MECHANICS
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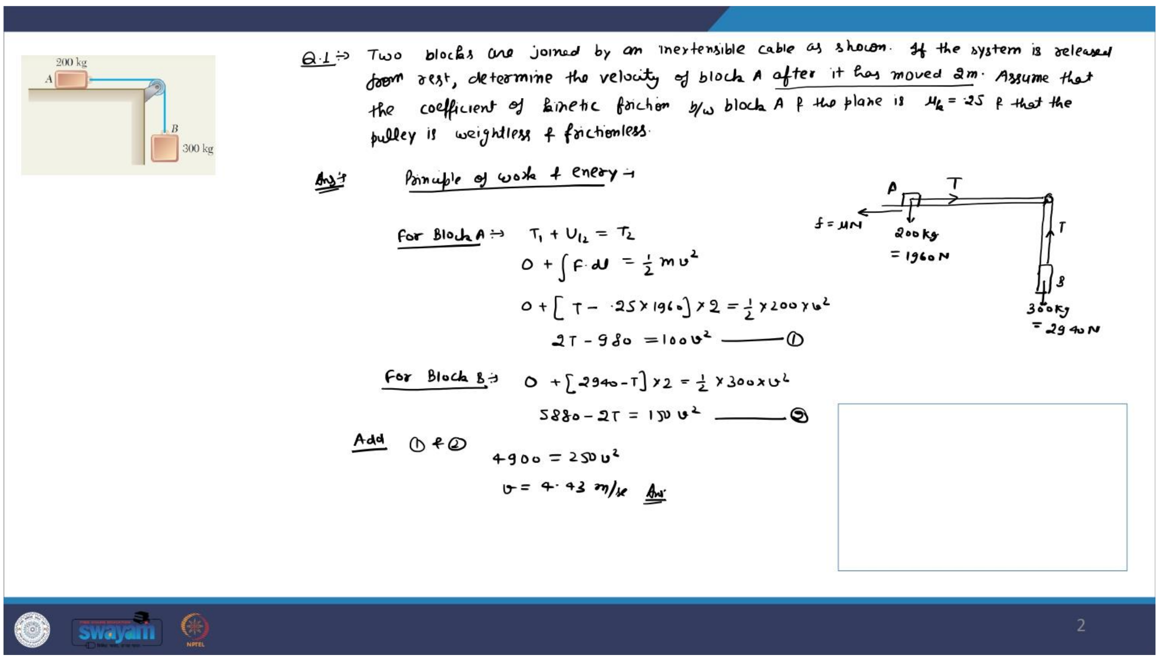
Lecture 38
Work energy method: examples

Hello everyone, welcome to the lecture again. In the last class, we looked at the work energy method and saw a couple of examples. Today, we are going to see more examples on the same concept.



So, the first problem statement is following. Two blocks are joined by an inextensible cable as shown. If the system is released from rest, determine the velocity of block A after it has moved $2\ m$ and assume that the coefficient of kinetic friction between block A and the plane is $\mu_k = 0.25$ and that the pulley is weightless and frictionless. So, in this question again, we can use the principle of work and energy and we can apply that for both block A and block B. So, let us apply the principle of work and energy for block A. So, we have to identify all the forces that are acting on block A. So, for that, let me make the free body diagram. So, we have this plane and then we have the block. Its weight is $200\ kg$, which is equal to $1960\ N$ and the tension T will be away from this block. So, let's say it is T, we have a pulley here and then we have the block B, its weight is $300\ kg$. So, therefore, this is equal to $2940\ N$ and again the tension T will be upward. Now, because the table has friction, therefore, the frictional force will be in the opposite direction of the motion. So, it will be $f = \mu N$ and N is given by the weight of this block. So, let us write down the principle.

So, we have $T_1 + U_{12} = T_2$, where T_1 is the initial kinetic energy of block A. So, $T_1 = 0$, the work done is force times the displacement $F.dl$ equal to final kinetic energy is $\frac{1}{2}mv^2$.

$0 + \int F.dl = \frac{1}{2}mv^2$.

Therefore,

$$0 + (T - 0.25 \times 1960) \times 2 = \frac{1}{2} \times 200 \times v^2$$

$$2T - 980 = 100\ v^2 ------(1)$$

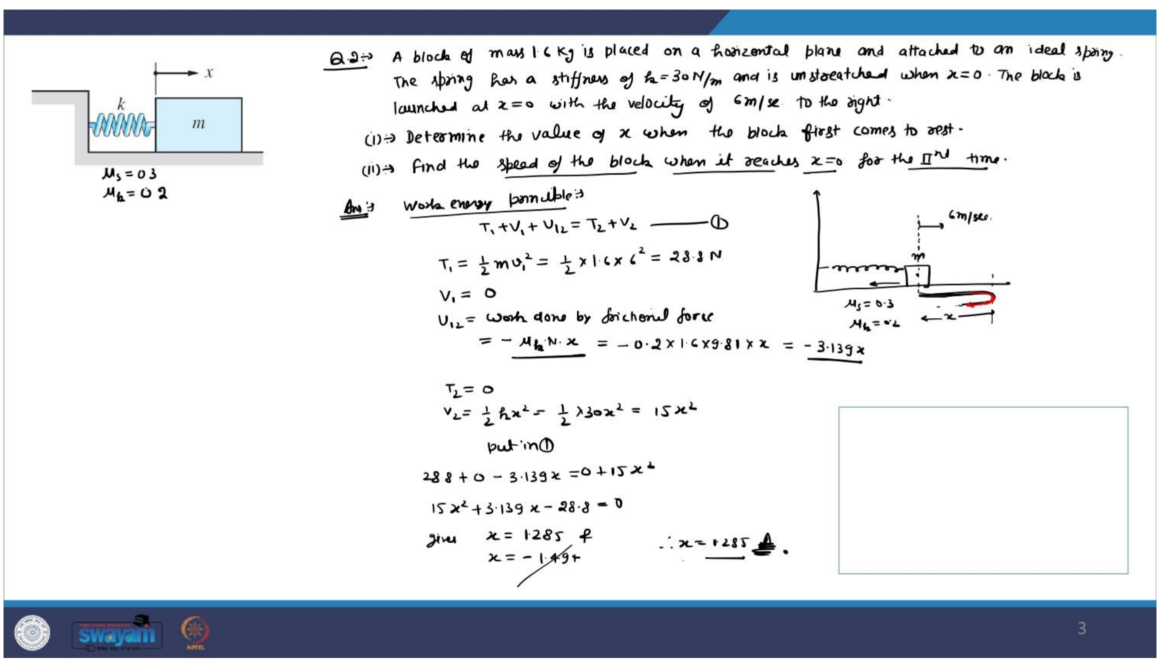
Now, let us apply the conservation or the principle of work energy for block B. So,

$$0 + (2940 - T) \times 2 = \frac{1}{2} \times 300 \times v^2$$

$$5880 - 2T = 150v^2 ------(2)$$

And we have to find out what is the value of v. Therefore, let us add equation number 1 and 2. So, $4900 = 250v^2$. $v = 4.43\ m/s$.

So, note that using the principle of work and energy very easily, we were able to find out the velocity of the block. We do not need to solve the Newton equation of motion.



Now, let us look at the another problem statement. A block of mass $1.6\ kg$ is placed on a horizontal plane and attached to an ideal spring. The spring has a stiffness of $k = 30\ N/m$

and is unstretched when $x = 0$. The block is launched at $x = 0$ with the velocity of 6 m/s to the right. And we have been asked to determine the value of x when the block first comes to rest. And in the second part, we have been asked to find the speed of the block when it reaches $x = 0$ for the second time. It is also given in the question statement that the static and kinetic friction of the plane is 0.3 and 0.2. So, it is given that $\mu_s = 0.3, \mu_k = 0.2$. So, to find out the value of x when the block comes into rest, let us use the work energy principle and the situation is following. We have this block of mass m and it starts with a velocity of 6 m/s. The static friction and kinetic friction are given. And then this block moves and then comes into the rest. We have to find out what is this distance. So, we have $T_1 + V_1 + U_{12} = T_2 + V_2 - - - - - (1)$. Now, the initial kinetic energy T_1 of the block is $\frac{1}{2}mv_1^2$.

$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2} \times 1.6 \times (6)^2 = 28.8\ N.$

V_1 is the potential energy. Note that there is no potential energy because at this location, there is no stretch in the spring. So, therefore, $V_1 = 0$ and U_{12}, note that it is the work done by the frictional force. So, whatever is the force except for the gravity and the spring forces, they comes into U_{12} and let us find it out. So, the frictional force is going to work in the opposite direction of the motion. So, they are going to act in a $-x$ direction. Therefore,

$U_{12} = -\mu_k . N . x = -0.2 \times 1.6 \times 9.81 \times x = -3.139x$

and T_2 is the final kinetic energy, it comes to rest. So, therefore, $T_2 = 0$. In this case, $V_2 = \frac{1}{2}kx^2 = \frac{1}{2} \times 30 \times x^2 = 15x^2$. Let us put all this value in equation number 1.

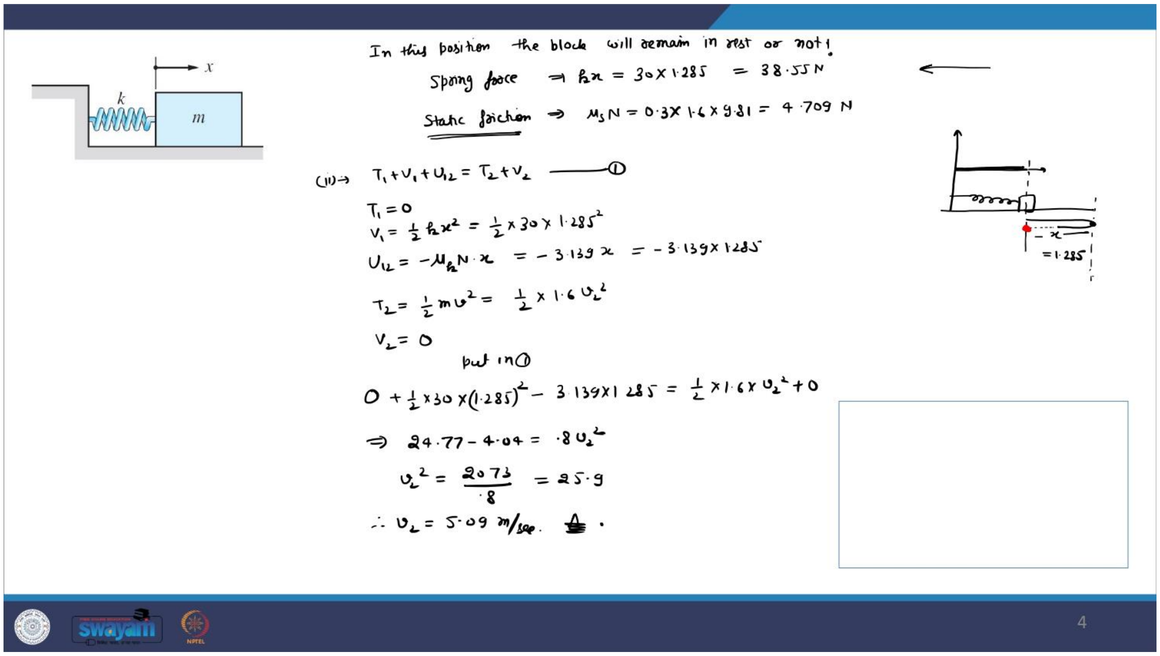
So, $28.8 + 0 - 3.139x = 0 + 15x^2$

$15x^2 + 3.139x - 28.8 = 0$

$x = 1.285$ & $x = -1.494$

Note that this value ($x = -1.494$) is not possible because the block is moving in the $+x$ direction. Therefore, negative value is not acceptable. So, we have $x = 1.285$.

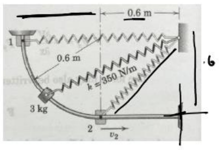
Now, let us see that for this value of x, whether the block will remain in the rest or not. So, we are going to see that in this position, the block will remain in rest or not. So, let us look at the spring forces for that value of x. So, the spring force is kx. Therefore, the spring force will be $30 \times 1.285 = 38.55\ N$ and it is going to act in the $-x$ direction. And what are the other forces that are acting on the block. Well, they are static friction. So, the static friction, its maximum value is $\mu_s N$. The value of μ_s is given, it is $0.3 \times 1.6 \times 9.81 = 4.709\ N$. Therefore, the frictional forces are not sufficient to overcome the spring forces.



Therefore, after the block reaches that maximum value of x, it is going to come back. It is not going to stop there. Now, in the second part, we have been asked to find out the speed of the block when the block reaches at $x = 0$ for the second time. So, just now we have seen that the spring forces are larger than the frictional forces. Therefore, the block is not going to stay over here. It is going to come back and the question is what is the velocity at $x = 0$. So, let us again use the work energy relation.

So, we have $T_1 + V_1 + U_{12} = T_2 + V_2 - - - -(1)$

and let's say this is the initial point. Therefore, the kinetic energy $T_1 = 0$ because the block is at rest over here and potential energy $V_1 = \frac{1}{2}kx^2 = \frac{1}{2} \times 30 \times (1.258)^2$. And U_{12} is the work done by the frictional forces. So, $U_{12} = -\mu_k N.x = -3.139\,x = -3.139 \times 1.285$. Now, let us look at kinetic energy T_2. $T_2 = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.6 \times v_2^2$ and the potential energy because of the spring is $0, V_2 = 0$ because its natural length is when the spring is at $x = 0$. Now, let us put all these values in equation number 1. So, we have $0 + \frac{1}{2} \times 30 \times (1.285)^2 - 3.139 \times 1.285 = \frac{1}{2} \times 1.6 \times v_2^2 + 0$ and from here we can rewrite it as $24.77 - 4.04 = 0.8v_2^2$. This gives you $v_2^2 = \frac{2073}{0.8} = 25.9$. Therefore, $v_2 = 5.09\ m/s$.



Q.3 → The 3-kg slider is released from rest at position 1 & slides with negligible friction in a vertical plane along the circular rod. The attached spring has a stiffness of 350 N/m & has an unstretched length of 0.6 m. Determine the velocity of the slider as it passes position 2.

Ans: Work energy eqn $T_1 + V_1 + U_{12} = T_2 + V_2$ ——— (1)

$T_1 = 0$

$V_1 = mgh + \frac{1}{2} k x_1^2$

$= 3 \times 9.81 \times 0.6 + \frac{1}{2} \times 350 \times (.6)^2$

$= 17.66 + 63 = 80.66 J$

$U_{12} = 0$

$T_2 = \frac{1}{2} m v_2^2$

$V_2 = \frac{1}{2} k [.6\sqrt{2} - .6]^2$

$0 + 80.66 + 0 = \frac{1}{2} \times 3 \times v_2^2 + \frac{1}{2} \times 350 [.6\sqrt{2} - .6]^2$

$80.66 = \frac{1}{2} \times 3 v_2^2 + 10.81$

$v_2 = 6.32$ m/sec. Ans.

Now, let us look at another problem statement. The 3 kg slider is released from rest at position 1 and slides with negligible friction in a vertical plane along the circular rod. The attached spring has a stiffness of $350\ N/m$ and has an unstretched length of $0.6\ m$ and we have been asked to determine the velocity of the slider as it passes position 2. So, in this question again, we can use the work energy equation between point 1 and point 2 to find out the velocity at 2. So, work energy equation is

$$T_1 + V_1 + U_{12} = T_2 + V_2 - - - - - (1)$$

At point A, because the mass is released from the rest, therefore kinetic energy will be $0, T_1 = 0$. Now, V_1 is the sum of the gravitational potential energy plus the spring energy. So, $V_1 = mgh + \frac{1}{2}kx_1^2$, where x_1 is the extension in the spring. And to find out what is mgh, we have to fix the reference axis. So, let me fix my reference over there. So,

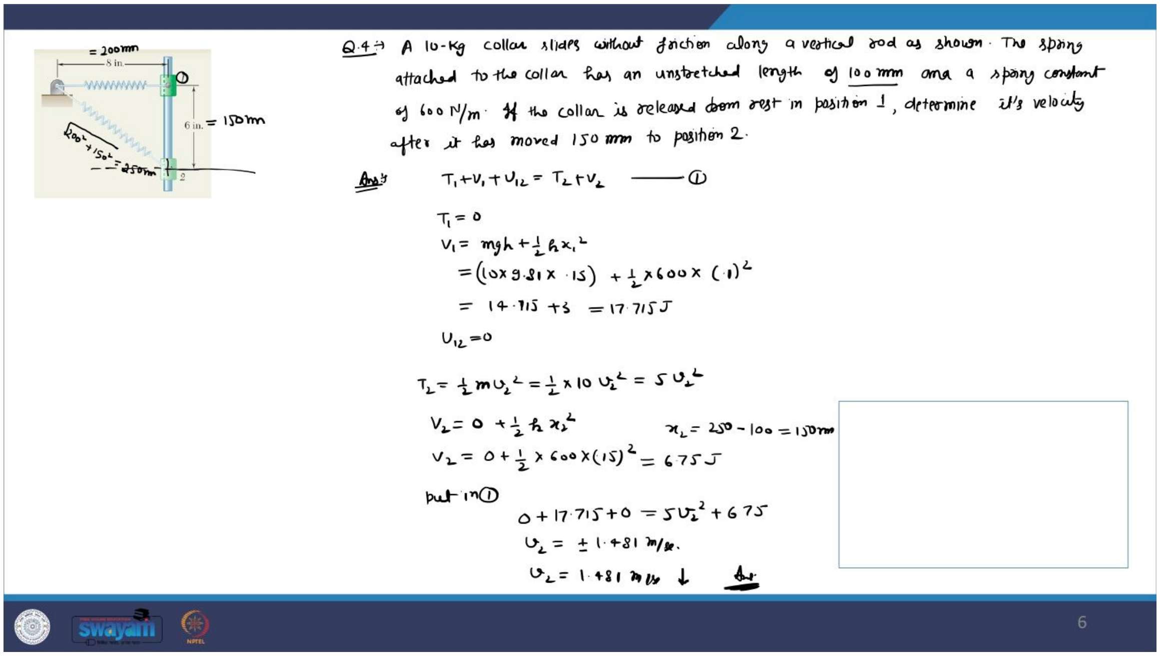
$$V_1 = 3 \times 9.81 \times 0.6 + \frac{1}{2} \times 350 \times (0.6)^2$$

$$= 17.66 + 63 = 80.66\ J$$

And U_{12} is the work done by the other forces. Since the friction is not present, therefore, there are no other work. So, $U_{12} = 0$. Now, let us look at point 2. Here, $T_2 = \frac{1}{2}mv_2^2$ and $V_2 = \frac{1}{2}kx^2$. So, x is the extension in the spring because this is 0.6, this length is also 0.6. Therefore, this length will be $0.6\sqrt{2} - 0.6$ and $mgh = 0$ because our reference is over here. $V_2 = \frac{1}{2}k\left(0.6\sqrt{2} - 0.6\right)^2$. So, therefore, let us put everything in equation number 1. So, we have $0 + 80.66 + 0 = \frac{1}{2} \times 3 \times v_2^2 + \frac{1}{2} \times 350\left(0.6\sqrt{2} - 0.6\right)^2$.

$$80.66 = \frac{1}{2} \times 3v_2^2 + 10.81$$

$$v_2 = 6.32\ m/s.$$



Now, let us look at another similar problem and the problem statement is following. A 10 kg collar slides without friction along a vertical rod as shown. The spring attached to the collar has an unstretched length of 100 mm and a spring constant of 600 N/m if the collar is released from rest in position 1, determine its velocity after it has moved 150 mm to position 2. So, this 6 in is given. It is 150 mm and 8 in is 200 mm. Therefore, I can find out how much is this length. So, this length will be $\sqrt{(200)^2 + (150)^2} = 250\ mm$. Now, let us write down the work energy equation to find out the velocity at position 2. So, we have

$$T_1 + V_1 + U_{12} = T_2 + V_2 - - - - - (1).$$

And in position 1, because the collar is released from the rest, therefore, $v_1 = 0$ and therefore, $T_1 = 0$ because it is kinetic energy and V_1 is the potential energy which is equal to the gravitational energy plus the spring energy that we have. So, to write down the value of mgh, I have to fix the reference.

$$V_1 = mgh + \frac{1}{2}kx_1^2$$

$$= (10 \times 9.81 \times 0.15) + \frac{1}{2} \times 600 \times (0.1)^2$$

$$= 14.715 + 3 = 17.715\,J$$

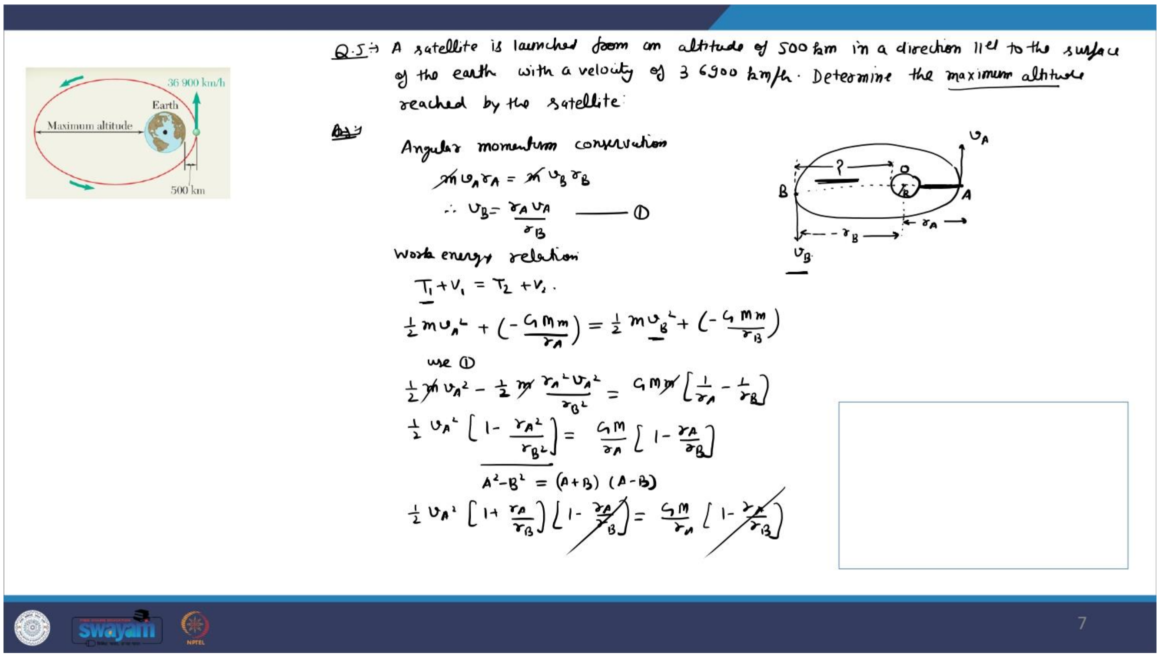
And again, because the friction forces were not there, therefore, $U_{12} = 0$. Now, let us analyze position 2. At 2, we have the kinetic $T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2} \times 10 \times v_2^2$. And $V_2 = 0 +$

$\frac{1}{2}kx_2^2$. Now, let us look at how much is x_2. $x_2 = 250 - 100 = 150\ mm$. Therefore, $V_2 = 0 + \frac{1}{2} \times 600 \times (0.15)^2 = 6.75\ J$. Let us put everything in equation number 1. So, we have

$+17.715 + 0 = 5v_2^2 + 675$

$v_2 = \pm 1.481\ m/s$

$v_2 = 1.481\ m/s \downarrow$ downward because the collar is moving down.



Now, let us look at one more problem on the same concept. And the problem statement is following. A satellite is launched from an altitude of $500\ km$ in a direction parallel to the surface of the earth with a velocity of $36900\ km/hr$. Determine the maximum altitude reached by the satellite. So, we have the following situation. We have this orbit of the satellite, we have the earth, its centre is let's say O, its radius is capital R, then the satellite is launched from point A, its velocity is v_A and at point B, let's say its velocity is v_B, then we have been asked what is this distance, its maximum altitude from the surface of the earth. Let's say the distance of point A is r_A from the center of the earth and the distance of point B is r_B. To find out the maximum altitude, let us first find out what is v_B and for that I can use the angular momentum conservation. So, at point A, the angular momentum is $mv_Ar_A = mv_Br_B$. m will get cancelled. So, therefore, $v_B = \frac{v_Ar_A}{r_B} - - - - - (1)$

Now, I can use the work energy relation between point A and B. So, we have

$$T_1 + V_1 = T_2 + V_2$$

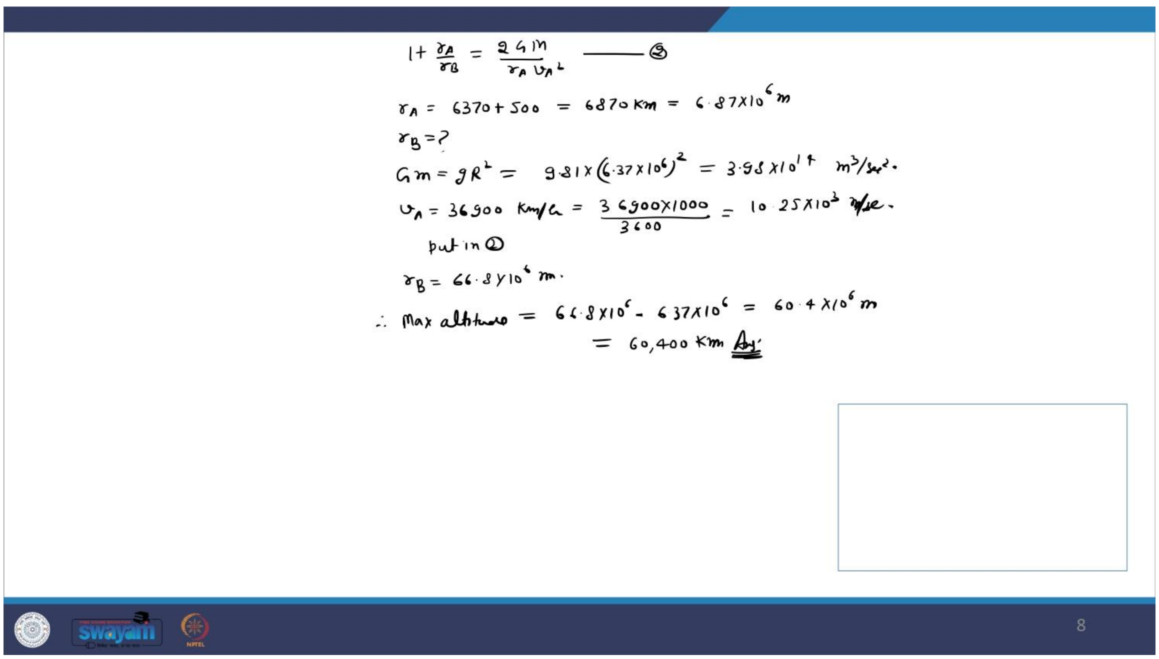
$$\frac{1}{2}mv_A^2 + \left(-\frac{GMm}{r_A}\right) = \frac{1}{2}mv_B^2 + \left(-\frac{GMm}{r_B}\right)$$

$$\frac{1}{2}mv_A^2 - \frac{1}{2}m\frac{r_A^2 v_A^2}{r_B^2} = GMm\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$\frac{1}{2}v_A^2\left(1 - \frac{r_A^2}{r_B^2}\right) = \frac{GM}{r_A}\left(1 - \frac{r_A}{r_B}\right)$$

$$A^2 - B^2 = (A+B)(A-B)$$

$$\frac{1}{2}v_A^2\left(1 + \frac{r_A}{r_B}\right)\left(1 - \frac{r_A}{r_B}\right) = \frac{GM}{r_A}\left(1 - \frac{r_A}{r_B}\right)$$



Therefore, I have $1 + \frac{r_A}{r_B} = \frac{2GM}{r_A v_A^2} - - - - -(2)$

Now, all the values are given in the question. So, r_A is this distance plus the radius of the earth capital R and this is given it is $500\ km$. Therefore, $r_A = 6370 + 500 = 6870\ km = 6.87 \times 10^6 m$. r_B is unknown and $GM = gR^2$ where R is the radius of the earth. So, that becomes $9.81 \times (6.37 \times 10^6)^2 = 3.98 \times 10^{14}\ m^3/s^2$.

And $v_A = 36900\frac{km}{hr} = 36900 \times \frac{1000}{3600} = 10.25 \times 10^3 m/s$.

Let us put everything in equation number 2. So, we get $r_B = 66.8 \times 10^6 m$. But in the question, we have been asked the maximum altitude. So, the maximum altitude is r_B minus the radius of the earth. So, therefore, the maximum altitude is $66.8 \times 10^6 - 6.37 \times 10^6 = 60.4 \times 10^6\ m = 60400\ km$.

With this, let me stop here. See you in the next class. Thank you.