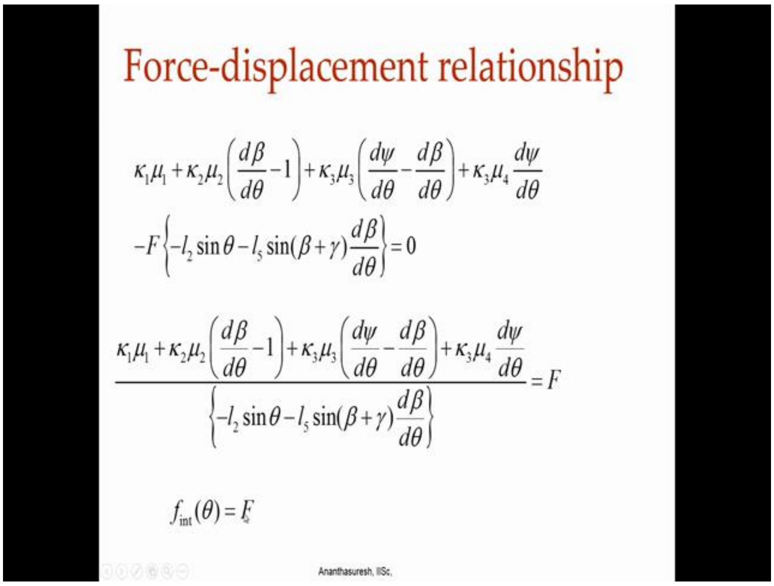

So, you can look at the force displacement relationship or equilibrium equation as something that equates internal force to external force these what we are linearized when we do this incremental iterative numerical method to solve for given F what is theta.

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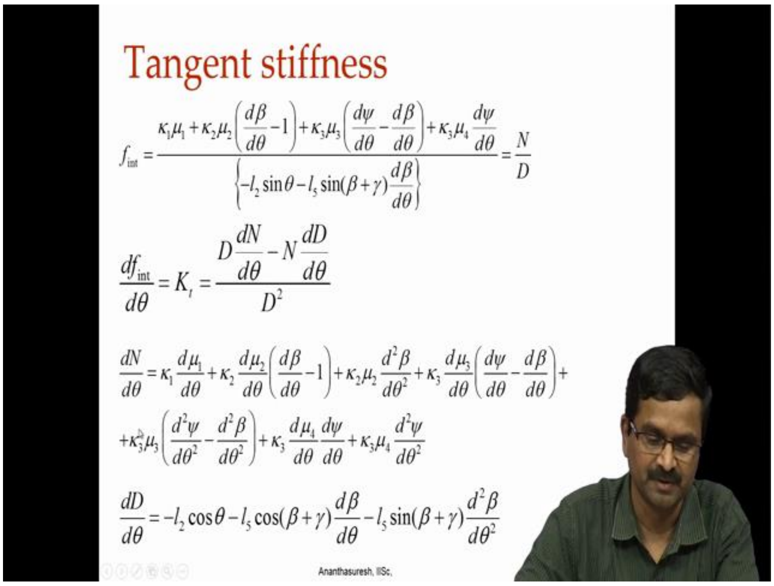

So, for that we use this what is called tangent stiffness, tangent here means that for force displacement relationship you draw the curve the slope of that the tangent if you draw that is what is needed for linearization around any point that you take on that curve. So, that f internal derivative with respect to theta here displacement is angular rotation. So, you have the angular displacement you are taking derivative you get this K t, which is tangent stiffness matrix which you can find any given position.

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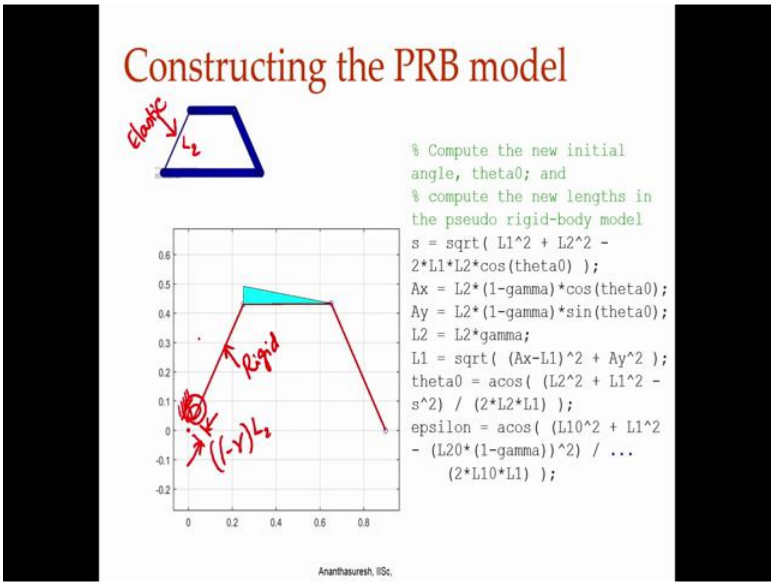

If you know theta everything that is there in the numerator denominator and their derivatives all are computable. So, you can actually get this tangent stiffness matrix now with which we do that iterative method and get the thing, this was the example one we consider in the last lecture where there is a elastic element which is the elastic element here and the other three are rigid here. So, this is the elastic element let us go back.

Let me get the pen, this is the elastic element other they are rigid elements. So, here what we have done again to recall what we did last time the originally here is where the origin is, but we have moved it by what we call 1 minus gamma times the original length if this is a L 2 we multiplied by L 2 that gives us how much you have to move and fix it over there that is the pseudo rigid body model concept now this also becomes rigid element and we put a torsion spring there. Here it was elastic, but now we did rigid that is by pseudo rigid body model and all the information is there in supplementary file is there for you to run this program.

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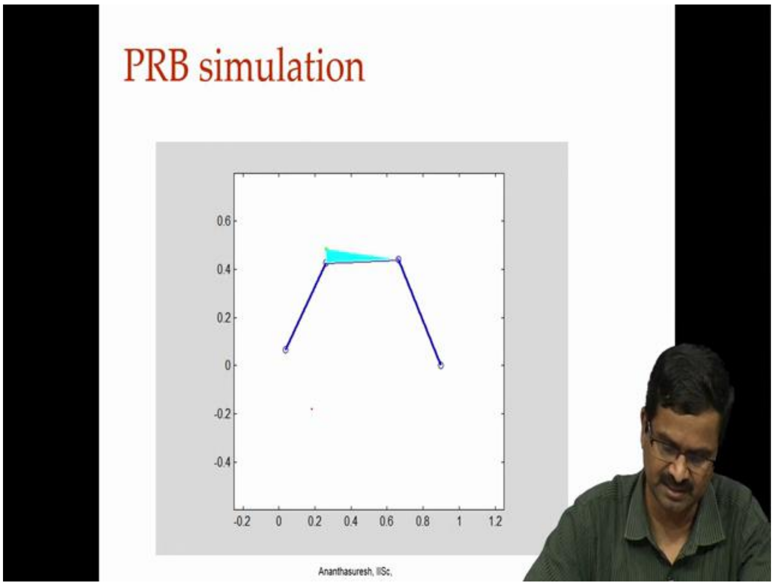

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Let us just look at that simulation last time we did see that, now let me also show you the abaqus simulation which we did see in the last lecture. So, let us run this simulation. It shows here how the pseudo rigid body model is working, now let us also look at for comparison how finite element simulation works for this elastic segment there is a pin joint here and there is a pin joint here, there is a pin joint here it is fixed as you will see when we run. So, it is deforming others are simply rotating and you can actually see that a point here does not really move much and that is where we have put our pin joint.

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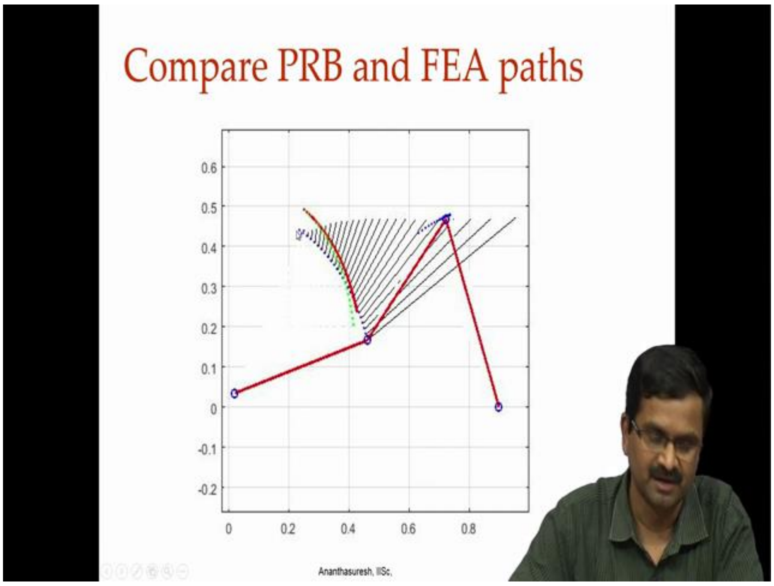

Let us say where my cursor is that point is essentially not moving that is the pseudo rigid body model rest of it moves like a rigid body with a torsion spring, that is the FEA simulation we had compared and as you can see the curves here the green curve is what PRB gave and the red one here this red one is given by FEA they are not coinciding the reason for that is that the force which is show me the black lines here is not necessarily satisfying our n value that we have taken for this n average, we took some value and where the gamma was 0.92, but yet if you see the force in different positions it is more along the axial direction than the transverse. So, any changing proposed into position that is what you leads this in accuracy as it starts rotating from where it was that is from here to here to where it is then there is this difference.

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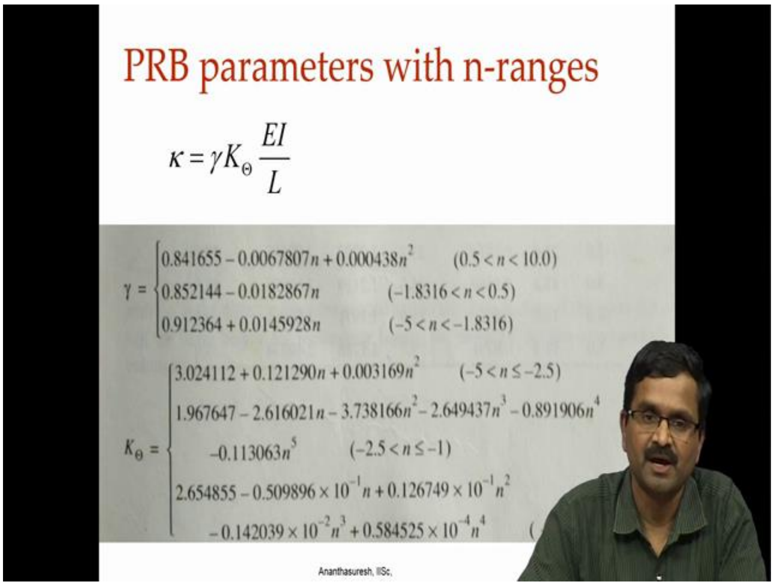

And we have to pay attention to these ranges that are given and choose them sometimes your mechanism may go out of this range this minus 1 to 10 minus 5 to minus 2.5 and so forth different ranges are given for both for gamma also for K theta which decide the torsion spring constant even outside the range you have to take the best guess there and it will work accurately for some range of motion after that you go as we said last time it is still good even if it deviates, because compared to finite element model here we are dealing with only 1 degree of freedom model having this accuracy is still useful in many practical situations when you want to analyze the compliant mechanism quickly.

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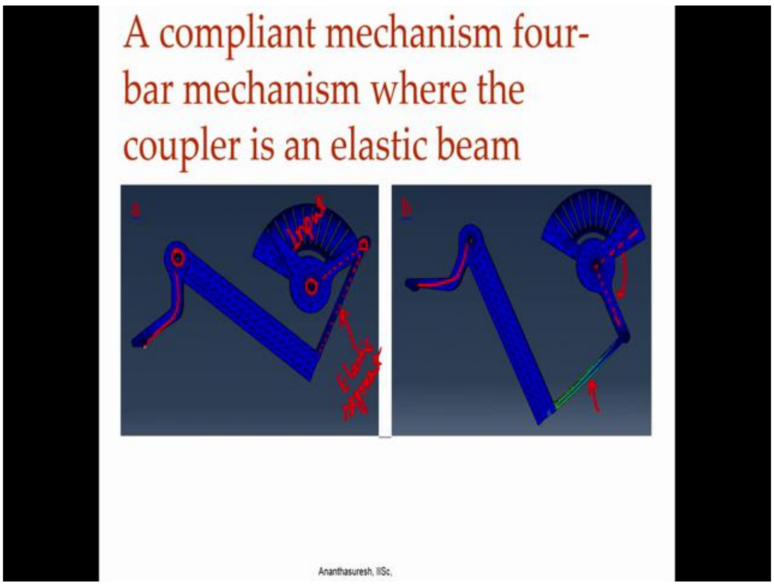

Let us look at another example today in this example we take something which is more practical it looks like a practical device where there is a hinge here. So, let me point those out. So, there is a hinge there and there is a hinge here and there is a hinge there and here is where we have in elastic segment there is a beam, all right.

Now if you see the first example we had that as the input link that is this if I call this the input that was the elastic segment, now the coupler or the middle one is the elastic segment it is fixed over here there is a pin joint is. So, now, we are taken an example where the middle one is elastic segment rather than the pivoted to the ground body. Now this figure shows how when the input that was here as been rotated what I have done there is rotated here, this is the input way this was let say over there this is being turned. So, we have applied either a torque or rotational displacement to this. So, this is the input body here and that as you can see that this elastic one. So, there is some stress shown there rest of it this also is rigid this is just an extra appendage I do not think that matters so much here that is just for the output something is added. So, this is the original configuration there is a deformed configuration. In fact, it is a special characteristic that we will discuss.

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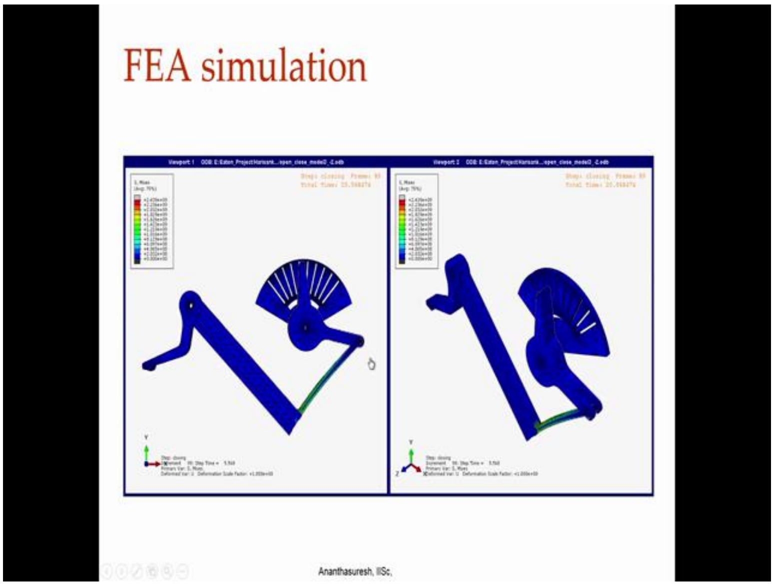

So, let us look at how it deforms, now we can see that this is moving and goes and settles two views of the mechanism and it is now coming back we are applying the torque in the opposite direction to bring it back by the torque or the displacement, let us play this again. So, it slowly would be turning it is turning and it kind of accelerates in the middle and then goes and settle somewhere else. In fact, it is a bi-stable device. Now you undo it, it is going to come back let us see how well the pseudo rigid body models will work for you this is the finite simulation done in abaqus.

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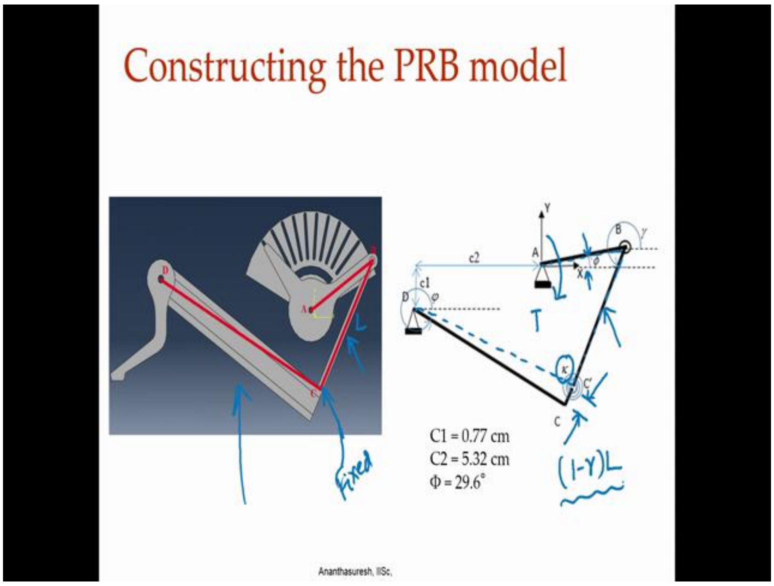

Now, if I want to construct a PRB model that is shown here. So, we have the elastic element here and that is now converted again this is the fixed joint. So, we have moved it this distance again will be that 1 minus gamma that characteristic length factor that defines a position of the pivot with the torsion spring times the length here which will be the length of this original beam segment. So, this torsion spring also the kappa of which we know how to get. And in this case, some of the numbers the c 1 c 2 and this initial angle that fee are some numbers if you want to try out and we have the information with other ones you can try.

So, this particular practical looking device you can construct your body model and here effectively the new output length our output body length becomes that this one remains the same and this reduces a little bit because, you have moved from here to here and we will apply either a torque there or a displacement of the fee itself is given some value and then see how it would deform, we get a 4 bar linkage where one of the joints has a torsion spring, again remember that now the coupler link is elastic segment and that is what we have done from the fixed side that we should remember this is the fixed side from that side you have to move it a little bit which is given by this value.

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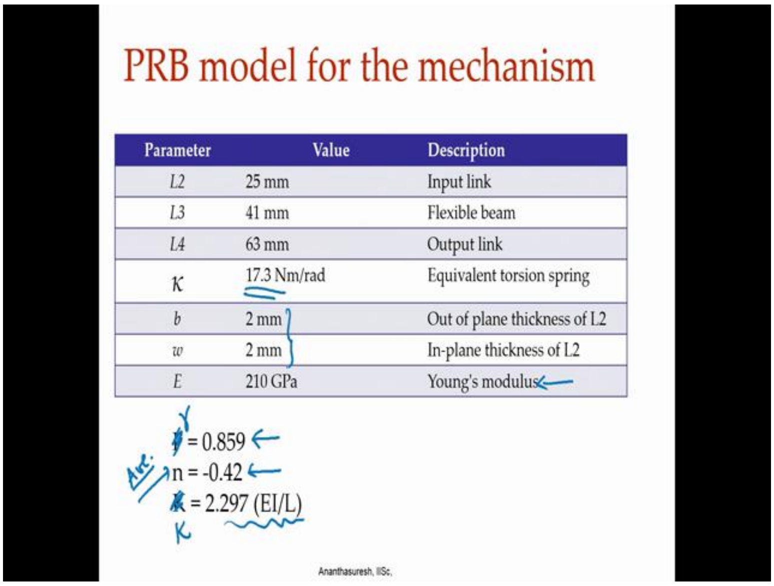

And that is done we get these parameters for L 2, L 3, L 4 is consistent with what we had used in the first example and a kappa and here this is not a capital gamma it should be small gamma it is taken as 0.859 based on the observed value I should say this n is kind of average value. As it mechanism is turning you have to see what force comes there that we do not know ahead of time that is one thing that we should note to begin with we do not know what n is, but you can do static analysis at different positions and then see what were the direction of the force that comes from the cantilever beam that you have and get that m value based and value you can estimate this gamma as well as this is actually kappa, that is calculated to be 17.3 Newton meter per radian here that is 2.297 times EI by L for that elastic segment. Other information is here including what was used as young’s modulus and cross section dimensions which are these two are there. So, with all of this we can make this simulation based on the pseudo rigid body model.

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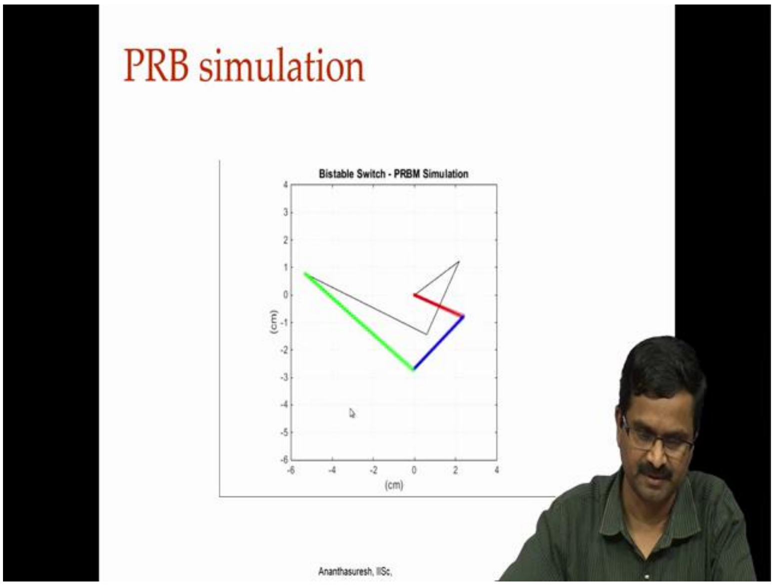

So, let us run this what it happens it is going and coming back, now the practical device the geometry is all hidden we only are dealing with four lines that is the 4 bar linkage the torsion spring being there we are trying to see this.

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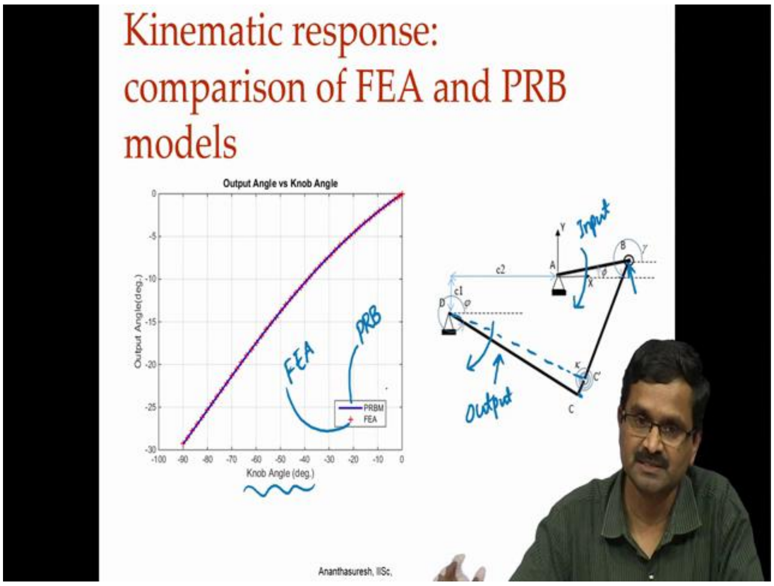

Now, if you look at the output angle versus knob angle. Knob angle is this one. So, this rotation is what is called here as the knob angle, that is the input as if there is input knob if you remember that simulation of the device which we saw and this is the output now here this rotation is output. So, those things if you see the blue colour is PRB, the blue here is pseudo rigid body model and the plus a red plus that is there is finite element analysis then in abaqus and they are agreeing quite well unlike in the first example.

If you ask why that is so? It has to do with the end that is if you look at the pseudo rigid body model of this if you look at this one and this is the elastic segment fixed here to here the force that comes at this point it is n value is pretty much constant. So, what we have taken n value in the previous slide which was minus 0.42 seems to be quite good and hence the pseudo body model and finite element analysis results are kinematically matching almost perfectly. So, we have to choose the right n value if n changes in position to position one thing is to change n and gamma and this, but that you are not going to do unless you do finite element analysis that is a little drawback here that you have to know what is the end that you should take, but better it is to take one and throughout rather than changing n from position to position.

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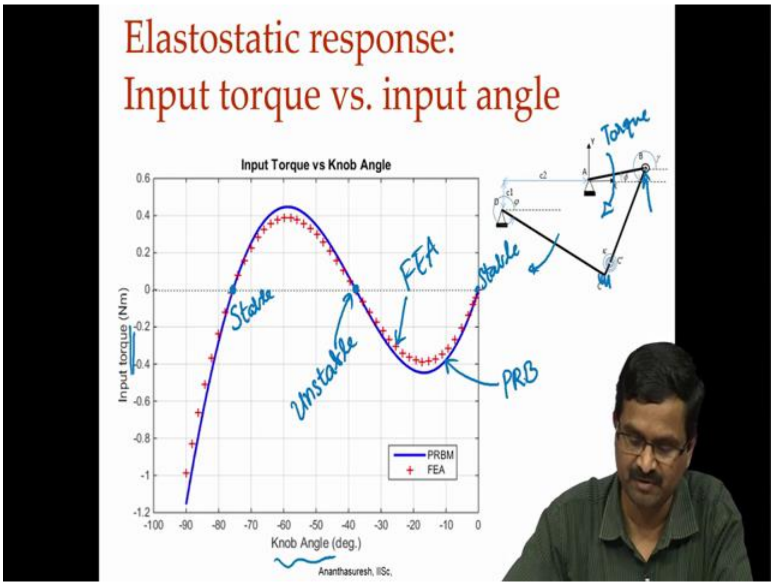

Even if it is inaccurate as we will see in the synthesis we will try to keep that n the same, that is the orientation of the force at the loaded tip of the cantilever, and if you did not notice how the force versus displacement. Here the force is actually the input torque, that is; the torque that we are applying right, that is the input here which is a torque versus the knob angle if you see we could also have plotted output angle, but here input angle is plotted and you see this as this by stable behaviour again the blue one is the pseudo rigid body model and the red one is abaqus finite element analysis right, again there even though it is highly non-linear we start out let us say 0 0 it goes up and comes down become 0 again and then become 0 again. So, there are three equilibrium positions where the torque is 0, that is you leave it there it will stay that is what it means it is a quasi static analysis.

So, we say that this is stable and this is also stable that is why it is a bi-stable system in between we have an unstable, what they say what goes up and comes down should have a middle point right. So, we have unstable equilibrium point. it is a bi-stable, even though it is a bi-stable mechanism we can see how well the PRB model is able to capture that behaviour it is only 1 degree of freedom which is this phi for given torque we can find this phi right. So, that pseudo rigid body model actually works well in this particular case. Again the reason for that is the assumptions made under to the body model modelling are true hold true here, because the force that comes or the loaded tip of the cantilever it is fixed here and the load comes there that load is not changing it is orientation the whole lot that is what we can conclude from the very good agreement that is they between FEA and PRB.

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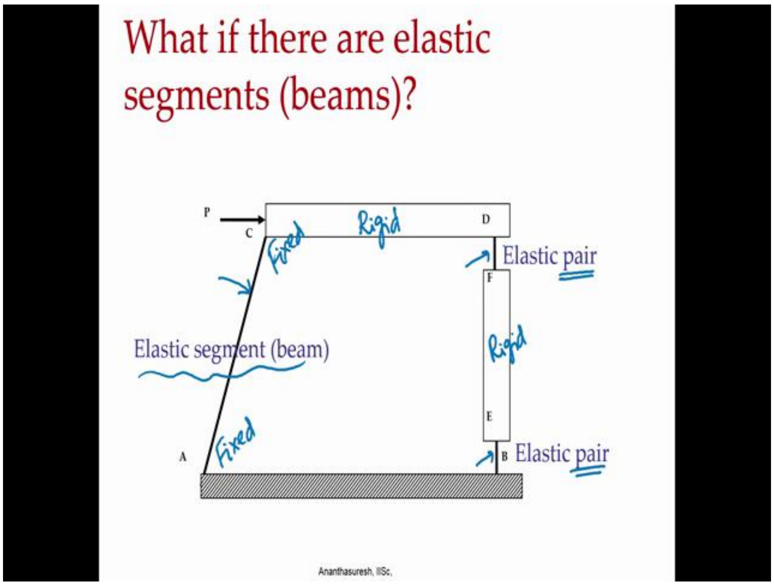

Let us look at one more example and here we have taken an example where we can call it a fully compliant mechanism; that means, that there are no joints then there are no joints at all it is actually a challenge for pseudo rigid body modelling, because now you have an elastic segment over here this first one is elastic segment and then there is a rigid segment and there is another rigid segment even then it is fully compliant because, now we have a fixed connection at point a as well as at point c and then here we have what we called elastic pairs or what are called small length flexural pivots. So, these are elastic segments, but short once, we do not do PRB modelling that we do for a long beam, there is a different modelling that we had already discussed and you have an elastic pair that is what we are going to do.

Now, this one can be put into finite element analysis without having to deal with hinges or anything although finite element analysis software today can deal with those as well, but here it is a plain old traditional finite analysis where everything is constrained like an elastic body no kinematic joints right. So, when you have this what we see is that let us recall that we have a beam with a force that is the n that I mentioned more than once today that is the ratio of the axial component to the transverse component.

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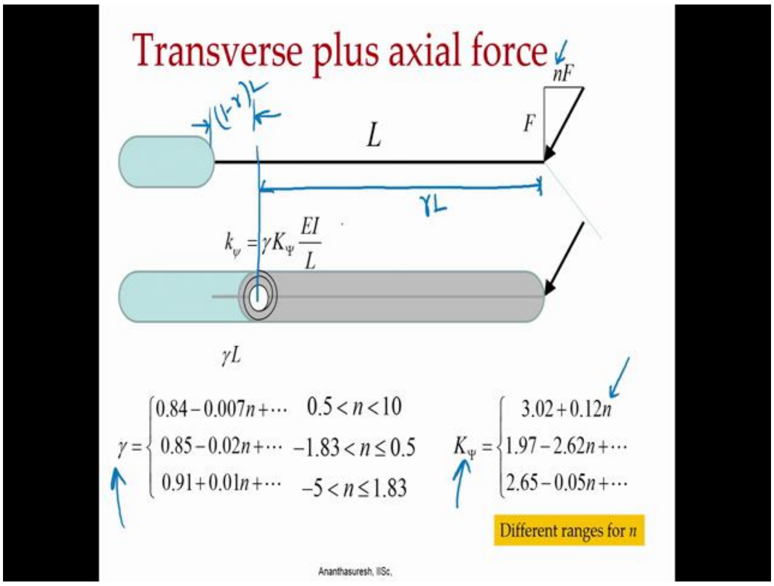

Based on the end you have to choose this gamma that is gamma is if it is so much you move that over here and that length is 1 minus gamma times L and the rest of it here from here to here it will be gamma times L right. You can choose appropriate gamma also k psi k theta whatever they all depend on the n, if n is changing from position to position then we have a problem as I have already highlighted in the first two examples for different ranges of n different values for gamma and this kappa. Once you know k psi and gamma you can see EI by L the properties you can get the torsion spring constant.

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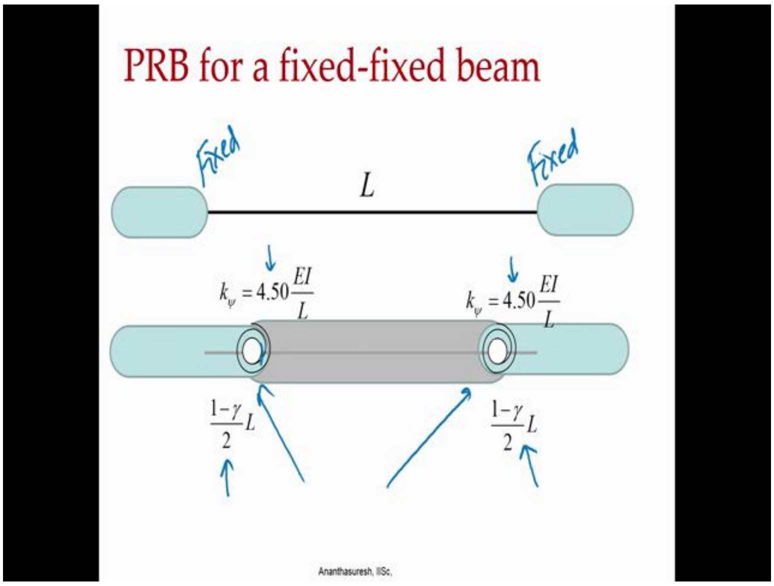

Now, if you have a fixed-fixed beam that is what we have in this example three, the elastic beam that we have is fixed on both sides. So, it is fixed here and it is also fixed here, in which case you have to put two torsion springs one over there one over there which we had already discussed once this is moving by 1 minus gamma by 2 times L and the same thing here and this becomes twice instead of 2.25 we have 4.5 torsion spring constant there torsion spring constant here, that is how we model that fixed-fixed beam that we have in the third example.

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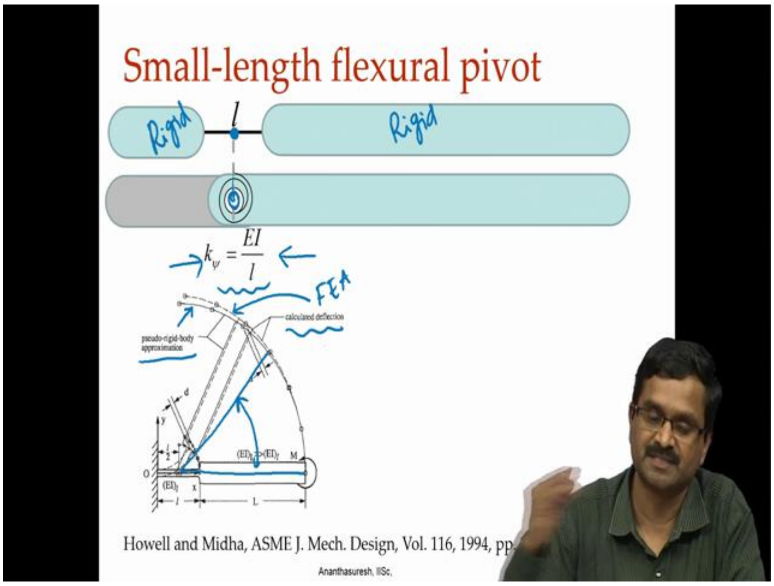

We also have this small-length lecture pivot. So, if I have a rigid body here that is what we have in this example and they are connected by this small flexural beam then it is gone elastic pair and it is torsion spring is constant is given by EI by l, where l is length of that segment and where do you put the hinge you put it the middle that is what we have shown you put the hinge there and torsion spring is given by that expression you can model that. As it is shown, you should play an attention to this figure which is taken from this paper of 1994.

If you were to do PRB approximation is the inner curve. So, this is the PRB approximation the outer curve is calculated deflection that is done either finite element analysis or some other numerical method. Now, you can see again kinematically the two curves are coincident for quite a bit and then they start deviating. So, you should expect that if the rotation effective rotation here compared to the original one that is this angle if it is anywhere in this range you can expect this small length flexural pivot approximation of the torsion spring here then you would get reasonable results that compare well with a (Refer Time: 23:22). If you go outside the deviations already the kinematical it and of course, the spring caution given here also would deviate it is not going to be the same throughout, you should expect that there will be discrepancies.

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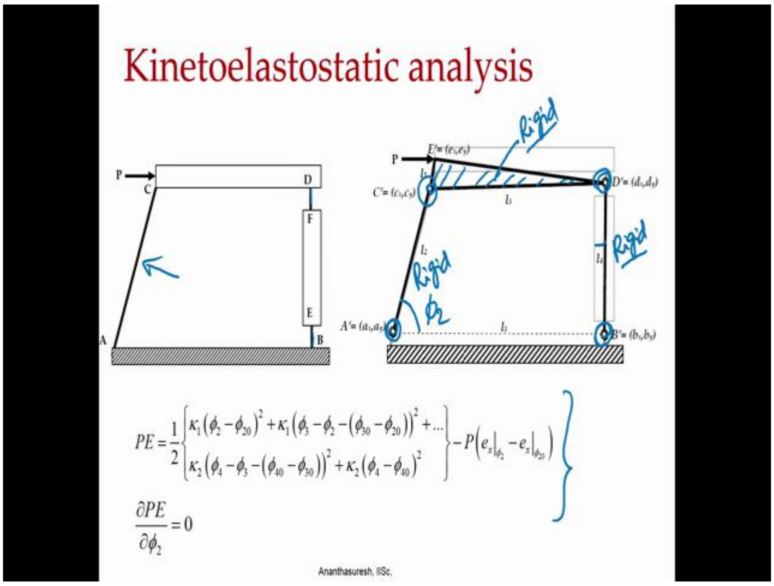

With these things now we have the elastic beam. So, we have a torsion spring here and torsion spring here and the small length flexural pivots are between d f and e b we have a torsion spring and torsion spring rest of them never the becomes rigid. So, this is rigid and this is rigid and it was already rigid that comes rigid again these two are already rigid earlier.

Now, we have now this rigid, now we have 4 bar linkage on the force which was over there then becomes like a ternary link here they connected to you know this body that body right, and then there is a force here acting now you can analyze this 4 bar linkage again we can use this potential energy strain energy plus work potential and if I call this angle as phi 2 here you take derivative with respect to that equal to 0 and you can get the force displacement relationship they can solve like we did in the first example and the second example.

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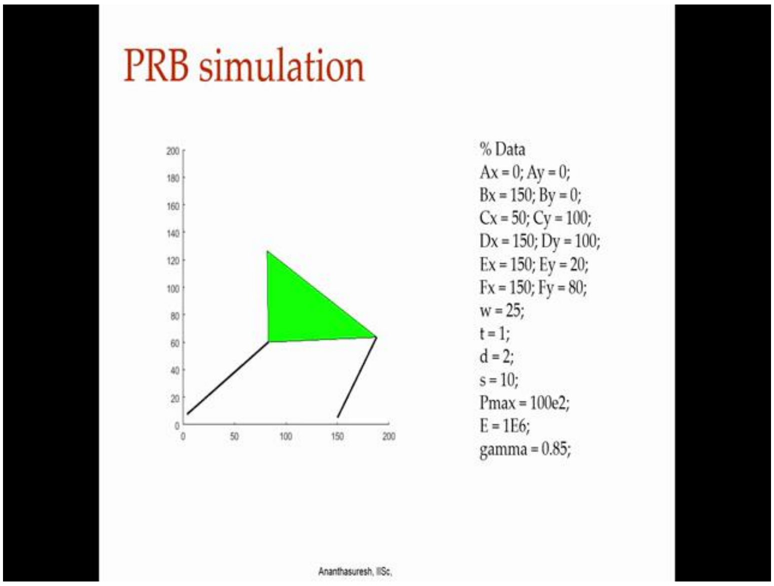

This is the PRB simulation we are applying some torque at the input thing let us run it again and you can see that when I am applying force there how it is actually moving right. So, you observe the simulation again it is goes fast and then you kind of slows down that behavioural we will see when we plot force versus this angle or angle versus the force this is the data used in this example we have A x, B x they are the coordinates of a b c d e f that are there and their thickness cross section parameters and the force maximum force applied and the gamma we have taken 0.85 here young’s modulus and everything.

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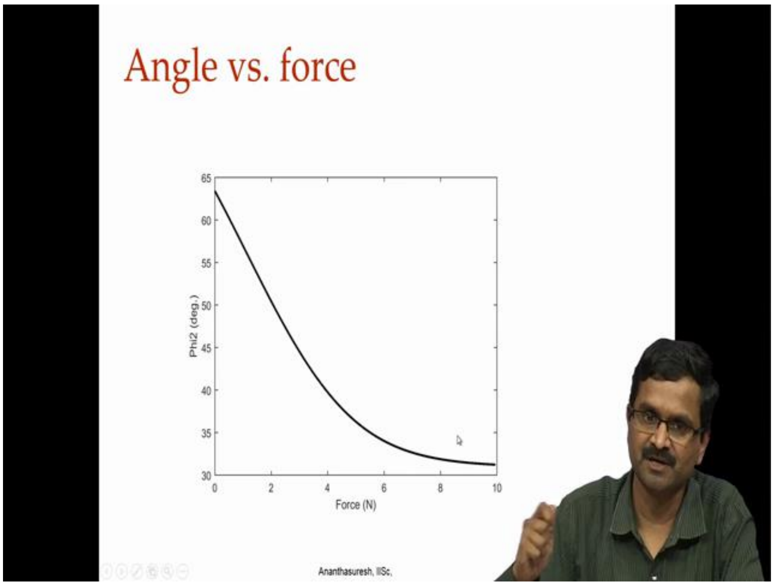

So, there will be supplementary file pertinent to this example there you can learn a MATLAB file and see how we get the simulation as well as this curve we are applying we are showing here the angle versus the force this was called for a given force what is the angle. So, we see as I increase the force the angle initially sharply decreases and afterwards it kind of saturates meaning that, even though increase in the force at steady rate the angle is not changing much that is the characteristic of a stiffening mechanism. Initially apply that is a 1 Newton it would move a lot, 2 Newton's move more and then 3 4 5 after that even though an increasing by 1 Newton the change in angle is not much meaning that the mechanism becomes stiffer as it is deflected more and more.

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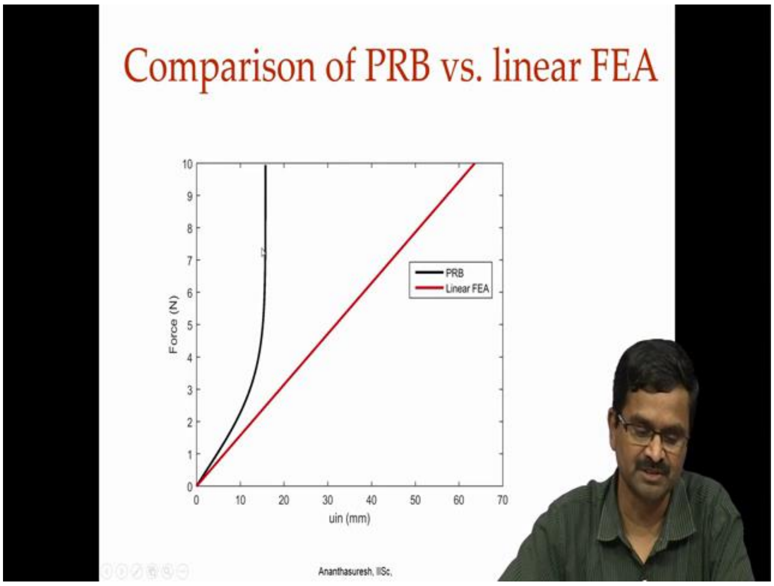

And we see that here force versus input displacement this uin here is the input displacement and you can see how the slope suddenly, becomes very large they almost infinite meaning that becomes very stiff after that you have to apply a lot of force to move by it is minute distance and here the red one is linear FEA not non-linear here I have done only the near analysis initially they agree and then it deviates a lot. If I were to do non-linear analysis here then it would have this non-linear behaviour will be closer to what PRB gave which is the black line here.

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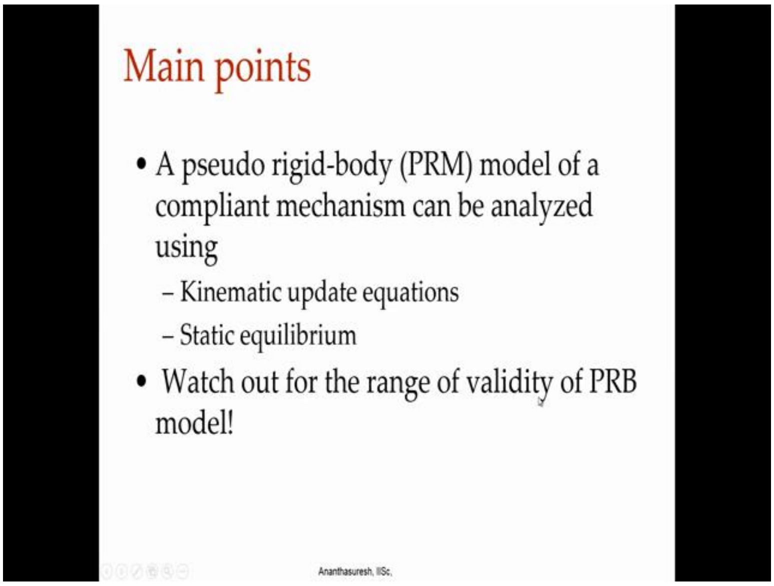

So, to summarize we discussed two more examples today, one of them had the coupler link to be an elastic segment and the third one and that was a bi-stable compliant mechanism even then PRB modelling captured the behaviour very well the third one was a fully compliant mechanism there was an elastic beam and two small length flexure pivots and we constructed a PRB for that not PRM, it is pseudo rigid body here PRB. There the non-linear and linear the beginning linear was comparing quite well we have two non-linear to really compare that you can do based on MATLAB code that is going to be supplementary file for this, but final caution here is that you have to watch out for the range of validity of the PRB model that n value if you do that then we can see you will get an answer very quickly that is the strength of PRB model, but then whether it is accurate or not depends on whether the conditions for PRB are true or not in our example.

What matters there is the orientation of the force if there is also a moment whose example we considered example three that is fixed-fixed one there we just took some parameters, but there is going to be at either end they are going to be forces as well as moment when that is there PRB model actually breaks down and even if we were to compare non-linear quantum analysis of the example three we would see considerable deviation, that we have to watch out for where PRB is valid where it is not valid what it strengths are, the strength of PRB actually place out well when you consider synthesis as we will discuss in next few lectures.

Thank You.