MECHANICS

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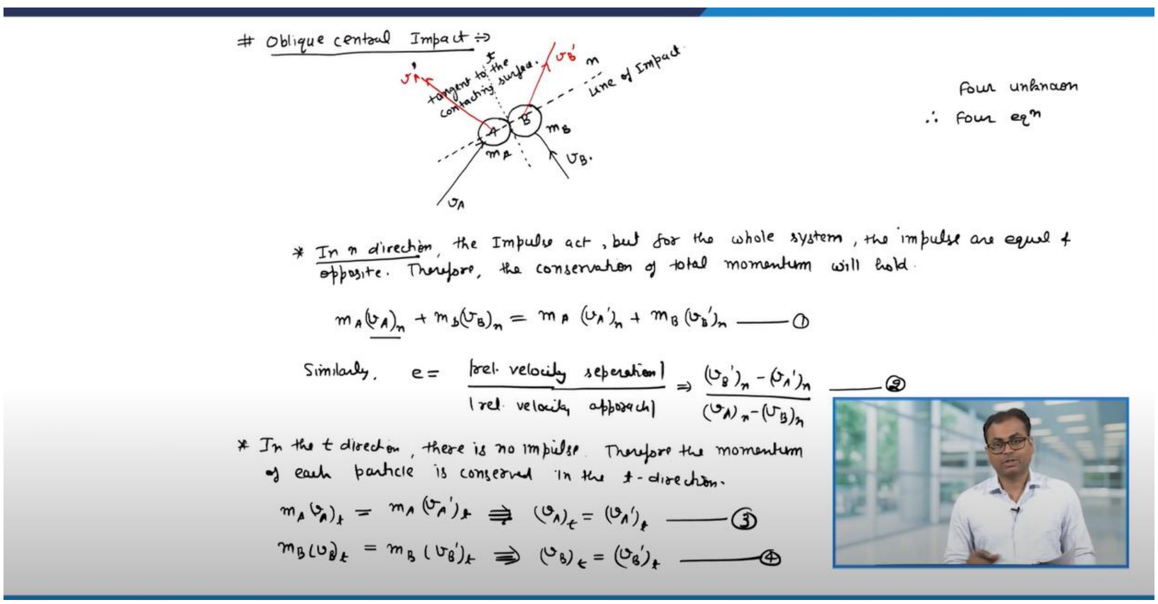
Lecture: 42

Oblique central impact

Hello everyone, welcome to the lecture again. In the last class, we discussed about the direct central impact wherein the direction of the motion of the particles were along the same line that was the line of the impact.



Today, we are going to discuss the oblique central impact wherein the direction of motion of the particle is at an angle to the line of impact. So, therefore, the situation is following. We have particles, let us say, two particles, and they are moving like this.

So, this is particle A, and its velocity is, let us say, like this, and then we have another particle B, and let us say it is moving like that. So, the common tangent is this, and the line of impact is like that. Let us call this direction as n and this is the line of impact. This is the tangent. So, let us call it t direction and this is the tangent to the contacting surface. And what we want to find out is, after the impact, what is the velocity of particles A and B, and at what angle they are going. So, basically, we need four equations because there are four unknown velocity and the angle of both the particles. So, four unknowns Therefore, we need four equations.

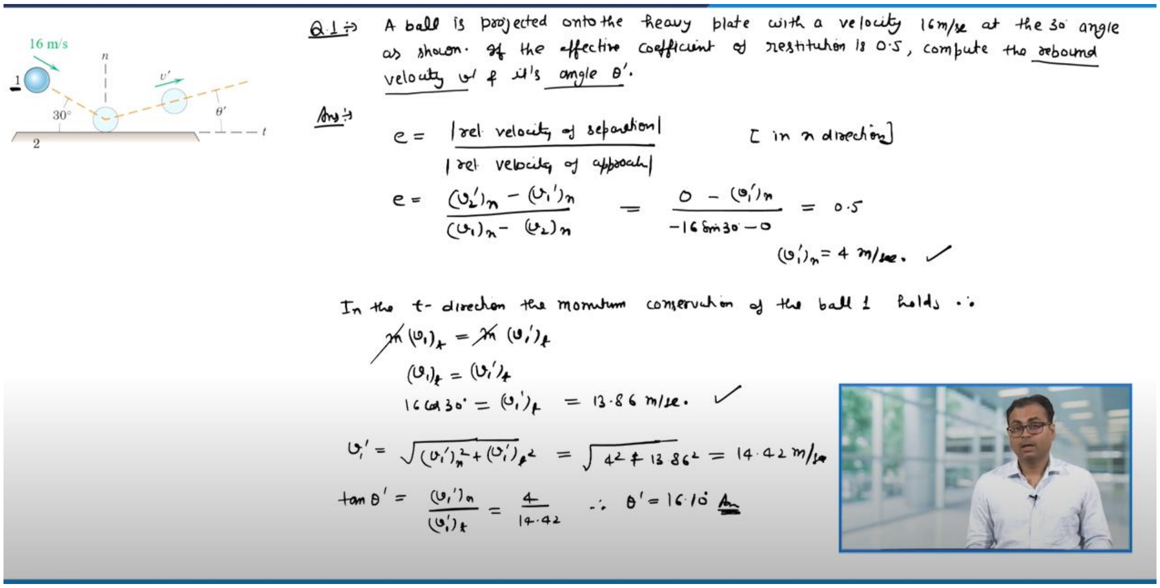
Now, in the n direction, so that is along the line of the impact, the impulse act, but for the whole system, the impulse are equal and opposite. Therefore, the conservation of total momentum will hold. So, we are talking about the n direction. So, let us apply the conservation of angular momentum for the whole system.

So, initial velocity is v_A, mass is, let us say m_A, and this has a mass of m_B. So, we have $m_A v_A$. Note that it holds only in the n direction. So, let me write down n here. So, that means the velocity of particle a in n direction plus $m_B v_B$ in the n direction equal to m_A. Let us say after the impact, the velocity becomes v'_B and v'_A. So, we have $m_A (v'_A)_n$ in the direction n and $m_B (v'_B)_n$ in the direction n. Let us call it equation number (1). Similarly, we have the definition of e which is the relative velocity of separation divided by the relative velocity of approach that will also hold in the n direction. So, we have a relative velocity of separation divided by the relative velocity of approach that will hold in the n direction. So, this will be equal to $\frac{(v'_B)_n - (v'_A)_n}{(v_A)_n - (v_B)_n}$. Let us call this equation number (2).

In the t direction, that is, in the direction of the tangent to the contacting surface, there is no impulse. So, if the impulse is not there, then the momentum of both the particles will be conserved separately. So, we have no impulse. The momentum of each particle is conserved in the t direction. So, we have mass $m_A v_A$ in the t direction.

Initial momentum of particle A equal to the final momentum of particle A. So, $m_A (v'_A)_t$. So, m_A will get cancelled, and this will give you $(v_A)_t = (v'_A)_t$. So, let us call this equation number (3). similarly, for particle B also, the momentum will conserve in the t direction. So, we have $m_B v_B$ in the t direction, this is the initial moment equal to the final moment in the t direction. So, m_B will get cancelled.

So, we have $(v_B)_t = (v'_B)_t$. Let us call it equation number (4). so, now, we have 4 equations, and as I said, there are 4 unknowns. So, in principle, the problem can be solved mathematically.



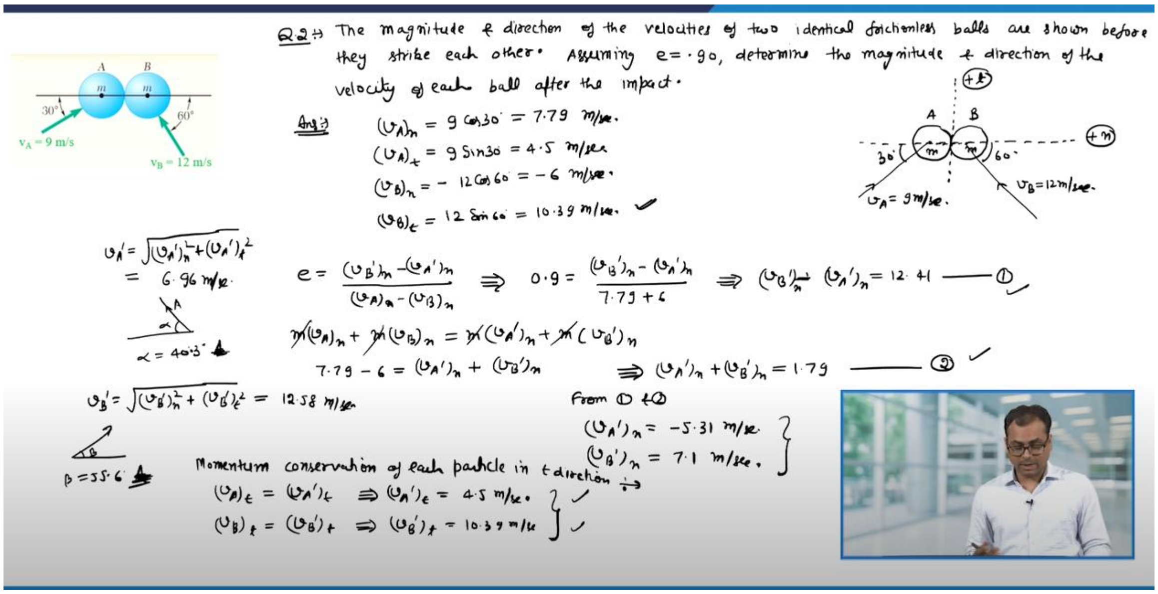
Now, let us look at some of the examples based on this concept. This is the first problem statement. A ball is projected onto the heavy plate with a velocity of 16 m/s. At the 30° angle as shown. If the effective coefficient of restitution is 0.5, compute the rebound velocity v' and its angle θ' . So, clearly, this is the example of oblique angle impact, and we have to find out what is the rebound velocity v' and what is the angle θ'.

It is also given that the plate is heavy. So, therefore, this plate does not move during the impact. Let us use the definition of e. So, e is relative velocity of separation divide by the relative velocity of approach. And this holds in the n direction.

So, let us find out what is the relative velocity of separation and approach in the n direction. So, e is $\frac{(v_2')_n-(v_1')_n}{(v_1)_n-(v_2)_n}$. So, this is in n direction. So, v_2 is or v_2' or v_2 is 0 because the plate is heavy, it does not move. $\frac{-(v_1')_n}{(v_1)_n}$. So, $(v_1)_n$ is the velocity of particle 1 in the n direction before the impact.

So, it is $-16Sin30°$, and $(v_2)_n$ is 0. And it is given that this is equal to e, and the value of e is 0.5. So, from here, I can find out what is $(v_1')_n$ and $(v_1')_n$ comes out to be 4 m/s. Now, in the t direction, the conservation of momentum of particle A holds. So, in the t direction, the momentum of conservation of let us say ball 1 holds. Therefore, we have $m(v_1)_t$ in the t direction t equal to $m(v_1')_t$, and m will get cancelled. So, we have $(v_1)_t = (v_1')_t$. Now, $(v_1)_t$, we can know from the diagram that $(v_1)_t$ is $16Cos30^{\circ}$ equal to $(v_1')_t$ and $16Cos30°$ is 13.86 m/s. So, we know what is $(v_1')_n$ and what is $(v_1')_t$. Therefore, I can calculate what is v_1'.

So, v_1' will be $\sqrt{(v_1')_n^2 + (v_1')_t^2}$. So, that will be $\sqrt{(4)^2 + (13.86)^2}$, and that will be 14.42 m/s. Now, let us calculate the angle θ'. So, $tan\theta'$, $Tan\theta' = \frac{(v_1')_n}{(v_1')_t}$. Now, $(v_1')_n$ is 4, and $(v_1')_t$ is 14.42. Therefore, θ' comes out to be $16.10°$.



Now, let us look at another problem statement. The magnitude and direction of the velocities of two identical frictionless balls are shown before they strike each other assuming e equal to 0.90, determine the magnitude and direction of the velocity of each ball after the impact. Okay. So, the situation is following. We have ball A and its mass is m and it is moving with velocity v_A equal to 9 m/s, and it makes an angle of $30°$. Then we have ball B, its mass is also m because the balls are identical, and velocity of B is 12 m/s, and it makes an angle of $60°$ from the horizontal. And we have been asked to find out the velocity and direction of each ball after the impact. So, this is the common tangent to it.

Let us take it as a plus t direction, and this is the line of the impact. So, let us take it as plus n direction. Now, we can, you know, from the component, we can find out the initial velocities of particles A and B in the t and n directions. So, let us see v_A in the n direction is $9Cos30°$.

So, that is 7.79 m/s. Velocity of Ball A or particle A in the t direction is $9Sin30°$. so, that is 4.5 m/s. For particle B or ball B, v_B in the n direction will be $-12Cos60°$, which is $-6\ m/s$.

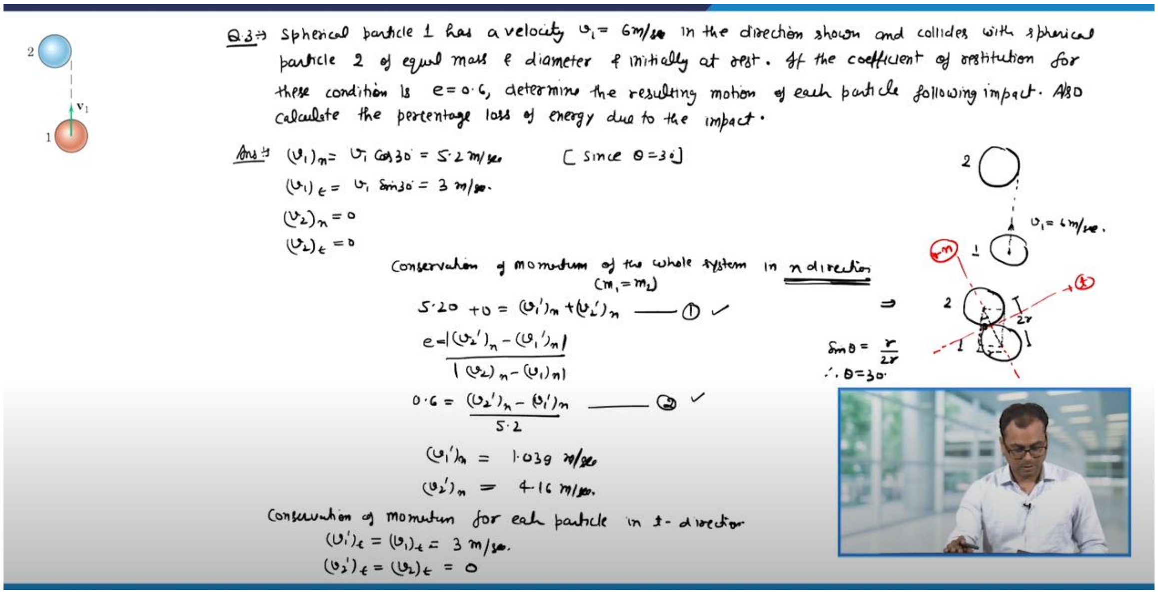
And $(v_B)_t$ will be $12Sin60°$. So, that is 10.39 m/s. Now, let us use the definition of e. So, e is the velocity of separation divided by the relative velocity of approach. So, it is $\frac{(v_B')_n-(v_A')n}{(v_A)_n-(v_B)_n}$. And e is given.

So, e is $0.9 = \frac{(v_B')_n-(v_A')n}{(v_A)_n-(v_B)_n}$ we already know. So, it is $7.79\ -(-\ 6)$. So, that becomes plus 6. So, that gives you $(v_B')_n - (v_A')n = 12.41$.

Let us call it equation number(1). Now, the total momentum will be conserved in the n direction. So, therefore, we have $m(v_A)_n + m(v_B)_n = m(v_A')_n + m(v_B')_n$. now, m will get cancelled because the masses are identical. So, we have $7.79 - 6\ = (v_A')_n + (v_B')_n$, or I can just rewrite this as $(v_A')_n + (v_B')_n = 1.79$. Let us call it equation number (2) and from (1) and (2), you can find out what is $(v_A')_n$ and what is $(v_B')_n$. So, from (1) and (2), I get $(v_A')_n = -5.31 m/s$ and $(v_B')_n = 7.1\ m/s$. Now, in the t direction, as we have discussed, the momentum of each particle will be conserved.

So, we have momentum conservation of each particle in the t direction. So, we have $(v_A)_t = (v_A')_t$ because m will get canceled. So, therefore, we have $(v_A')_t = (v_A)_t$, we already know, it is 4.5 m/s. And similarly, $(v_B)_t = (v_B')_t$, again m will get cancelled. So, we have $(v_B')_t = (v_B)_t$, which is, again, we know it is 10.39. So, this is 10.39 m/s. And from here, I can find out what is v_A'. So, $v_A' = \sqrt{(v_A')_n^2 + (v_A')_t^2}$, and that will be 6.96 m/s.

Now, the angle α that particle A makes after the impact can be find out by using $Tan\alpha = \frac{(v_A')_n}{(v_A')_t}$. So, angle α comes out to be $40.3°$. And v_B' that is the velocity of a particle B after the impact, is $\sqrt{(v_B')_n^2 + (v_B')_t^2}$. So, that will be 12.58 m/s. And the angle that it makes from the horizontal can be find out again by taking $Tan\beta = \frac{(v_B')_n}{(v_B')_t}$. So, β comes out to be 55.6° . So, using these four equations, we have calculated the velocity and the direction of particle A and B after the impact.



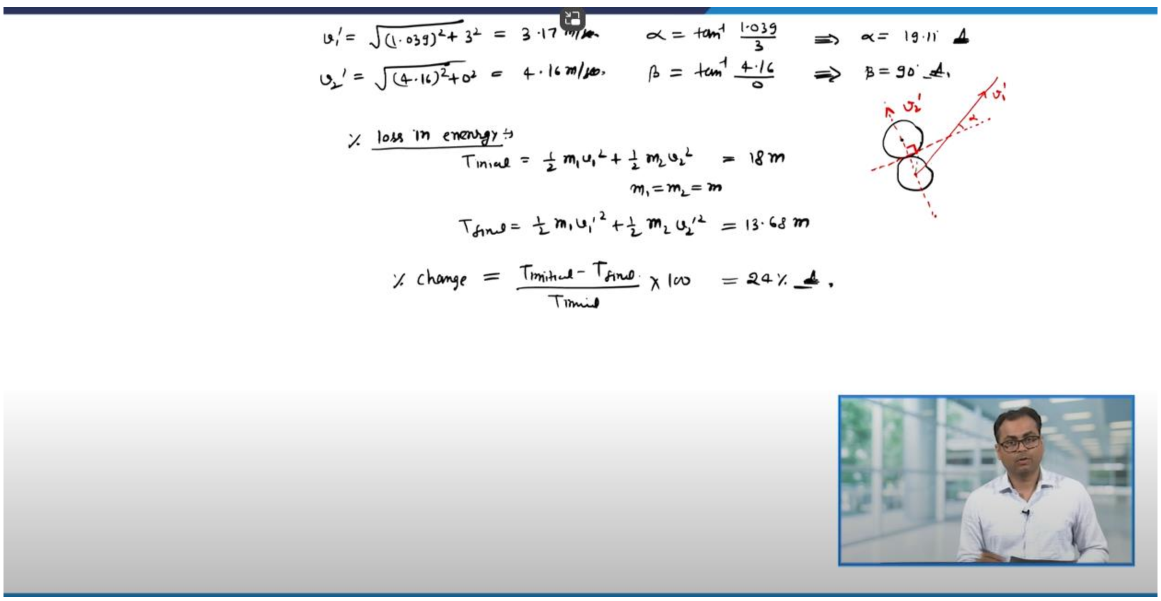
Now, let us look at another problem statement. A spherical particle 1 has a velocity $v_1 = 6\ m/s$ in the direction shown and collides with a spherical particle 2 of equal mass and diameter and initially at rest. If the coefficient of restitution for these condition is $e = 0.6$ determine the resulting motion of each particle following impact. Also calculate the percentage loss of energy due to the impact.

So, let us analyze this. We have particle 2 and particle 1. The outer surface of particle 2 is aligned with the center of particle 1. Particle 1 is moving with a velocity of $6\ m/s$. So, during the impact, we will have particle 2 and particle 1 like that. The centre-to-centre distance should be 2 times r. So, this distance is 2r, if r is the radius of the particle or the sphere. And since this is r, therefore, I can find out what is this angle θ . So, from the geometry, you can see that $Sin\theta\ = \frac{r}{2r}$. Therefore, θ should be 30°.

Now, the common tangent is, is this and the line of impact is that. So, therefore, let me take this as plus n and this one as plus t direction. Let us find out the velocity of a spherical particle 1 and 2 along the t and n direction. Since θ is 30° , therefore, $(v_1)_n$ is will be v_1, which is $6cos30^0$. So, that will be $5.2\ m/s$. And $(v_1)_t$ from the geometry, it will be $v_1 Sin30°$, that will be $3\ m/s$. And since particle 2 is at rest, therefore, $(v_2)_n$ will be 0 and $(v_2)_t$ will also be 0.

Here, it is given that $m_1 = m_2$, and we know that the conservation of momentum for the whole system will be valid in the n direction. So, let us apply the conservation of momentum of the whole system in n direction. And $m_1 = m_2$, they said, it is given. So, therefore mv , but m will get cancelled.

So, we have 5.2 in the n direction $(v_1)_n + 0 = (v_1')_n + (v_2')_n$. Let us call it equation number (1). Now, also in the n direction, we have $e = \frac{(v_2')_n - (v_1')_n}{(v_2)_n - (v_1)_n}$. So, e is given, it is $0.6 = \frac{(v_2')_n - (v_1')_n}{5.2}$. Let us call it equation number (2), and from equation number (1) and (2), we can find out both the unknowns, that is, $(v_1')_n$ and $(v_2')_n$, and it comes out to be 1.039 m/s, and $(v_2')_n$ comes out to be 4.16 m/s. Now, the conservation of momentum for each particle will hold in the t direction, so conservation of momentum for each particle will hold in t direction.



So, since the masses are equal, so m will get cancelled. So, $(v_1')_t = (v_1)_t$ and $(v_1)_t$ is already known. So, therefore, $(v_1')_t$ becomes 3 m/s, and $(v_2')_t = (v_2)_t$, and $(v_2)_t$ is 0. So, therefore, this is 0.

So, from here, I can find out the magnitude of v_1' and the direction of, you know, v_1. So, the magnitude is $\sqrt{(v_1')_t^2 + (v_1')_n^2}$. So, v_1' becomes square root $(v_1')_n$ is 1.039 and $(v_1')_t$ is 3.

So, therefore, it is$\sqrt{1.039^2 + 3^2}$. So, that is 3.17 m/s and the angle $\alpha = \tan^{-1}\frac{1.039}{3}$. so, from here, you get $\alpha = 19.11°$. Similarly, v_2' will be $\sqrt{(v_2')_n^2 + (v_2')_t^2}$.

So $\sqrt{(4.16)^2 + (0)^2}$. So, square will get cancelled with the square root. So, we have $4.16\, m/s$ and the angle $\beta\ = \tan^{-1}\frac{4.16}{0}$. So, that will be 90°.

Okay. So, the situation will be like that. So, we had this particle there and the then this one here after the impact. So, this is our common tangent, this is the line of impact, this is $\beta°$.

So, therefore, particle 2 will move with a velocity of v_2' in that direction, and particle 1 will go in a direction α with a velocity of v_1', where α is 19.11 degrees. Now, let us calculate the percentage loss in the energy. So, here the energy is the kinetic energy. So, let us find out what is $T_{initial}$, the initial kinetic energy.

It is $\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2 2$. And v_2 was initially 0, and m_1 is equal to m. So, let me put $m_1 = m_2 = m$ because both of them has the same mass. So, that comes out to be 18 m, where m is the mass. And T_{final} comes out to be $\frac{1}{2}m_1(v_1')^2 + \frac{1}{2}m_2(v_2')^2$. And, you know, the values of v_1' and v_2' you can put and $m_1 = m_2 = m$. So, that comes out to be 13.68 m. So, therefore, the percentage change in the kinetic energy is $\frac{T_{initial}-T_{final}}{T_{initial}} \times 100$. So, you know the $T_{initial}$ and T_{final}. so, that comes out to be 24% . With this, let me stop here. See you in the next class.

Thank you.