Compliant Mechanisms: Principles and Design
Prof. G. K. Ananthasuresh
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture- 15
Frisch-Fays Approach to Large Deformation of Beam

Hello, in the last two lectures of this week we looked at elliptic integral based solution of large displacement analysis of beams in particular a cantilever beam is an analytical solution that you elliptic integrals, today what will do is to consider those equation that we wrote in the last two lectures and implement it and mat lab. So, that we understand the concept of elastic similarity that we discussed in the second lecture of this week. So, looking at this elastic similarity and how we can do those calculations in mat lab.

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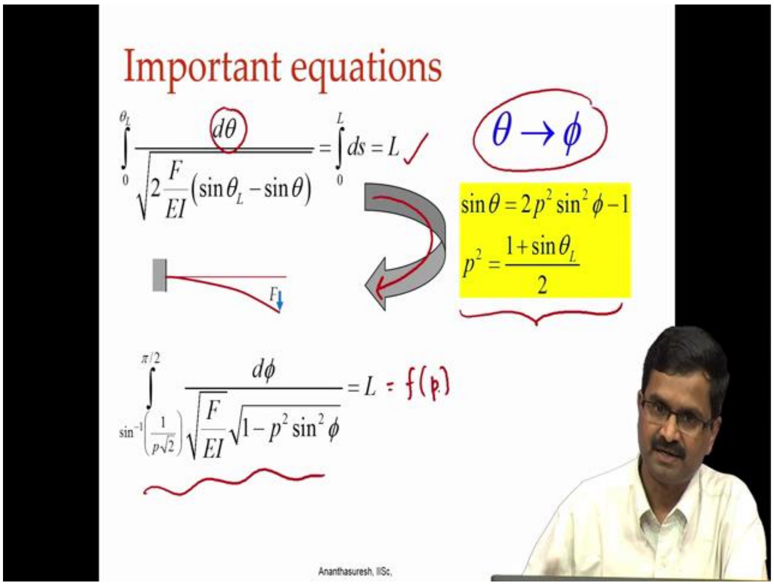

Let us first look at the important equations which are shown here, the first equation shows how the length of the beam is obtained by integrating in the theta of variables.

So, here we are looking at d theta. So, theta variable we also looked at a transformation or a change of variables from theta to phi using these equations, so that we will transform that to this equation were the phi were is from sin inwards 1 over p over square root of 2 to pi by 2. So, this equation is basically an equation in terms of p. So, p is a variable once we know p we can get everything else that is what we have discussed.

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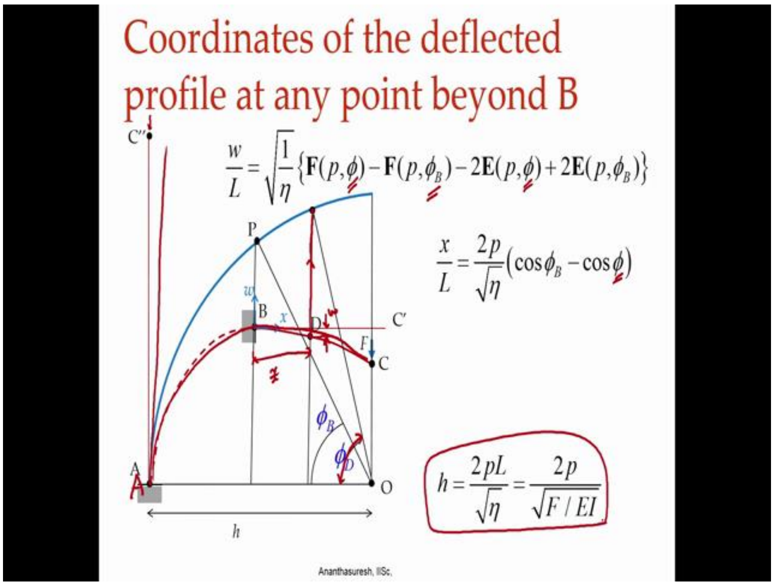

Now, if we look at the geometric significant of that phi when we change the variable theta to phi we know that is the angle that is shown here for point B we go up to p and then connect that gives me phi B. So, only for point D B go there and point C phi equal to 90 degrees because if a go up and come down and look at the angle that is 90 degrees and for this point which had called A earlier phi is equal to 0.

So, if I want know W by L or X by L that given by these equations, what is w? W is the displacement in the w axis and x is the displacement of x axis if I take a point D at that point if I calculate this angle phi D by going up and finding this point and then joining it to O then that in inclination here is your phi D, if I want to know w of this meaning that displacement that is w and this will be from here to here will be x, if I want to get that then I have to instead of this angle phi B that will remain as it is instead of this phi we had to put phi D and same thing over here same thing over here for every angle we get this displacements and we also talked about extending it all the way were phi goes to 0, as if there is a vertical stretch that is there we apply a force on it that is going to be if this the vertical stretch if there is a force there, that is going to buckle into this shape that we have, that is what we had discussed.

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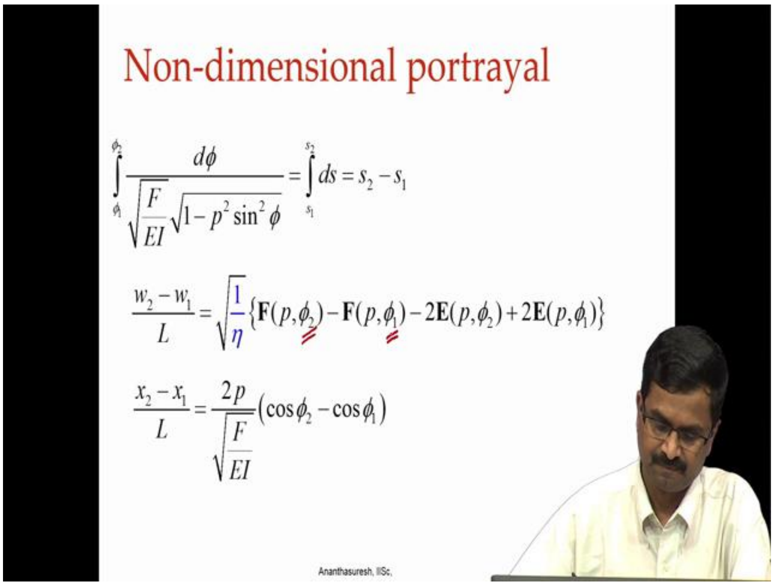

Now, this h is given by this one which you had also derived in the last lecture. Non-dimensional portrayal, if I take two different angles in that curve phi 1 and phi 2 the difference in the w direction x direction are given by these two equations again the angles here that we have taken that is where they figure in the elliptic integral solution.

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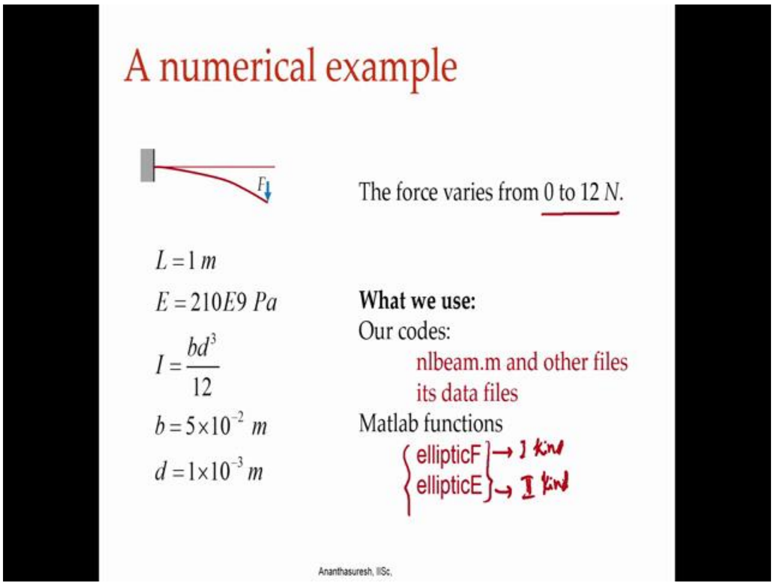

So, now let us look at a numerical example to understand how to implement these. So, we have taken a beam of length 1 meter and young's modulus that of steel 210 gigapascals 10 power minus 9 Pascal's and were taking rectangular cross section, that is why second movement of area is given as b d cube to 12 were b is the width of the beam cross section which is 5 centimeters here, and it is 5 10 power minus 2 meters and d is depth of the beam 1 millimeter are 1 10 into power minus 3 meters. The force were bearing from 0 to 12 Newton's, and what we will use here will be our own code in house there up code for non-linear beam element using co-rotational beam element and some other data files that needs and also use our elliptic integral solution in mat lab. There are two functions elliptic F and elliptic E this is the elliptic integral of the first kind and this is the elliptic integral of the second kind that is what we need in the formulae there available in mat lab.

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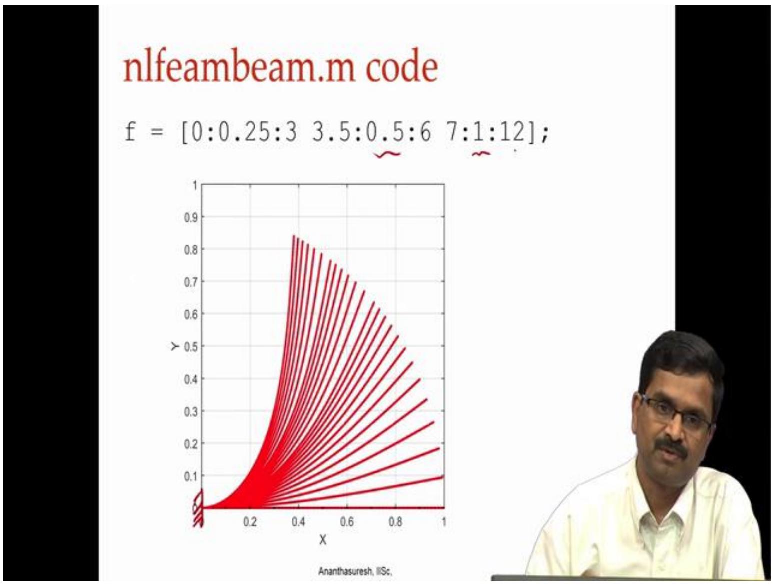

Now, here is the result of running non-linear beam code that you will all have a supplementary material with this lecture, were we have taken the beam which is fixed over here and a force is applied there, but different values of forces starting from 0 in steps of 0.25 up to 3 steps of 0.5 up to 6 steps of 1 Newton up to 12, beyond about 14 or 15 this fails to converge because, there will be some kind of in stability in the equations or the beam and finite elements analysis does not converts; however, as we will see elliptic integral solution has no problem in converging.

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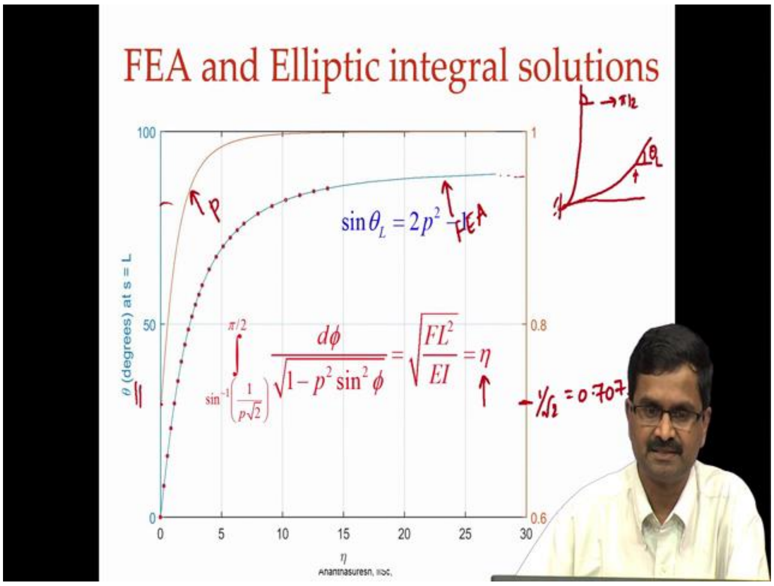

This is with the finite element cod, now from the finite element code we have gotten d theta at the fluent that is; if I have a beam when it is going to bend like this when we apply force there this here is theta L that is shown in this blue curve this is finite element solution.

The dots here are the elliptic integral solutions we can get this theta l from the elliptic integral solution also on the other access. So, theta varies from 0 to 90 degrees theta l and if the beam completely goes like this, this angle will be approaching almost 90 degrees. This angle tends to pi by 2 we see that 90 degrees it goes there that is this side; on this axis we have shown another curve which is the p curve that is the p meaning what we had in the equations what is called the modulus elliptic integral that is p. In the elliptic integral of the first kind, we have 1 over square root of 1 minus p squares and square phi that is the p and that is also done and you can see the ranger of p it is about from 1 over square root of 2 which is 0.7071 all the way to 1 that is the range of p that is what will be unknown in on this problem.

And theta l and p are related, were we have sin theta l is 2 p square minus 1 and in order to find that p have to solve this equations for given theta which involves F L L E and I. So, those are things we need to solve, but here were comparing what we got from finite element analysis and from elliptic integral solution and there agreement is pretty good as you can see that dots which correspond elliptic in to solution or write and the curve which was drawn using finite element analysis result these another way looking at a w L and u L or x L.

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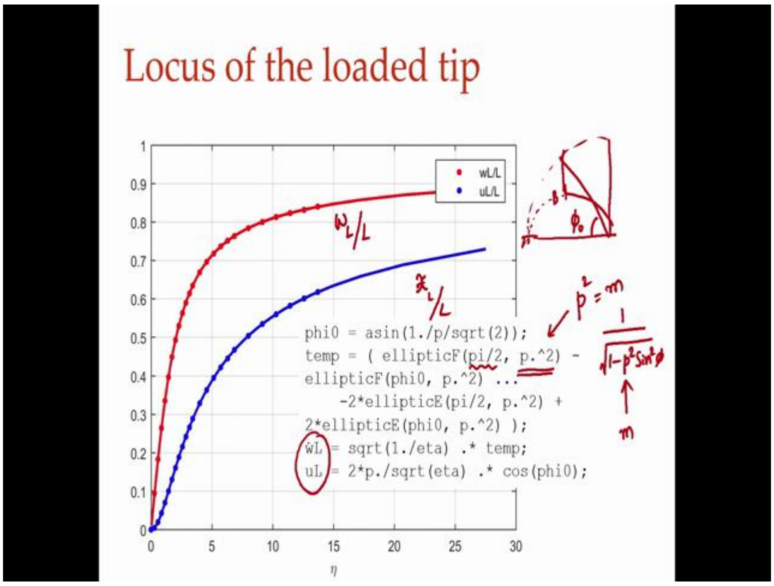

So, this is X L by L this is W L by L, W l meaning w what x equal to l loaded tip of the beam and likewise X L is x cover theta loaded tip the beam again the curves are using finite element analysis and the dots are from elliptic integral solution there agreeing very well here, and this is the code for that; we need to have this phi 0 when we had this cantilever that goes like this we extended back over there and we also have this circular arc it was there and if I go like this I get this angle that is phi 0 or phi b. If I call this point b refer in to the previous that we had we have phi 0 that is in terms of p and then we have this elliptic integral first kind and second kind that are given, what you notice is this thing.

Elliptic integral function elliptic F first kind in mat lab takes the angle first and then the modulus square, some books have this p square denoted as m meaning that in the elliptic integral solution they integrand is 1 minus p square sin square phi some people write it as instead of p square they write m and that m is what should go there then you get the correct a values elliptic integral we do that we can get w L and u L or x l and we get this curves.

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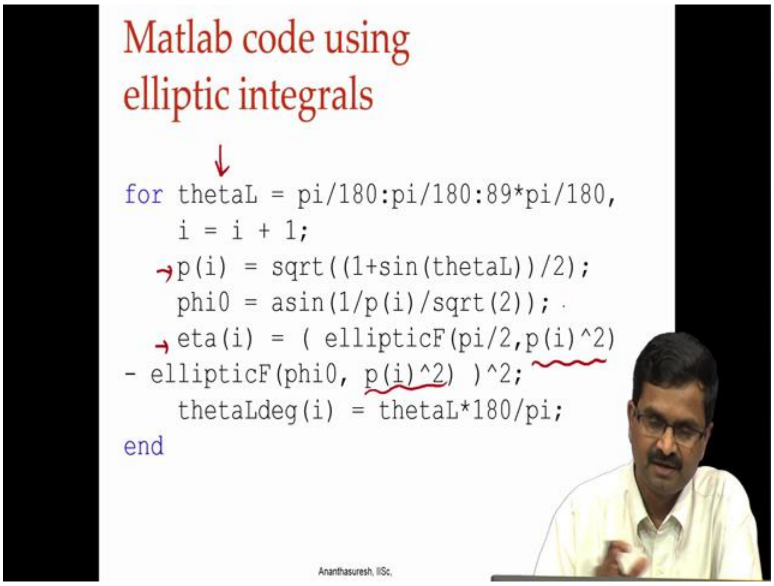

This is the code in mat lab how it to do it a little close of you can see the commands again do not forget that you to put p square there and whenever you call elliptic integral first kind or second kind, here if you notice we are putting a for look were we wearing theta L and trying to get eta and p of course, p and theta it will le to related eta needs some competition. So, doing the other way is more difficult that is were given right hand side that is eta which involves l and f and e and i getting p and then theta l is a inverse thing mat lab. Some versions have elliptic integral inverse thing some of them do not have, but I will discuss how to do that in the later part of this lecture.

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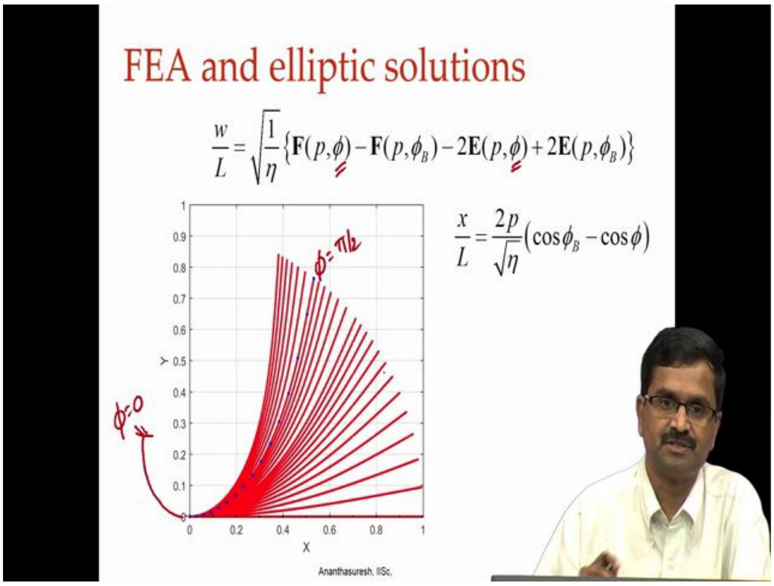

Now, here are the finite element analysis and elliptic integral solutions were we are showing for different general values of phi which were is from, if I want to go all the way back here right, when it goes like this here is where phi equal to 0 equal to pi by 2 there, this is phi equal to 0 here is where phi equal to pi by 2 if vary that then I will get w everywhere and x everywhere. So, I can put all these dots again the blue dots refer to elliptic integral solution and the red curves are from finite element analysis for various forces that we have, now if I go all the way back to phi equal to 0 I generate this curve that is what we want.

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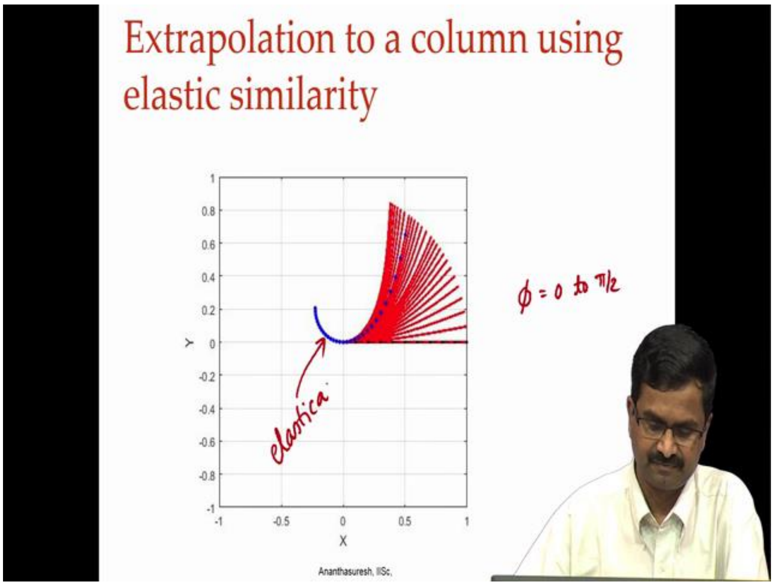

So, as we vary phi from 0 to pi by 2 we get this entire curve for some force values the other one it is a different one that will be accordingly some other curve which we call elastica curve and this elastic similarity is in exploiting this elastica curve.

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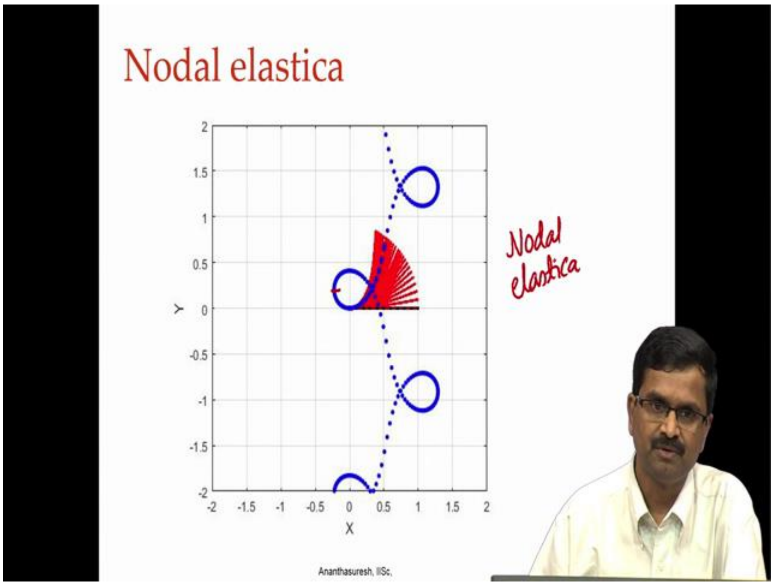

In fact, if we were to continue that angle from phi equal to 0 here, if I go negative I go back and place this entire curve if I go positive beyond this I trace this other curve. So, we get this what we had called nodal elastica. We will look at mat lab code now to see how we can see the code generating all these curves that we have seen nodal elastica by changing the phi angle let us which over to mat lab now.

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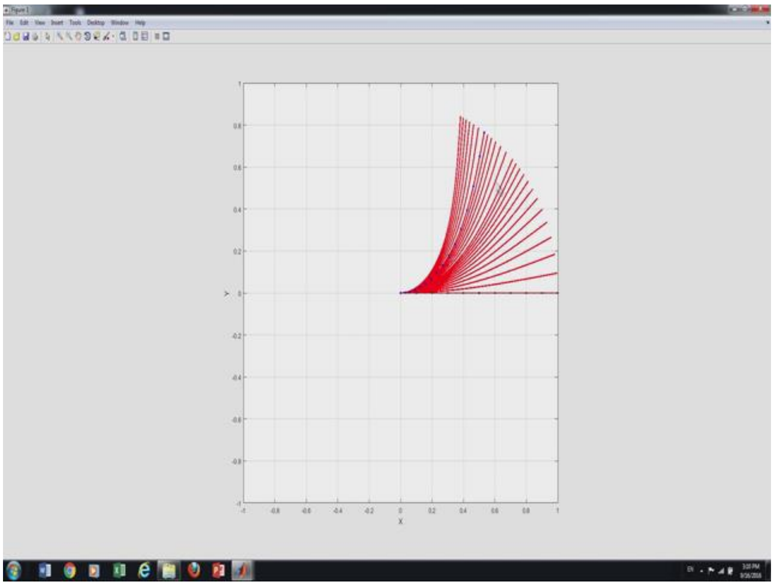

Here is the code that illustrates elliptic integral solution for large displacement analysis of cantilever beam; it uses non-linear finite element beam code written by my student P

Sivanagendra back in 2006, this particular script of mat lab combines the code had written along with the elliptic integral solutions, that we can compare what we get.

So, it calls a few m scripts that are required for the finite element core intentional beam analysis and it has some data files that we are already familiar with when we discuss this small and large displacement analysis and all this code we are familiar will be have the usual processing preprocessing. And then the code calls this n l f e n beam for the displacements for given forces the boundary condition, there is post processing there actually plots the deformed one when it is done it is going to look like this. So, here we are changing forces let me magnified we are changing force from 0 value to 2.25.5 Newton's some so forth, for the example that we have considered it is a 1 meter long beam that you can see this 1 meter origin is where it is fixed here and you can see how it deflects, now let me also show the elliptic integral that is implemented here elliptic integral solution again we have taken length one equal to 1 meter next one that have steel second point of area 5 centimeter by 1 millimeter cross section and force as much as 14, but we are actually changing the force per various values above when I shown this finite element analysis and show you were the force is force is actually change in a loop.

Here is the for i equal to size of f and small f is where 0 in steps of 0.25 up to 3 Newton's and then 3.5 steps of half a Newton up to 6 and then 7 to 12 in steps of 1 Newton. We have all that now elliptic integral 1 in this particular case I have take a particular case of 14 Newton's and try to get the w and x L here I am using symbol u L how we get elliptic element solution for the deformation of the a beam if I a look at that.

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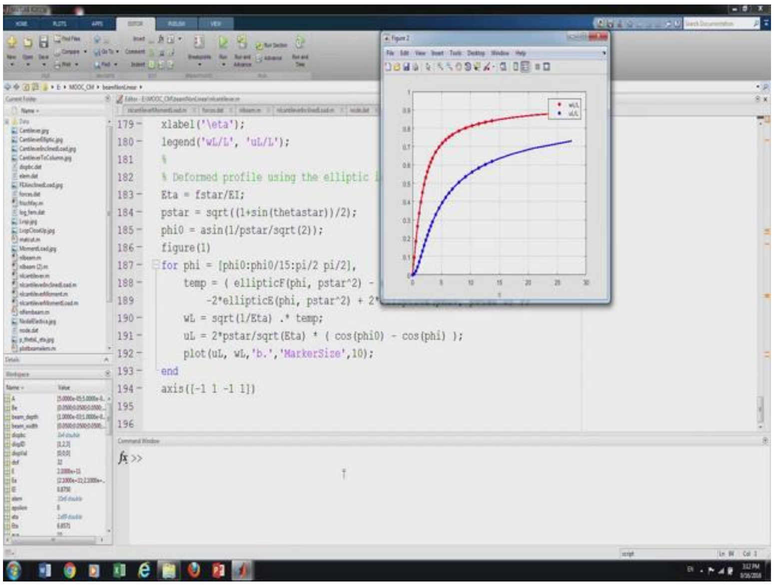

So, we can see that this particular a thing here what I am show in here are this dots are the elliptic integral solution let we make it bigger that dots are elliptic integral solution that curves are from finite element analysis. So, this is the u l or axial displacement this red 1 is the transverse displacement we can see how well they agree with each other the non-linear beam code using co rotation beam element our own in house code and elliptic and solution that we implemented over here in the same code they both are agreeing with a each other.

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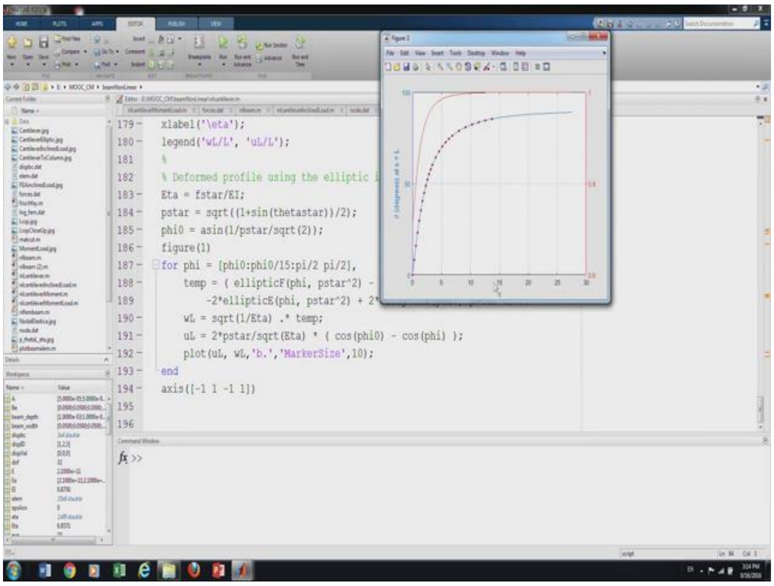

And let us also look at this particular plot were we are plotting this blue curve is that theta l that is the slope at the free end are the loaded end of the cantilever beam verses eta is very small, but x axis here is eta which is f l square by e i this plot is there in the presentation you can see the numbers clearly there, eta in the x axis y axis we are showing were this theta will goes from 0 to about 90 degrees here, slightly less than that as increase force more eta increases eventually it is going to become 90 degrees.

The red one is shown on the right side axis using that a mat lab command which gives you a way of plotting two things in the same graph.

So, these showings in the p value, the p that we have were the modulus of elliptic integral equation. So, there if we notice p starts from some were here; which is actually 1 over square root of 2 which is 0.7071 all the way to 1. So, p range is limited that we said in the lecture that we are discussing here you can see that really how p varies with eta, eta is nothing, but force once e i and l are chosen.

So, that is what we have in this particular a case. So, by looking at elliptic integral again I would like to point out that we have to put this p square there here, I am calling p star for a particular f that we are doing compare with the non-linear beam code. If I where to change this value instead of phi 0 if I were to put let us say 0 itself right and run this portion. So, let me do that and let me also because it is going to little bit out of this thing let me put let us say 2 star that let me run this portion now elliptic and solution I would like to plot from 0 to that.

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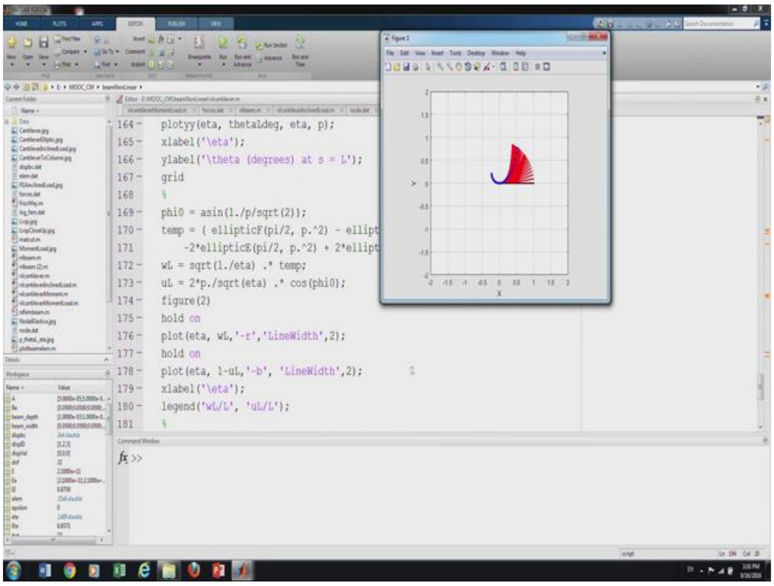

So, if you look at this solution were we are going from let me make it bigger it is previous was phi b 2 pi by 2 now we have phi equal to 0 2 pi by 2 we extended beyond this what we are said this elastic similarity if this blue dot curve, if I were to make it vertical here apply a load you will to buckle to that shape that what we see that phi 0, let us actually run this again from a much smaller value that is go negative e let I say minus 6 into pi by 2 and then here also I would go as far as 6 pi by 2 then I will remove this pi by 2 that is not necessary now will part of that. So, now, if I were to a plot it let us not make it.

So, large as 2 here let us just take this much now, if I were to run this only evaluate only the selection a let us see what happens now the phi is increasing much less than 0. So, it is still running code is running because at a value at elliptic integral we have taken about 15 that phi 0 that we have 15 step we are going from a very large a that is 3 pi 2 minus 3 pi 2 3 pi it is taking time to plot it.

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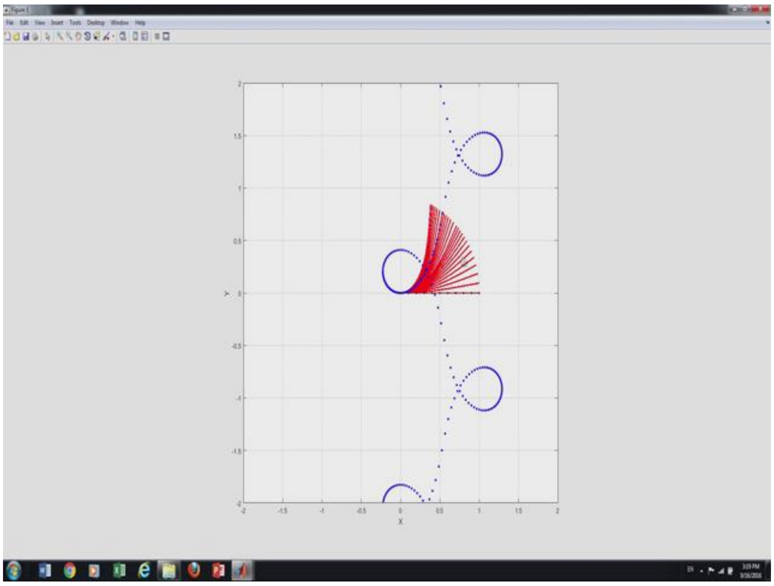

Now, you see it is showing us what is called a nodal elastica.

So, let me change this axis 2 see if there is more 2 star minus 1 1 minus 1 1 that is x mean y mean. So, let us see if that figure shows more yes we have seen this. So, or cantilever beam only this much when I go up to phi equal to 0 went when I went minus 3 pi it bend like this and similarly we are went to up here we have this we see this nodal elastica that we had mentioned when were discussing this elastica problem.

So, we can play with this and we can see all of that that all the solution that we see here depends only one value p that once we have we can get this and the p changes from curve to curve here that is if I what to take this one that nodal elastica the p is different from for that as oppose to this one and so forth.

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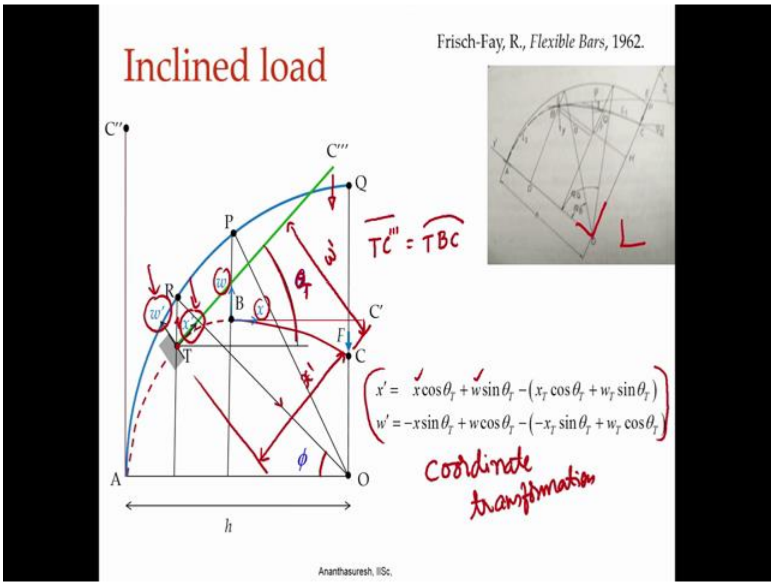

Now, let us consider inclined load case, what if there is a inclined load any beam the way we had discussed was we can take this primary beam that we have that is B C prime and get then elastica then choose a point which we called t and draw a tangent line the green line whose length will be equal to the arc length T B C.

So, the length straight lined length T C triple prime that green line here length of that is equal to arc length T B C that is the length property of this elastica curve. So, if I do that now if you see on this 1 the force is vertically down meaning that to the beam that is inclined direction, but we already know the solution this elastica which depends 1 value of P, if I have a horizontal beam of length B C prime it bends like this if extended that a phi B that have taken take it all the way to 0 to rest of the curve any point t have take that has a particular phi at go up and then this join this line I get this angle phi from phi to pi by 2 if I go I get the length T C double T C triple prime.

So, this all we can do, but if you want to know the deflection of this beam in this coordinate system that is w prime and x prime now, whatever we have this w l x l that we saw in the previous slides they are in this coordinate system x and w now, instead of this x and w if I want to go to a x prime and w prime let me switch out another color that is x prime and w prime.

So, if I do that then we will have to see how in this coordinate system we get these things with the transformation what we have here, is nothing but coordinate transformation as

Frisch Fay in his book shows as if you can turn around this coordinate system he has turned what was like this is turned here we are keeping that and using analytical equations to transform, so that x prime w prime will be deflection of this point over here that is now from here to here will be x prime and from here to here will be w prime that is what you want that we can get provided we know w and x in the original coordinates system of x and w. So, you can handle and inclined load case.

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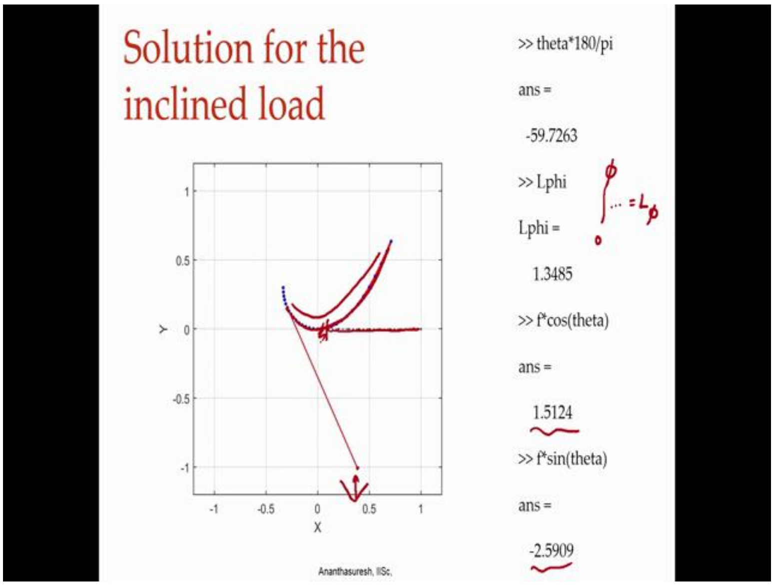

Now, if I look at the solution inclined load if I take this beam here and take that tangent you easily obtained. So, how to do that here we have the angle theta from there we get this phi from there we can get this loads.

So, if we go back up to the thing here we had this angle this angle here which is theta T if I know this theta T, I will know how this curve is and from there I can obtain the corresponding phi value because a this theta to phi change a variable we know the value, we can get that phi and once you get that phi L phi can be obtain by integrating from 0 to at phi that we got that will if I do the integration of that elliptic of the first kind were the eta is also FEI square root is there then I will get this L phi I can call it that is length from here to arc length all the way this whole length or length of the original beam I get that now, if there is a force p here then we need to resolve a component along this in perpendicular those are these.

So, if apply this forces that will same it will be deform in to the shape it will be deforming to this shape same thing that this beam which is fixed here would to there is a elastic similarity.

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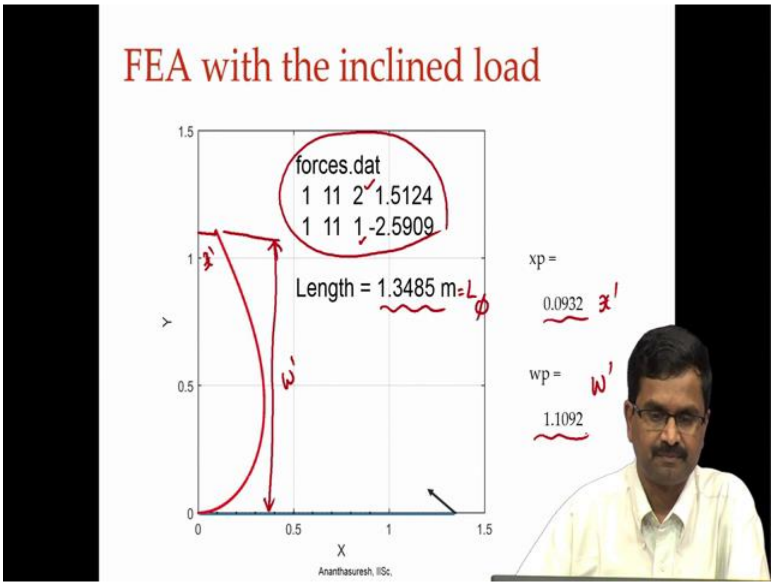

So, to see that we will see this mat lab code in a movement here if I have a beam of length now is different that is what we had called L phi in the previous slide and here is the force which was actually straight for that beam, but the straightly vertical, but here is inclined load based on that alpha that we have when we take that we have to put these new forces inclined forces these a second degree of freedom first degree of freedom I put those forces then phi bends like this just like that now, what is x prime and this is actually x prime this is w prime what are those if I take this point this would be x prime and from there to here will be w prime because transformation we have that now we look at the mat lab code.

So, we can see how if I what to take this beam I get the same values for x prime and w prime as we give the elliptic integral solution were we had rotated the thing and found out, let us look at the mat lab code now.

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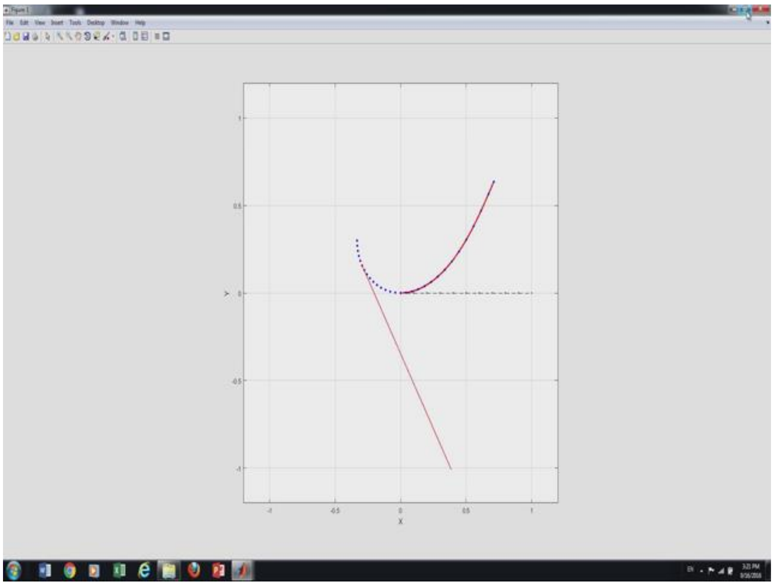

So, in this particular mat lab code we are dealing with an inclined load, we are going to show that with elliptic integral you can handle inclined load as well and again this code also combines the non-linear beam finite element code. So, we can compare the results of this. So, if you remember in this particular case if I take this, we have plotted this elastica from phi equal to 0 phis equal to pi by 2.

So, this is our beam of length one with some transverse force it is has deformed to this and this the part of the nodal elastica, in over to choose a point over here and draw a tangent if I were to fix this right there and apply a vertical force here it would deform from here to here, length of this is more than the length of this because as we know any two points I take when I do that integral that involves phi, I get the arc length.

So, I get the arc length from here to here which in un deformed case will be tangent at that point if you look at this beam there is a vertical load and that vertical load for this inclined beams an inclined load. So, we can rotate this whole thing as we just saw in the slides we can solve inclined load problem using a horizontal beam of a smaller length we have to find that length that is what we discussed in the slide this is in implementation that we see. The inclined one and one more thing is what I need to point out here which we had in the slides is getting the displacements in the local coordinate system of that inclined beam, inclined beam T what we had use the letter those x T and w T are computed once we know the value of p which I am calling here p star in the coordinate system of the original beam meaning.

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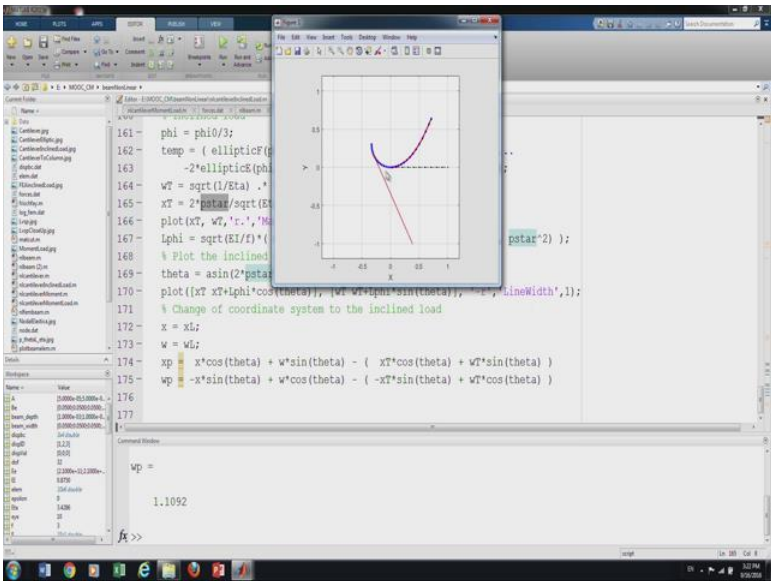

Let me point out that we have a coordinate system here this x and w we know the coordinate this point there as well as this point T were we drawn the tangent now, for this inclined a beam with an inclined load in this context I put x in this direction w in this direction, then what is it and that is given by actually this transformation that we have discuss in the slide that is what it is and these are the values this x p let me make it little bigger here so anyway, X p 0.0932 w p is 1.1092 which we can check with a regular non-linear beam code which does not have a elliptic integral, but one more thing we need to know we are the forces we put those forces then we can verify that this beam would actually deflect to that shape.

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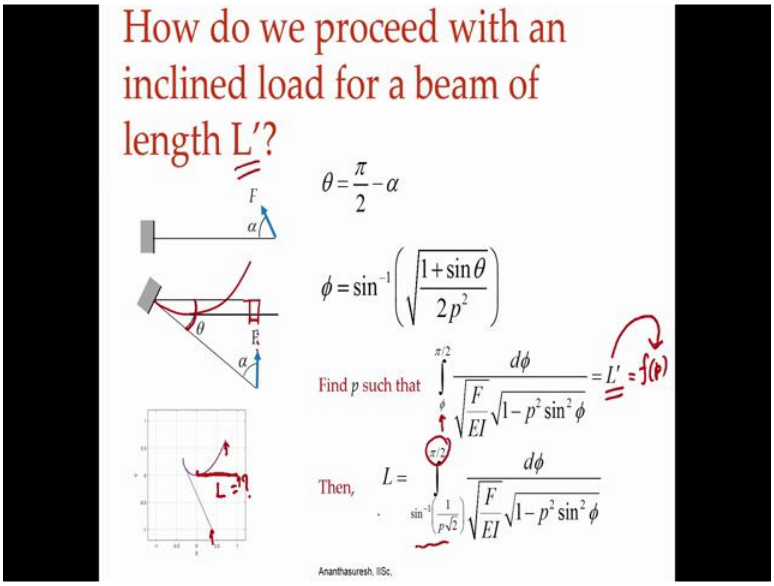

Now, if we see in general; if you are given a beam of length L prime and the load is given at an angle alpha how do we proceed in order do that what we do is we have to imagine that we have to turn it around. So, the beam inclined load should be like this. So, that we get the equivalent beam which is horizontal and that has a length L we want to know what that L is for that what imagine that we rotate this beam.

So, that this force becomes vertical what is inclined rotate it until this force becomes vertical, then we have this curve that we want to find what we know this angle theta and theta and alpha are related how are the related they related as theta equal to pi by 2 minus alpha, because if I complete this triangle if this is 90 degrees here this is 90 degrees that is theta alpha is pi by 2 minus alpha, theta is pi by 2 minus alpha. If I know theta I can get phi using this change available equation once I get phi, we had to find phi such that this integral from phi to pi by 2 is equal to L prime that is a given length of the beam, we have to solve this were we have to have this inverse elliptic integral function which all versions of mat lab does not have.

But I will show you how we can solve this equation never the less, we find p because p is here sin square phi that also is p and over all integration we do and this has from phi to pi by 2 that is in pi p everything this whole thing is a function of p now given L prime we need to find p that is equation we need to solve, once we do that we can get the length l that we wanted that length l were the force will be vertical.

So, we know that force acting here is same as force acting here to get to that and we get this elastic similarity or elastica solution, we can find l which goes from this angle theta equal to 0 corresponding phi which is shown here going all the way to pi by 2 gives us l once we know l we know how to proceed the problem that is how we do it. Now, let us consider a moment load also now there is a transverse should we have consider to be discussed both the straight (Refer Time: 33:39) vertical transverse load as well as inclined load now let us consider the moment load, as we had discussed second lecture we have this extension of the rigid lever.

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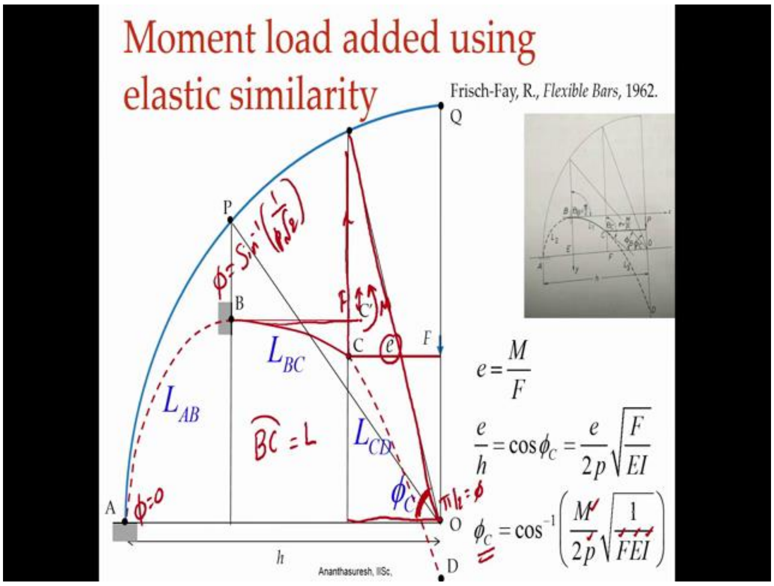

So, we have this extension of the rigid lever e and that e is chosen such a way that there is a movement and the force are both felt at the point C now, here is where the large displacement analysis concept comes to clearly in that in large displacement analysis we do the equilibrium the deform configuration. So, if I the beam has bent like this at point C if there are about to be transverse force F as well as a movement for this point we want apply a transverse force as well as a movement, if I want to do that that movement should be such that there is a force which is displaced little bit to the right.

So, that F times e will be as the movement at this point the deform configuration at this point this movement will be acting that is a reasoning behind this e and from the geometry here we can see that e by h this is e and h is this 1 e by h is cosine phi c and cosine h is equal to 2 p by square root of F over E I. So, we basically get cosine phi are we can calculate phi C from here what where saying is that when a beam is like this here we know that phi is equal to sin inverse 1 over p square root of 2 were as and here it is pi by 2 here it is pi by 2 that is fierier that point A phi equal to 0 what is it for point c were we go up like this and join that is my phi C.

How do you I write phi C here this is how we compute based on the movement value that is given force value young’s modulus second point of area and p that we found, as we have to find were to find p what is the equation for that we know the length of the beam from B to C. So, arc length from b to C is the given length L that is the problem we want to solve there is a beam of length L with force F and movement m and then we get this in terms of phi C will see how we can compute.

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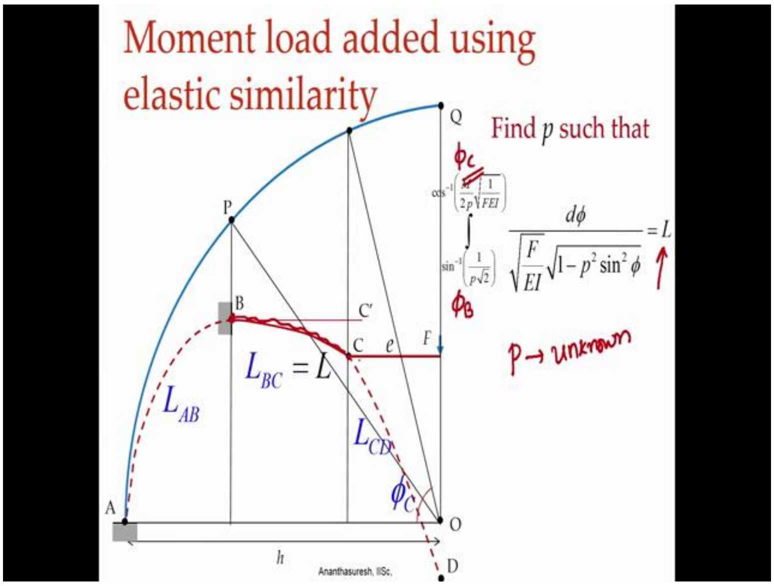

That is if I go back and see this now we have to find p in such a way that when I go from phi B that is what this is to phi C is I go from here to here along this thing here when I do this elliptic integral of the first kind I should get back l again we are given l we have to find p here, P is the unknown for every Elastica here is this characteristics modulus p that decides the elastic is going to look like and p there p end load and that can of decides everything here.

So, we have to find the p normally at the end would have been p equal to pi by 2, but now we have a different expression phi C based on the equivalent movement that will have force P force F here another movement which is F times e were to find this p that you point p such that this equation satisfied if we do that, let us take an example.

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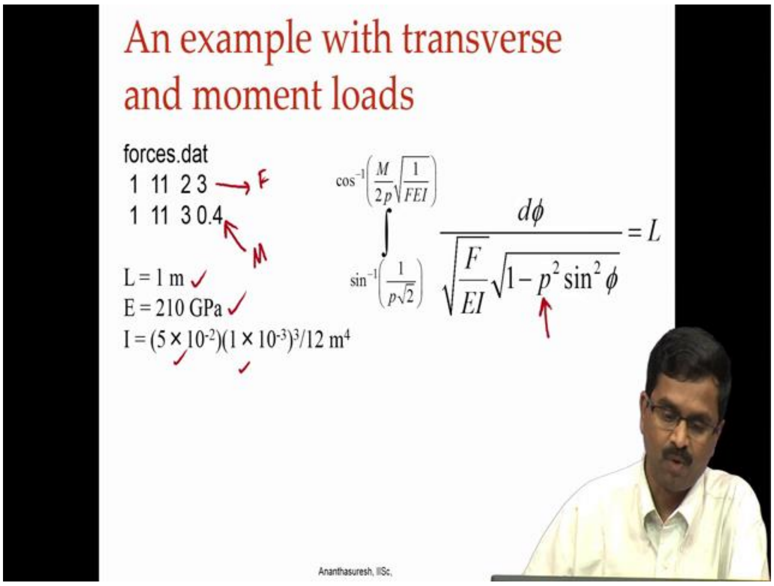

So, were we have the force F here and then movement M here 3 Newton's 0.4 Newton meter the same length same young's modulus same cross section that we have taken we find the value of p, how do you do it, it is actually quite simple because range of p is only from 1 over square root of 2 2 1 in that range solving this equation is non-linear we plot it and see how it looks and we can calculate the value of p were length is in our case 1. So, let us plot it have plot it using mat lab so from here to here.

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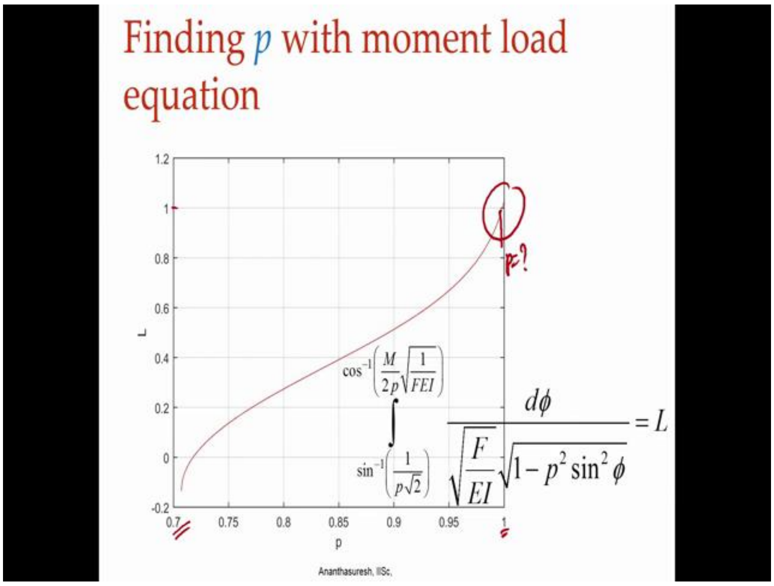

So, we can see it varies p from 707.707 and 2 1 and length is here and it go here that is will length equal to 1 at find the corresponding P value this so close to that this part we will zoom in and try to see and it becomes 1. So, here is where it became 1 and then have to find the corresponding value of p which is 0.9984, it is very sensitive at go up to four decimal places to get it right and you can see that it is only the fifth decimal place that is changing.

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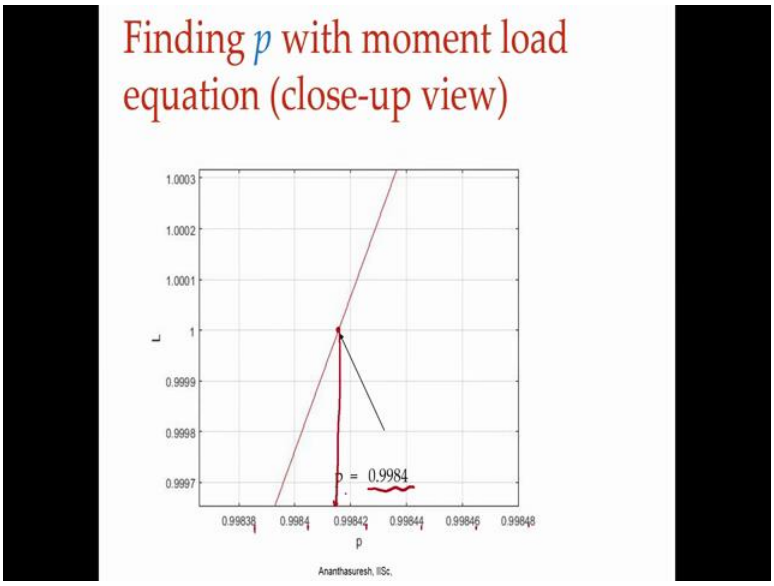

So, we take that P value here that is how graphically we were solving, but you can also solve it computationally mat lab you can use a find command and see when l is closes to one meter that the length here what is the corresponding p if you do it in a loop that is what we will look at to solve this problem using mat lab.

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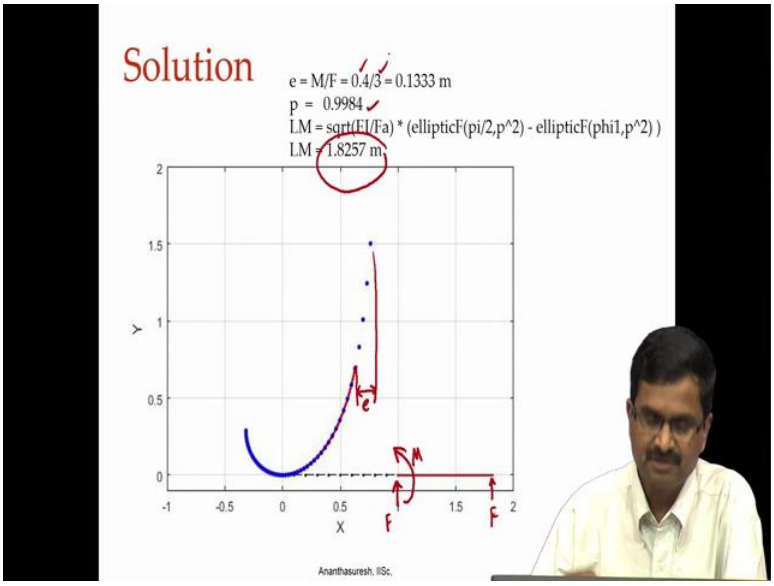

This is a solution were that moment load one gives you this curve that is at this point apply a force f and the moment this force and moment that is equivalent applying a beam of length 1.8257 as we will see in mat lab, if I have to apply force F here it be like this and what is e here e is this horizontal span this is our e because that force F times e is what will be the movement that will do felt here in the deform configuration of the beam were the beam is straight with these is equivalent to a longer beam this length we can compute as we seen mat lab this will be 1.82557 meters, then p equal to this value m is 0.4 f is 3 we get this solution, let us look at this example in mat lab to understand what is happening.

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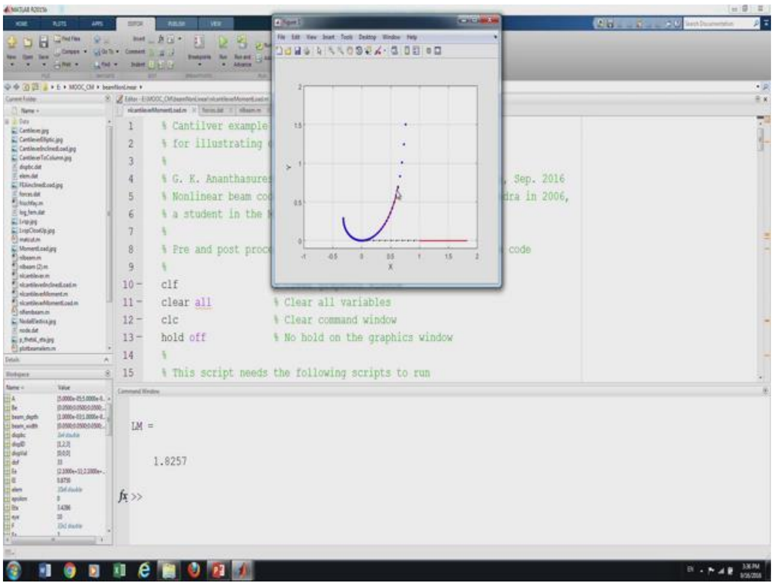

Now, let us look at the case of a transverse load as well as moment load using an elliptic integral solution, once again will compare with non-linear beam code and what we see here this l M is 1.8257 that is if we look at the solution will find out what it is. So, here elastica is blue dots it is starts from phi equal to 0 and our interest is from where this phi b were theta is equal to 0, we can see that red one is a finite element one we are plotting from here to here were the finite element one, if I look at the forces data file we have second degree of freedom 3 Newton’s transverse force and then this the theta degree of freedom that is the transverse displacement here we have 0.4 Newton meter.

So, the 2 degrees of freedom have the loads here for a beam of length 1 meter that we had. So, now, if I look at this we see that the equivalent force here there is if aware to apply a force over there for something that is as long as 1.8257 meters, if apply a force here at this point I would feel that force of 3 Newton’s as well as force times this one in the deform configuration this e that we have taken that is whatever that we have here that e will be the movement felt and we chose that e. So, that the movement will be 0.4 Newton meter that we have taken in this particular case, the p is for the elastica curve we need to solve that equation that we had seen in the slide.

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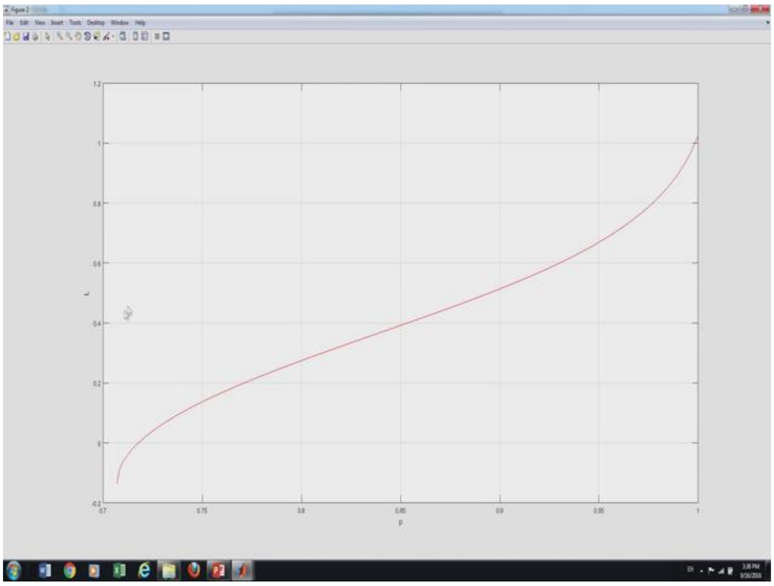

Here we have plotted l verses this p when l were as is 1 meter. So, 1 is over there. So, let me zoom in to see what p value that we need to take. So, if I what to zoom in around 1 this is graphical way of solving an equation were we know the right and side we are get the modulus of the elliptic integral write do this few times then I would get that.

So, here up to 4 decimal places 9 9 8 4. So, let us go back and looking the code of this moment one to see what p value we need to take for the elliptic integral solution, as said we can actually solve it use in bi section method, but now we are doing this graphically that p value is 9 9 and 8 4 we put it we get this if I were to change it let us say I take of 2 decimal places and now let me re plot it to see what the effect is this one that a valid that part of selection when it is done will see how it would deviate. In fact, it does deviate now we can see I will make it bigger.

So, what was like this now moves slightly away a from the finite element solution which is the correct one. So, we need to take P 2 high accuracy in this case we have taken up to 4 decimal places were the finite element solution and elliptical solutions a agree each other.

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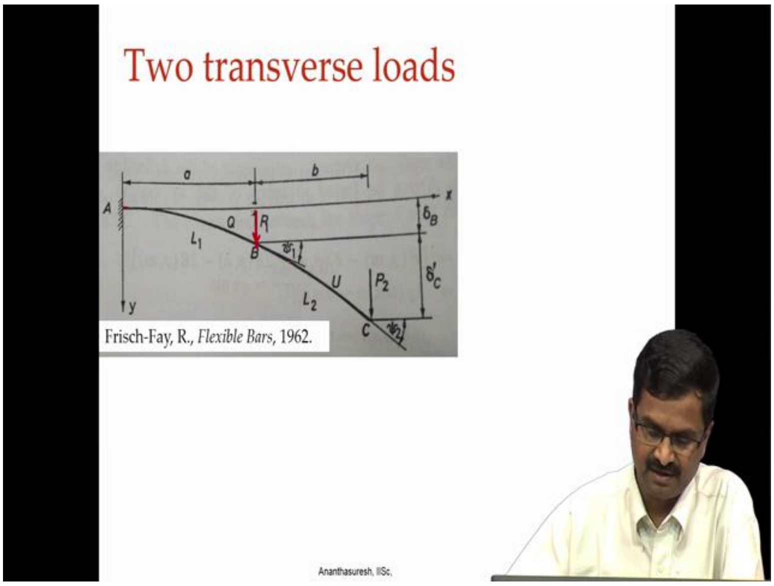

If there are two transverse loads then we have to basically have at P here that P 1 that we have on the movement we to P 2 should felt here rather at this point b there will be affect of forces P 1 and P 2 and the movement, were as this one feels only P to except that these cantilever with slope equal to 0 here the cantilever that the slope which is non 0.

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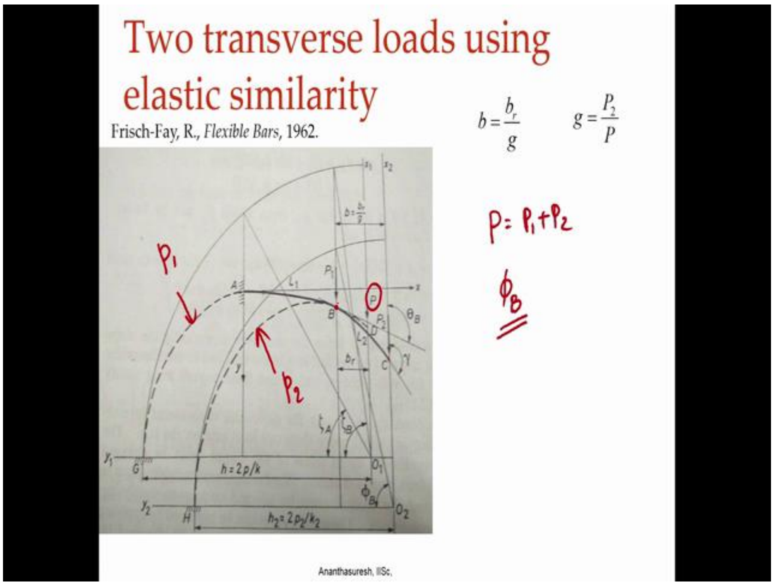

So, what we have here in this case would be an equivalent force P which is equal to the sum of two forces that are applied on these beam P 1 plus P 2 for this portion for this Elastica that calls these there is a modulus P 1, for this Elastica there is a modulus P 2 because the rest of the portion behaves differently because, this one we just a transverse force and a movement this one is only transverse force from here to here another elastica for this one we know the final angle phi is pi by 2 were as for this one we have to find phi b.

So, correspond to each force there is a p modulus that is not known for except the last one this phi end will not be known. So, we have the, this is phi b in this case we do not know phi end is not known.

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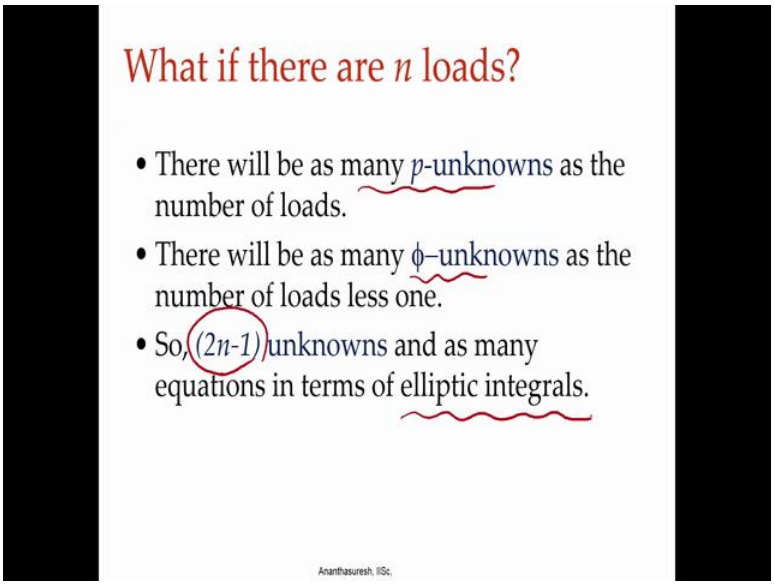

So, if have let us say n loads on a cantilever beam there will be as many p unknowns as there are the number of loads and number of phi unknowns will be one less because for the last 1 phi equal to pi by 2 will be 2 n minus 1 unknowns we have that many equations in terms of elliptic integrals and we can solve.

So, what we need to note here is that a differential equation if you what to solve using finite element analysis there will be many degrees of freedom as many as number of nodes times, because a 2 d b involvement will have x displacement y displacement and a rotation. 3 times a number of nodes are the number of degrees of freedom in finite element number of analysis, were as here we can reduce it to 2 n minus 1 equations and variables were n is a number of loads even if there are 5 loads will only have 2 times 5 minus 1 that is only 9 equations.

So, 9 variables and equations then elliptic integrals, but note that the p factors p modulus values are only going to were from 1 over square root of 2 2 1 that is not difficult to find and the angle phi also is not going to change too much will be usually between 0 to 90 degrees when a beam bends with multiple loads of course, there will become complications when one force is down other force up the beam develop points we can inflections which will discuss in other later lectures.

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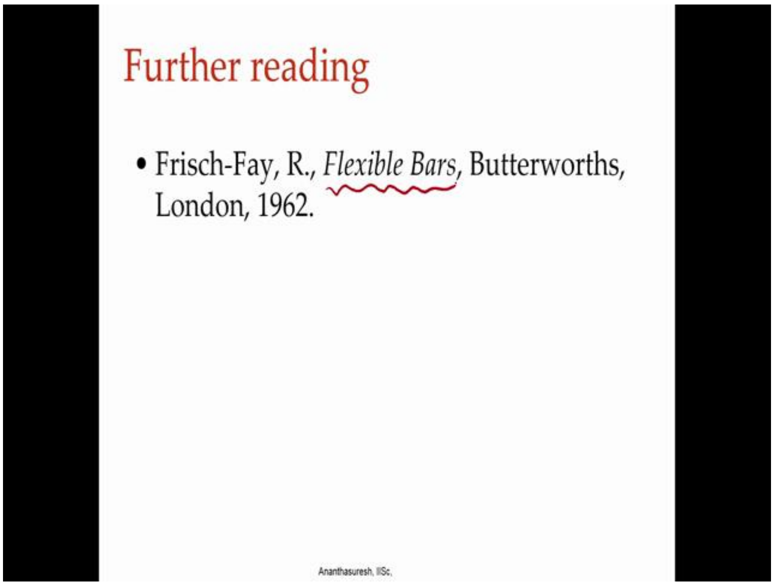

If you want to know for the all of what I have said this wonderful book by Frisch Fay con flexible bars where you can find lot more information done what we have discussed in the last two lectures and today's lecture in this week.

Thank you.