**MECHANICS**

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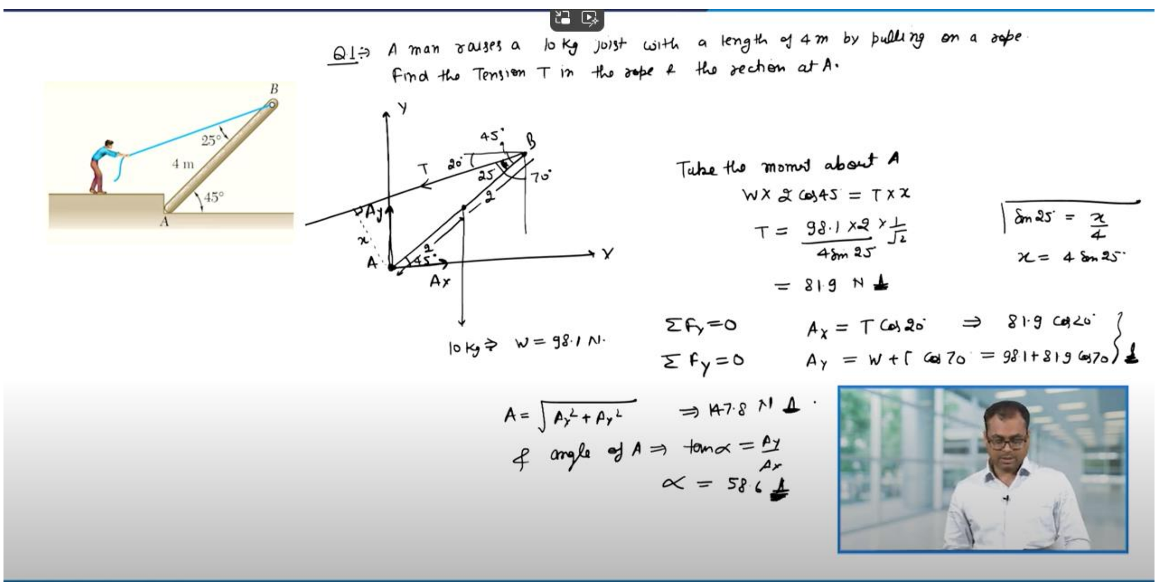
**Lecture 08**

# Examples: Equilibrium of rigid bodies in two dimensions

Hello everyone, welcome to the lecture again. In the last lecture, we looked at the equilibrium condition for one-dimensional object, two-dimensional object and three-dimensional object.



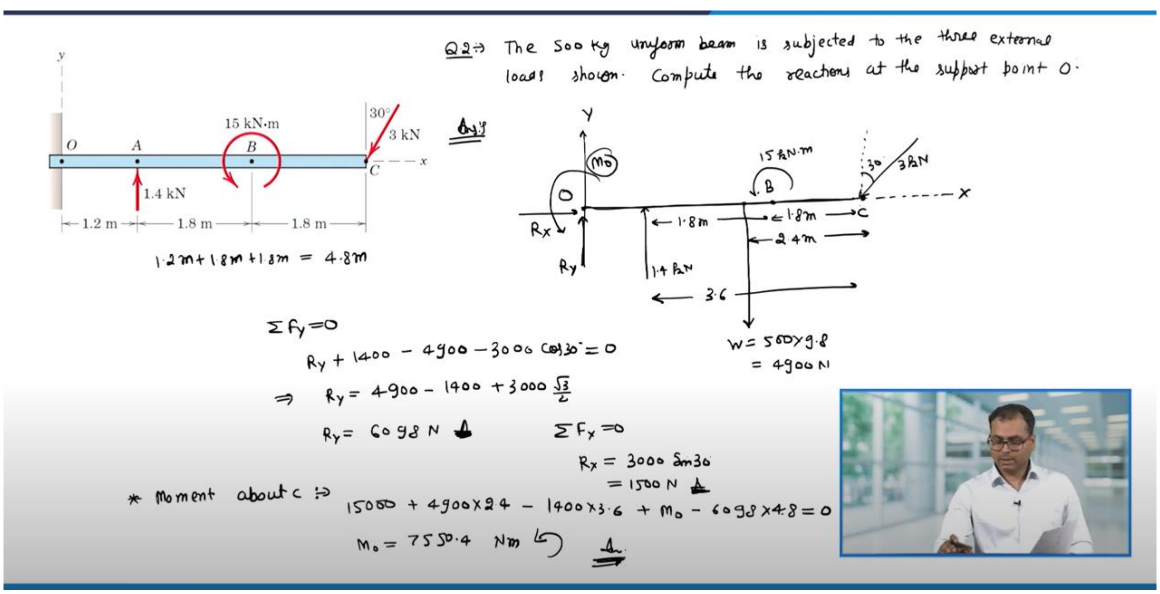

Today, we are going to do more examples on the equilibrium of the rigid bodies in two-dimensional systems. So, let us look at the following problem statement. Question number 1, a man raises a 10 kg zoist with a length of $4m$by pulling on a rope. And in the question, it is asked to find the tension T. in the rope and the reaction at A. So, first let us make a free body diagram of this. So, we have the x-axis, the y-axis and we have this joist whose weight is going to act at the center of mass and it is 10 kg. So, W is $98.1N$ and this is going to act at a distance of $2m$ because joystick is $4m$ long.

At A, we have the reaction forces $A_x$ and $A_y$. It will not support the moment because it can rotate about A. Now, the tension T will be away from the object and to find out the value of T, let us take the moment about A. So, for that, I need to know how much is this distance? This is perpendicular on T. So, let us say this point is P. This point is B, this point is A and so on.

So, this is $45^o$. This angle is given. It is $25^o$. Therefore, this angle will be $70^o$ and this angle will be $45^o$. Now, let us take the moment. about A. So, W into the horizontal distance which is $2cos45^o = Tx$. Now, I have to first find out how much is $x$. Now, you can see here that this angle is $90^o$ and this angle is $25^o$. Therefore, $sin25^o = x/4$ or $x = 4\ sin25^o$. So, T will be $98.1 \times 2 \times \frac{\frac{1}{\sqrt{2}}}{4sin25^o}$ and this comes out to be $81.9N$.

Now, the $A_x$ and $A_y$ , I can find out using the force balance along the x and y direction. So, let us look at $\sum F_x = 0$. So, this gives me $A_x = Tcos20^o$ because this angle over here is $20^o$ and $\sum F_y = 0$ will give me $A_y = W + Tcos70^o$ and since the values of T are known, it will be $81.9cos20^o$ and this will be W is 98.1 plus T is $81.9cos70^o$.

These are the values of $A_x$ and $A_y$. Now, total A will be $\sqrt{A_x^2 + A_y^2}$ and the angle that A makes from the x-axis will be $tan\alpha = A_y/A_x$ and this comes out to be $147.8N$ and this comes out to be $58.6^o$.




Now, let us look at this question, here the problem statement is following. The $500kg$ uniform beam is subjected to the three external loads. As shown compute the reactions at the support point O. So, the first thing is to make the free body diagram of this. So, let us make the free body diagram. Let us say this is y-axis and we have the x-axis and this is the uniform beam. Now, at point C, we have a force of $3kN$ which is acting at $30^o$.

At point B, which is $1.8m$ away from C, we have a moment and its value is $15kNm$. At $1.8m$ away from B, we have another force of $1.4kN$ that is acting now, this beam has a weight of $500kg$ and the length of the beam is $1.2m + 1.8m + 1.8m$ which is $4.8m$. Therefore, at $2.4m$ from C, its weight is going to act.

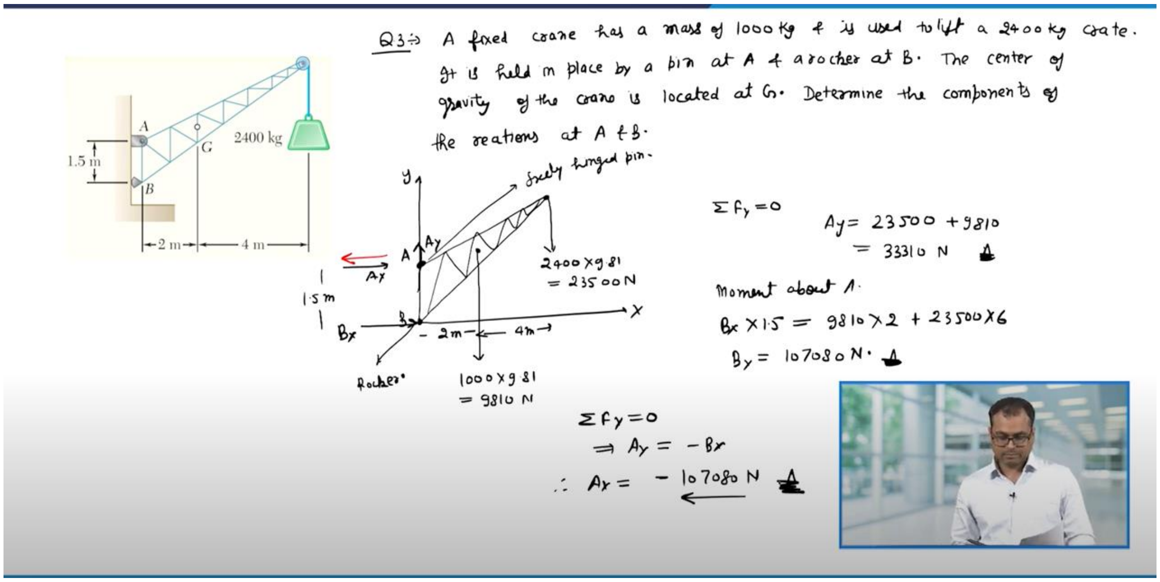
So, this will be W equal to $500 \times 9.8$ which is $4900N$. Now, at point O, we have a built-in support. Therefore, the reaction forces will be $R_x$, let us say $R_y$ and a moment $M_O$ . To find out the unknown parameters, let us first take the force equation and see $\sum F_y = 0$.

So, let us look at what are the forces that are acting in the y direction. So, we have $R_y +$ $1400$ because $1.4\ kN - 4900N$ acting downward $-3000cos30^o$ because we have $3kN$ force which is acting at C. So, this gives us $R_y = 4900 - 400 + 3000\sqrt{\frac{3}{2}}$. So, $R_y$ comes out to be 6098. Now, we can use the force equation along the x direction.

So, $\sum F_x = 0$. Now, the forces in the x directions are $R_x$ and the component of the $3kN$. So, force in the x direction. So, we have $R_x = 3000sin30^o$ and this gives us $1500N$ force along the x direction.

Now, to find out the moment $M_O$, let us take the moment about C. So, here you have to note that the moment is a free vector. Therefore, this moment, $1500kN$ or $M_O$, they are about B and 0.0 respectively, but it does not matter. So, moment about C will be $15000 +$ $49000 \times 2.4$. So, this is the moment of the weight $-1400 \times 3.6$ because this distance is 3.6 plus $M_O$ plus because the moment is in the anticlockwise direction $-6098 \times 4.8$ and this would be equal to 0.

Now, from here, you can find out the moment. It comes out to be $7550.4Nm$. Now, note that this moment comes out to be positive. Therefore, the direction of the moment of $M_O$ that we have taken is correct. If this would have been negative, then we would have changed the direction of the moment. So, this is correct.




Now let us look at the third example and here the problem statement is a fixed screen has a mass of $1000kg$ and is used to lift a $2400kg$ crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G and in the question, it is asked to determine the components of the reactions at A and B. So, first let us make the free body diagram. So, we have the x-axis, we have the y-axis and we have this screen. And its weight is acting at $2m$ from point B and this is $1000kg$. So, $1000 \times 9.81$ gives you $9810N$. This is $2m$. At $4m$, we have a weight of $2400N$. So, let us multiply it by 9.81.

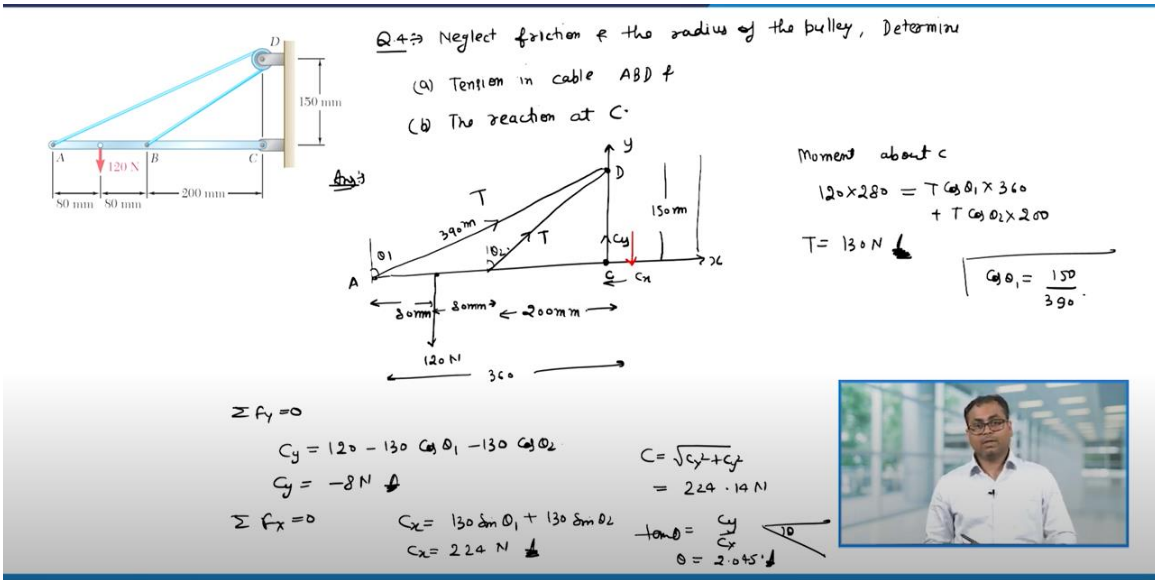
So, it will be $23500N$. Now, let us see if this point is A and this point is B. Now, at B, we have a roker support. So, this one is roker. So, therefore, this will support only the horizontal forces. So, let us call it $B_x$.

It cannot support the B y component. At A, we have $A_x$ and $A_y$ forces since this one is a freely hinged pin. Okay. Otherwise, this crane cannot go up and down. So, therefore, this has to be a freely hinged pin and therefore, it cannot support any moment.

And it is also given that this is $1.5m$. Now, to calculate the values of $A_x$, $A_y$ and $B_x$, let us first use the force balance equation along the y-axis. So, $\sum F_y = 0$. So, this gives me $A_y$ equal to 23500 plus 9810 equal to this much Newton.

Now, to find out $B_x$, let us take the moment about A. So, we have $B_x$ multiplied by the perpendicular distance, which is $1.5 = 9810$ and from A, the horizontal distance is $2 + 23500$ and the distance is 6. So, this gives you $B_x = 107080N$. Now, to find out the $A_x$, let us take the equation $F_x = 0$.

So, this gives you $A_x = -B_x$. Therefore, $A_x$ will be $-B_x$ is $10708.0N$ and the minus sign indicate that the direction of $A_x$ is not correct. So, we have to change the direction of $A_x$ and the correct direction should be along the -x direction. So, therefore, this force is acting in this direction.




Now, let us look at the next question and here the problem statement is following. Neglect friction and the radius of the pulley. Determine A, the tension in cable ABD and B, the reaction at C. So, to solve this question again, let us first make the free body diagram. So, let us see, this is x-axis and we have y-axis.

Here at point D, we have a pulley. This point is C and this point is A. Now, from A to B, we have a same cable. At this point, a force of $120N$ is acting. So, this is $80mm$.

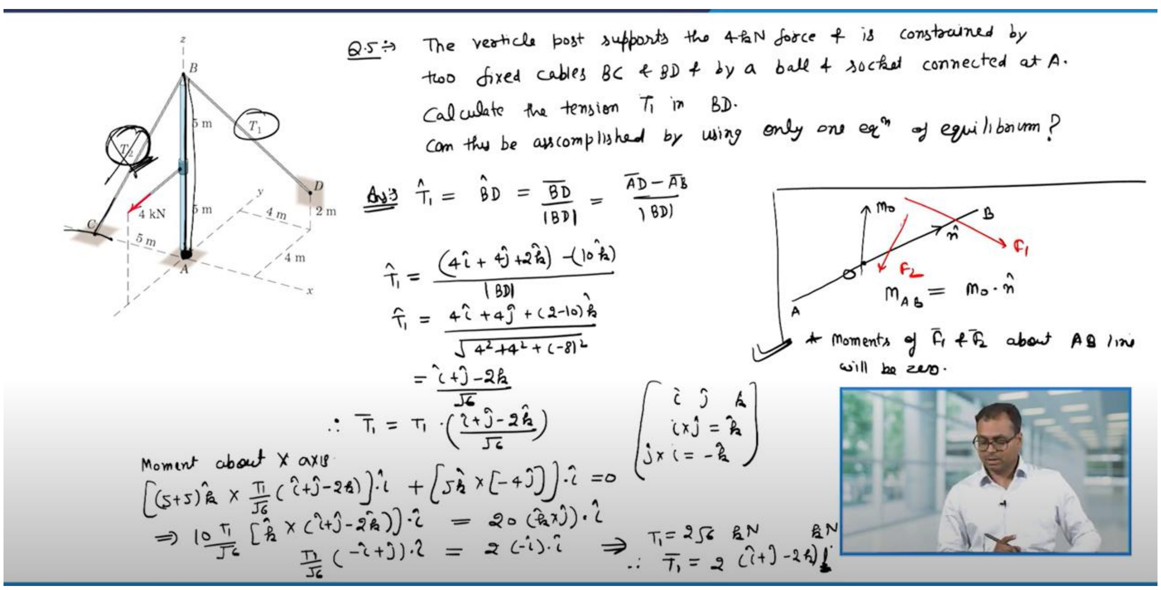
This is also $80mm$ and this distance is $200mm$. Now, since the cable is 1, therefore, the same tension is will act. So, on both AD and BD. Now, at C, we have two forces $C_x$ and let us say it is acting in this direction and $C_y$.

This is given $150mm$. Let us say this angle is $\theta_1$ and this angle is $\theta_2$. Now, to find out the tension T, let us calculate the moment about C because there are two unknown forces, so they will go away. So, moment about C. So, we will have 120 into the distance, the horizontal distance, it is $280mm = Tcos\theta_1$. So, I have taken the vertical component, multiply by the distance, which is $200 + 80 + 80$. So, this is $360mm + T\,cos\theta_2$ into

Now, $\theta_1$ and $\theta_2$ are known because this is 360 and this is 150. So, therefore, this distance will be $390mm$ and cos $\theta_1$ will be $150/390$.

So, we can put the value of $\theta_1$ and $\theta_2$ and this will give you $T = 130N$. Now, we can balance the forces along the y-axis. So, let us say summation F y equal to 0. So, this gives us $C_y$ equal to all the vertical forces. So, we have $120 - 130cos\theta_1 - 130cos\theta_2$ and this gives you $C_y = -8N$. This means that the direction of $C_y$ we have to reverse. So, this is the correct direction of $C_y$. Now, let us balance the force along the x direction. So, $\sum F_x = 0$, this gives you $C_x = 130sin\theta_1 + 130sin\theta_2$.

Again, $\theta_1$ and $\theta_2$ are known. So, therefore, sine $\theta_1$ and sine $\theta_2$ is also known. So, this gives you $C_x$ equal to 2 to 4 Newton. Now, total $C$ will be $\sqrt{C_x^2 + C_y^2}$ and this comes out to be $224.14N$ and $tan\theta$ will be $C_y/C_x$. This gives you $\theta$ equal to $2.045^o$ and its direction will be downwards.




Now, let us look one problem which involves the calculation in three dimensions. So, the problem statement is following. The vertical post supports the $4kN$ force and is constrained by two fixed cables BC and BD and by a ball and socket connected at A. calculate the tension $T_1$ in BD, and further can this be accomplished by using only one equation of equilibrium.

Now, to solve the three-dimensional problem, we require the following concept. Let us say there are forces which are acting on the object and you calculate the moment about O and let us say this moment is $M_O$. Now, you want to know this moment about a line. Let us say this line is AB. In that case, you take the projection of $M_O$ along the line.

So, let us say the unit vector along the line is AB. In that case, the moment about line AB will be the moment about O and then its projection along AB. So, here n cap is the unit vector along the direction AB. Now, if there are various forces, which are passing through the line.

So, they intersect this line in that case. So, let us say this is $F_1$, this is $F_2$. If the forces intersect this line AB, then their moment of $F_1$ and $F_2$ about AB line will be 0. We are going to use this concept.

So, since in the question it has asked to find out $T_1$, so first let us look at the direction of $T_1$. So, the direction of $T_1$ will be $\widehat{T_1}$ which is $\widehat{BD}$ and this will be $\overrightarrow{BD}/|BD|$ and this can be written as $AD - AB/BD$. Now, AD is $4i + 4j + 2\ k$. You can look at the diagram and AD is $10k$ divide by the magnitude and this comes out to be $4i + 4j + (2 - 10)k$ and its magnitude which is $\sqrt{4^2 + 4^2 - 8^2}$ and this will be $i + j - 2k/\sqrt{6}$. Therefore, $T_1$ vector will be the magnitude which we do not know and which we have to find out and the unit vector which is $i + j - 2k/\sqrt{6}$. Now, to find out the magnitude of $T_1$, let us calculate the moment along the x-axis.

So, we are going to calculate the moment about x-axis. Now, note that this force T 2 is passing through the x-axis. Therefore, this force will not contribute into the moment. Now, for the x-axis, what I will do is I will calculate the moment about A and then take the dot product with I, okay?

So, the moment of the force $T_1$ will be its distance. So, this distance, it is $(5 + 5)\widehat{(k)} + \frac{T_1}{\sqrt{6}}(i + j - 2k)$ and for the x-axis let us take the dot product with i. For the $4kN$ force the distance is $5k$ and the force is$-4j$ and let us take the dot product with i. Now, this can be simplified. This is $10T_1/\sqrt{6}$ and we have $\widehat{(k)} \times (i + j - 2k).i = (k \times j).i$ and this is $T_1/\sqrt{6}$ and we get $(-i + j).i = 2(-i).i$. Here I have used that $i \times j = k$ and $j \times i = -k$. And this gives you $T_1 = 2\sqrt{6}\ kN$. Therefore, $\underset{T_1}{\rightarrow}$ will be $2i + j - 2k$. So, we can see that just by using one equation, we were able to accomplish the value of $T_1$. With this, let me you know, conclude this lecture here, and thank you for your patience.