Indian Institute of Science

Variational Methods in Mechanics and Design

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NPTEL Online certification Course

Hello again so we discussed the connection between optimization and variation methods and finite very optimization problem and calculus of variations and this being an alternative to what we call factorial methods or force balance for statics and Newton's second law for dynamics right but then just as there is this optimization in mechanics we also optimization in design or structural optimization a typical problem looks like this is what we discussed minimize an objective function with respect to some variables objective function is going to be function of variable optimization variables as well as some what we call state variables they are governed by some equations which will be constraints and there will be constraints and resources.

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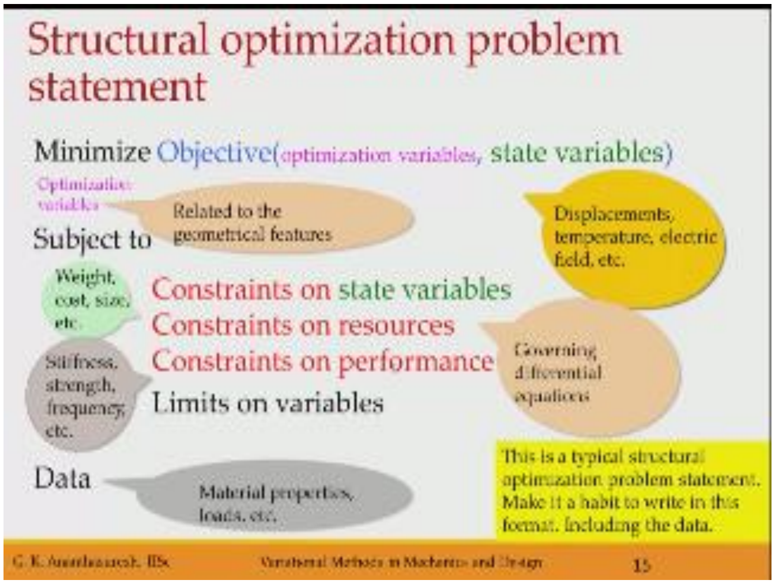

And then constraints and performance their limits on the variables all of these make up a optimization problem where there is conflict between the objective function and constraints are different terms within the objective function and so forth okay.

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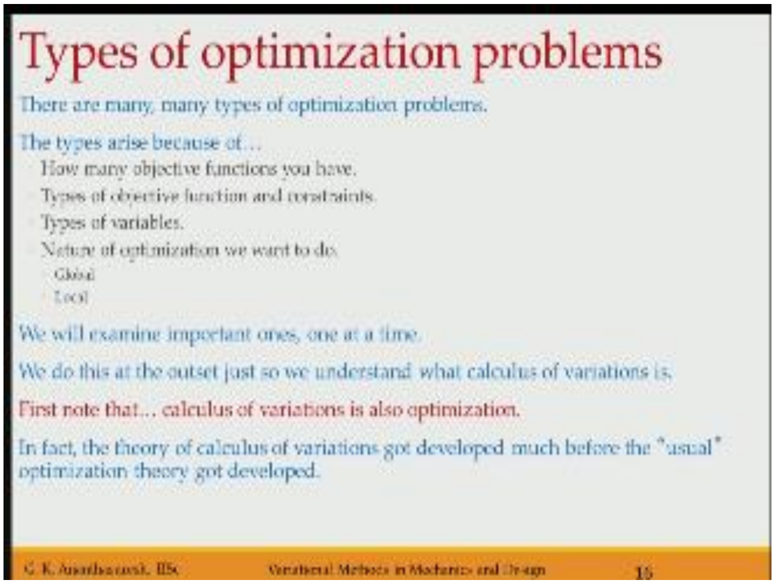

Now let us look at different types of optimization problems there are many many many types of problems in operations you can pose the difference among all these types comes based on objective function based on constraints based on variables and the nature of the problem itself whether it is global or local okay a number of problems will examine all of these one at a time okay.

But we also need to focus on the difference between calculus of variations and optimization in finite number of variables that is our objective today to understand how calculus of variations is different from finite variable optimization both are optimizations but there is a difference that is what we want to understand in their process we also want to look at different types of optimization problems.

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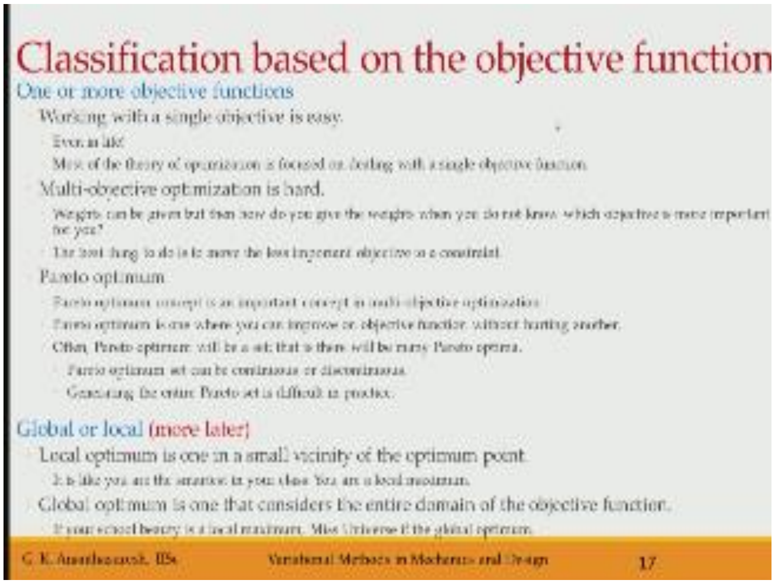

You can classify based on the objective function okay. Objective function if there is only one single objective that is very straight forward you have only one objective one goal you can achieve it but it is not always the case we have multiple objectives just like in life we have many objectives you want to do that we want to do this and to 12 to several things at the same time there we want to define the notion of optimum further there is a concept called Perryton optimum those of you who would study multi-objective optimization will definitely have to understand what this Perry to optimize optimization or pirate optimum is okay.

Overall in this course will be sticking to only single objective that is only one objective will not deal with multiple objectives but just know that there is a single of the optimization there is also multi-objective optimization okay the other is whether your optimum is global or local okay what does it mean or you first in your school or your first in entire country okay if you are first in entire country or even entire world then you are a global maximum if you are only first in your class then you are a local maximum right that is what we mean by global and local will be discussing more of it in later parts of the course okay.

Local optimum is an optimum only the vicinity in your neighborhood you are the hero you are the best okay global optimum means that you are the best among all the people who existed let us say all the time right then you will be a global optimum right so you have to understand the difference between global and local optimum okay, that is there are global optimization problems there are local optimization problems there are single a bit of optimization problems there are multiple objective optimization problems that is one type of classification.

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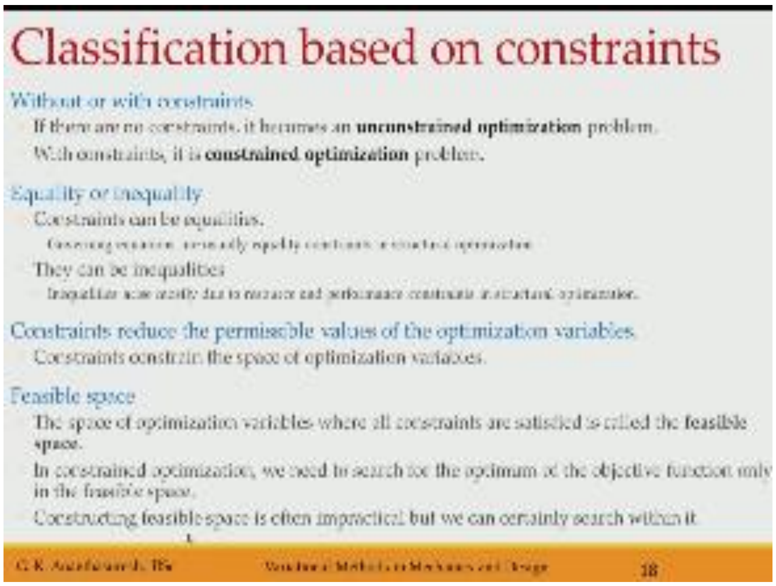

You can also classify based on constraints you can have optimization problems without any constraints in which case we call it constrained optimization there are no constraints okay when you have constraints it is constrained optimization that is a classification based on constraints whether you have constraints or not you get two different types of problems when you have constraints and B equality okay.

There is an expression equal to something usually we write it as 0 that will be equality constraint problem you can also inequality constraints less than equal to 0 greater than or equal to0 of an expression okay so constraints can be equality constraint inequality constraint and then you have constraints we talk about what is called a feasible space meaning the space you have a search space whenever you are optimizing your variables will be defining a search space if there are two variables x1 andx2 your search space will be a plane a2d plane okay.

Or a surface like a spherical surface or a conical surface where you have two variables x1 and x2okay but then when you have constraints part of that space will not be permissible will not be available to you then we define this concept of feasible space that is every point in the feasible space is going to satisfy your constraints be them equality's are inequalities okay whether they are equaling equalities all the tension satisfied by all the points in a space which we call feasible space okay. So first you have to construct a feasible space and search in it to find the best that is what we need to do in constraint optimization.

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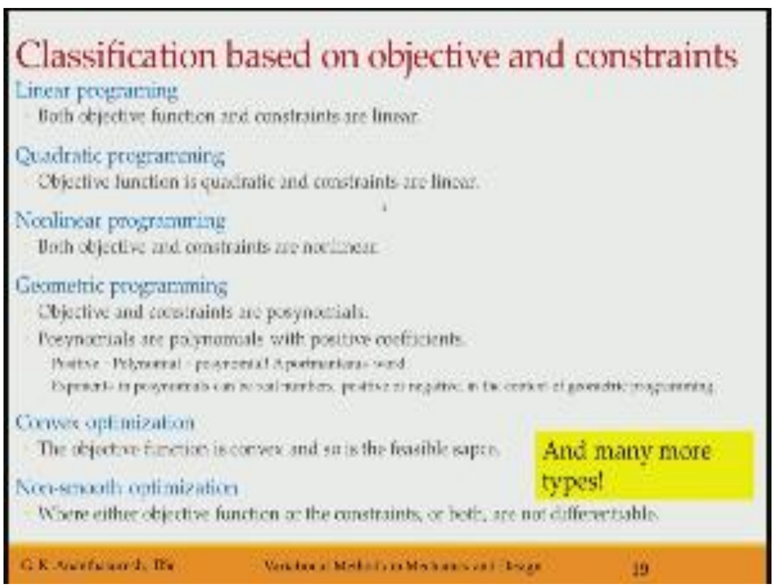

You can classify them based on the nature of the objective functional constraints for example if you were constraints let us say objective function and constraints all of them are linear we call it linear programming problem in operations research most of the problems are linear that means that objective function depends on the design variables in linear fashion okay you only have X 1 X1 X 2 X 2 X 1 X 2 X 3 as linear variables and you do not have terms like x 1^2 R X 1 times X 2 will not be there okay.

That is a linear programming there is something called quadratic programming where your objective function is quadratic that means it will have terms like $X 1^2$ $X 2^2$ or X 1 X 2 things like that they are all quadratic okay but constraints are still linear in a quadratic programming problem the word programming itself is not computer programming although up to manage problems are solved using computers that is not computer programming here it is just that optimization process is called programming it is a it is a historical word that just was coined and got stuck.

It all happened before the actual programming computer programming started okay linear programming has nothing to do with IT in a way okay it is a still need to do using computer to solve this problems but that programming is different from coding that we do in computer science okay there is no linear programming where objective function and constraints are nonlinear in terms of variables and then something called geometric programming where the objective and constraints are these nota spelling mistake it is it is not a polynomial it is a posse Nomial posse nomial is a polynomial with positive coefficients if you have $ax^2 + BX + C$ where ABC are all positive such a polynomial called polynomial if all of them are polynomials that is positive polynomials then such a thing is called geometric programming it has some special properties that one can exploit to solve such problems.

there is something called convex optimization where your objective function constraints are all convex okay then you have convex optimization there is also non-smooth optimization where objective function or constraints are not differentiable meaning that they do not have a well derivative or gradient at all points such problems are called non-smooth optimization problems there are many more categories that one can have for operation problems based on the nature of the objective function the constraints that are there in the problem.

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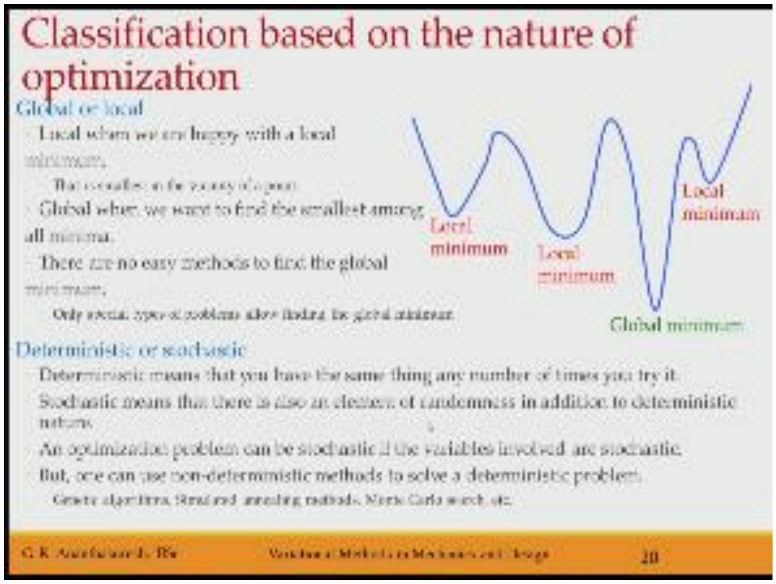

This thing about global and local we discussed it earlier but here we are showing in the pictorial fashion if I have a function this blue curve coming down going up reaching a maximum reaching a minimum another minimum another maximum another minimum another maximum another minimum all these minimum points are what we call minima as a plural there are local minima only the vicinity they are minimum if I look only in this region that is a minimum if I look only in this way that is a minimum if I look at this region this is the minimum.

If I look at the entire region here then these are not the best minimum the best one is here that is a global minima global minimum is also a local minimum just like if you are the first in the country you are also first in her class right it is also a local minimum it is a global minimum it is the least among the local minima be called global minimum so you can optimize globally or locally okay, and the variables that you have can be deterministic or stochastic there is another classification okay.

So your variables are all deterministic meaning that they have definitive values you have deterministic optimization most of the engineering problems will have deterministic optimization okay but if you look at practically because any dimension that you take any metal property you take they are not going to be precisely some value there is a little bit of uncertainty then you can do stochastic variable optimization so that is another classification or deterministic or stochastic.

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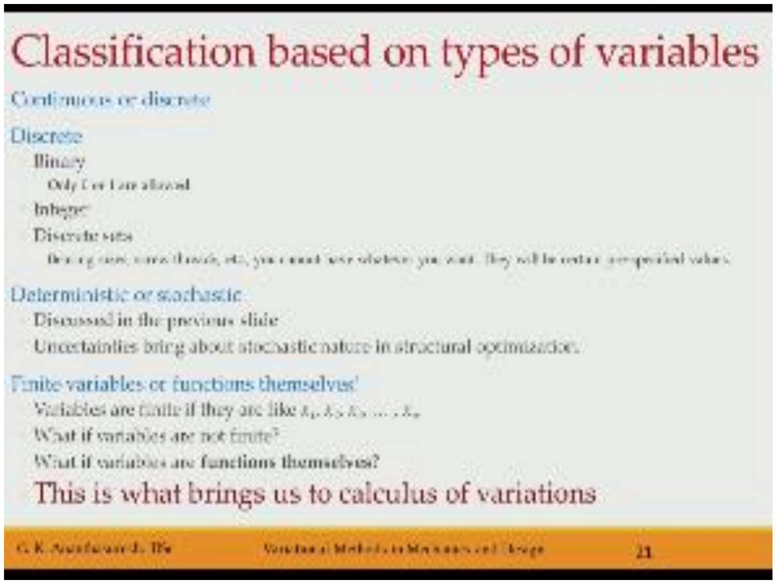

And the variables themselves can be binary for example they can take values only 0 or 1then you have binary optimization are you can have the variables taking only integer values and then you have integer optimization are they will take discrete set of values but not necessarily integers if you have such a thing you will have discrete optimization right if you are doing let us say mechanical design you cannot have bearings of any diameter that you want what is available in the market will be a few diameters or screws length of a fastener or gears whatever you take there will be a certain variables values then that is a discrete optimization okay.

So variables themselves can be as we already said deterministic or stochastic okay all these are finite variables meaning you have x1 x2 x3 up to x n okay these variables are finite in number okay there is in this case n that we have shown okay this is called finite variable optimization and all these other classifications local global constrained unconstrained discrete binary integer quadratic programming stochastic terminus take all those are various types of optimization problems

But now what we want to focus on this calculus of variations how is it different from the rest okay.

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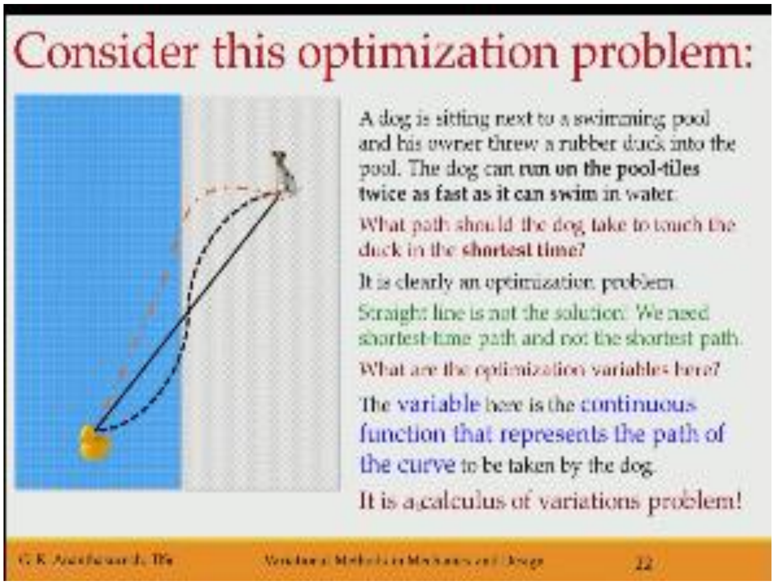

For that let us take this fun problem if I may say there is a swimming pool in the water there is a rubber duck that a kid has thrown in and then the child's dog is here which is asked to fetch that rubber ducky okay so and the child is impatient he wants this rubber duck as quickly as possible so the dog has to reach this rubber duck in least time okay, least I meaning the lowest time or the quickest right there is a sense of optimization which path should this dog take in order to reach the rubber duck will it go along the straight line that is the solid black line which is the minimum distance path from the other dog is to be the rubber duck is or should we take a different path why will you take a different path why not the least distance path because dog can run on the swimming pool floor at some speed.

And it can swim in the water at some other speed which is lower than what you can do by running on the swimming pool removable flow right so if it takes like this it will be taking a long distance in water so instead dog comes here and then goes this way you can be shorter right so for example this dashed line one this path looks shorter in water compared to this line this line segment from here to here I do not think dog would do this unless dog issue that it can run fathom in water faster than running on the flow okay but there are lot of possibilities right.

So such a problem is different from finite variable optimization problem here we do not have x1 x2 x3 x4 + xn okay we want to find a particular path if the dogs take that takes that path it will reach the rubber duck in the quickest time the least time okay the variable in such a problem is a continuous function that represents this curve okay that is the essential difference between finite variable optimization and calculus of variations okay.

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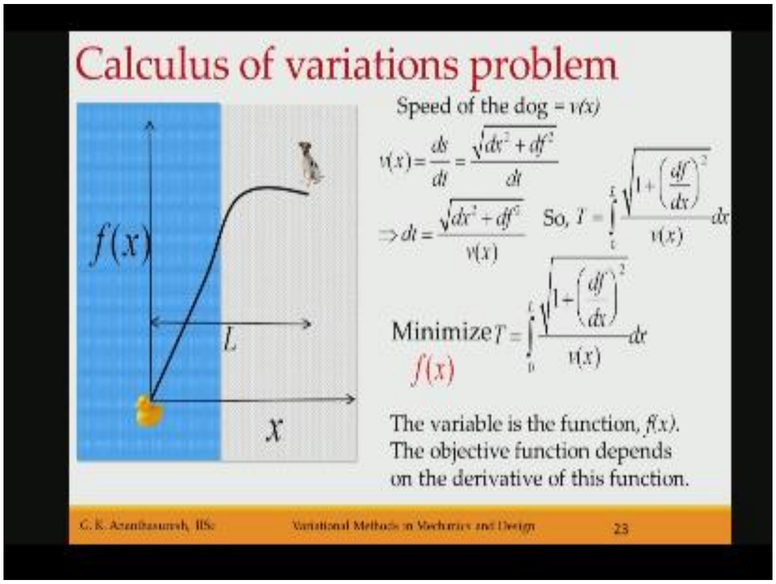

If I put it mathematically like I said lost like in the last lecture that optimization is mathematical everything an analytical right we call it analytical mechanics or analytical dynamics optimization is very analytical very here we can pose problems mathematically okay for this if I denote speed of the dog as V X okay that is as a function of X at different values of x it has different speeds there is some speed in the flow some swimming speed in water that V X is given let us say that VX is DS by DT the speed right so S is the value that goes along the curve okay.

This way or that way DS by DT that DS can be written as using Pythagoras theorem square root of DX square plus D F square that is f is a function that I am taking if I take any little thing there will be vertical DF and then DX that D s is that okay so square root you take this speed and try to find that time DT to traverse a small DS okay we have the DS that we have here divided by the speed that we have then that equation basically gives us DT so I have taken DT that way and try to get this now they have the distance that I have divided by that speed gives me time okay.

This is a distance DS divided by the speed gives me the time DT I want the total time for the dog to go from here to there then I integrate that so we get these integrals in calculus of variations now I am integrating this DT from 0 to capital time T capital T the time t that is equal to integral of this quantity and I have done little thing here I have divided the two terms by DX square okay that means this is going to become one and it is going to become DF by DX square and then the square root of DX cochin the thing that we have their web multiplying dividing if I say that has come out here the DX.

Basically I have taken DX out of this and VFX is still here I got an integral okay so now what do I need to do I want to minimize this time of going from where the dog is where the rubber duck is so I have 0 to L square root of 1 plus DF by DX square by V X DX and what is my variable the function itself what we showed in the previous slide right there are many possibilities of paths from where the dog is to wear the rubber ducks and that function is my and that is the nature of calculus variations okay.

It look it may look at the first sight formidable but in a dog apparently does that people have done this fun experiments go into the beach and throwing a Frisbee into ocean and the dog was observed to take the quickest path rather than the least distance path which is the straight line okay so dog knows how to solve calculus variation problems such as this okay that means there is sense of purpose for the dog and there is a sense of purpose here and that is what we are going to discuss.

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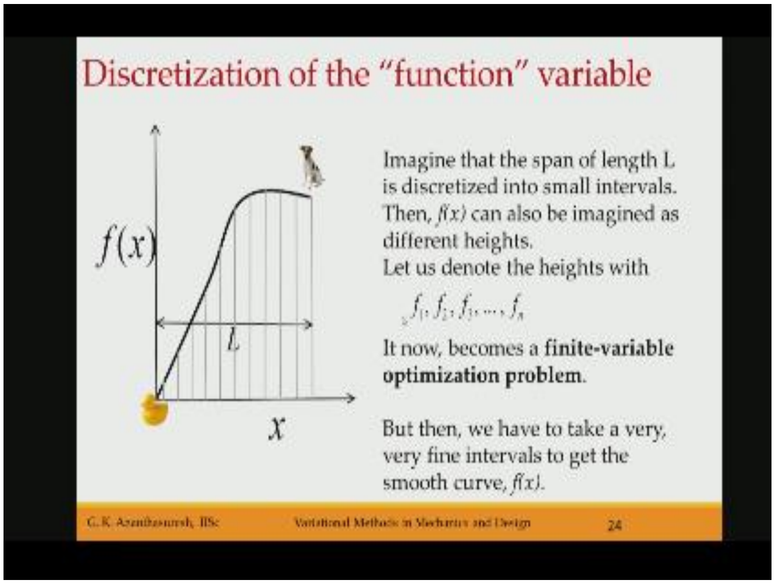

How do you solve the problems like that a problem that a dog can solve okay one way is to convert that into discrete problem that is we want to know where the dog should be for different values of x as it starts from here and goes to the rack which is at the origin let us say so I want to know what is this point it is here what is that point that point I am discretizing along the x-axis so I want to know not the continuous function but I want to know what it is here, here, here, here what is that function what path should I take when I take that then this f1 f2 f3 all these hides of this become my variables then that becomes finite variable optimization problem.

As opposed to calculus revelation problem where the function itself is an unknown okay that difference has to be understood very clearly at the outset of this course calculus of variations has unknown variables or optimization variables as functions are not as discrete variables.

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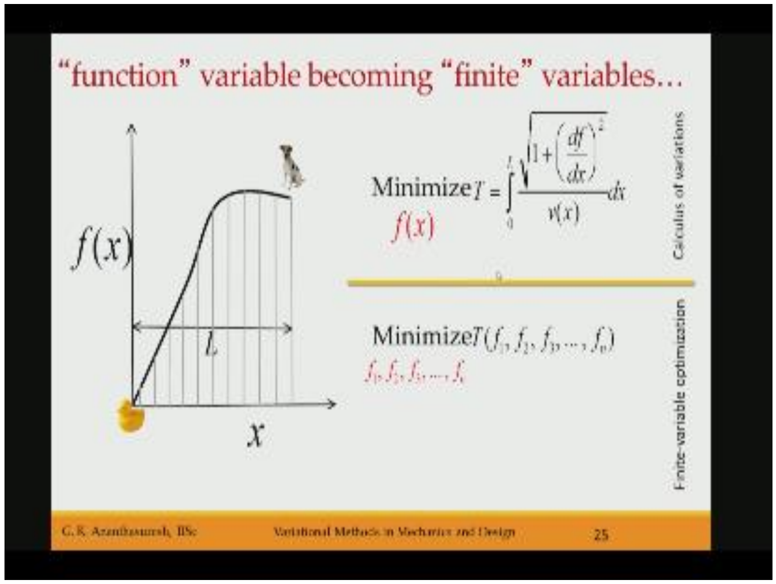

So here if you take calculus facial problem we have an integral minimize T which is defined in this particular manner it depends on the derivative of this function f + V of X is given to us we are optimizing this are minimizing this with respect to the function itself as opposed to discretize problem we are minimized respect to finite number of variables f1 f2 FN and you an objective function with a function of all of those just like here it is function of the objective function WT function here is a function of f of X here objective function T is a function of these finite set of variables this is normal optimization what we call finite variable optimization.

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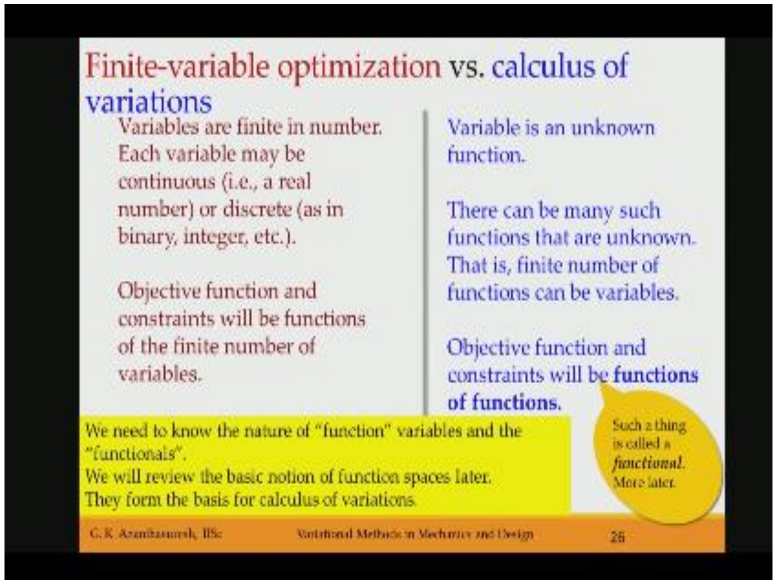

This is calculus of variations okay so let us look at the similarities and differences in finite variable optimization you have variables that are finite in number that is a few of them 1 2 up to n each variable may be continuous it can be real number or a complex number a discrete set of values like binary integers of whatever here unknown is a function okay there can be many such functions that are unknown just like we have finite value variables many here also you can have many functions if you want okay and in this case objective function constraints are functions of this finite variables here there are functions of these functions okay.

Objective function and constraints are functions of functions which you call a functional okay but calling them functions of functions will be actually incorrect as we will learn later but know that not only it depends on the unknown function but like in our previous example in the dog duck problem it can also depend on the derivative of the problem here it does not depend on f of X depends on DF, DF by DX okay so objective function constraints here can be functions of the function and its derivatives okay the notion of a functional which is what we call this integral we will deal with it later. But understand that it can depend on the unknown function and its derivatives okay.

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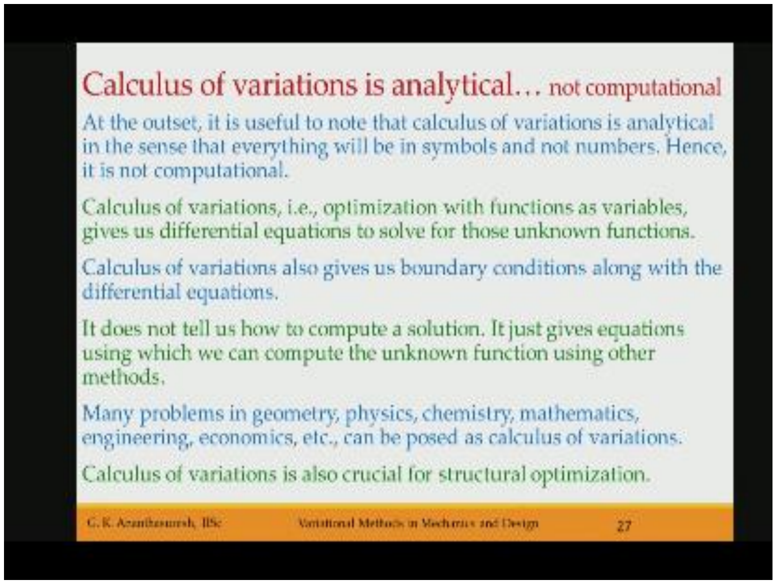

And again I want to emphasize that calculus of variations is analytical it is not a computational technique it is an analytical technique where you are going to write everything in terms of symbols okay because the function is unknown the function is unknown you are going to have a lot of symbols okay in order to solve it using computer we have to establish some conditions for optimality necessary condition sufficient conditions and that is how we will move on okay.

Calculus of variations when you establish the conditions of optimality it will lead to differential equations whereas if you do an optimization problem in terms of finite number of variables you will get algebraic equations okay whereas when you deal with calculus of variations we are going to get differential equations as the necessary conditions and it will lead to boundary value problems and initial value problems with boundary conditions and initial conditions okay.

And if you want to compute a solution then you have to solve those equations okay the computer when you do it you might discretize that is a different matter but what calculus of variations gives you is a set of differential equations they can be order differential equations they can partial differential equations many problems in geometry mechanics physics chemistry economics in all branches of science and engineering and even humanities you can deal with optimization problems and calculus of variations problems also and for structural design or design in general calculus of variations is very important as we will see in this course okay.

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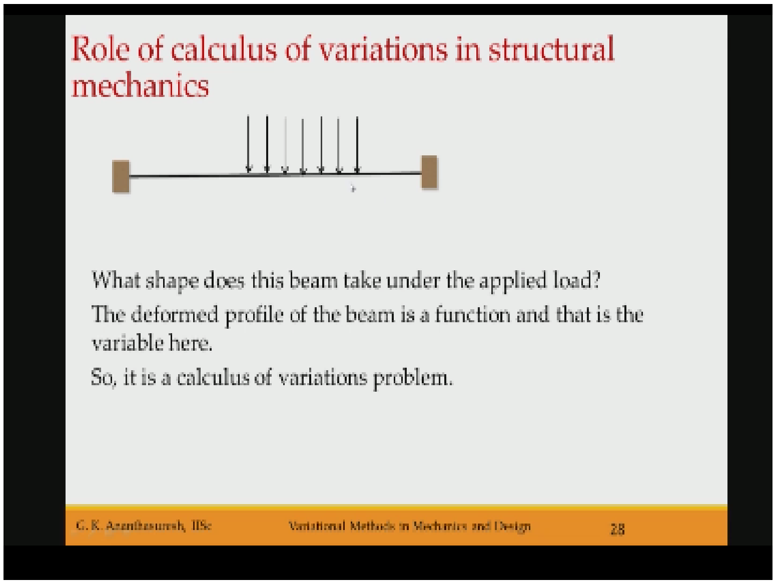

Let us look at these variation methods in mechanic's first okay so if you have abeam like this which is fixed on this side as well as on this side there is some loading on it, it is going to deform and take a deformed shape okay the beam has the freedom to deform to any shape but it does deform to a particular shape because that particular shape minimizes the potential energy just like we saw a spring with a mass it displaces by a quantity which minimize the potential energy.

Similarly this beam will deform to a particular shape which minimize the potential so it is a calculus of variations problem because the deformation of the beam we call the W X there is an unknown function whenever a function is unknown it is a calculus of variations problem .

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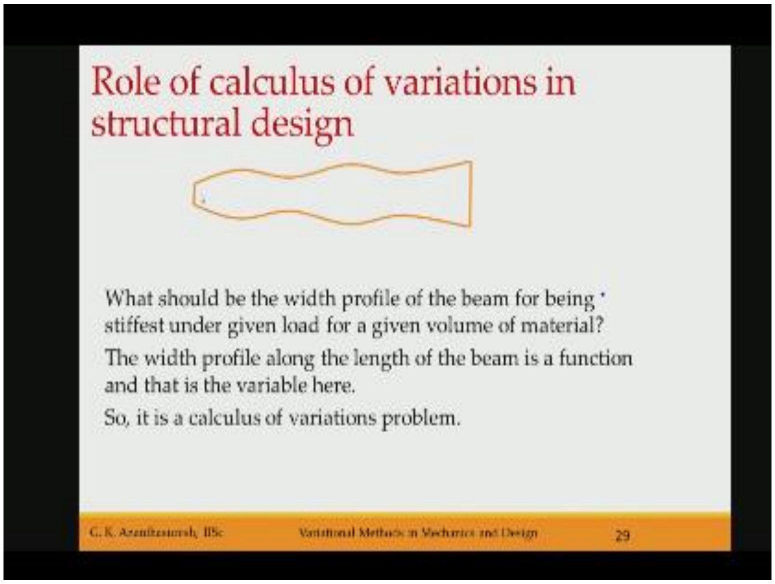

In the case of structural design let us take the same beam fixed on both side sit cross section area profile can be different from point to point this is not how we see normally beams in any room there will be uniform cross section okay but one can make variable cross section beams and that is a structural design problem there what is an unknown the area of cross section a of X if I say these x axis from 0 to some value L A of X is an unknown function.

That is my calculus of variation problem again because it is not the finite set of variables that are optimization variables but the function itself is an unknown that is the calculus of variations problem okay.

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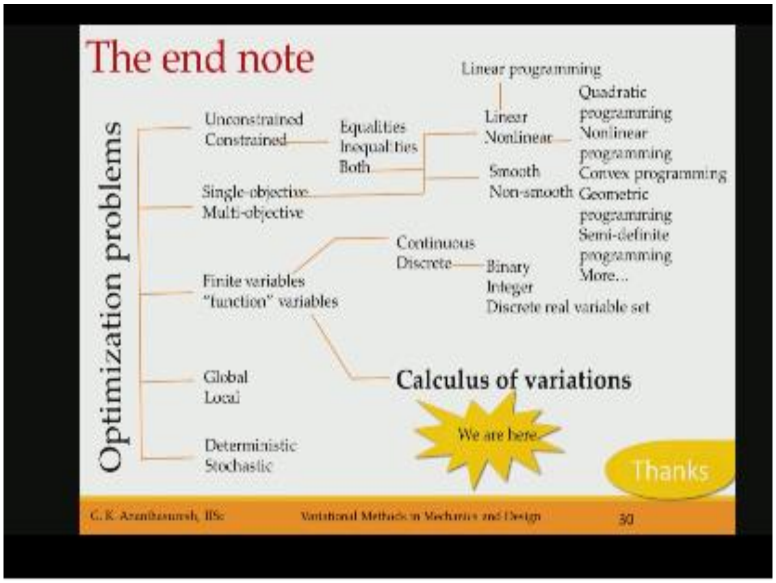

So we saw both in mechanics and design there is calculus of variations and there is optimization in general and this is an alternative to our classical way of looking vectorial way of doing mechanics and that way of doing design in this course we are going to use calculus of variations that is we are going to spend a lot of time and understanding collective variations now let us see where it lies in the panorama of optimization problems we have unconstrained and constrained problems with the qualities inequalities or both and there is linear programming there are nonlinear programming.

There is quadratic programming convex programming geometric programming dynamic programming all of those a semi definite programming it can be non smooth or smooth that is differentiable here all the things are differentiable here they may not be differentiable meaning they do not have unique gradient and derivative and there can be single objective or multiple objectives and you can have functions of V as variables then you get calculus of variations as opposed to finite variables which are continuous are discrete with the discrete binary integer or discrete real variable set as opposed to that now you have function variables in you get calculus of variations that is what we will discuss later again will be global or local optimization feministic or stochastic and many other types of problems are there okay thank you.