Compliant Mechanisms: Principles and Design
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Lecture - 18
Pseudo Rigid-body (PRB) Modeling

Hello, in the last two lectures we looked at kinematic as well as electostatic approximation for the large displacement behavior of a cantilever beam, and at the end of last lecture we also talk briefly about pseudo rigid body modeling for beams a slender beam that a existing compliant mechanisms. Let us look at that in detail today with some example then we work out in next week lectures.

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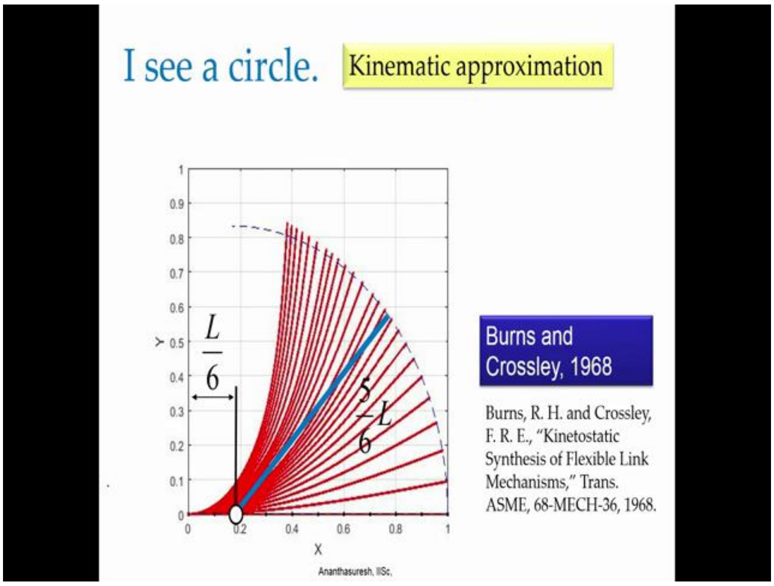

So, this pseudo rigid body modeling as also known as acronym PRB pseudo rigid body modeling pertains to approximating the behavior of cantilever beam undergoing large displacements. The first thing is kinematic approximation, there a circular ark is taken as approximation to the locus of the loaded tip of the cantilever beam that was done by Burns and Crossley as we have discussed already.

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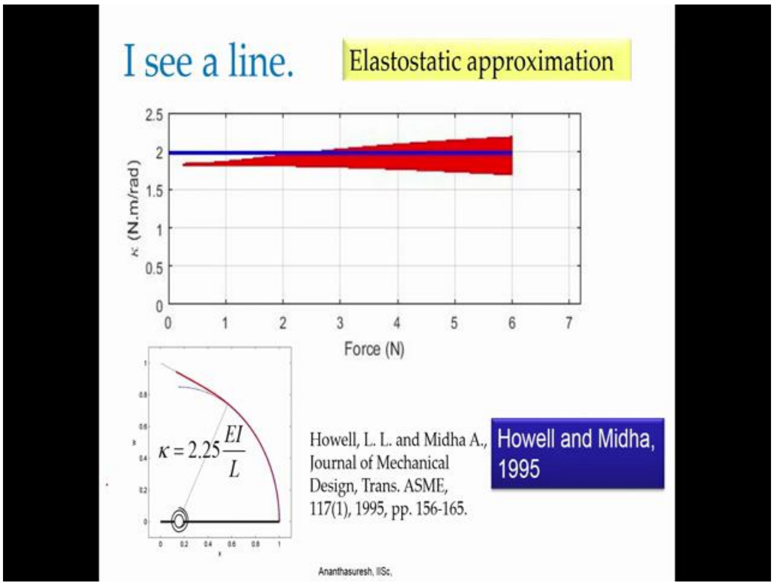

And we also have another important approximation, which is elastostatic approximation that is under the action of the static load the deflection of the cantilever beam. The resistance that the beam offers can be capture with the torsional spring as was done by Howell and Midha in 1995, where they saw that this in the spread which is very narrow of this equivalent spring constant, torsion spring constant that can be approximated with some number in this particular example it was 1.98 newton meter per radiant, so having these approximations for kinematics as well as elastic, elastostatic behavior.

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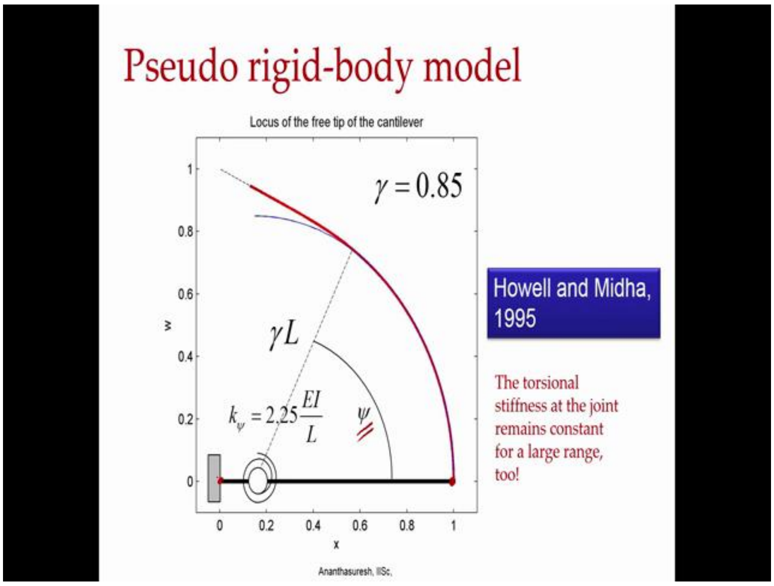

At the end of the last lecture, we also talked about the pseudo rigid body model that is if I have a cantilever beam of length L fixed at one end free at the other end meaning that it can have load in any direction that is there will be transfer component to that load as well as the excel component which can be tensile or comprehensive then we can replace that cantilever beam with rigid crank of length gamma times L, where gamma is called the characteristic length factor for the beam, which is on the average which taken as 0.85 it was 0.833 based on the kinematic approximation that Burns and Crossley done, lets refined Howell and Midha 2.85 and the average where you maximize the angle what shown as psi here that angle as large as possible.

So, that this approximation with circular ark holds for the locus of the loaded tip. And this one if you used that is PRB pseudo rigid body model if you use, you can forget about the cantilever beam meaning that the effect of the cantilever beam is captured at the free tip, one end is any where fixed with the beam as part of a compliant mechanism. If one end is fixed other end is free mean that there is a no movement then we have this nice pseudo rigid body model, where it can be between two bodies that is; connected to the connected to the beam in question. So, we can replace that beam with a rigid body torsion spring then we can go back to the literature of kinematics and do our analysis and synthesis comfortably and then come back when you want to have compliant mechanism replace that rigid body torsion spring, back to in elastic beam and that is how this is going to work that is the concept of the PRB.

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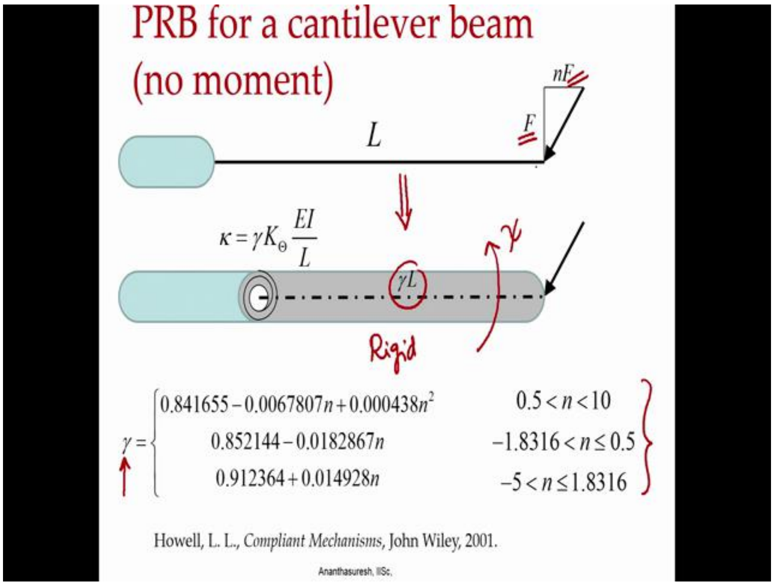

So, let us look at that, which is this beam over here is replaced by this rigid crank. So, this is the rigid crank where the force can be (Refer Time: 04:31) there will be a transfers component as well as an axial component n is the one that relates both of them n can vary as it is shown over here per various values of n you have different value of this characteristic factor the gamma L, that is you do not take the rigid body as long as elastic beam, but only take f fraction of its which is the fraction is controlled by this gamma for various values of n that is depend on the force here. You have this approximation that was walked out by looking at integral solutions and also verified parent element analysis for this large deflection which is really non-linear, but now we have a 1 parameter model here as we also saw with elliptic integral they are object this modulus p as controlling the behavior of the entire elastic segment modeled with, in this case there 1 parameter is the rotation angle of this which we can call psi which will be the angle at the tip of the of the elastic beam.

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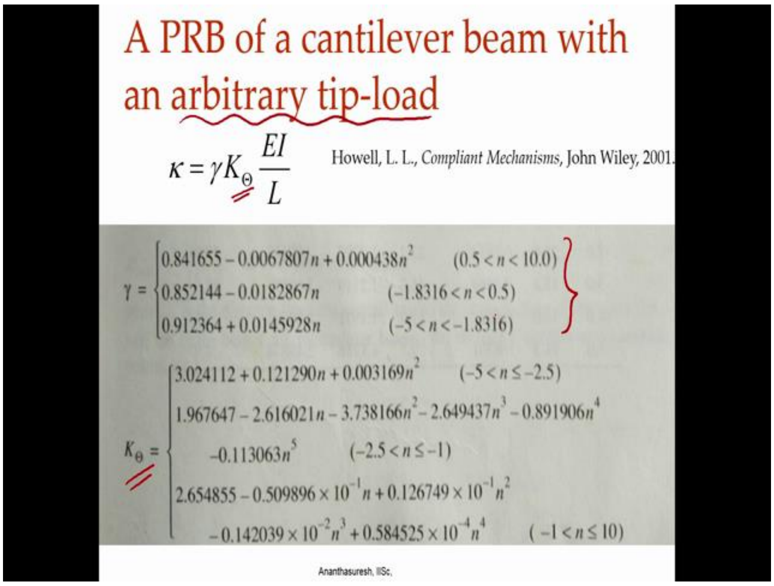

If you take the elastic beam as it is for your analysis and this was a detailed modeled when we have an arbitrary tip load only force, but no movement then you have the gamma which we just saw in the previous slide, but also this k theta which will determine our kappa that is our torsion spring constant takes into account property cross section property as well as the length of the b.

And the gamma came here where take this gamma multiply by theta and E I by L to get this kappa. So, this is for a beam that has an arbitrary load that is transfers that axial load combination in which way within this range, that is what happens when this beam is part of a mechanism a compliant mechanism has both rigid body and elastic segments there you can use this comfortably and quite accurately as we will see where you walk out examples. Now let us take beams of different boundary conditions what if have a fixed guided beam? What do you mean by guided? Its guided in such a way that it cannot rotate fixed is like a cantilever or enchase condition where the beam cannot move let us say 2d cannot move in either translational direction and at the same time it can also not rotate where as guided one you allow the translational motions, but do not allow to rotate it look like this.

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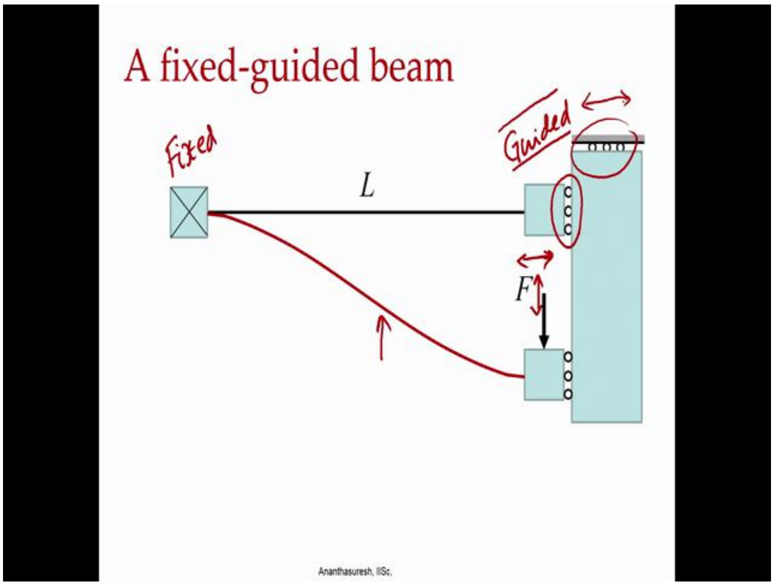

So, we have a beam this is the fixed connection the other end here is the guided connection that is what we call it guided. As you can see in this model this point can move in this direction, because we have shown that roller over here and because of this roller over here it can also move in this direction, but it cannot rotate because it has to be in contact with this the way it is shown here, but in reality there are lot of conditions where can realize this guided condition. So, we have fixed guided beam how do you come up with in equivalent pseudo rigid body model that is what will discuss now.

Because we discussed at length of cantilever beam, now we move on to other boundary conditions if you have fixed guided beam then it is going to deflect like this the beam over there when you apply this force it is going to d form like this, that the same beam will deform in the process looks like here it is stretched because that length and this length is more that is because, in the slide have not shown the movement of this in the axial direction. So, will see that properly rights.

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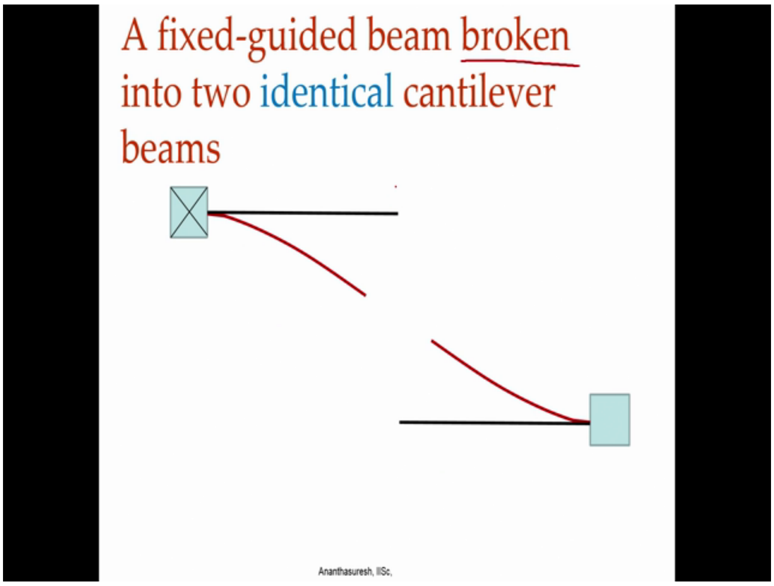

So, if you have 2 beams like this let us say there are two identical cantilever beams just for the sake of illustration are put one below the other, but imagine that they are right next to each other. So, both of these if we fix here at this end this and this end apply force its go in to d form like this. Now again we have broken them down just for the sake of modeling it or imagining how it works otherwise actually together right. If you have that you can think of these as two identical cantilevers where there is a load acting any which way that is it can be axial transfers any combination that is the n value that we had in the previous slides can be any value.

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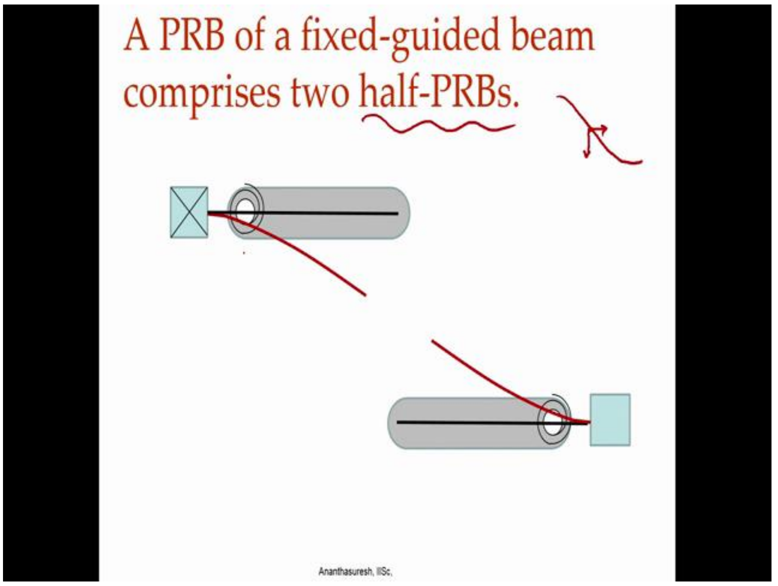

So, that is minus 1 to plus 1 and even minus 2 plus 2 whatever, value depends on the ratio of the axial force to the transfers force. When you have this we can model each one of this cantilever beams as a pseudo rigid body model as we have because, we are broken down a fixed guided beam into two cantilevers fixed and other one is free meaning that load can be in which direction, because when a beam bends like this over here that is like a cantilever because it has a finite slope.

So, if apply a force in some direction there will be a transfers component and axial component. So, when you have that is going to deform like this and that is what we are using to model using PRBs to have two half PRBs that we put together, the PRB means that we take this length to be a characteristic value gamma times the length of that in this case l by 2 it will be gamma by 2 l for these and they will be kappa for this that also we have to see, because earlier if it a cantilever beam there was only one torsion spring, but now we have one here one here based on the force we have to be take that also as twice of that value it turns out over here. So, we can replace the cantilever deformation with a pseudo rigid body model since there are two of them here we make a model like this.

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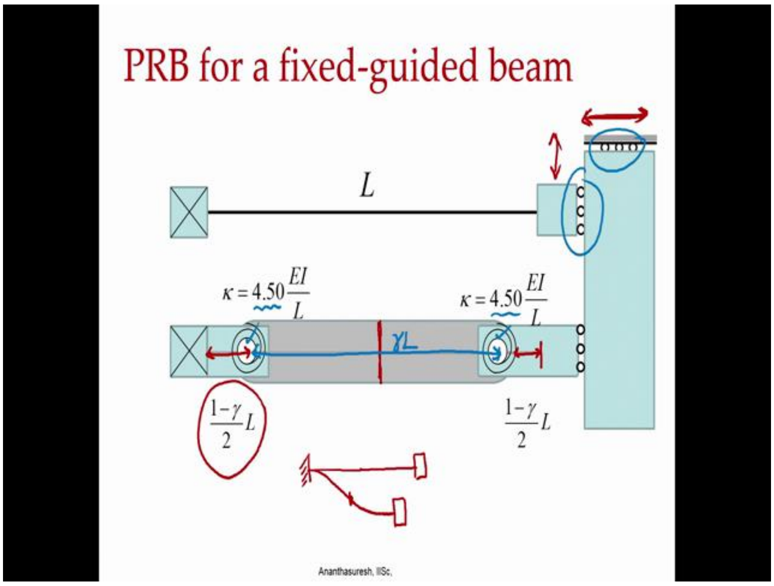

So, we have the beam which is fixed and guided and vary place. In fact, what I am not shown here is the breaking of this that is why the slopes of these two are going to be equal, because as we saw this beam when it deforms let me draw that beam fixed let us say fixed here and guided over here, when this moves let us say somewhere here again remember that it us going to have motion in this direction as well as this direction then it us going to move like this. So, when that happens we said that we break it into two parts and there is this cantilever there is that cantilever, because this is other than the little bit of motion in the axial direction rigid body translation this is like a cantilever beam we have broken that is what shown here, two of those we put together that becomes the pseudo rigid body model for this fixed guided beam.

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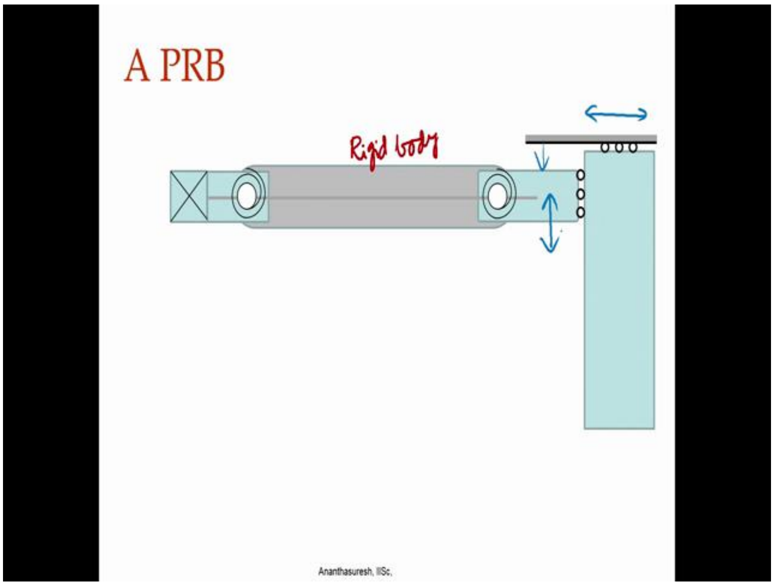

And using this we can think of for something like this where we have replaced a fixed guided beam with a rigid body, with this one in the middle is a rigid body and there are two torsional springs. As we saw earlier lets go back here to you see that now this distance from here to here is 1 minus gamma by 2 l and same thing over here from here to here in that is where beam is fixed and then in between that is from let me change the color. So, you can see clearly let us say, from here to here that will be again we have this is gamma by 2 gamma by 2 are subtracted this will become gamma l like what we had a under kappa here transfer to be 2.25 is what we had now it becomes 4.5 for either of those, because they share the load in a way because they break it here there is a force coming on both of them.

So, it becomes twice of what we had earlier, because this is like a two springs in series where in the spring constant becomes half that become 2.25 eventually, but both places there is we have a torsion spring that is over here as well as here let us see how this works in a complain mechanism context where we do not have to have this type of you know sliders. So, if we have something like this when we apply force over here that is where we will apply some force, when we do that it is going to deflect like this you notice that we are showing both the motion in this direction as well as this direction when you apply force and that has to happen right.

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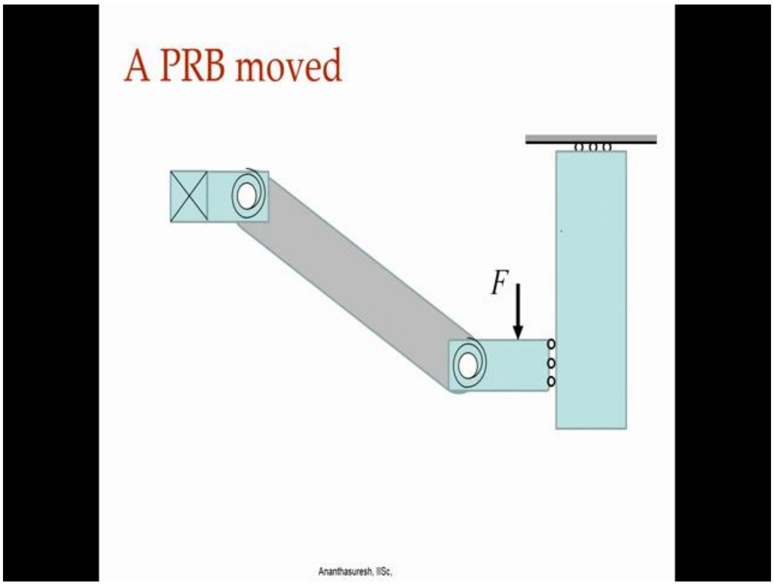

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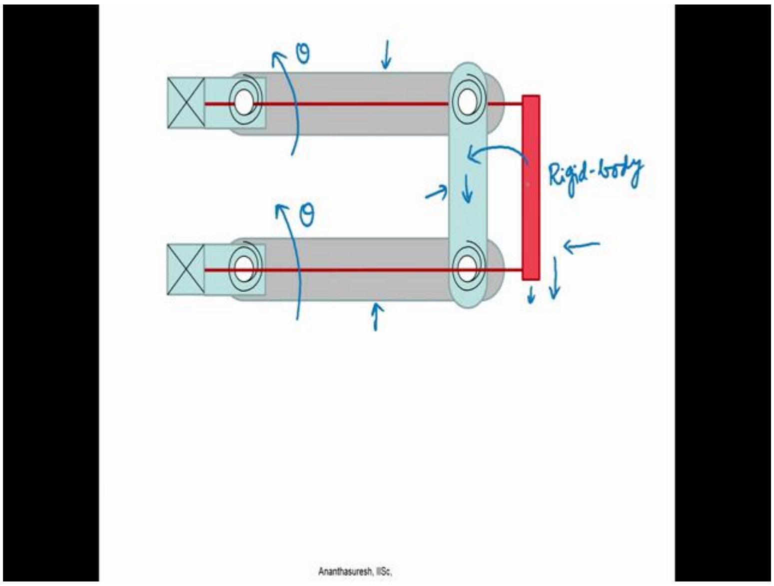

So, it is going to be from there it comes down here when it moved we can also imagine if we have the original elastic beam its actually called a parallel motion compliant mechanism or parallel guided compliant mechanism. Where the red ones or 2 elastic beams and this one is a rigid one connecting both of them if this is a rigid body, if you have that when you apply force over here are the corresponding pseudo rigid body model where you are apply a force there it is going to deform like this, where the beams are bending and you can see that this red rigid body moves in that direction as well as in that direction, that is how going to deform and that is cover parallelogram linkage also is going to move right.

So, our parallelogram here moves in this diagram it looks like the rigid body stretched a little bit, but that is just a mistake in drawing, but it is a parallelogram linkage that rotates and moves closer to the fixed and that we can model with again 1 parameter, because if you see this parallelogram mechanism there is only rotation of this that is same as rotation here right, that if you call this theta, theta and that is 1 parameter for this mechanism which consist of 2 elastic beams undergoing large displacement. So, PRB model enables us to analyze things that are complicated this is not that complicated in the first example, it can be complicated and we can analyze with a 1 parameter model by making a 1 degree of a freedom pseudo rigid body model of the mechanism.

So, here the red ones are real mechanism the one that we have with rigid bodies that is this one, and this one, and this one, anywhere was there is a replacement of this red thing and four joints and there are torsion springs at all four of them if you put the values that we have indicated then your behavior of the pseudo rigid body model we very close to that of the elastic body one if you what to do finite element analysis and compare that is what we want to do and see for ourselves that this is a good model to approximate the large displacement it is non-linear behavior of a compliant mechanism comprising slender beams. Now let us also move to a case where there is a cantilever beam with a moment load, only a moment load.

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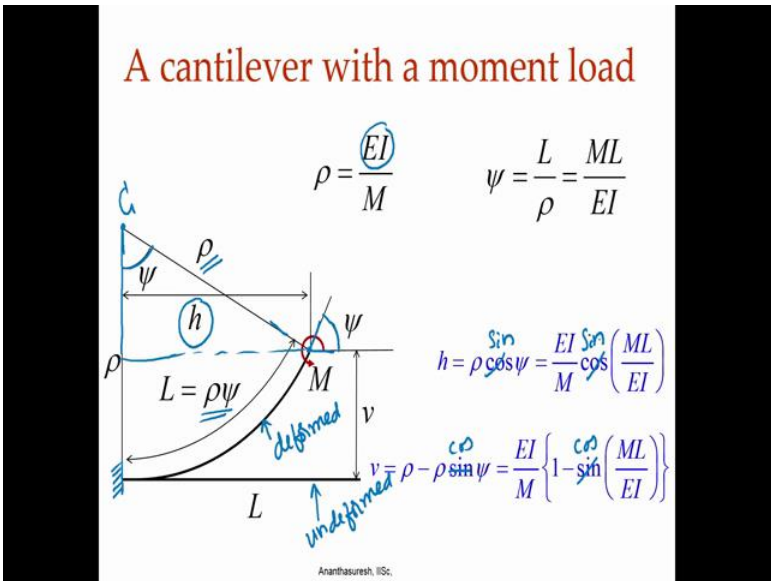

When we have that which we had discussed earlier and its particular case there is a pure moment the beam which is fixed over here when there is a moment M acting this is the un-deformed beam and this will move to this one which is deformed. Since movement is bending movement inside the beam is proportion to curvature if there is only moment load throughout the beam the bending movement will be constant, because if we draw the free by diagram wherever you cut there are no forces, pure moment throughout the beam. So, moment is constant and hence curvature is also constant or radios of curvature that is our rho is constant for the entire deform profile of the beam and the curve that has constant curvature is a circle of arc.

So, here the beam of length L is deforming into an arc whose length is equal to L, because in this whole theory. So, far that we have discussed we do not consider the stretching of the beam. So, here L becomes rho times psi where psi is the angle subtended by this arc at the center of curvature which is constant here, because rho is constant right, L equal to rho times psi that give us a way of computing psi when you know rho or rho when know psi and psi again is the tip depletion, because this angle here is same as this angle there. If you see this vertical line is this horizontal line this normal here is a tangent here so that angle is a same. So, that is also the end slop of a cantilever beam when the movement M applied we know that rho is equal to E I by M because M is positive curvature are the M is proportion to 1 over rho. 1 over rho is a curvature when rho is a radius of curvature and constant proportionality is this E I youngs modulus and second moment of area.

So, from here we can get psi because its L by rho, rho we get from here that becomes M L by E I which what we had for this mitosis method and everything else we have done with small displays analysis that wholes here even though it is large displacement, because curvature is constant we do not have to solve the differential equation is readily solved for us we can get this h that is when it deflexed the free tip we want to get the locus again. So, that h is given by rho cosine psi that is this distance that is rho the psi t rho cosine actually this should be psi so it should be rho sin psi. So, not cosine there should be rho sin psi.

And the v here this will be cosine, because this is sin and this is cosine the vertical one rho minus rho cosine psi. So, let us correct it and the slides that will be put up a supplementary file will have this correction done. So, sin and cos we have we get this h and v. If you what to plot here do we see a circle again. So, here what have taken is our elliptic integrals are finite element analysis solution to plot for various moments.

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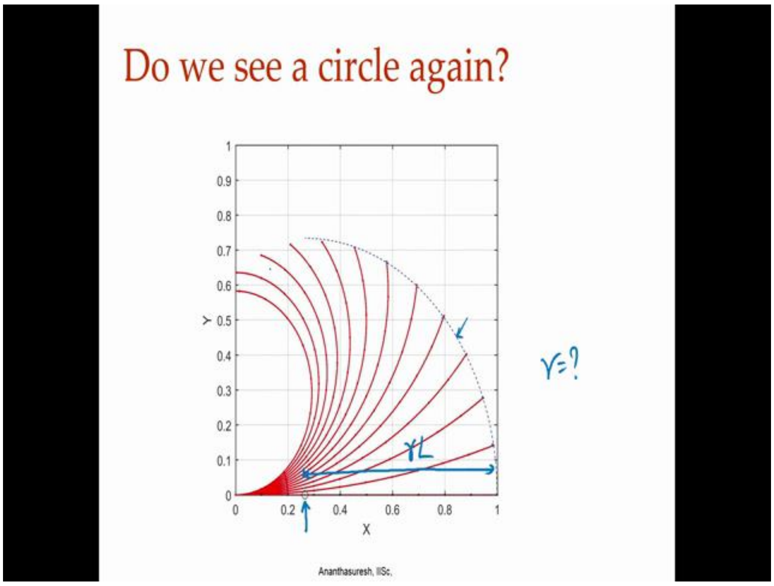

So, moment here is 0 here is some value and so forth, again the values taken for this are what we had taken previously is for steal length is 1 meter cross section is 5 centimeter by 1 millimeter strip of steal and the load here was moment load varied up to 3 newton meter there is moment. Now if you see there is a gain is circle right, you see this if we draw this dash line here that approximates the tip locus of the tip quite well here it is almost rapping around right. So, for that kind of a deformation which is practically good enough for most compliant mechanisms with there is a pure moment load again there is circle the circle if you see the characteristic length as what we called gamma, we need to find out what that gamma here is gamma L here L is equal to 1.

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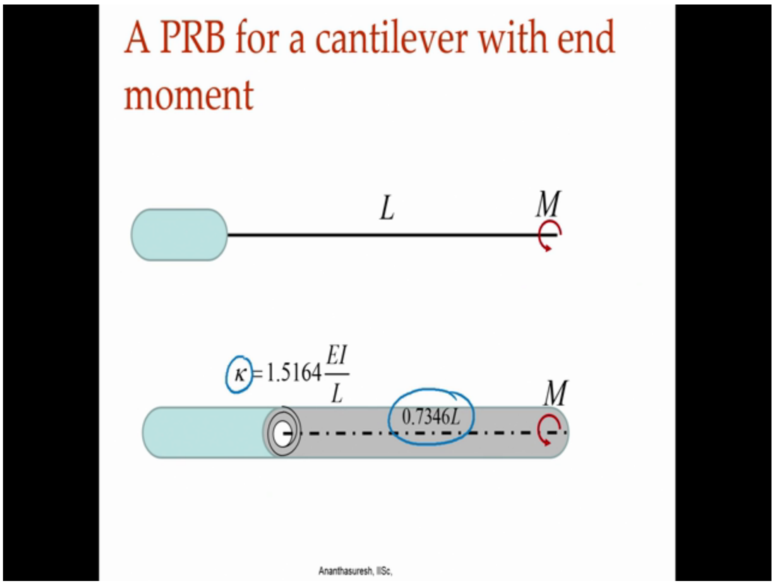

So, gamma is basically that length of that this is not our 5 over 6, because this is more than 0.2 what is that y should be take a smaller radius that we can again see now that we know that there is a circle we can fit a circle and find out what that is that transfer to be 0.7346 times L, where we have to do the optimization that is you have to minimize the deviation between what you get from the large disc analysis of a beam undergoing a moment load another moment load and then circular arc where to see what this gamma is and if you minimize the error between the circular ark and this locus that you get from elliptic integral solution or here where you actually have analytical solution right.

So, we do not have to go for numerical solution and if you minimize that you will get gamma to be this value and likewise if you look at the torsion spring constant that once to be 1.5164 E I by L. So, we can replace that with this pseudo rigid body model when there is only a moment load.

Now, what about a fixed-fixed beam? Fixed-fixed beam is a little complicated unfortunately this is also the most common thing in compliant mechanisms and pseudo rigid body model for that is a little bit tricky, because arriving at what gamma we need to take and what kappa we need to take at another end is dependent on the load.

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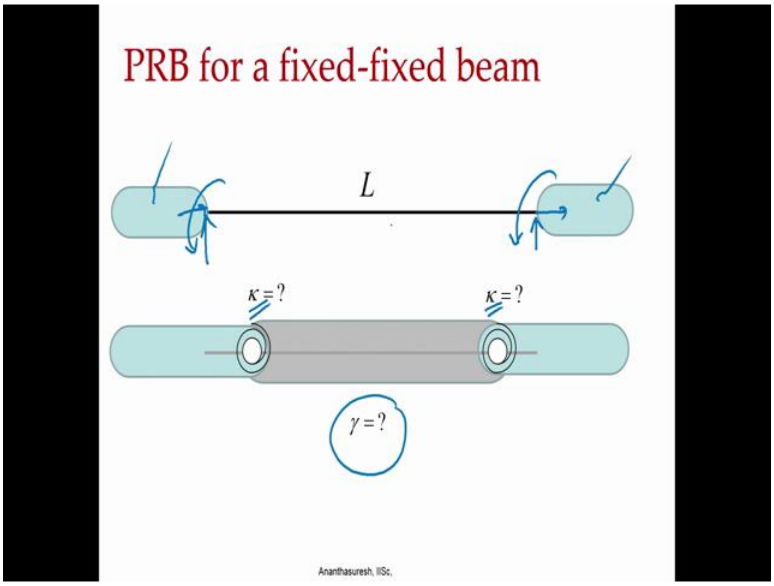

So, when these a rigid body is connected to other things at this end they will be transfers force axial force and moment and here also there will be axial force transfers force and moment they need not be the same they can be whatever loads at you have depends on where these two are connected. So, you can certainly model like this, but get in the parameters that hold good for large displacements are deformations are the fixed-fixed beam is little bit difficult.

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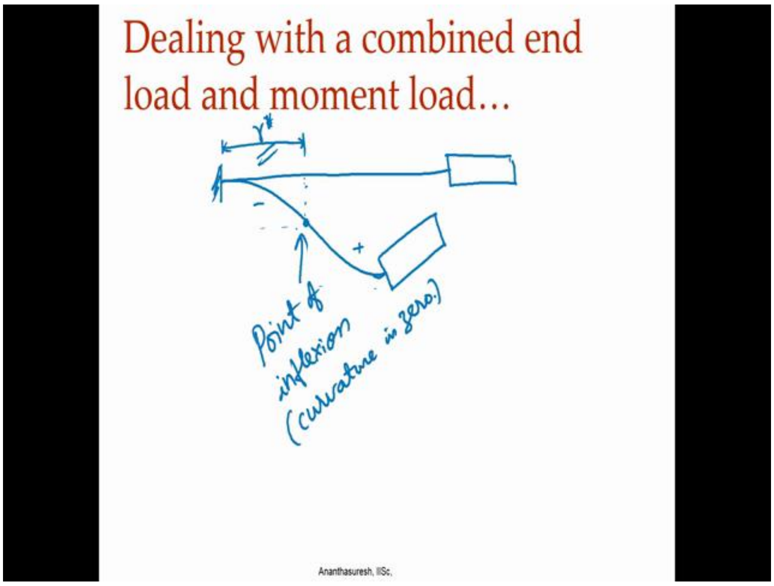

Because we have let us say what we need if I take a beam that is fixed and another one let us say the rigid body that was connected it is not guided any more that we already discussed, if I have a beam that has another fixed connection here let us say this one I take it like this.

So, when it goes like this certainly the curvature is 0 somewhere, it changes sin that is say from negative to positive curvature what here, but then that does not happen necessary the midpoint as it happened in the fixed guided case. So, when we allow rotation and translation for this, whichever way it goes now you take this where it happens do not know once we know at the that point we can say these a cantilever beam of not equal length unequal length, but we can still do it.

So, the trick would be when you have a fixed-fixed beam we need to know where this point of inflection. So, this is the point of inflection where the curvature is 0, instantly means here curvature is 0 for there if you have a such a situation then we need to know at what distance we have that point (Refer Time: 25:30) if we know that then there will be cantilever this side and that side and we can develop a model, but then unfortunately in a compliant mechanism where there are this fixed-fixed beam segments this length let us call this gamma star or something.

Let gamma star changes from position to position in one mechanism if it constant we can get it and be done with that, but that is not the case as the mechanism moves these gamma star also will be vary and then we will have an extra parameter, we have gamma star is a parameters and then we have a cantilever beam the slope at that point which will be the same for this part of the cantilever beam and the other part left side and right side then there will be another parameter and that is what we need to do. So, it will not be as elegant as the PRB that we have discussed so far. So, fixed-fixed beam is a little bit of a problem actually not a little bit a lot of problem to deal with, lets note that and let us see if there is a compliant mechanism like this will considered some examples can we do this can we analysis this.

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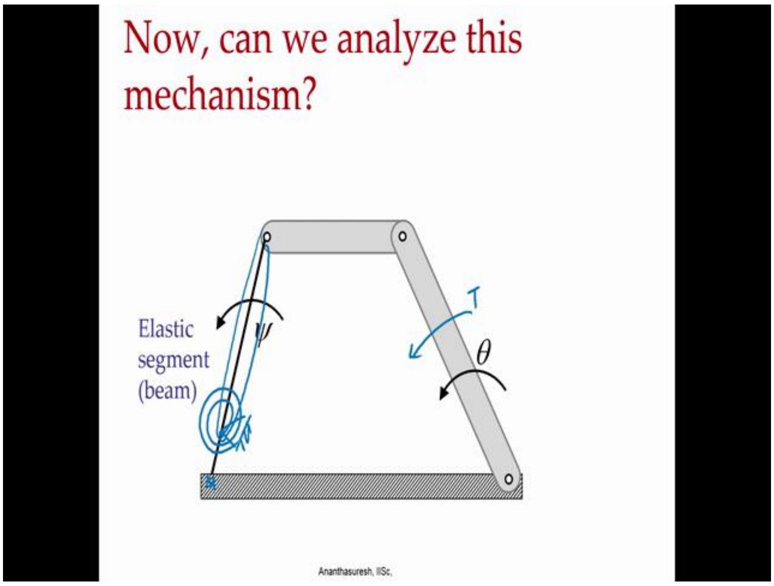

So, if we say that there is some tarc over here, can we analyze it meaning that can be see how this deforms how the coupler link deforms and so forth, answer is yes, because this is fixed here and it is movement free over there load can come any which direction when we how deal with that we can replace this with a pseudo rigid body model where the length is shorter and put that beam there put a torsion spring and you can analyze it.

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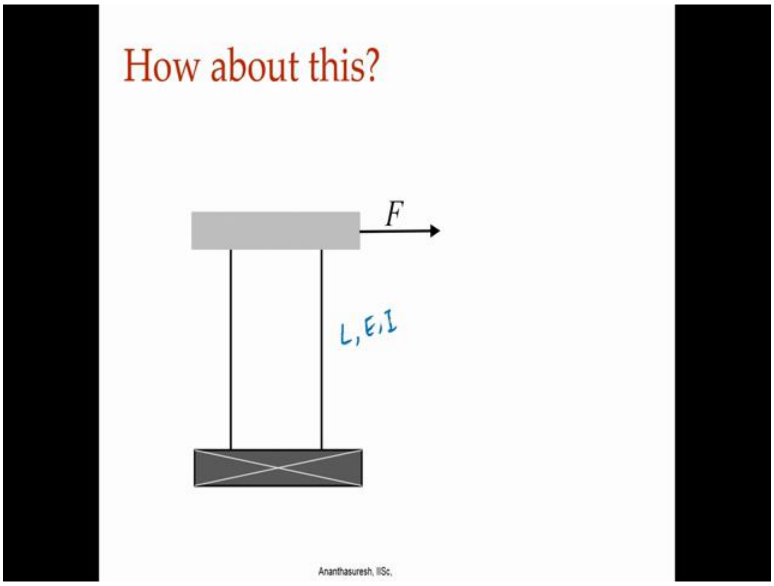

How about this? We just discuss this accept that was horizontal we immediate vertical, we have parallel guided motion compliant mechanism force is given length youngs modulus I have given we can analyze this which will consider in the next lecture. And what about this now, I had put this in the middle like a coupler link again I have put pined here meaning that that will be there the movement will be absent.

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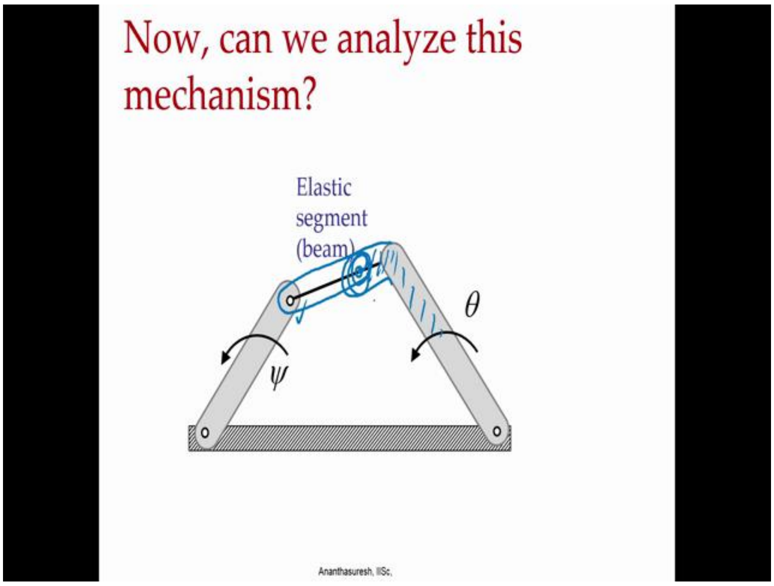

So, we can deal with this because its fixed and force any direction this also can be analyzed by putting a rigid body over here, there I am moving based on that gamma factor and put a joint there and do this and extend this body like this. So, this one now gets extended right. So, then we have a regular four bar linkage with the torsion spring we can analyze it.

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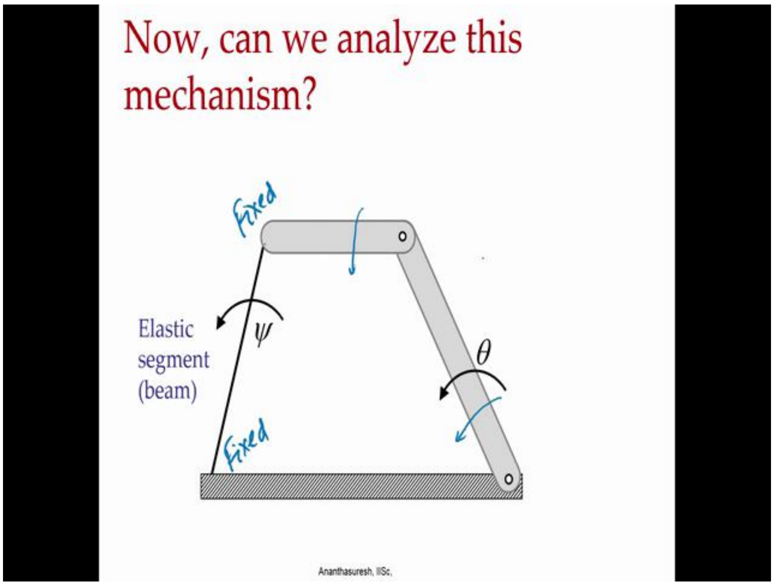

What about this? This is where we have now fixed and fixed and this will be a problem for different positions of this crank or this coupler that gamma star I wrote in the one of the previous slides where the point inflection occurs that is not know the keeps changing. So, predicting that and doing it amounts of lot of work.

So, fixed-fixed one is difficult to deal with in the pseudo rigid body model concept, but when get an example to see how close it will be make an approximation right, just forget about the fact is fixed-fixed take 0.85 or (Refer Time: 28:47) factor or some other value and see how close we can get if you what analyze it using its a finite element analysis and compare it.

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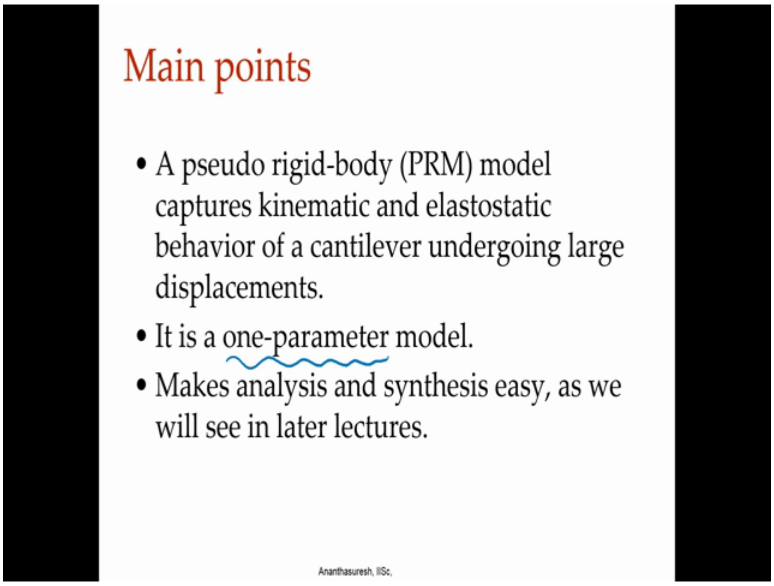

So, we will look at these few examples in the next lecture, just to summarize what we have discuss now this pseudo rigid body model captures kinematic and electrostatic behavior of a cantilever undergoing very large displacements, quite well. And it is a 1 parameter model, that should not surprises because elliptic integral solution enable does to solve the differential equation governing the large displacement we have a cantilever beam we just 1 parameter p, which is a modulus have the end slope theta there all related or eta F L square by E I there all equivalent 1 parameter model, that is changed physically in PRB as that psi which is the rotation rigid crank of reduced length.

The other thing we should realize is that; pseudo rigid body model enables us to analyze as well as synthesis compliant mechanisms that comprise of a few beam segments that have this different boundary condition, we discussed the cantilever beam, fixed guided beam and a beam with end movement only and fixed-fixed beam is little bit of a problem will deal with that when we discuss the examples.

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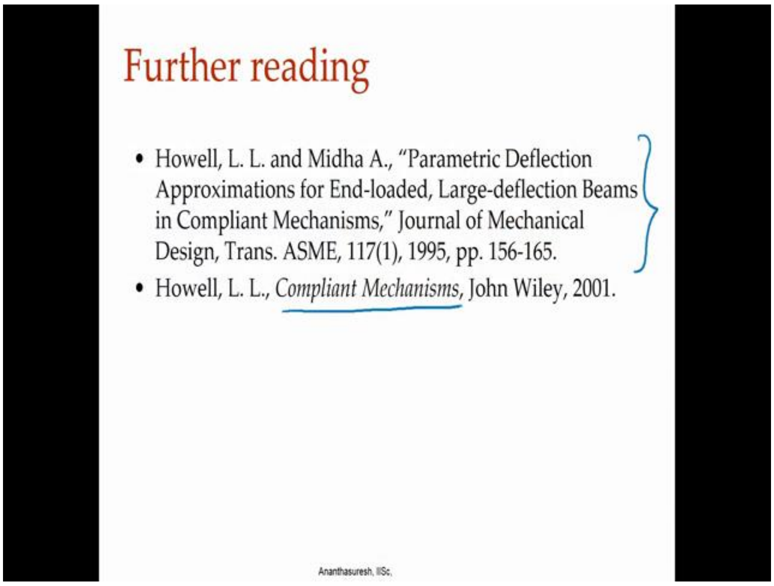

And further reading for this lecture you can look at the paper by Howell and Midha, as well as the first text book on compliant mechanisms by Larry Howell.

Thank you.