MECHANICS

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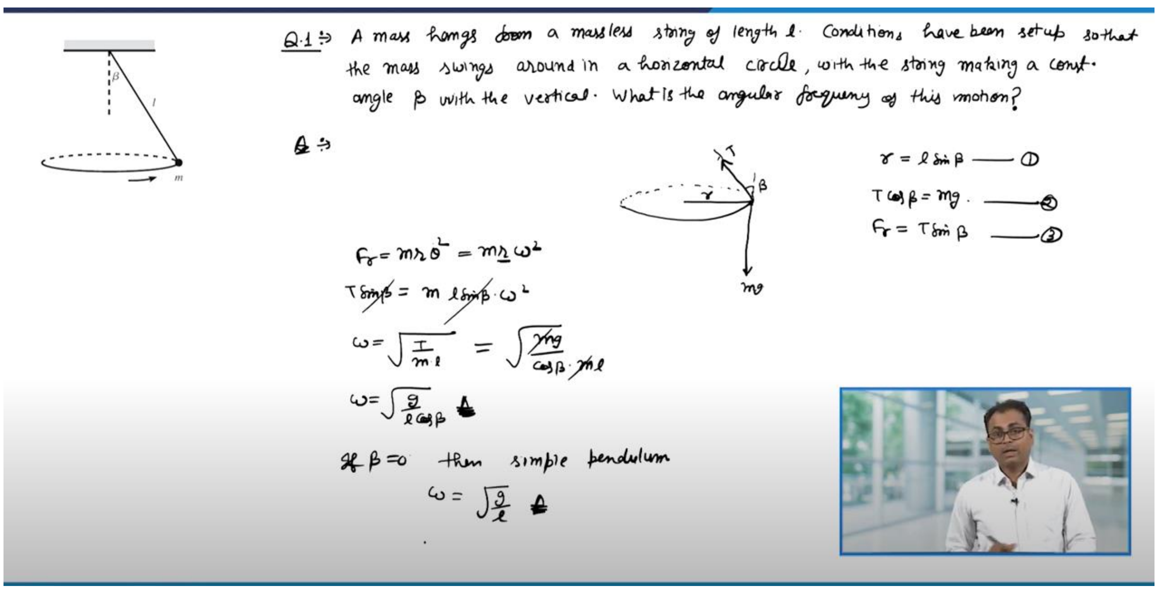
Lecture: 36

Equation of motion: examples

Hello everyone, welcome to the lecture again. In the last class, we look at the equation of motion in different coordinate systems and solve couple of examples for Cartesian coordinate.



Today, we are going to look at examples in other coordinate systems. So, this is the first problem statement. A mass hangs from a massless string of length l. Condition have been set up so that the mass swings around in a horizontal circle with the string making a constant angle beta with the vertical. What is the angular frequency of this motion? So, let me make the free body diagram and look at the forces that are acting.

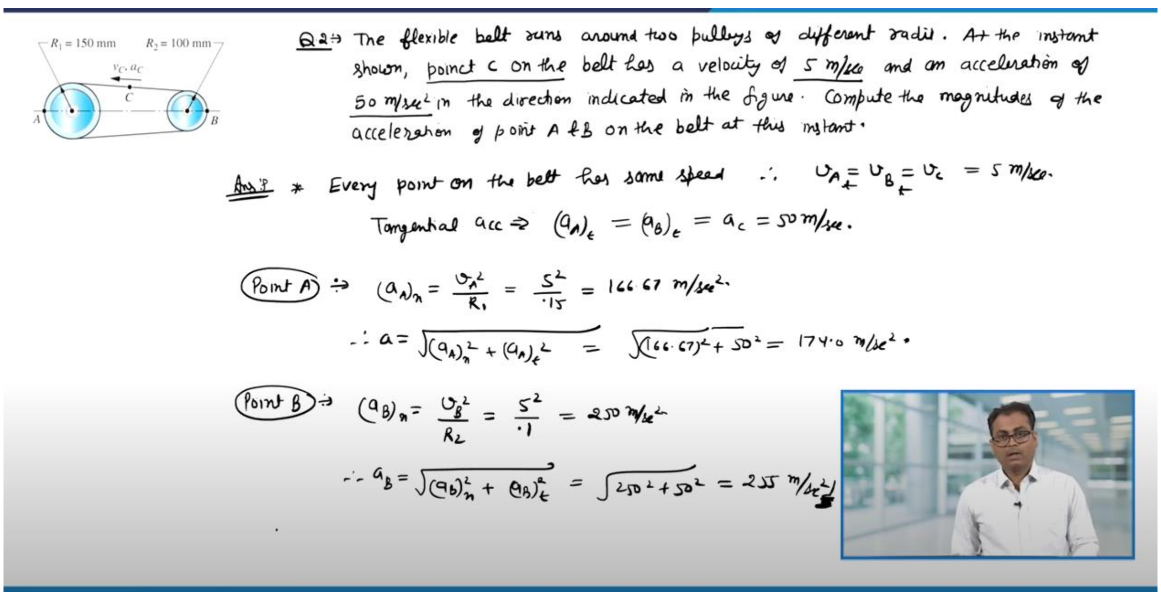
So, we have this mass which is going in a horizontal circle. Its mass is of course m and the tension T is going to act along the string and and it is making an angle beta from the vertical.

Now, the relation between the radius of the horizontal circle that it is making and the length of the string l can be find out from the geometry. So, you can see that $r = lsin\beta$.

Let us call it equation number 1. The force balancing along the y direction gives you $Tcos\beta = mg$. And the force along the r direction, F_r is $Tsin\beta$. Now, let us look at equation of motion in the r direction. So, we have $F_r = mr\dot{\theta}^2$, which can be written as $mr\omega^2$.

And F_r, I already know, it is $Tsin\beta = mr$ is $lsin\beta\omega^2$. So, $sin\beta$ will get cancelled. You get $\omega = \sqrt{\frac{T}{m}l}$. Now, this I can rewrite because from equation number 2, T is $\frac{mg}{cos\beta}$. So, T is $\frac{mg}{cos\beta \times m \times l}$ and m will get cancelled.

We get $\omega = \sqrt{\frac{g}{lcos\beta}}$. Now, note that for a simple pendulum, we have $\beta = 0$. So, let us see what happens when $\beta = 0$. If $\beta = 0$, then let us put there $cos\beta$ will be 1. So, we have the condition of simple pendulum. So, we get $\omega = \sqrt{\frac{g}{l}}$. This is something that we already know.

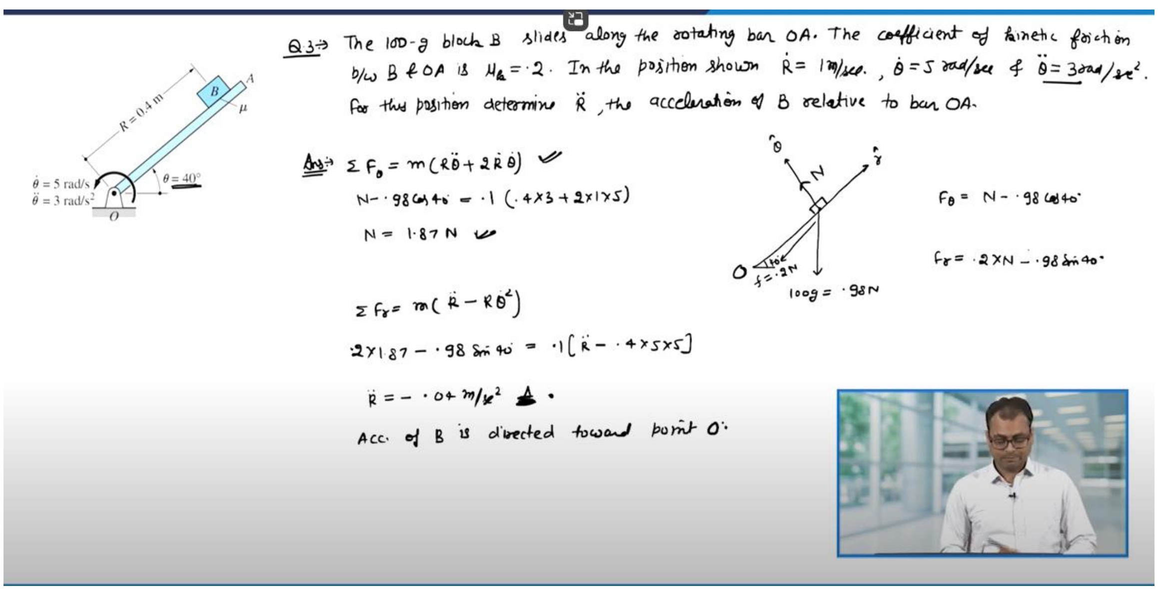


Now, let us look at another problem statement based on flexible belt. The question statement is following. The flexible belt runs around two pulleys of different radii. At the instant zone, point C on the belt has a velocity of 5 m/s and an acceleration of 50 m/s^2 in the direction indicated in the figure. Compute the magnitudes of the acceleration of point A and B on the belt at this instant. Now, in this question, it is given that point C on the belt has a velocity of 5 m/s and acceleration of 50 m/s^2. Therefore, the tangential component of the velocity and acceleration will be 5 and 50 throughout the belt.

So, every point on the belt has same speed. Therefore, velocity at point A, velocity at point B, and velocity at point C, its tangential component will be 5 m/s. Similarly, the tangential acceleration will let us say acceleration at point A, its tangential component, and acceleration at point B, its tangential component, or the acceleration at point C, it will be 50 m/s. So, this is again A_t and B_t.

Now, let us analyze point A. at point A, the normal acceleration will be velocity square divided by its radius. So, that will be $5^2/0.15$, which gives you 166.67 m/s^2. Therefore, total acceleration a will be a normal component square plus a tangential component square. That means it is $166.67^2 + 50^2$ that gives you 174 m/s^2.

Now, let us look at point B. At point B, we have the normal component equal to v_B^2/R_2. So, that will be $5^2/0.1$, which will give you 250 m/s^2. Therefore, the total acceleration of point P will be square root a_B^2 normal component plus a_B^2 tangential component that will be $\sqrt{250^2 + 50^2} = 255\ m/s^2$.



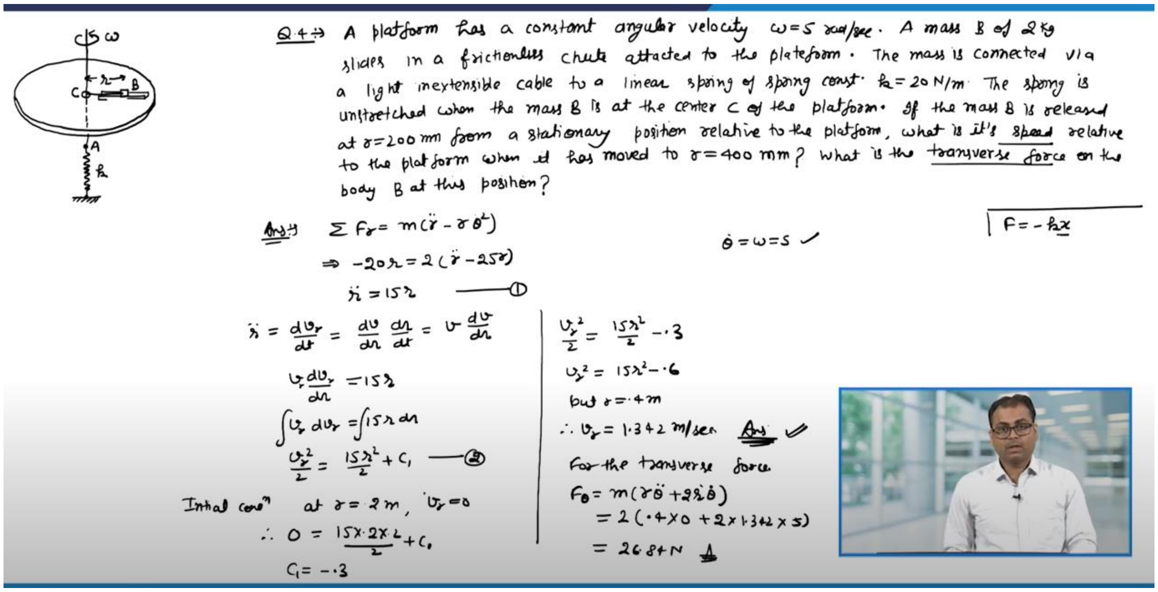
Now, let us look at another problem statement and it is following. The 100 gm block B slides along the rotating bar OA. The coefficient of kinetic friction between B and OA is $\mu_k = 0.2$.

In the position shown, $\dot{R}$ is $1\ m/s$, $\dot{\theta}$ is $5\ radian/sec$. And $\ddot{\theta}$ is $3\ radian/s^2$. For this position, determine $\ddot{R}$, the acceleration of B relative to bar OA. For $R = 0.4\ m$, the value of $\dot{R}, \dot{\theta}$, and $\ddot{\theta}$ is given, and we have to find out the value of $\ddot{R}$. Let us look at the free-body diagram of the block B. So, along the rod, let us take $\hat{r}$ and $\hat{\theta}$ therefore will be perpendicular to it.

The weight of the block which is $100\ gm$ .Which is equal to $0.98\ N$ is going to act downward, and the direction of the friction will be opposite to the motion. So, let us take it in this direction, and it will be μN, μ is given, and N is the normal reaction, which will, of course, act along the $\hat{\theta}$ direction. So, let us write down the equation of motion along the $\hat{r}$ direction and $\hat{\theta}$ direction. So, we have $F_\theta = m(R\ddot{\theta} + 2\dot{R}\dot{\theta})$ and we have $\sum F_r = m\ddot{R} - R\dot{\theta}^2$.

From the free body diagram, let us find out what are the forces in the F_θ direction and what are the forces in the F_r direction. Now, you can see that the F_θ or the force along the $\hat{\theta}$ direction is N minus the component of the weight along the $\hat{\theta}$ direction, which is 0.98 cos, and we have to determine all this at θ equal to 40^0. So, this angle is 40^0; therefore, $cos 40^0$ and F_r, the force along the R direction is the frictional force, which is $0.2N$, and N is something that we do not know right now. So, let us write it as $N - 0.98 sin 40^0$. So, let us use this equation.

So, we have F_θ, which is $N - 0.98 cos 40^0$ equal to m is $100\ gm$. So therefore 0.1, R is0.4, and $\ddot{\theta}$ is given. It is $3\ radian/sec$. So, therefore, multiplied by $3 + 2 \times 1 \times 5$. So, this gives you $N = 1.87\ N$. And now, let us use the second equation. So, we have F_r, which is $0.2\ N$. So $0.2 \times N$. N is something that we know now. So, it is $1.87 - 0.98 sin 40^0$ equal to m is 0.1. $\ddot{R}$ Is a something that we have to find out $-R$ is $0.4\dot{\theta}^2$. So, $\dot{\theta}$ is 5. And this gives you $\ddot{R} = -0.04\ m/s^2$. And the negative sign here indicate that the acceleration of B is directed towards point O.



Now, let us look at another problem statement, question number 4. A plate form has a constant angular velocity $\omega = 5\ radians/second$, a mass B of 2 kg slides in a frictionless tube attached to the platform. The mass is connected via a light inextensible cable to a linear spring of a spring constant $K = 20\ N/m$. The spring is unstretched when the mass B is at the centre C of the platform. Now, it is given that if the mass B is released at $r = 200\ mm$ from a stationary position relative to the platform. What is its speed relative to the platform when it has moved to $r = 400\ mm$?

And we have been also asked what is the transverse force on the body B at this position. In this question, we have been asked to find out the speed and the transverse force. To find out the speed, let us look at the equation of motion along the r direction. So, we have $\sum F_r = m\ddot{r} - r\dot{\theta}^2$. Now, the force along the r direction is because of the spring.

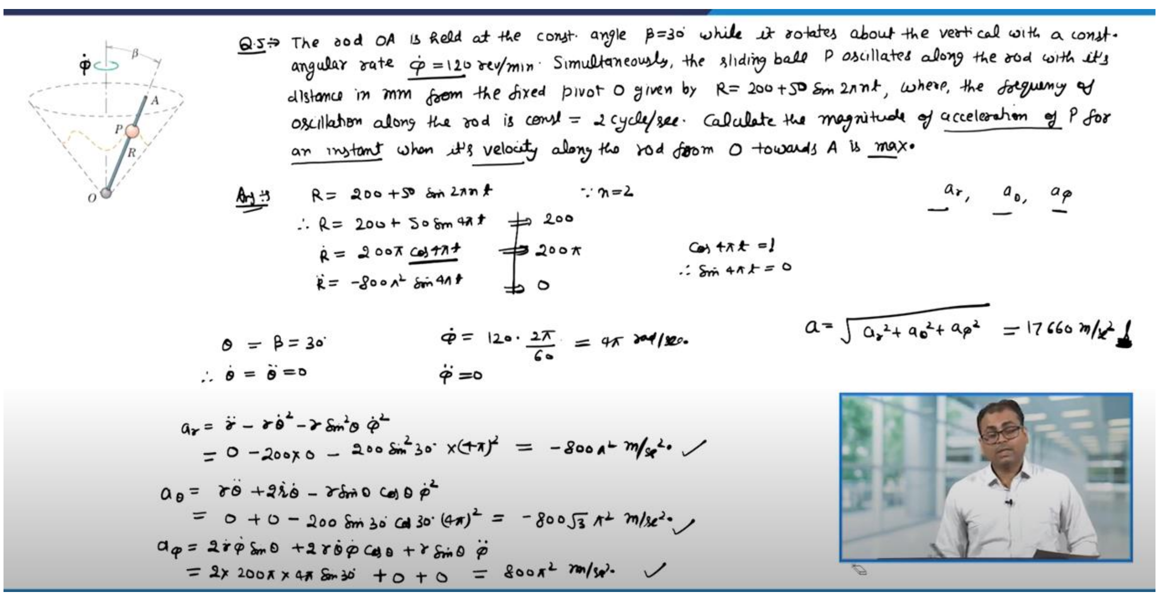
And the spring force $F = -kx$ where x is the extension, which in this case is r, and k is given. Therefore, F_r will be $-20r$. m Is also given $\ddot{r} - \dot{\theta}$ is also given. $\dot{\theta} = \omega = 5$. Therefore $5^2 r$. And from here, we can get $\ddot{r}$.

So, $\ddot{r}$ is $15r$. Now, this $\ddot{r}$ is acceleration. So, it is dv_r/dt rate of change of velocity. And again, I can write down dv/dr and dr/dt, which I can rewrite as vdv/dr. Therefore, from this equation, we have vdv/dr, and everything is about the radial component equal to $15r$ from equation number 1. Now, I can integrate it to find out what is v_r.

So $v_r dv_r = 15rdr$, let us integrate it. So, we have $\frac{v_r^2}{2} = 15r^2/2$ plus constant. Now, this constant I can find out from the initial condition that were set up in the problem. So, the initial conditions is at $r = 0.2\ m$ $v_r = 0$, okay.

So, therefore, $0 = 15 \times 0.2 \times \frac{0.2}{2} + C_1$ or $C_1 = -0.3$. I can put this in equation number 2. So, we have $\frac{v_r^2}{2} = \frac{15r^2}{2} - 0.3$. This I can rewrite as $v_r^2 = 15r^2 - 0.6$ and let us put $r = 0.4\ m$.

So, we get $v_r = 1.342\ m/s$ because we have been asked to find out the velocity at $r = 0.4\ m$. Now, let us look at the force for the transverse force. Because we have been asked to find out the transverse force, we know that $F_\theta = mr\ddot{\theta} + 2\dot{r}\dot{\theta}$. Now, all the values are given m is 2, r is 0.4, $\ddot{\theta}$ because it is rotating with a constant velocity. Therefore, $\ddot{\theta}$ will be $0 + 2\dot{r}$, $\dot{r}$ is 1.342 that is the velocity along the r direction which we have just find out multiplied by $\dot{\theta}$. So, $\dot{\theta}$ is given 5 and this gives you 26.84 N.



Now, let us look at another problem statement. The rod OA is held at the constant angle $\beta = 30^0$ while it rotates about the vertical with a constant angular rate $\dot{\phi} = 120\ revolution/minute$ simultaneously the sliding ball. P oscillates along the rod with its distance in mm from the fixed pivot O given by $R = 200 + 50sin2\pi nt$ where the frequency of oscillation along the is constant and it is equal to 2 $cycle/second$, and we have been asked to calculate the magnitude of the acceleration of P for an instant when its velocity along the rod from O towards A is maximum.

In this question, the road O is rotating with a constant angular velocity about point O, and the ball P is making an oscillatory motion. We have to determine the acceleration of P. So, first of all, we have to identify the right coordinate system and for this problem, the right coordinate system is the spherical polar coordinate. Therefore, we can find out the acceleration if we know what is a_r, what is a_θ and what is a_ϕ. So, let us see the situation. In the question, we have been given that $R = 200 + 50sin2\pi nt$ and $n = 2$.

R Becomes $200 + 50sin4\pi t$. And therefore, $\dot{R}$ becomes $200\pi cos4\pi t$, and $\ddot{R}$ is $-800\pi^2 sin4\pi t$. Now, we have to find out the acceleration of point P. for an instant when the velocity is maxima. So, velocity maxima means $\dot{R}$ has to be maxima and it can be maxima if $cos4\pi nt = 1$. Therefore, $\dot{R}$ will be 200π and if $cos4\pi t = 1$, then $sin4\pi t$ will be 0. Therefore, $\ddot{R}$ will be 0 and R will be 200.

Now, in the question, we have been given $\beta = 30^0$. Now, β is the angle that it makes from the z axis and this angle in our formalism, we have taken it as θ. So, therefore, $\dot{\theta}$ and $\ddot{\theta}$ will be 0 because θ is constant. In the question, we have also given $\dot{\phi} = 120\, revolution/minute$. So $\dot{\phi} = 120\, revolution/minute$.

So, let us convert it into $radian/second$. So $2\pi/60$, which is $4\pi\ radian/s$. Again, it is constant. Therefore, $\ddot{\phi}$ will be 0. Now, we are in a situation to find out what is a_r, what is a_θ and what is a_ϕ.

So, a_r will be $\ddot{r} - r\dot{\theta}^2 - rsin^2\theta\dot{\phi}^2$. Now, let us put these values. So, $\ddot{r}$ is $0 - r$ is $200\dot{\theta}^2$. So, $\dot{\theta}$ is $0 - r$ is $200\sin^2\theta$ is 30^0 and $\dot{\phi}$ is 4π. So, $4\pi^2$ and this you can find out this is equal to$-800\pi^2\ m/s^2$.

Now, let us look at a_θ. a_θ is $r\ddot{\theta} + 2\dot{r}\dot{\theta} - rsin\theta cos\theta\dot{\phi}^2$ and again r is 200 but $\ddot{\theta}$ is 0 and $\dot{\theta}$ is also 0. $-r$ Is $200sin30^0cos30^0\dot{\phi}^2$. $\dot{\phi}$ is $4\pi^2$, and this you can find out it is equal to $-800\sqrt{3\pi^2}\ m/s^2$. Now, let us look at a_ϕ.

A_ϕ Is $2\dot{r}\dot{\phi}sin\theta + 2r\dot{\theta}\dot{\phi}cos\theta + rsin\theta\ddot{\phi}$. So, now let us put the values. This is equal to $2\dot{r}$. $\dot{r}$ Is 200π. $\dot{\phi}$ Is 4π. And $sin30^0 + 2r\dot{\theta}$, $\dot{\theta}$ is $0 + rsin\theta\ddot{\phi}$ is also 0. So, therefore, this comes out to be $800\pi^2\ m/s^2$. Now, total a will be $\sqrt{a_r^2 + a_\theta^2 + a_\phi^2}$ plus which is equal to square root of this square plus this square plus that square and this is equal to $17660\ m/s^2$.

With this, let me stop here. See you in the next class. Thank you.