# MECHANICS

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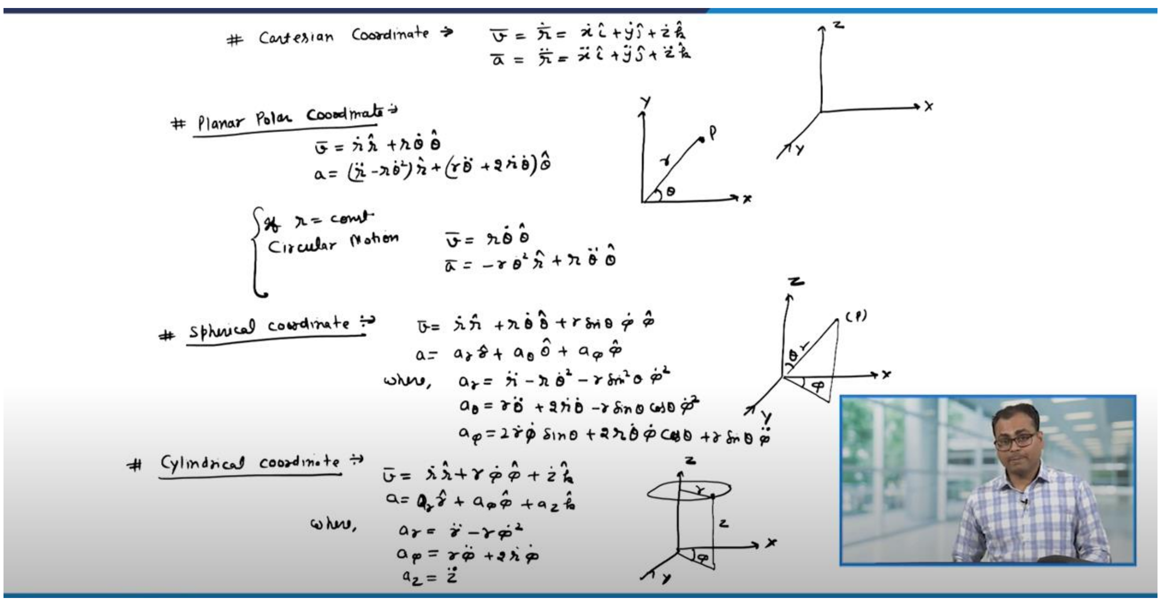
**Lecture: 33**

## Cartesian and planar polar coordinates: examples

Hello, everyone; welcome to the lecture again. In the last few classes, we find out the expression of the velocity and acceleration in Cartesian coordinate, planar polar coordinate, spherical coordinates and cylindrical coordinates.

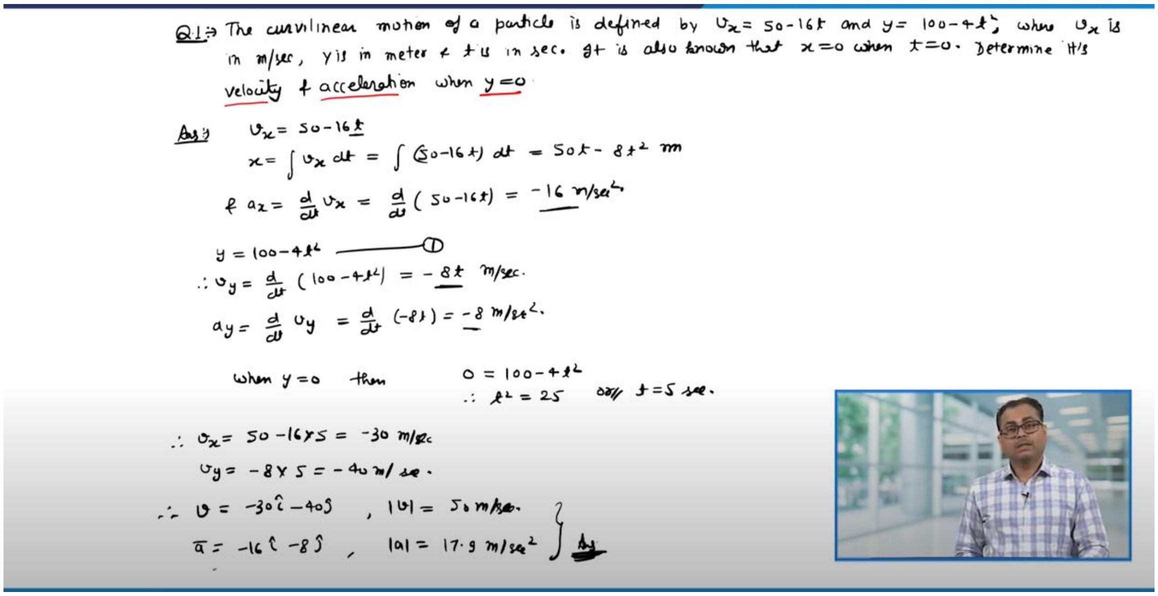



So, let me summarize these results first and then we will look at some of the examples. So, we saw that in Cartesian coordinate, the velocity vector which is $\dot{r}$ is $\dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$ and the acceleration $a$ is $\ddot{r}$ and it was $\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$.

And of course, in Cartesian coordinate, we have x-axis, y-axis and z-axis. In planar polar coordinate, we have $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$ and the acceleration $a$ was $\ddot{r} - r\dot{\theta}^2\hat{r} + r\ddot{\theta} + 2\dot{r}\dot{\theta}\hat{\theta}$, wherein the coefficient of $\hat{r}$ is the velocity and acceleration in the radial direction and the coefficient of $\hat{\theta}$ is the velocity and acceleration in the $\hat{\theta}$ direction. Here, let us say this is x-axis, this one is y-axis, and then we have some point P. So, this is your r and the angle that it makes from the x-axis is theta. We also saw that if r is constant, then it becomes circular motion. And in this case, because r dot and r double dot will be 0, Therefore, velocity vector v becomes r theta dot theta cap and acceleration a becomes minus r theta dot square r cap plus r theta double dot theta cap. In spherical coordinates,

We have velocity vector v is equal to r dot r cap plus r theta dot theta cap plus r sine theta phi dot phi cap. And the acceleration a can be written as a r r cap plus a theta theta cap plus a phi phi cap where again a r a theta and a phi are the acceleration along r theta and phi direction. Here a r was r double dot minus r theta dot square minus r sine square theta phi dot square and a theta was r theta double dot plus 2 r dot theta dot minus r sine theta cos theta phi dot square and a phi was 2 r dot phi dot square sine theta plus 2r theta dot phi dot cos theta plus r sine theta phi double dot. And again, here you have x axis, y axis and let us say z axis.

Then r is the, you know, position vector of this point P and the angle that it makes from the z axis is theta. So, this is the z axis, this one is the y axis and its projection on the xy plane, whatever angle it makes from the x axis is your phi. Now, in cylindrical coordinates, we have velocity v equal to r dot r cap plus r phi dot phi cap plus z dot k cap and again the acceleration a is let us say a r r cap plus a phi phi cap plus a z k cap where this A r is r double dot minus r phi dot square. A phi is r phi double dot plus 2 r dot phi dot and a z is just z double dot. All these expressions we have derived in the previous classes. Let me again say that in cylindrical coordinate system, you have let us say a point P that you want to represent. So, you draw a cylinder of radius r and its projection, the projection of r on the x and y plane, whatever angle it makes from the x axis was $\phi$ and the height of the cylinder was $z$.




Now, with this background, let us look at some of the examples. So, the first problem statement is following. The curvy linear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$. $v_x$ Is in $m/s$, $y$ is in meters, and $t$ is in seconds.

It is also known that $x = 0$. When $t = 0$ and we have asked to determine its velocity and acceleration when $y = 0$. Now, the first thing that we have to identify is the right coordinate in which this problem can be solved. Now, you can see that $v_x$ is given and y is given. Therefore, the natural choice is the Cartesian coordinate.

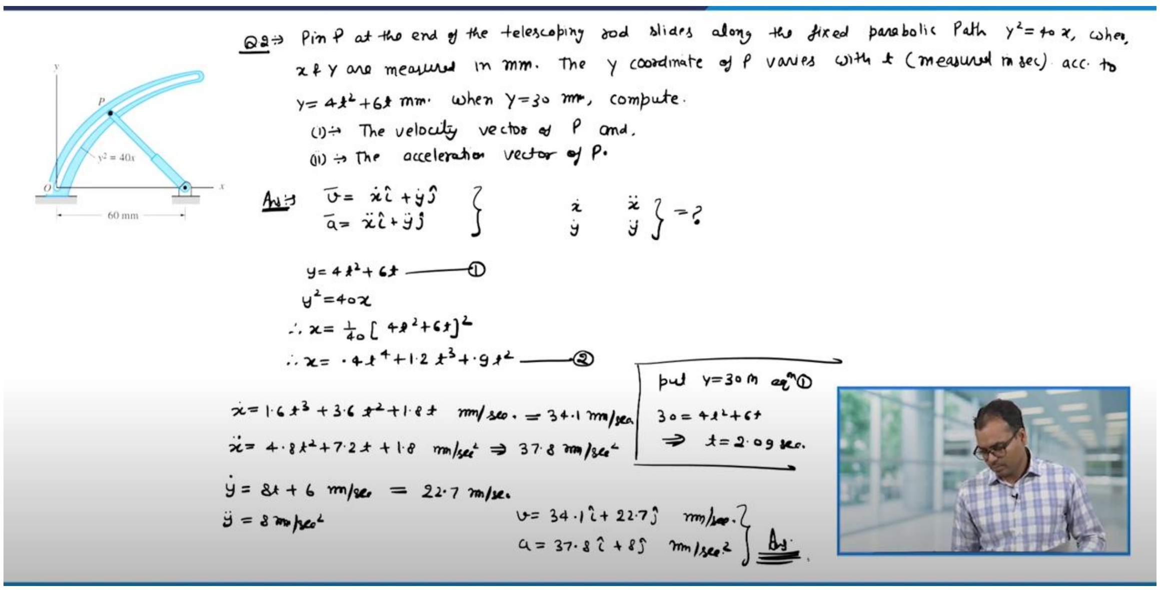
So, let us look at $v_x$ is given. It is $50 - 16t$. Therefore, from this, I can find out what is the $x$ and what is the acceleration along the x direction. I can find out by integrating this $v_x$ with respect to $t$. So, we have $50 - 16tdt$, that is equal to $50t - 8t^2\ m$. And the acceleration $a_x$, we can find out by differentiating $v_x$ with respect to $t$.

So, that will be $d/dt$, $v_x$ is $15 - 16t$ and this comes out to be $-16m/s^2$. So, now we know $x$ and $a_x$. Let us now find out $v_y$ and $a_y$. For that, $y$ is given. $y$ Is $100 - 4t^2$.

Therefore, $v_y$ will be $d/dt$ of $y$, which is $100 - 4t^2$. Therefore, this becomes $-8tm/s$. And acceleration ay we can get by differentiating $v_y$ with respect to $t$. So, it will be $\frac{d}{dt}(-8t) = -8\ m/s^2$. Now, in the problem statement, we have been asked to find out the velocity and acceleration when $y = 0$. Therefore, let us see the value of velocity when $y = 0$.

So, when velocity $y = 0$, then you can see from equation number 1, let us call this equation number 1, that $t$ comes out to be 5 second. So, let us put $y = 0$ equal to $100 - 4t^2$. Therefore, $t^2 = 25$ or $t = 5s$.

Now, I can find out what is $v_x$ when $t = 5\ s$ or when $y = 0$. So, that will be $50 - 16 \times 5$. So, that is $-30\ m/s$ and $v_y$ will be $-8t$. So, $-8 \times 5$ which is $-45 m/sec$. Therefore, $v$ which is $v_x i + v_y j$ becomes $-30i - 40j$ or its magnitude is $\sqrt{30^2 + 40^2}$ which is $50\ m/s$. Now, the acceleration $a$ will be $a_x$ which is $-16i$ plus $a_y$, which is $-8j$ and you can find out its magnitude. It will be $\sqrt{16^2 + 8^2}$, which will be $17.9\ m/s^2$.




Now, let us look at another problem. Here, the problem statement is pin P at the end of the telescopic road slides along the fixed parabolic path $y^2 = 40x$, where x and y are measured in $mm$. The $y$ coordinate of $t$ varies with $t$ which is measured in second. According to $y = 4t^2 + 6t\ mm$. When $y = 30\ mm$, compute number 1, the velocity vector of $t$.

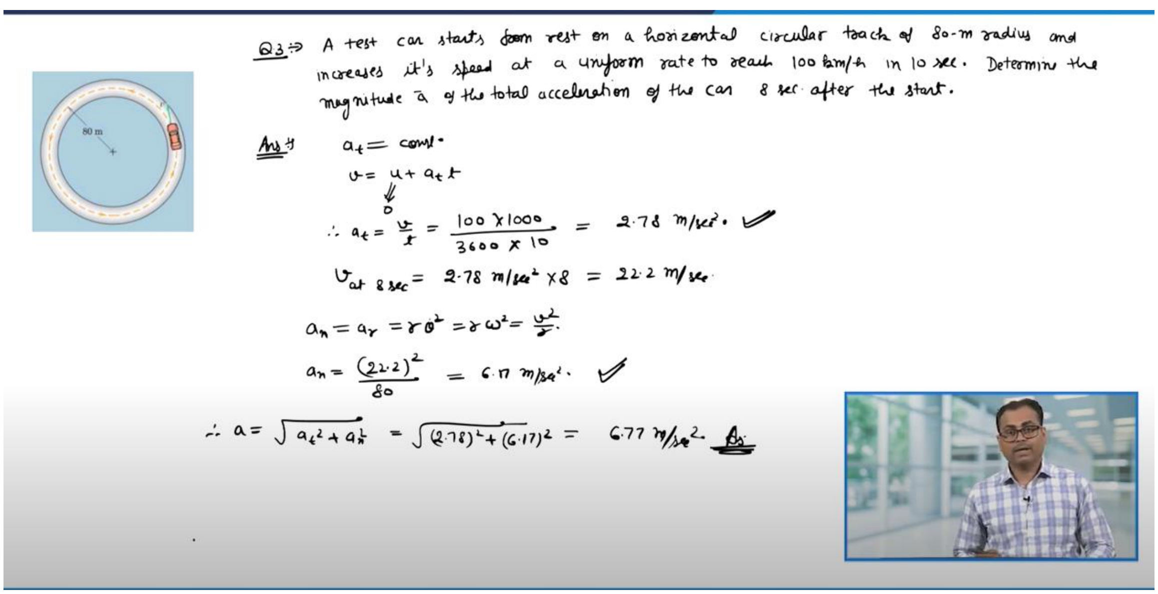
And the acceleration vector of P. So, again in this problem, you can identify that the problem can be solved easily in the Cartesian coordinate because the relation between x and y is given. So, we know that In Cartesian coordinate, the velocity v is $\dot{x}i + \dot{y}j$ and the acceleration $a$ is $\ddot{x}i + \ddot{y}\ j$. Therefore, to find out the velocity and acceleration, we should know what is $\dot{x}, \ddot{x}, \dot{y}$ and $\ddot{y}$. Okay. So, let us see the information that is given in the question.

It is given that $y$ varies as $4t^2 + 6t$ and the relation between $y^2$ and $x$ is $y^2 = 40\,x$. Therefore, I can find out how $x$ depend upon $t$. So, $x$ will be $1/40$ and $y^2$. So, $y^2$ will be $(4t^2 + 6t)^2$. Let me call this equation number 1. Therefore, $x$ becomes $0.4t^4 + 1.2t^3 + 0.9t^2$.

So, now we know what is $x$ and what is $y$ in terms of $t$. So, I can easily find out the $\dot{x}$ is $1.6t^3 + 3.6t^2 + 1.8t\ mm/sec$. Now, to find out the value of $\dot{x}$, I should also know what is $t$ at what instant I am calculating the $\dot{x}$. So, to find out the $t$, it is given that you have to find out when $y = 30$.

So, let us put $y = 30$ in equation number 1 to find out the $t$ when this happens. So, we have $30 = 4t + 60t$ and this gives you $t = 2.09\ sec$. I can put this value over here and you will get $\dot{x} = 34.1\ mm/sec$. Now, let us look at $\ddot{x}$ will be $4.8t^2 + 7.2t + 1.8\ mm/s^2$ and you can put $t = 2.09$, you will get $\dot{x} = 37.8\ mm/s^2$.

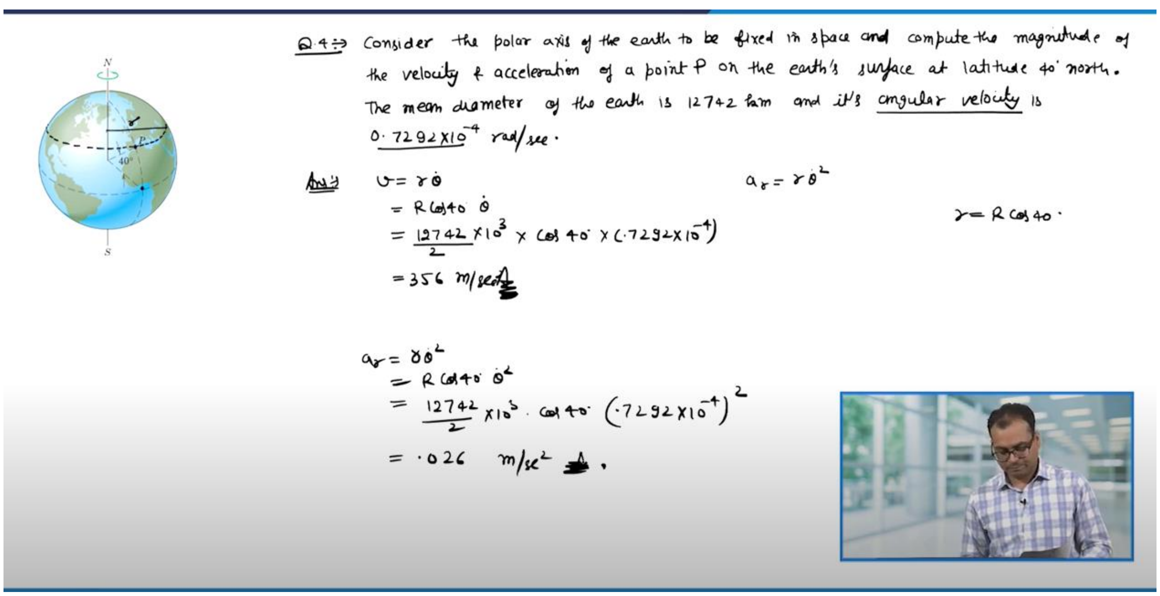
Similarly, $\dot{y}$ will be, you can use equation number 1, it will be $8t + 6\ mm/sec$ and put $t = 2.09$, you will get $22.7\ mm/sec$ and $\ddot{y} = 8\ mm/s^2$. Let us put that in this equation. So, you get $v = 34.1i + 22.7j\ mm/sec$ and acceleration $a = 37.8i + 8j\ mm/s^2$.




Now, let us look at another problem statement. Question number 3. A test car starts from rest on a horizontal circular track of 80 $m$ radius increases its speed at a uniform rate to reach 100 $km/hour$ in 10 seconds. And we have asked to determine the magnitude $a$ of the total acceleration of the car 8 seconds after the start. So, first of all, it is clear that the motion is circular and it is also given that the transverse acceleration is constant. So, it is given that $a_t$ is constant. Therefore, we can use the simple equation $v = u + a_t t$, and it is also given that initially, the car is at rest. Therefore, $a_t$ will be $v/t$ and $v$ is given.

It is 100 $km/hour$. So, $100 \times 1000$ to convert it into meter and hour let us convert it into second multiplied by $t$. So, it attains this in10 $sec$. So, therefore, 10 and you get the transverse acceleration which is 2.78 $m/s^2$. Now, the tangential velocity at 8 $sec$ will be $2.78 \frac{m}{s^2} \times 8$ which is 22.2 $m/sec$.

And now, I can calculate what is the normal acceleration or acceleration along $r$. It is $r\dot{\theta}^2$ or we can write down $r\omega^2$ or $v^2/r$. Now, $v$ is known, $r$ is also known. Therefore, $a_n$ will be $\frac{22.2^2}{80}$ which is6.17 $m/s^2$. So, Therefore, I know the acceleration in the tangential direction. I have calculated the acceleration in the normal direction. Therefore, $a$ total will be $\sqrt{a_t^2 + a_n^2}$ which is equal to $\sqrt{2.78^2 + 6.17^2}$ and it comes out to be 6.77 $m/s^2$.



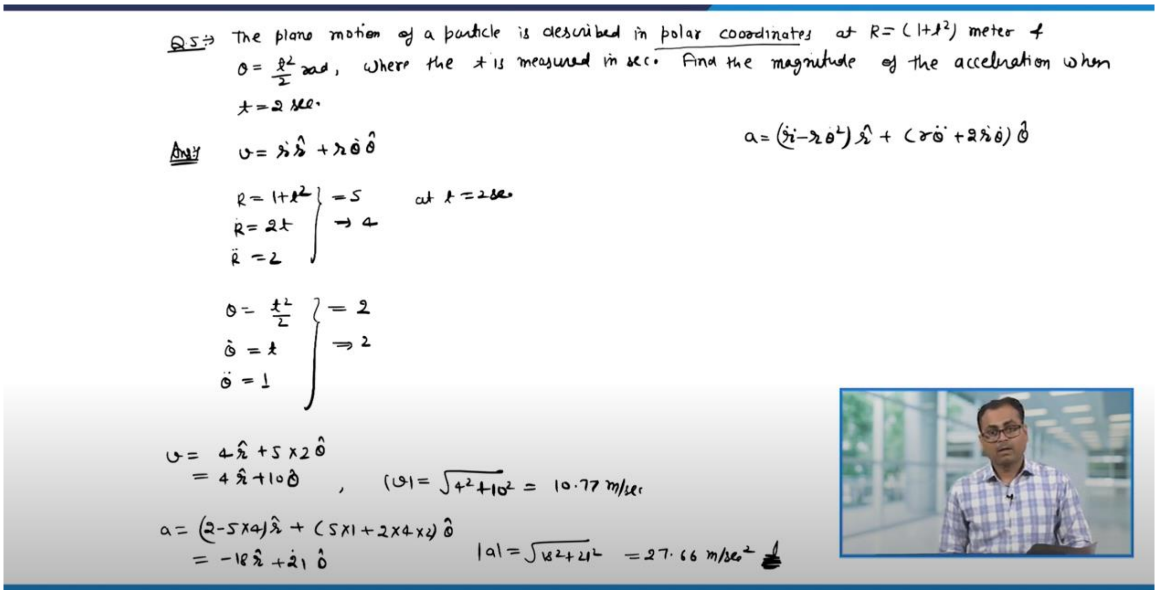

Now, let us look at another problem statement. Question number 4, consider the polar axis of the earth to be fixed in space and compute the magnitude of the velocity and acceleration of a point P on the earth's surface at latitude 40 degrees north, the mean diameter of the earth is 12742 $km$. Its angular velocity is $0.7292 \times 10^{-4} rad/sec$.

So, in this problem, first of all, you have to realize that point P is making a motion in circle and for circular motion, we have velocity $v = r\dot{\theta}$ and acceleration $a_r = r\dot{\theta}^2$, wherein this $r$ is the radius of the circle. And from the geometry, you can see that small $r$ is $Rcos40^0$.

So, let us put it over here. You get $v = Rcos40^0\dot{\theta}$ and this will be $12742$. This is kilometer. So, let us convert it into meter multiplied by $10^3$ divided by 2 because $12742$ was the diameter and multiply by $cos40^0$ and then $\dot{\theta}$. $\dot{\theta}$ Is given in the question.

Angular velocity is $0.7292 \times 10^{-4}$. So, let us multiply it by $0.7292 \times 10^{-4}$ and this comes out to be $356\ m/sec$. Now, let us see $a_r$ is $r\dot{\theta}^2$. So, $r$ is capital $Rcos40^0\dot{\theta}^2$.

This will be $\frac{12742}{2}10^3$ and then we have $cos40^0$ and then $\dot{\theta}$ which is $(0.7292 \times 10^{-4})^2$ and this comes out to be $0.026\ m/s^2$.




Now, let us look at one more problem and the problem statement here is following. The plane motion of a particle is described in polar coordinates at $r = 1 + t^2\ meter$ and $\theta = \frac{t^2}{2}\ rad$ where the $t$ is measured in seconds and we have asked to find the magnitude of the acceleration when $t = 2\ sec$. In this question, it is already given that the motion is happening in planar polar coordinates. And in planar polar coordinate, the velocity $v$ is$\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$. And the acceleration $a$ is$\ddot{r} - r\dot{\theta}^2\hat{r} + r\ddot{\theta} + 2\dot{r}\dot{\theta}\hat{\theta}$.

So, we have to find out what is $\dot{r}\dot{\theta}\ddot{r}$ and $\ddot{\theta}$. It is given that $r = 1 + t^2$ and all this thing we have to find out at $t = 2\ sec$. So, therefore, this becomes 5, $\dot{r}$ becomes $2t$, which is 4 and $\ddot{r}$ becomes 2. Now, $\theta$ is $t^2/2$.

Therefore, this becomes $2$. $\dot{\theta}$ becomes $t$. Therefore, let us put $t = 2$. So, this again becomes 2 and $\ddot{\theta}$ becomes 1. Let us put them above.

So, we get $v = 4\hat{r} + 5 \times 2\hat{\theta}$. So, we have $4\hat{r} + 10\hat{\theta}$. This is the velocity and its magnitude will be $\sqrt{4^2 + 10^2}$ which is $10.77 m/sec$ and acceleration $a$ becomes $\ddot{r} - r\dot{\theta}^2$. So, this is $2 - 5 \times 4\hat{r} + (5 \times 1 + 2 \times 4 \times 2)\hat{\theta}$, which is nothing but $-18\hat{r} + 21\hat{\theta}$.

Therefore, the magnitude of $a$ is $\sqrt{18^2 + 21^2}$, which is $27.66\ m/s^2$. With this let me stop here see you in the next class. Thank you.