Micro and Smart Systems
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Lecture - 23
Coupled Electrothermal-elastic Modelling

Hello, as part of the Micro and Smart Systems course today we go to one more topic related to the couple modeling and this has to do with electrothermal and elastic coupling. There are 3 different energy domains and all these 3 are coupled each other and we will describe how one can model such systems.

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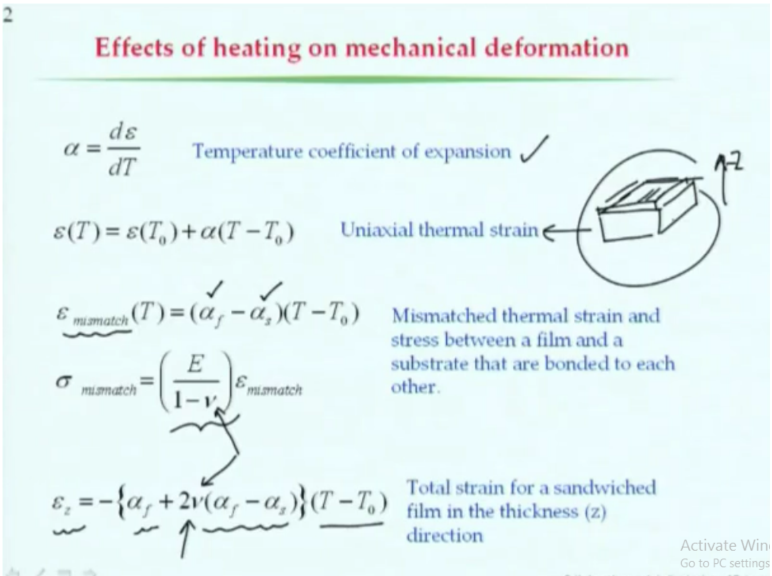

Earlier in the course we had discussed the thermal loading or effects of temperature and structures. We will just recapitulate what we had done. We had actually discussed at length about bimorphs. That is if we have 2 different materials that are attached to each other because they both have different thermal expansion coefficients the whole structure will deform. We had discussed that at length in detail.

We are just going to describe what we have discussed very briefly before we go on to the topic of coupled electrothermal elastic modeling. First let us recall the definition of thermal expansion coefficient. It is the rate of change of strain with respect to temperature. One can define it as d epsilon/dT where T is the temperature and epsilon is strain. So alpha is a symbol we use to denote the temperature coefficient of expansion of materials.

And because of thermal expansion there will be a strain created in the materials and that is given by the relation that we have shown over here. This is uniaxial thermal strain meaning that in one dimension what will happen. If I take rod and heat it, it is going to deform by certain amount and that amount leads to the mechanical strain and that is given by alpha times temperature difference.

If that happens in the material, let us say because of a substrate alpha s and alpha f, which is a film, that means that we take a substrate, all this we had discussed earlier. We are just recapitulating it. If I have a substrate, over that I put a thin film of some other material. So that is alpha f is the alpha of the thin film that we have put over that substrate. And the difference of those 2 will be the mismatch strain.

Both of them actually expand by 2 different extends and that is where there will be a mismatch strain which we have denoted, the difference of alphas between the 2 materials and the difference in temperature. And because of this there will be a biaxial stress mismatch. So epsilon mismatch is not multiplied just by E but it should be multiplied by E/1-nu for a situation such as this, which also we had discussed in one of the previous lectures.

And if we think about what will happen to the strain in the thickness direction that is our thickness direction, z direction what will be the strain there. The strain there is given by alpha f times of course temperature difference but also the Poisson effect coupled with this mismatch trade and because of x and y, there is a factor of 2 and nu is the Poisson's ration. The same nu that occurs here and here.

This will be the total strain for a beam that is sandwiched between a substrate and there may not be another material over there. But if you were to restrict it, that is a strain it will have in the said direction. If it is free, then you can simply deform up or down depending on the difference, the temperature and thermal expansion coefficients, but if you sandwich it, then this is a strain that is going to feel in the z direction. That is a thick z direction.

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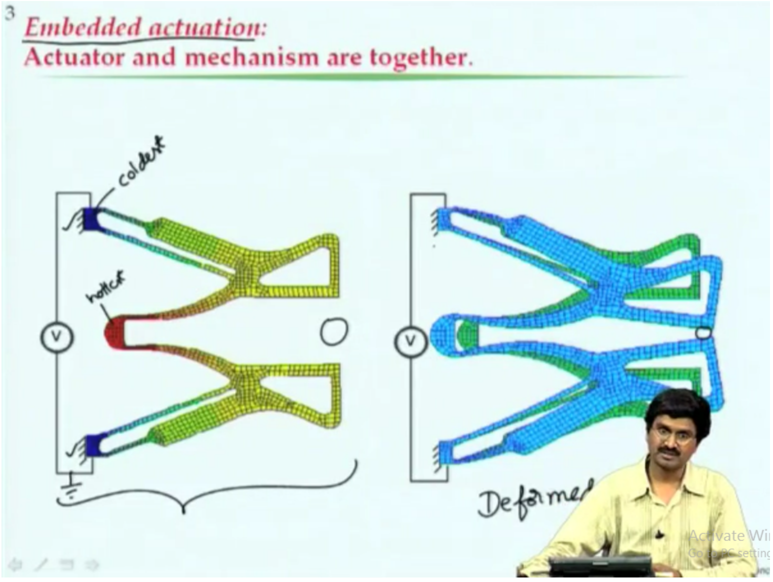

Now let us look at something very different from bimorph, so if it is a single material, we will have layers we just reviewed, but if we have 2 different materials that are attached to each other and if I heat it, one of them wants to expand more than the other, it will bend. That is a bimorph effect or bimetallic effect which we have discussed in detail. But now let us consider a single material such as the one that is shown here.

So let us say that what we are looking at over here is a single structure, the top view, if I have the wafer we are looking from the top. If it has certain geometry like this and if it is anchored over here and here and let us say they are thermally also anchored in the sense that the temperature here and here is equal to the room temperature and if you keep one of them at ground potential electrically and apply some potential difference between this point.

And this point, that portion and this portion, then the current starts flowing through this structure. And since there are holes within the structure, the current has to go, and then divide, probably little bit goes here and meets again, goes and splits and then it comes back here and then it grows and drains out. Now because of this non uniform current, the ensuing ohmic heating or joule heating will also be non uniform and because of that different portions of the structure will get heated up to different extends. If look at the color coding here.

Red is very hot, this is the hottest and blue of course the room temperate, that is relatively in this case is a cool dust. In between there is a temperature gradient, but specifically if you look at what is going to happen to this structure because of this temperature, non uniformity, there will be a differential thermal expansion. We just learned that alpha is D epsilon/DT, but now T is varying quite a bit within the structure.

So accordingly epsilon will also vary and the structure will deform to achieve equilibrium and in this particular case it will deform as it is shown over here. This is the deformed structure because under the thermal strain the structure tries to achieve static equilibrium and it is going to move like this. If you think of this a griper here is where you can put something and it can grab it or if you want to push on something that is also possible with this.

So this we call embedded activation meaning that we do not have the actuator and the griper like mechanism as 2 separate entities. The actuator is actually embedded with Hindi gripping structure, a grasper or a griper structure. So we can call it embedded actuation. It is not like an active material, this can be done with any electrically and thermally conducting material and the key for this lies in the way the shape is obtained.

Once you have this shape appropriately done then it can create behavior that you want. As opposed to piezo electric, electro restrictive, many restrictive and other smart materials. Here the material is not smart, but the design is what is giving it a smartness that it is showing, which is applied electrical continue between 2 points and it just acts like a mechanical structure with the actuation given in it. How does it work?

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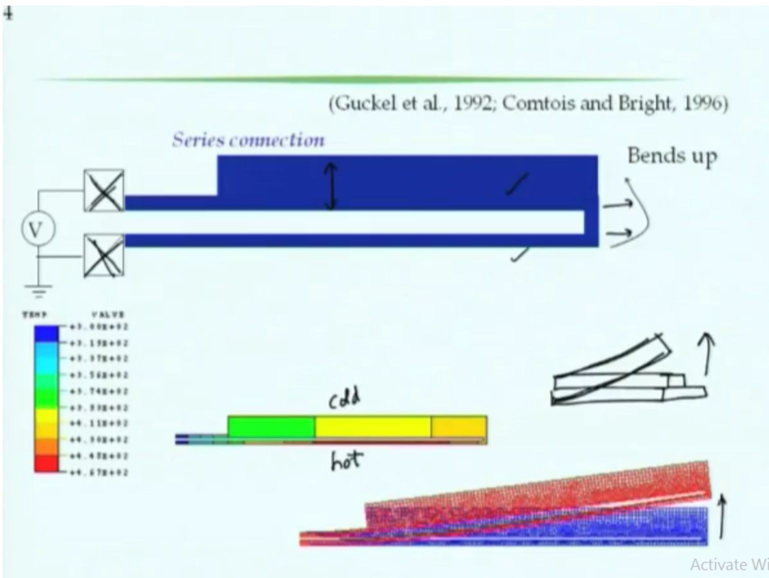

Let us take a very simple example, where we are taking 2 beams that are connected serially, we call it a series correction, you will see the reason for that in a minute. We have this structure like this and let us say this is mechanically anchored, and so is this, we applied a potential difference between these 2 points. Let us say that this portion and this portion are at room temperature.

So thermally grounded, what will happen when we apply, what is the difference between these 2. One is that, the current starts flowing, like this, of uniform width everywhere. The current density is going to be the same and it is going to flow through out, there will be ohmic heating, it do not heat up and that is not going to be uniform because these at room temperature this will be promoter.

Whereas heat is given at everywhere, that heat has to go back to the substrate or through convection and radiation to the environment surround it. Nevertheless, temperature profile is not going to be constant along this folded beam. But there will be something. What will happen to it? It will simply expand like this because how much ever this beam deforms this also will deform, simply expand.

Now if we introduce little bit of asymmetry into this as it is shown here, that is we make this wider. We are making this push and wider than this portion. Now if we look at the temperature distribution, it will be asymmetric. This part, this portion on this side will be hot, this will be cold. The reason is that, the current that is going through this portion and this portion is the same because whatever current goes here, the same thing has to go there.

But the current density here is higher. So this portion gets hotter than this portion. To achieve equilibrium, imagine that if I were to have a bimorph, if I put those 2 layers together, if this is going to expand more than this one, it has to bend like this because when it bends, this portion is going to have longer length compared to inner one, that is why it bends up like this. Here also it bends because this portion because it is hotter than this it will get expand more.

It expands less and it give you this kind of motion. So this is the principle. So we have to use the differential expansion, not achieved with different materials but with the same material but introduced an appropriate bias in the design by getting the geometry right, then you can create this mismatch thermal expansion and make it move. It is in plane. It goes in the plane of the wafer.

So like electrostatic actuation we can generate in plane forces very easily, if we want it out of plane we can just use bimorph and make it go or the same structure can be put in vertically and the structure can be made to move up.

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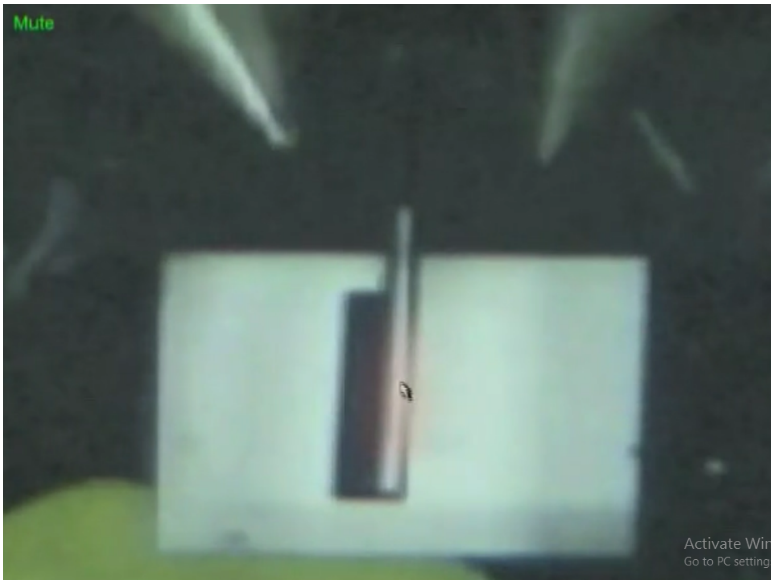

And now if we compare the previous one, here we are applying the potential difference between this point and this point whereas let us say that we do differently. We keep this at the same potential, that is this and this, mechanically grounded, selective potential, of course temperature is room temperature. But this side is where we can electrically ground it. If we do that, then it actually moves this way.

Previously slide you saw that it actually bends like this, here it bends down. The reason for that is clear from the temperature distribution that we see here. This is the temperature distribution. You can see now this portion has become hot as compared to relatively colder or less hot I should say because it does increase in temperature because you are passing current, it will be less hot compared to this side and so the hotter portion will expand more so it has to bend down like this. Why does this portion gets hot?

Now, when you look at this and this as 2 separate resistors, more current will go through this because resistance of this portion will be less than this because resistance is rho l/A. Here area of cross section is more than area of cross section here. So the top portions resistance is going to be lower. So that draws more current than this. So the total current that comes here has to split from here to here.

More current goes to this one rather this and that gets hotter than this other side bends down. What it tells us is that, we can take a structure, make holes wherever we want and try to get the type of motion we want. Again it is embedded activation and making this is rather easy in any micro fabrication process that has releasable structure layer, meaning that there should be one planar layer that is free to move in the plane.

If you have it and if you are going to have some electrode pads, we can apply potential differences and get the actuation that you want. Let us look at a movie that shows one of this actuators actually working. So just let end this and see that movie. Okay, here is what you see. Here is a little thin beam, that is corrected here and the wider beam, it is corrected here.

You see this one is one of the probe and the other probe faint stress so the other probe is here between these 2 we are applying the potential and because of that there is a PAT, so separating around, when you apply potential difference between these 2, you can see this moving like this. This is serial connection, meaning current is going from here and going like this. Okay, you can actually see vaguely that it is getting hot where the redness can be seen.

And by applying lot more voltage you can actually melt it, so you can go to very high temperatures in these up to the point of melting. You can clearly see it moving this way and that way. The force in this could be quite large as compared with let us say electrostatic actuation.

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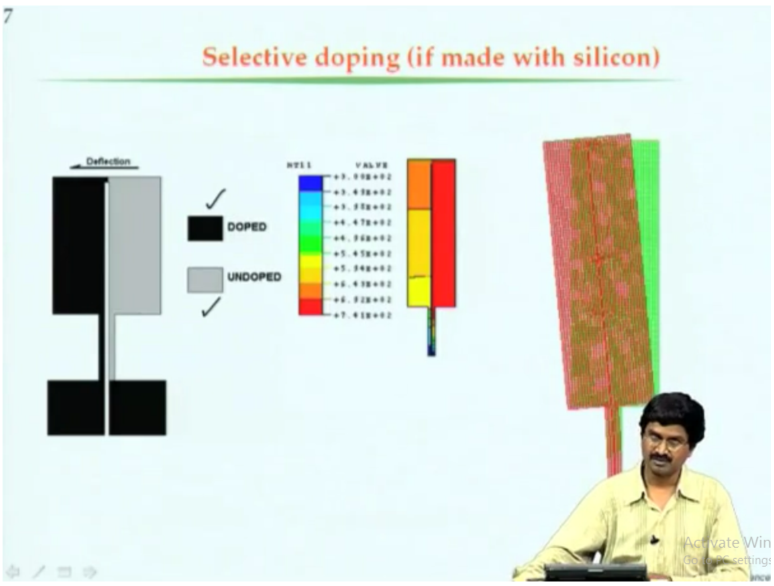

Now that we have seen a real device working, experimentally prototyped electrothermal actuator of this kind, let us look at some more issues related to it. Now we saw that we can take this basis thermal actuator and introducing a symmetry to take the linear expansion into the transverse domain. Instead of expanding just like this, it is going to expand like that. But this asymmetry need not be only in the geometry but can be, the structure can be symmetric.

One portion is doped and other portion is not doped. So this portion is doped, this one is not doped here. So that way also you can change the electrical properties because by doping meaning that introducing foreign species into a semi conductor material we can change the electrical conductivity by hardness of magnitude, if we do that here again we are creating asymmetry not in terms of geometry.

But in terms of material properties and we can make it deform as it is shown here. So there are lot of possibilities to define electrothermal actuators.

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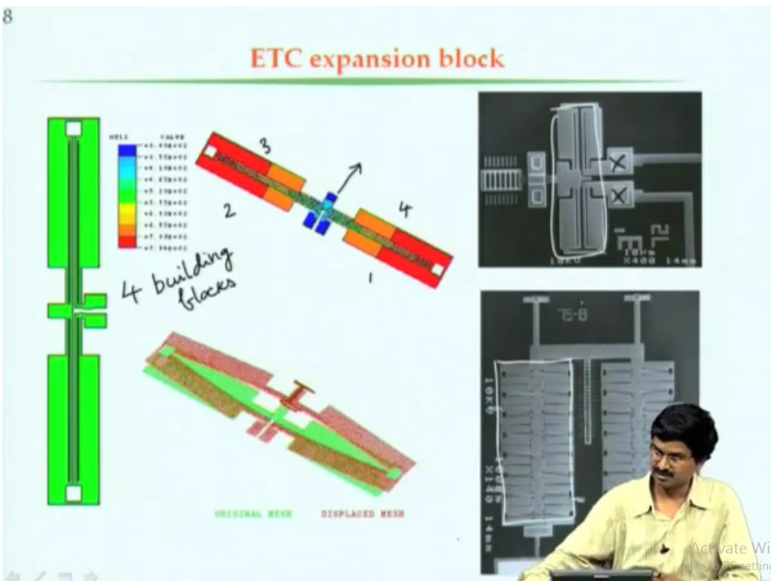

We can take that basic thermal actuator that we saw in the movie also which we observed just now that it actually works, now if we take 4 of those, so if we look at this one there are 4 building blocks within it which is shown here. So we have 1,2,3,4. They are connected in a serial fashioning. It comes from here, we apply this is mechanically anchored. When you apply voltage between this and this, the current has to go splits and then meets again, goes, splits, meets again, splits, goes there meets and then splits there and meets and then goes.

So if we have something like that, the way you arrange them, you can actually get, you can make this more like this. 1-2-3-4 are there, so here is 1,2,3,4 that makes a deform like this. You get lot more movement. In fact, what we could do is take a large block of this kind, let us say that if we look at this large block over here, make these holes, very tiny holes, if you see the holes, if I take this block over here. adjust to make this slits here.

If I do that several of them, it actually becomes like a linear actuator because you can see how much this can move, that is from here it is moving till here, that is the actuation and if you put several of them together, we get more force and more importantly got more displacement, lot of these. You put them in parallel, you get more force but then in series you get a large stroke or displacement with this expansion block.

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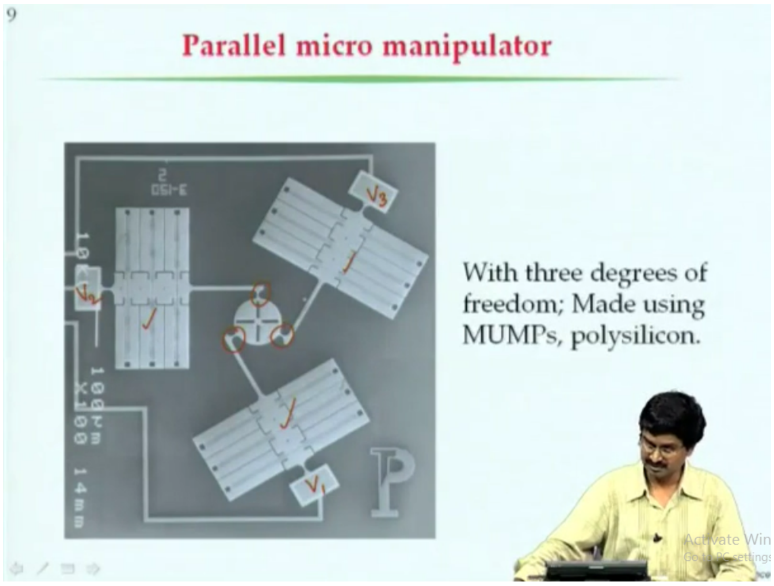

One can also arrange them to contract. The same blocks that you have arranged them differently there you can get contraction of this. Now we saw here those actuators, 1,2,3, the stack of the thermal actuators connected with some flexures, this is what we call, a central platform where we can apply different voltages, V1, V2 and V3, one of them grounded.

The other potentials above the ground, we can make this one move in x direction and y direction and also oriented in different directions. This again is a single layer, any process that gives a single releasable structural layer is eligible for doing this with pretty much any micro fabrication process. So once you have it, you can make these structures. It is a parallel manipulator.

Because it can position the thing anywhere in x and y within a workspace and also can rotate. That is orientation can be changed.

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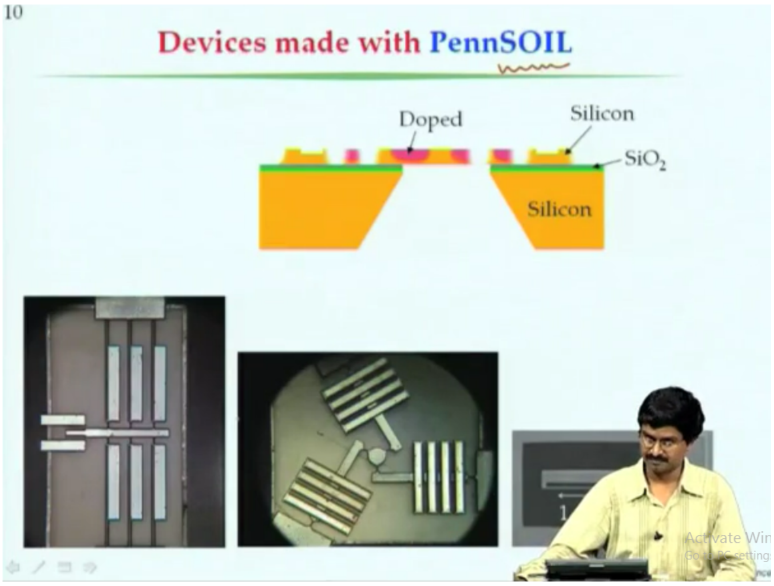

And it can be done with many process but one of the things that it is very easy to do is silicon and insulator layered process where you take a SiO wafer with silicon dioxide already there, silicon substrate is there, the top silicon structural layer device layer is there. You can dope it if you want and define different structures and get things like this. It is a thermal actuator, it is a platform, it is an individual thermal actuators such as the one whose movie we just saw.

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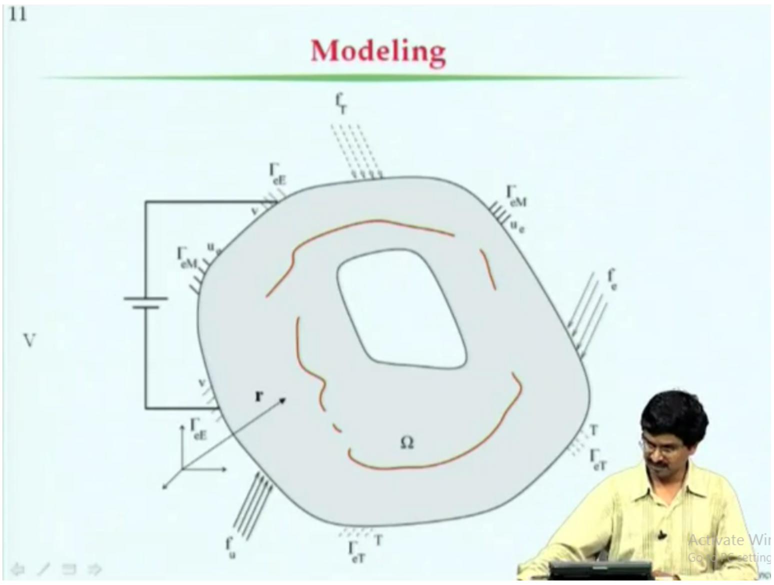

And now let us take a much more general scenario where we would like to consider a general object. This is a general object, we do not specify particular shape, it can be of any shape people interested in, it can be fixed as there can be 2 points that maybe different points where you apply voltage difference or potential difference and can have thermal loads that is flex is coming in or there are sources at different points.

You can have different types of bond equation thermally and then you can have elastic problem where you can have mechanical forces of whatever kind and also electrothermal kind of loads if there are any that can also be included and with all those things if you want to model such a structure what do we do. In fact, we did not talk about the modeling of the simple heat actuator or thermal actuator itself, but if you take a general situation what do we do.

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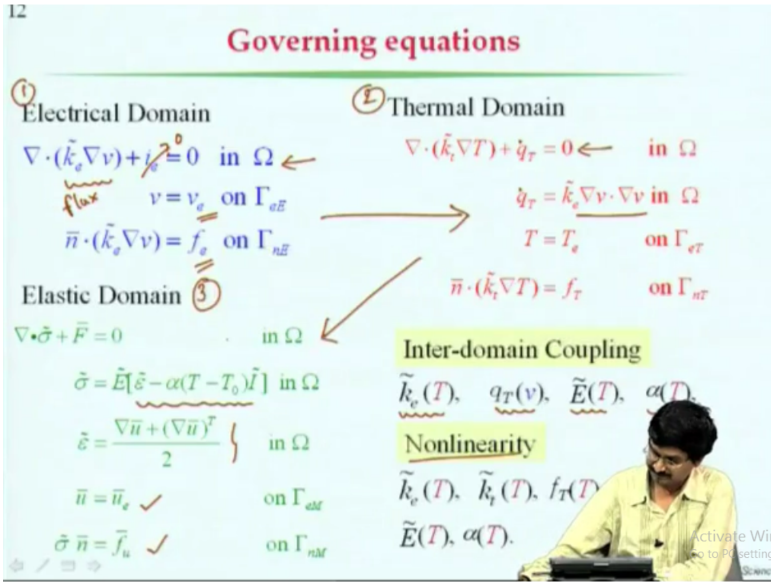

So for that let us look at the governing equations because as we noted earlier modeling of Microsystems is different because any Micro system device that you take, is going to involve more than one energy to me. It is definitely 2 always, otherwise it is not going to be that interesting and it is going to be more than that. This particular electrothermal actuation involves electrical thermal and elastic 3 different energy domains.

So we have actually 3 governing equations. There is a first one, electrical domain and the second domain thermal domain and elastic domain, the third domain. So first, second and third here. So now if you look at what is governing equation which at first sight maybe intimidating but there are all very straight forward. The first equation that we see is del. Means divergence of Ke times delta V, Vis the voltage and this is the electrical flux or current.

Ke is electrical conductivity or reciprocal of the electrical resistivity. And this is usually 0 because there is no current source in the domain. So essentially del. Ke del V=0 is what we solve here and then certain portions can have potentials specified, as it is shown, another portions maybe electrical current like is specified also and that is also indicated. Now once you write this equation and solve for this you will get the electrical current everywhere.

Once you know that we come back to thermal domain whose equation is similar because just the way electrons flow, the (()) (23:18) of flow and both mechanisms are through diffusion, so the equation look similar. So if you look at this equation here and this equation, we can see the similarity. Wherever V is there we have T, the temperature. So it will be del.Kt del T, gradient T and this kT here is thermal conductivity, ke is electrical conductivity.

And then q or q. one can say because the units of that will be powered. And that q. is ke times del v.del v. This is the joule heating. Electrical resistivity times del V. del V that we can put there and then T is equal to a temperature is specified and part of the boundary and other part you may have thermal flux that is shown here. Thermal flux can be convection or radiation where heat goes out. (()) (24:22) and is hotter heat can come in also.

So here base the coupling between this and this because the voltage here appears over there because that is the joule heating or ohmic heating by this problem, we have used that into the thermal problem and once you finished that, you have to go back to the elastic problem because our interest is to look to actuation. How much does it move? And for that we have to go to elastic move where divergence of stress plus anybody force that maybe there like gravity (()) (24:56) = 0 is our equation.

And the sigma=stress times strain and that strain as we discussed in the beginning of the class, there will be alpha times t-t0 and where epsilon is the strain that we need to determine and hence how it moves that is given by this definition, part of the thing we have specified, other parties because of this, if they are about to be any mechanical traction load, a few on the boundary of the mechanical structure that can also be taken into account.

So first we have to solve this problem and then solve that problem and then solve this problem. So these 2 are coupled to each other. So it is a sequential coupling meaning first we do electrical analysis and then we do thermal analysis and the last we do the elastic analysis.

And these 2 are sequential, so the coupling is only one way, but it can be other way also especially if you take the temperature dependent properties.

For example, we have electrical resistivity can be different on temperature and the qT is already depending on voltage, that coupling is already there between electrical and thermal domains and hence modulus can depend on temperature and alpha can depend on temperature. We saw that temperature is going to go very high in these things when we looked at the movie, under now linearity in this problem comes because of the properties depending on temperature, so we had to redo this.

If you did electrical and then thermal and elastic, after you get this and then if you find temperature is very high, then you have to go back and substitute it there and evaluate this properties and come back again, for example if resistive different temperature we will do this and this simultaneously or if Young's modulus depends on temperature that also has to be taken into account.

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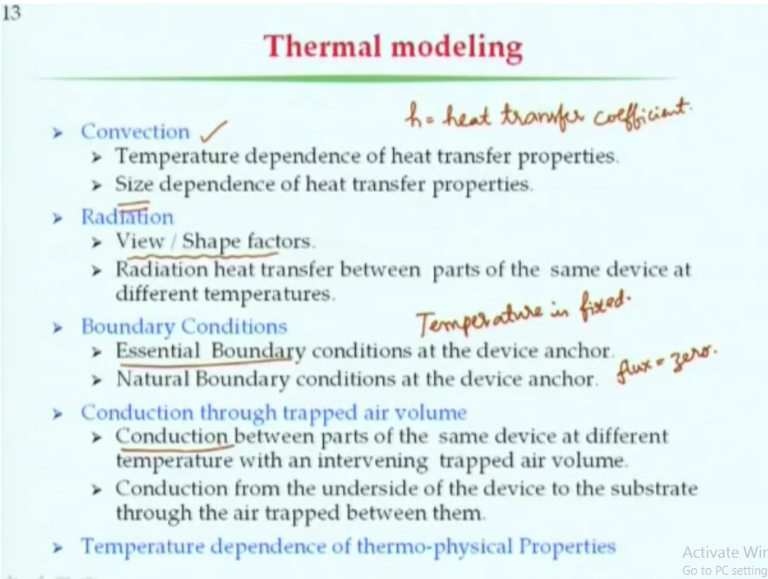

Now we discussed what the equations are for solving. These equations have to be solved using numerical methods unless the structure we are considering is very simple. When you are doing numerical modeling, we also have to remember about this mode of heat transfer from the structure to the surroundings. One is of course is the conduction. It will go through the structure, go through the substrate and get the heat conduct the way.

But we also have convention which is what is indicated here. And this convection heat transfer coefficients are highly temperature dependent and in fact, they also depend on the size of the structure. What one can do here is when you have a structure like a wide beam, we want to know is convection through the top or while in the surface we have to go to heat transfer handbooks sand find what is called heat transfer coefficient that is denoted with h heat transfer coefficient.

This can be for natural convection or forced convection, say pretty a fan I am trying to cool something. Taking this convection is very important and this h itself might depend on temperature and the size also may affect this h. If you go to handbook and try to get h from that for let us say a plate that is horizontal heated from above, heated from below whatever conditions, when you take that formula for use we have to make sure that the assumptions made in deriving the formula do not have anything with size.

If they have, then we need to pay careful attention to that point. And then the radiation, radiation there is this concept of view factors or shape factors depending on how the surface are oriented related to each other and the heat, how it is going from one to the other or to the surroundings based on the radiation properties and sometimes narrow structure such as the ones that we saw, that will be here, that will be small gaps.

So radiation can take place from one surface of this device to another surface. So it is going to be quite complicated to accompany radiation and the boundary conditions in this problem, can the essential boundary conditions meaning temperature is fixed, when we say essential or device layer is where temperature is fixed. Natural boundary conditions are those where temperature is not fixed but flex is 0. Then there will be different conditions.

Let us say that we have put an actuator, we have put a thermal mask, big substrate but in between we have put a very good thermal insulator. There is nothing like a perfect thermal insulator unlike an electrical insulator there are very good ones, thermal we are not that perfect, especially if there structures are very thin, interface layers are very thin, we cannot get a perfect thermal insulation.

But if we do have that in, heat cannot be conducted through that insulator to the substrate and it gets hot within the structure and one needs to worry about that as well. And sometimes in very narrow gaps the air that is there instead of convecting, it will start conducting. So that also needs to be taken into account. There are lot of other reasons why this convection is very important for this problem.

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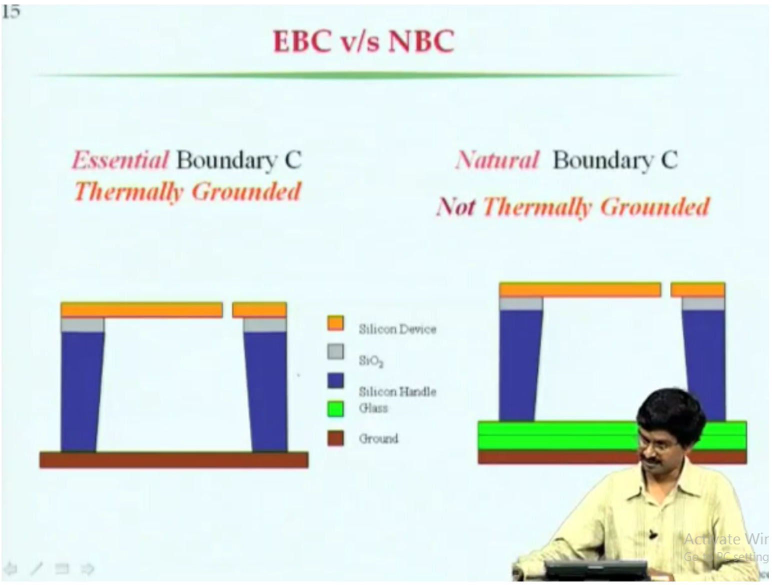

And we will consider one example to show how dramatic the situation could be whether you model convection or you do not model convection. The word here is model, real structures of course there will be convection, it will do. So if you take a structure that is, I think it is easier to see the thing, so again we have put a slit here, that is a narrow slit and this is the place where between these 2 where applying with electrical potential from heating.

And according to this is supposed to go up, meaning by the way this would bend that way, bend this way, should go up but actually it is coming down as you can see when you model without convection. When we model with convection, it is actually going up as intended. So we can see that whether you model convection or radiation, radiation effect will not be much until the temperature is very high.

But this example shows that with convection we can get qualitatively different behavior, not just a slight error in the problem, but error in the solution but the nature of the solution are nature of the deflection itself could be different, so that is why it is very important to model convection properly and also the boundary conditions. So here it is thermally grounded by putting an insulating layer or not thermally grounded.

So you can have different boundary condition and try to play the geometry according to boundary conditions and the voltage level, potential difference level, we can make these structures work.

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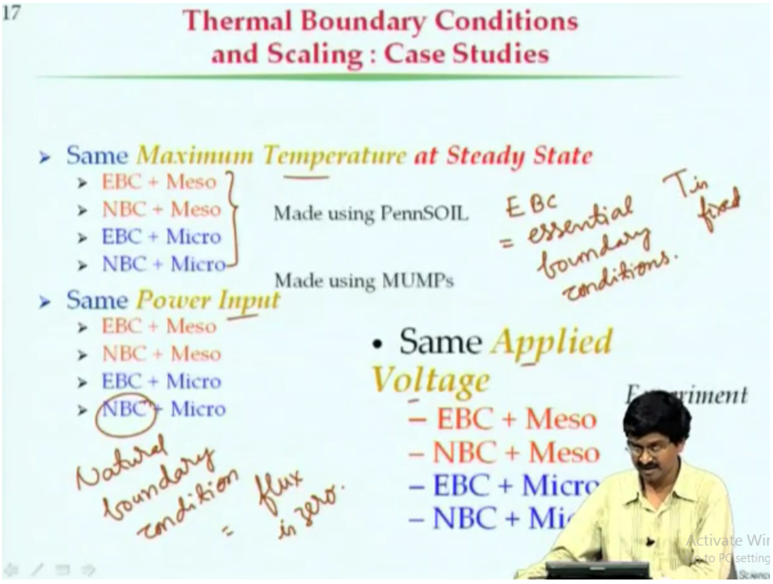

And how do you analyze these things. One way is to use finite element model. Here you can see a 3D mesh, 3 dimensional mesh, all of these are (()) (33:12) that we can see and it is fully couple electrothermal analysis and it is sequentially coupled to thermal elastic analysis meaning that this is very time consuming for simulation because of the complexity of the structure.

But it is sequentially coupled, first we have this electrical problem, thermal problem and then elastic problem. When you take temperature and material properties, it gets even more time consuming to simulate this kind of structures. A small case study can be done where we try out different boundary condition, EBC here stands for essential boundary conditions and NBC here stands for natural boundary condition.

Essential means temperature is specified or fixed at the boundary, here it means that flux at the boundary is 0, okay. That is thermal flux. Kt temperature conductivity times gradient of the temperature. That is the thermal flux, that should be 0, then you call it NBC. If T is fixed on another value, it is essential boundary condition. If we try with various thing and compare the maximum temperature power input to this thing and applied voltage.

If you take some parameters and fix them and compare, we will find that in each case the temperature distribution is going to be very different.

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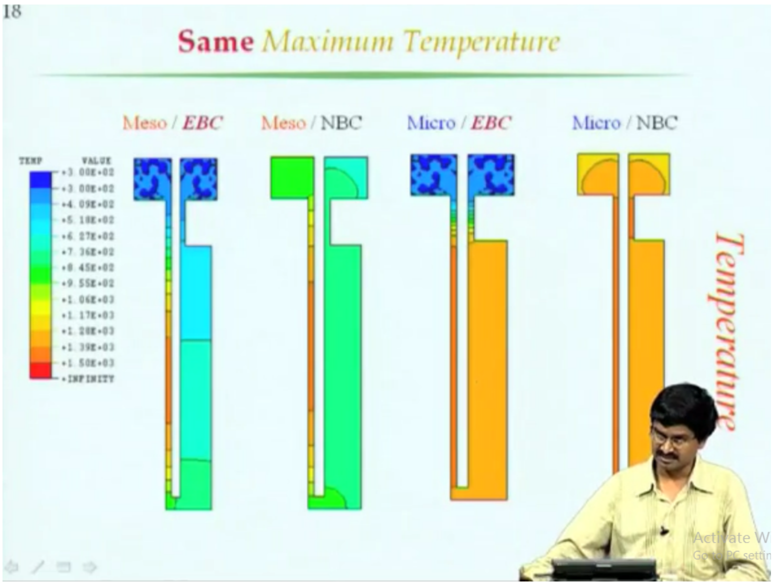

Each of them behaves differently and once can have a lot of design freedom in coming up with structures such as these.

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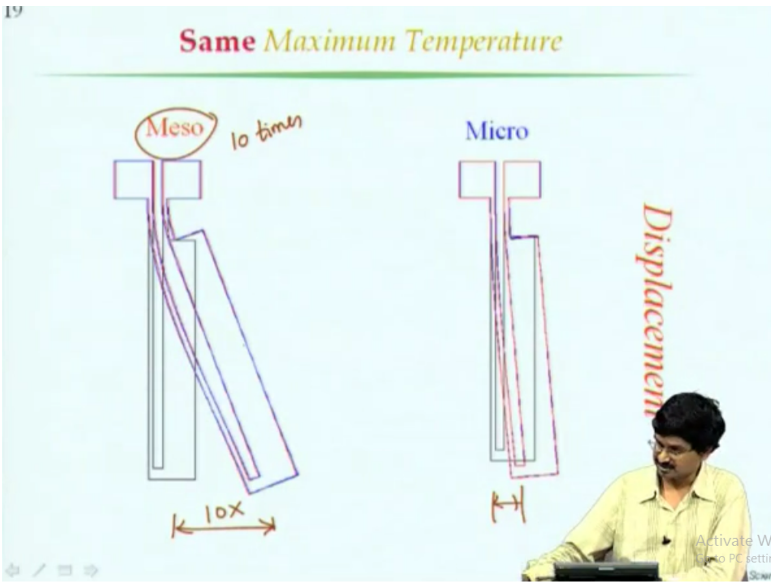

And one other thing we see here is that, if you analyze Meso is 10 times larger than, Meso is something in between, it is larger than micro. Here it is a scale drawing to show that at Meso scale you get much more relative deflection than what happens at the micro scale. This is 10 times larger in size and also gives 10 times larger stroke, a displacement for this case. And it can be done with other conducting materials also.

But you have to make sure that the electrical conductivity and the thermal expansion coefficient are appropriate to make them heat up to this temperatures and also hold up, the strength also comes in, so that it does not break because of this passing electrical current through this structure.

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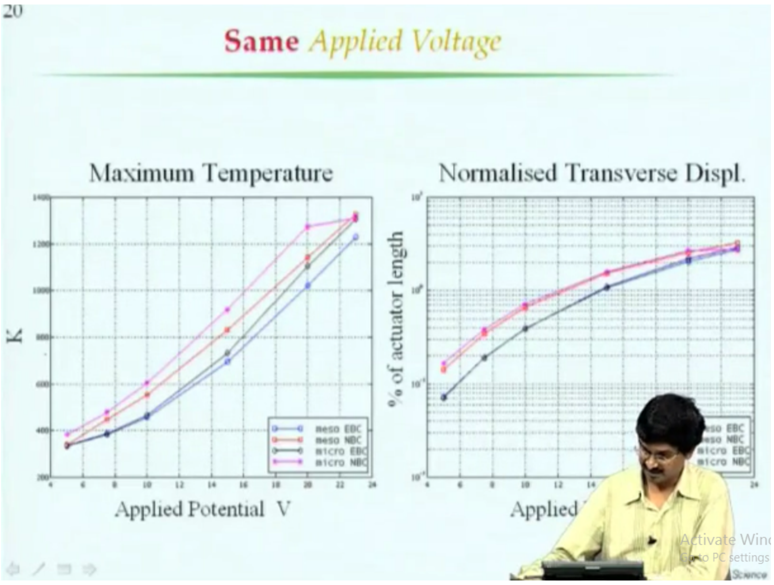

And lot of data of one of the device that are stimulated and is presented in a paper for different boundary conditions, different maximum temperature that will occur in the structure

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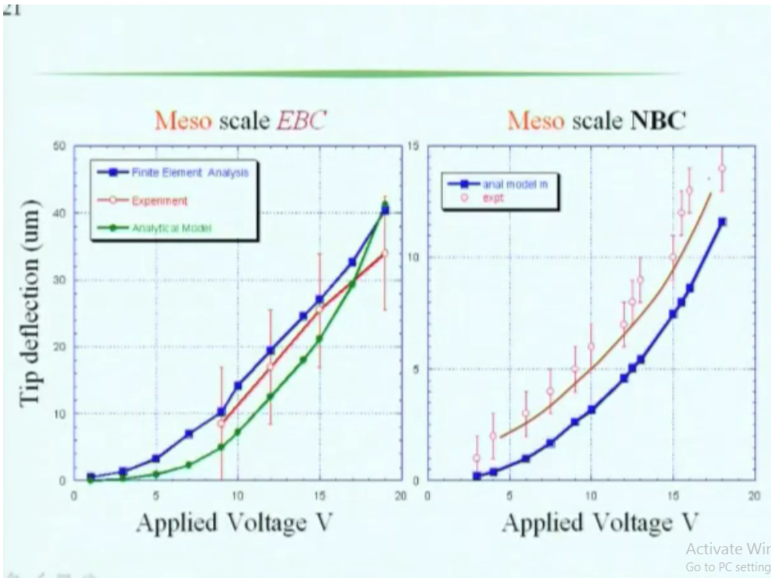

And some more data where the tip deflection is being shown for an actuator of this kind there is an x modulator with large error in measurement but never the less trend can be seen through the modeling. This was done with finite element modeling where we first do electrical analysis and then thermal analysis and then do the elastic analysis.

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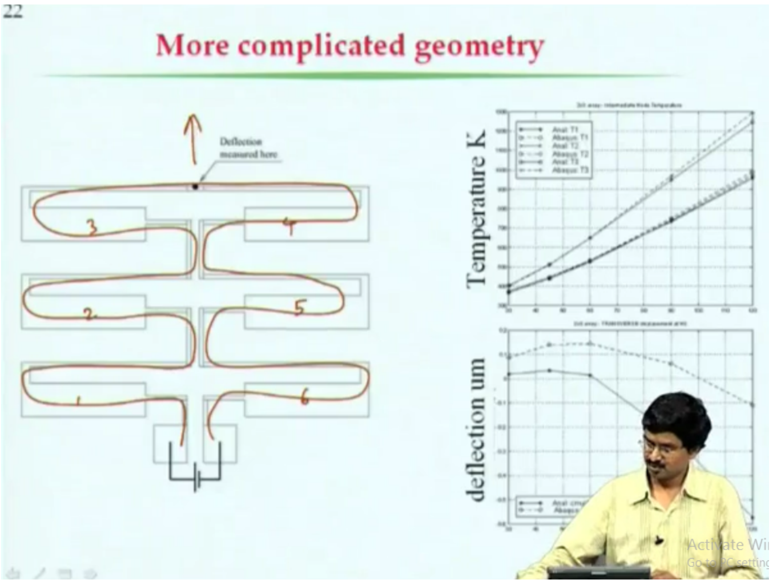

Now what happens, if we have a more complicated geometry. All this that we have shown, these complete data is for this single structure. So this one structure is what was done. Imagine that the number of elements that needs to have for very narrow structure, it is last structures nothing can be ignored. This can take quite a bit of time. But now what if I have this array, kind of thing that we had earlier.

So there is this 1,2,3,4,5,6 building blocks that are arranged, where the current will go like this and go like this, goes here and then comes back. There are 6 building blocks arranged into this fashion, so that we can overall see some deflection like this. We already saw that this was used in a micro machined platform. One can do the analysis for this again using finite element analysis but it will take enormous amount of time. So is there a way to do it in a simpler manner.

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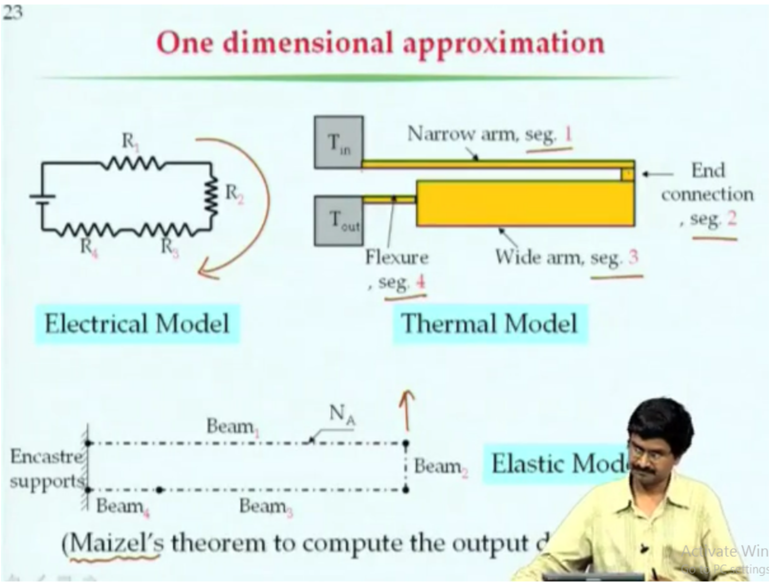

So let us look at a simpler model, because we have done that for mechanical structures, we have done that for electrostatic structures. Mechanical structures, however complexities we can make it like a spring, however complexities for electrostatics we can make it like capacitor, similarly for thermal problem we can make it into a thermal resistor. Before that first you need to think of them as electrical resistors.

So this particular thing we have 4 parts that is a segment 1 here and segment 2, segment 3 and segment 4. So we have 4 electrical resistors. R1 corresponds to resistance of this. R2 corresponds to resistance of this small element, then R3 is this wide beam segment and R4 is for this flexure which allows it to bend and there is a beam associated with that. Now we have to apply voltage, we can find the current through it and complete electrical heating or joule heating and this thermal problem.

Once we get that thermal problem, boundary condition can be put in, solve for temperature distribution and then due to elastic problem, again we take a beam, little beam here, another white beam, then the narrow beam. If we have that, we can use what is known as Maizel's theorem, we have an expression for it in the subsequent discussion. This is like Castigliano's theorem.

If our interest is only to find deflection at one point we can just use this Maizel's theorem to get that deflection because our interest now is what is the deflection here. Once I do this kind of lump modeling, I can do it for any complicated structure.

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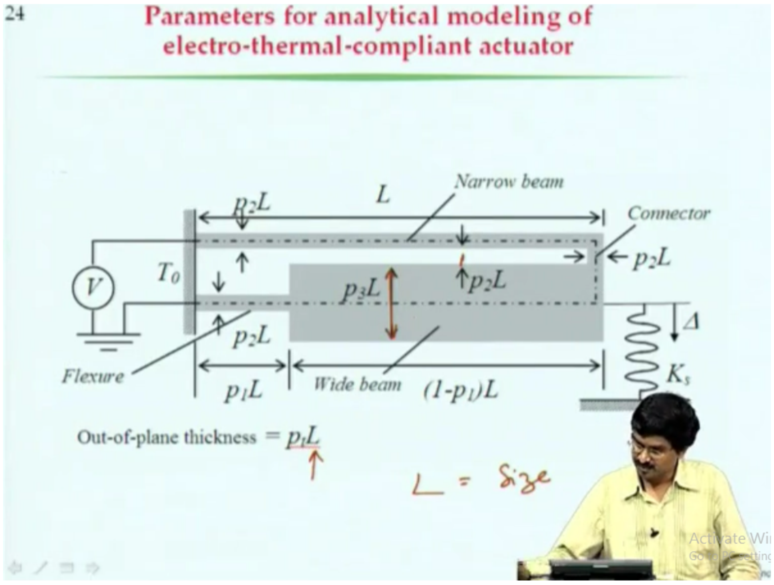

Okay, now we are showing this here. Again the narrow beam, connector beam, white beam and the flexure, 4 parts, parameters in various ways. So what we are basically saying this, let us take this size out, L is the size. That you can analyze micro scale structure and Meso 10 times larger or any other thing that we would like to have. So now in this case what we are saying is this, this dimension is P3 times L.

So we are keeping the P3 as a proportion parameter, and likewise P2 for this gap and this thickness also P2, this length is P1 and this in plane width is also P2 and so forth by introducing this proportion parameters we can analyze this structure.

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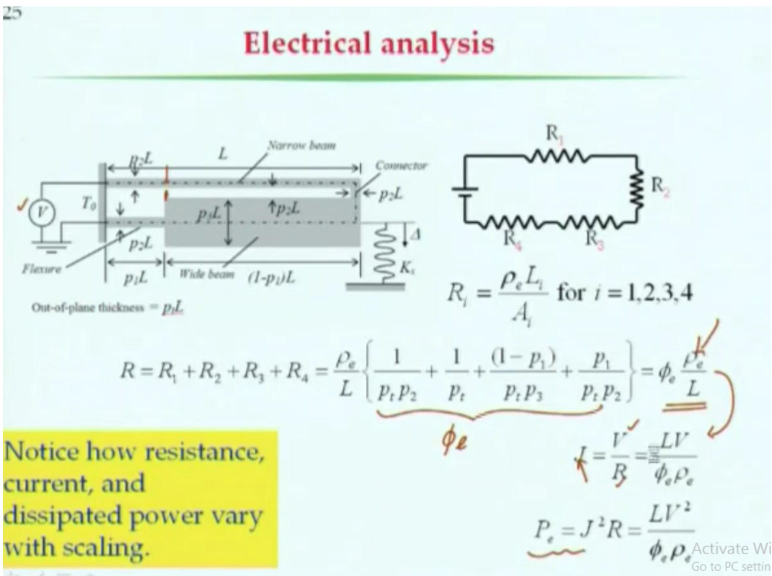

The first thing is the electrical analysis. If I take the first one, the resistance it is R1, that will be rho l/A. So here we have taken that because there is Pt, P2 meaning that there will be A, will be P2 times L that is in plane width, but also there is thickness which was Pt times l if you go back the thing, this is the Pt times L is the out of plane thickness, because there will be L square and then top will be 1 here.

So if we look at this expression for R, it is inversely proportion to the size L because phi e is simply the proportional part of it, this whole thing, we are calling it phi e, that if we keep the proportion on this, it is not going to change and rho e is electrical resistivity, Pt L property and if you look at the resistance that is inversely proportional to the size, the smaller the size the higher the resistance.

And the current here is potential difference, the v that we apply, that v over here divided by resistance and we get this by substituting that into there. And then we can get the joule heating, the Pe part of it, J square r, where J is the current density which is voltage divided by resistance R, this is the current density and times R will be the heating or q. term that we will have to put in the thermal problem.

So here we should notice that resistance inverse proportion to the size whereas ohmic heating is directly proportional to the size. So we have to remember the scaling effects. So we develop some intuition or problems of this kind.

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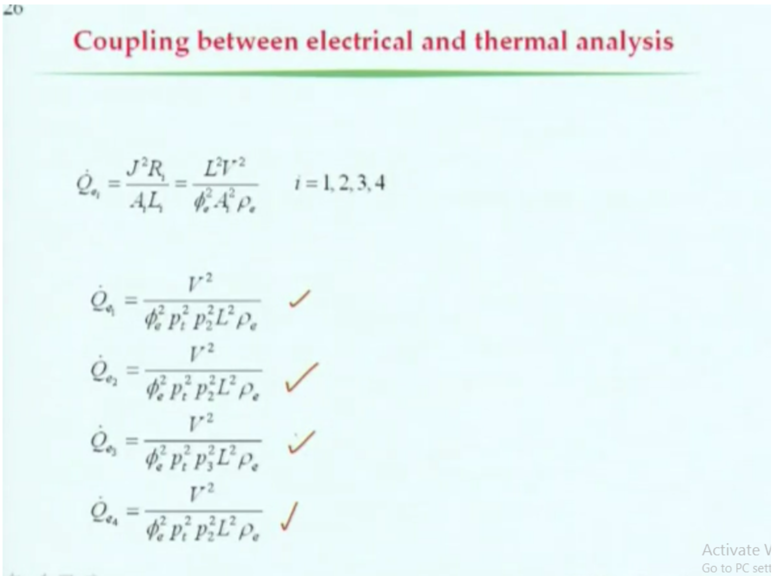

And now the q. the J square all that we have is heat dissipated per unit volume, so we are dividing by the volume also here, that goes as q. in the thermal problem. We have 4 segments, this is the heating sourced term in the first segment, second segment, third segment and 4th segment.

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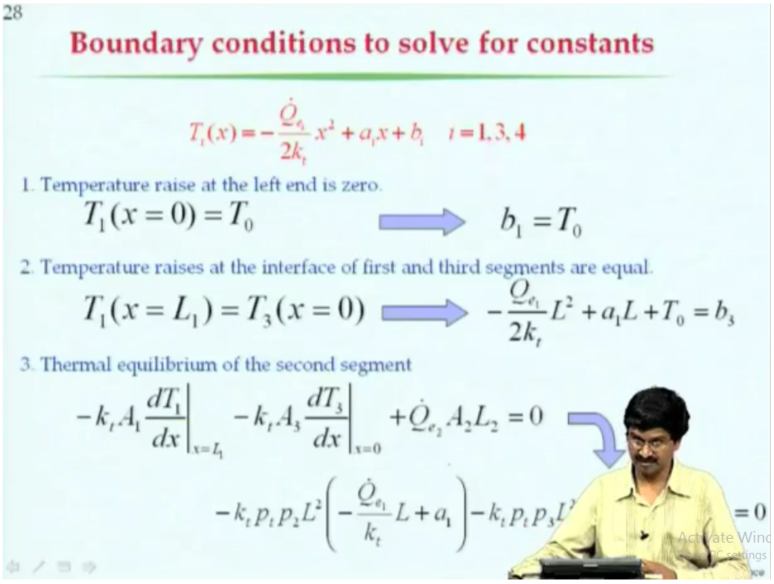

So we have to put them now and derive a thermal lump model, just as we lumped the first, second and third and 4th beams. Of course second one is very tiny little segment over there, but for all of those we did electrical resistor, so q. thermal resistor; it is exactly the same because governing equation we saw earlier is based on diffusion process. So electrons, the way they move and (()) (44:02) the way they move is both because of diffusion.

So we have the same governing equation and same lump model also. Now if you see, for each segment we can write a differential equation, make it go the second one, that is why we do not have 2 here, 2 is missing intentionally, it should be missing. It is not a typo, the 2 you do not consider, as such a small one, each of them will have an equation like this which is a diffusion equation.

Secondary way to d square t/dx square+q. term and the kt would multiply because that is how the 4ier law of conduction work. Now if you do this we will have integrating this equation for each segment, that is the first segment, third and 4th, we get an equation, there will be ai, bi,1,3,4. That is a1, a3, a4; b1, b3, b4, 6 constants, so we need 6 boundary conditions and we have them.

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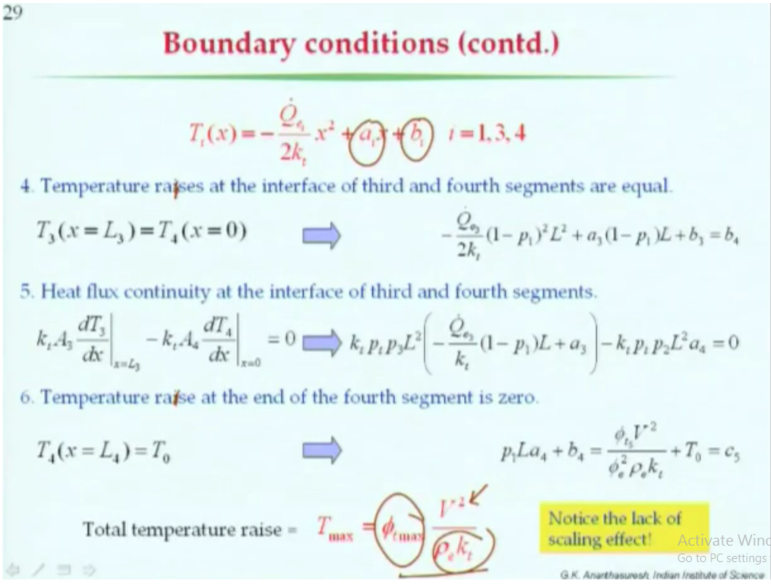

Temperature rise at the left end if 0, the one boundary condition, the second one the temperature rises at the interface of the first and third segments are equal because when our 2 things meet, from this equation, that equation have same value and thermal equilibrium of the second segment we need to have because that what we ignore there needs to be there is some heat generated within, some enters, some goes.

We need to account for it and that is the third equation. The 4th equation is temperature rises at the inter the third and 4th segments are equal and then same thing third and 4th segment's heat flux continuity and temperature rise at the end of the 4th segment is 0 is another condition. So we have 1,2,3; here 1,2,3, and then 4-5-6 boundary conditions to solve for a1, a3, a4 and then b1, b3, b4 we can solve.

And then if you look at the maximum temperature there in that, that is proportional to this and you can see that the size is missing here. It was not depending on l, the l we will say, it is proportional to the size, but here it is not depend, and the Tmax transfer to be this, depending on only the proportion of the thing and material properties and applied voltage. But remember that we have done this so far in an electrical and thermal coupled manner, but now we need to move on to the mechanical part of it.

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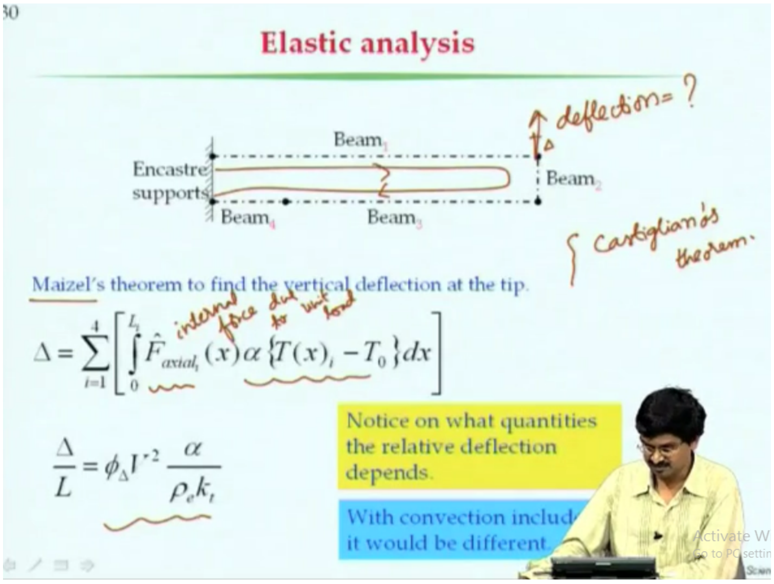

We would like to now, what is the deflection. It is going to bend up or down, how much is the deflection in this direction. Here, we use words called Maizel’s theorem which is the counterpart of Castigliano's theorem type of theorems that we have already discussed in our earlier lectures. Castigliano's theorem, is an energy method based theorem and that says that one deflection there, you need to compute first the force here.

If I have to apply unit force here, there will be internal force set up in that structure, that unit force causing internal thermal force in this, if you denote is the internal thermal force due to unit load, internal force. It is actually mechanical force due to unit load at the point of interest, when you multiply that with a mechanical strain, alpha Tx, delta T and integrate it over the entire beam from here, till here, that is this whole thing, we get deflection at that point, this delta we get that.

That is what the Maizel’s theorem gives just like Castigliano's theorem, one of the energy methods work. Now if you do that, delta relative to (()) (48:24) if you take, it is going to be here again, it does not depend on the size, but in reality it does, because design approximation, that is what we have done, reality is different.

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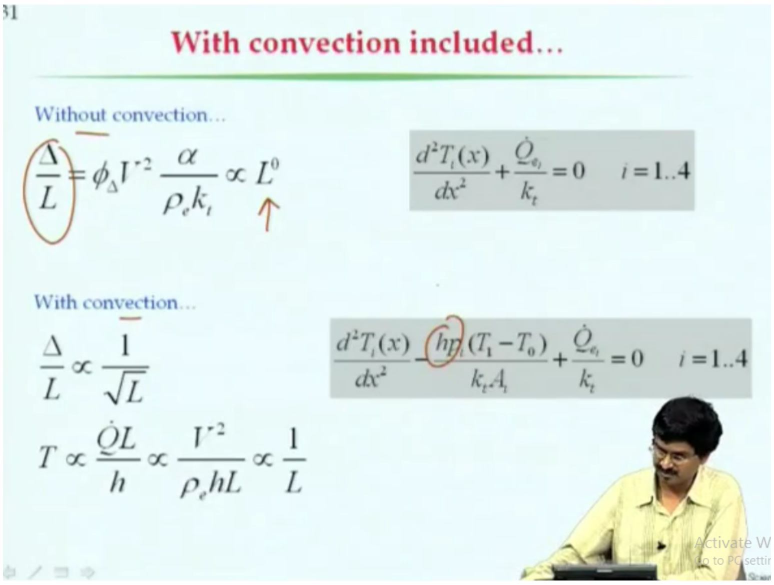

Here also if we do not include convection, it includes convection the electric reflection will change. Electric reflection here is only L0, it is a scale independent, where as if you take convection and look at the temperature, its inverse proportional to the size meaning, smaller the size, larger the temperature, so little thermal actuation actually works more at small sizes, micro or 10 times larger but not at macro scale.

Although thermal loading is huge in terms of leading to cracks in building and making the bridges collapse but when you actually want to make an actuator, its temperature difference is going to be inverse proportion to size, accordingly the deformation, everything will also be more at the micro scale and again it depends on the properties electrical resistivity, thermal expansion coefficient.

If you have the appropriate properties for these 2 medium can be used for thermal actuation. So again we have the equations 1 to 4, then we had neglected the segment 2 and took that for heat balance anyway without convection, with convection we get a slightly different type of expression and h, the heat transfer coefficient.

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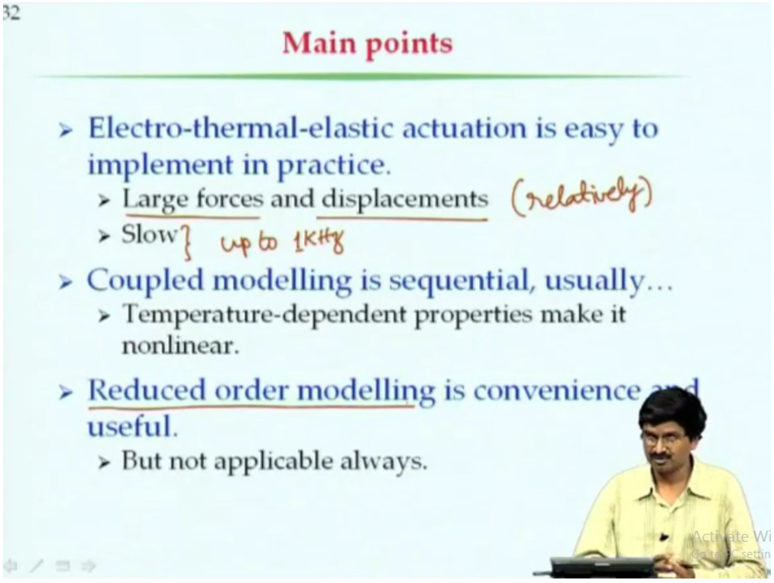

There is a lot to be modeled there to get the heat transfer coefficient properly because as we said a few minutes ago that if you go to handbooks on heat transfer, they will give you the heat transfer coefficient for a vertical plate heated from this side or a horizontal plate heated from bottom or above and all these different conditions and the usually natural convection, means that there is no fan to increase the convective heat transfer.

So once we get those coefficient properly and there will be scaling effects also. We will take that into account and then solve the problem. To summarize what we have discussed in the context of electrothermal actuators, one thing that we need to know is that electrothermal actuator gives us a way of embedding actuation into a mechanism, so that if have a gripper there is no actuator pushing or pulling the gripper rather the structure itself works both as an actuator as well as mechanism.

And then create a reasonably large displacement and forces as compared with, solely relatively, because we are not saying that we will get a large force like you know electromagnetic or IC engine, done commercial engine type of actuation, but for micro scale visible large forces and large displacement, but it will be slow, what do you I mean by that? If I want something to cycle, right, so I apply potential electrical, then thermal coupling and then elastic it will move.

Now, I want to move back and I want to do this fast and it cannot be done very fast because it takes time to heat-cool, heat-cool, that normally at the micro dimensions will take out a millisecond, so we can go up to 1 khz. That is one drawback of these systems. If you push it you can go to 10 khz, but not faster than that because it does take time to heat and cool. And couple modeling here is sequential.

Usually but when we include convection because when we take temperature depending properties things can become more complicated in that, we cannot do it sequentially, electrical first and then thermal and then mechanical. When you include convection, convection heat transfer coefficient also depends on temperature and that also leads to linearity and sequential coupling maybe destroyed.

And so we will have large structures, we want to use reduced order modeling. There can be a large dimensional structure but we will consider that as thin beams and make them as one dimensional electrical resistors, one dimensional thermal resistors and then beams and use energy theorems to quickly calculate the deflection. But this kind of reduced order modeling is not generally applicable.

Because if there is a slender structure, we can call it electrical resistor and then thermal resistor and beam and if it is a bulky structure and that is allowing for non uniform electrical current distribution and hence thermal and then mechanical strain, it may not be amenable for reduced order modeling where you have definitively start to finite element type of solutions and look at the entire behavior.

This kind of coupled modeling is the characteristic of micro electro mechanical systems, the micro systems in general. So in this course we only looked at electrostatic and mechanical coupling and now we looked at electrical thermal and elastic coupling, but there are lot more things. If you look at the piezo electric materials which we discussed briefly in this course, there is material level coupling that needs to be taken into account.

But in general any micro system device that you take there will always be coupling and it will be more than energy to be made and at coupling is what makes their modeling interesting also leads to lots of advantages. Whenever something is difficult to model, we immediately see an opportunity for design because it will lead to some interesting phenomenon and that we can take into account and take advantage of it to design interesting structures.

Such as the one that we saw in plane if you want to move, these single materials you can get different expansion and get actuation. Okay. I will write my email address. If you have any questions, you can send an email. It is suresh@mecheng.iisc.ernet.in and my name is G.K. Ananthasuresh. Thank you.