MECHANICS

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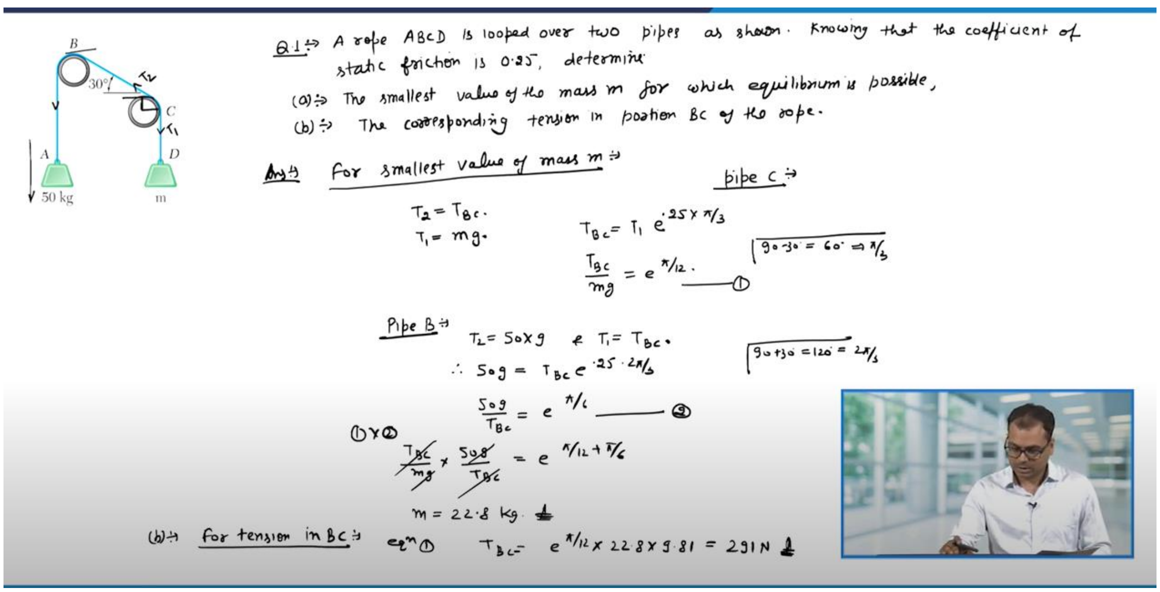
Lecture: 27

Rope and belt friction: examples

Hello everyone, welcome to the lecture again. In the last class, we derived the expression for the rope or belt friction. We also look at couple of examples.



Today, we are going to continue the discussion on the rope or belt friction and we are going to look at some more examples on the same concept. So, let me write down the first problem statement.

A rope A, B, C, D is looped over two pipes as shown knowing that the coefficient of static friction is 0.25, determine (a) the smallest value of the mass m for which equilibrium is possible and (b) the corresponding tension in portion BC of the rope.

In this problem statement, we have been asked to find out the smallest value of mass m and also the corresponding tension in BC. So, first of all, for the smallest value of mass m, the impending motion of the 50 kg mass will be downward. So, Let us look at the situation for the smallest value of mass m. And let us look at what happens with pipe C. Okay. Now, since the impending motion of the 50 kg mass is downward, therefore, the tension in BC part will be more than CD.

Therefore, this can be taken as T_2 and this can be taken as T_1. So, T_2 will be T_{BC} and T_1 will be mg for this pipe. Therefore, you can see that T_2 which is T_{BC} equal to $T_1 e^{\mu\beta}$. μ Is given, it is 0.25 and you have to see how much angle this rope is making on pipe C. So, if the rope would have been like that, then it would have been 90^0.

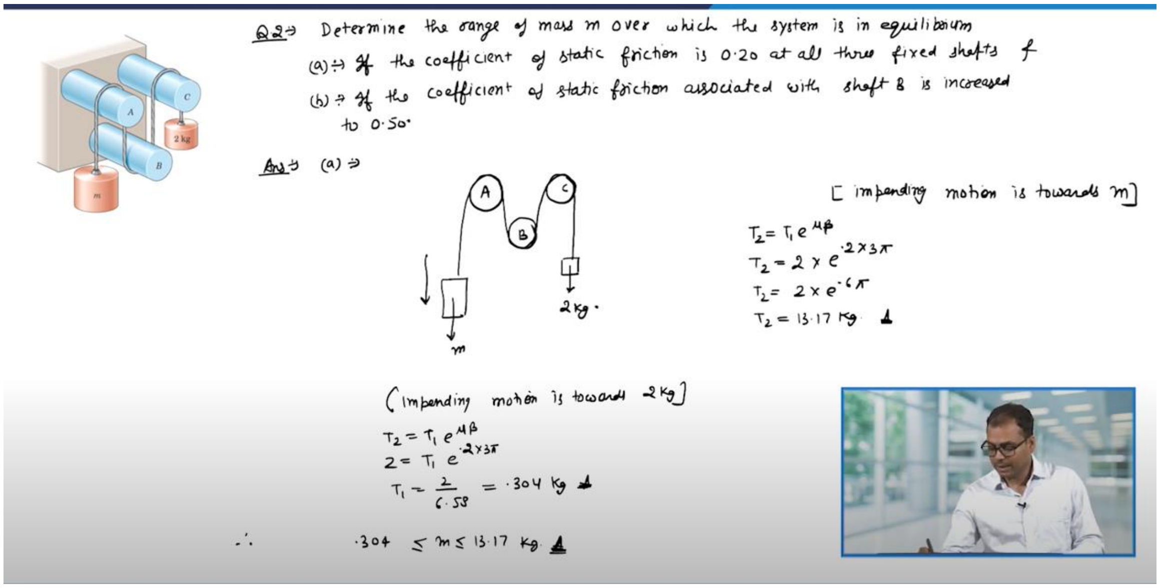
But since this angle is 30^0, therefore, the rope is making an angle of $90 - 30^0$, which is 60^0 and it can be written as $\pi/3$. So, therefore, you have to multiply it by $\pi/3$ or we get T_{BC}/mg because T_1 is nothing but $mg = e^{\frac{\pi}{12}}$. Let us call it equation number 1. Now, let us look at pipe B. For this, we have this tension to be larger than T_{BC}.

So, therefore, T_2 will be 50 kg. So, 50 into g and T 1 will be T BC. Therefore, we have $T_2 = T_1 e^{\mu\beta}$. Now, in this case, the total angle is $90 + 30^0$.

So, $90 + 30^0$ which is 120^0 and that is equal to $\frac{2\pi}{3}$. So, into $\frac{2\pi}{3}$. Therefore, I can write down $\frac{50g}{T_{BC}} = e^{\frac{\pi}{6}}$. Let us call it equation number 2.

And since we want to calculate m, therefore, let us multiply equation number 1 and 2 so that T_{BC} will get away. So, $\frac{T_{BC}}{mg} 50g/T_{BC}$. So, we are just doing 1 into 2 equal to e^($\frac{\pi}{12} + \frac{\pi}{6}$). Therefore, we will get $m = 22.8\ kg$.

Now, to find out the tension T, you can use equation number 1 and you can put the value of m. So, for part B, this is for tension in BC. Let us use equation number 1. So, we get $T_{BC} = mge^{\frac{\pi}{12}}$, m is 22.8 and g is 9.81. So, you get $291\ N$.



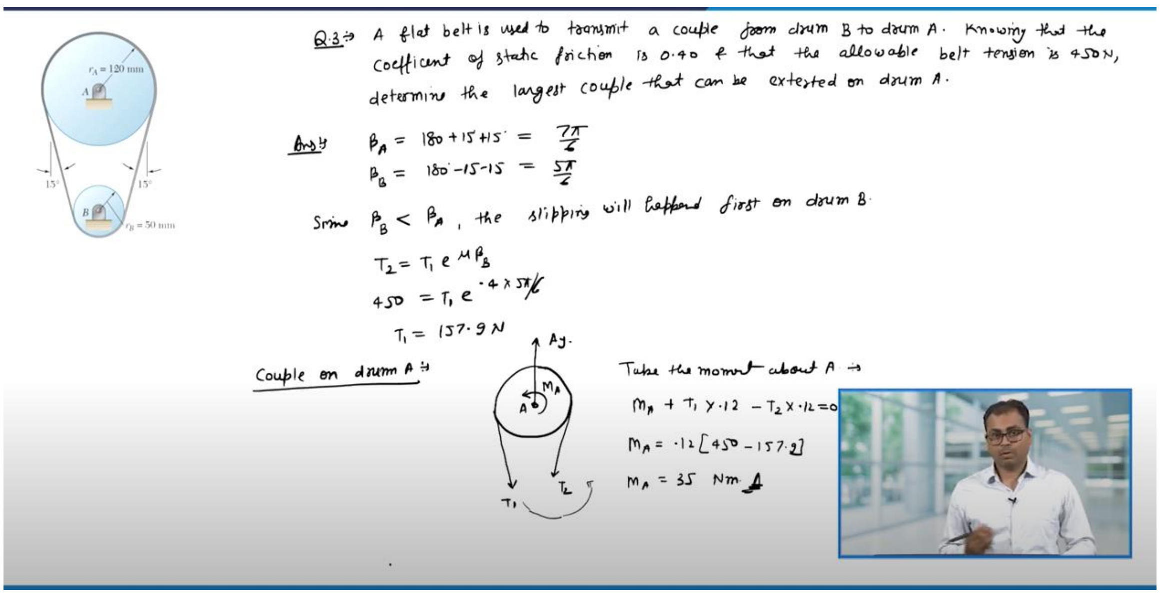
Let us look at one more problem on the similar concept and the problem statement is following. Determine the range of mass m over which the system is in equilibrium (a) if the coefficient of static friction is 0.20 at all three fixed shafts and (b) if the coefficient of static friction associated with shaft B is increased to 0.50.

So, again the concept is same. Let us look at part A. So, you have three shafts. This mass is m, you have one shaft here, one shaft there and one shaft here. This is A, B and C and here i have 2 kg mass. So, we have to find out what is the range of masses over which this will remain in equilibrium.

Let us say that first the impending motion is towards m. We will consider the other case later. So, let us first think as if the impending motion is towards let us say m. So, if the impending motion is like that, in that case, this will be our T_2. So, we will have $T_2 = T_1 e^{\mu\beta}$ and T_2 will be T_1. T_1 Is $2e^{\mu}$. μ Is given. It is 0.2 for all the shaft. So, 0.2β. Now, we have to see how much overall angle it is making. So, here it is an π angle plus π angle plus π angle.

So, it is 3π angle. Therefore, T_2 will become $2e^{0.6\pi}$ and this gives you $T_2 = 13.17\ kg$. So, this is one range for which the mass m is going to go downward. Now, let us look at the second case wherein we assume that the impending motion is towards 2 kg. In that case, 2 kg will be your T_2.

So, again we have $T_2 = T_1 e^{\mu\beta}$, but now T_2 will be 2 kg. So, 2 equal to $T_1 e^{\mu}$ is 0.2 and β is $\pi + \pi + \pi$, which is 3π. So, from here we get $T_1 = 2/e^{0.6\pi}$ which is 6.58 and this gives you 0.304 kg. Therefore, the ranges for mass m for which it will be in equilibrium is m should be less than equal to 13.17 kg and it should be larger than equal to 0.304 kg. Now, in part (b), it has been asked to again find out the range of masses for which the equilibrium is there. But now, the static friction associated with shaft B has been increased to 0.5.The friction with shaft A and C, it remains 0.2.



So, let us look at part (b). Again, let us say that the impending motion is 0.5. Towards m first. So, therefore, we will have $T_2 = T_1 e^{\mu\beta}$. So, let us say it is $\mu_1\beta_1 + \mu_2\beta_2$. So, T_2 will be mg. So, it will be $T_2 = T_1$ is $2e^{\mu_1}$. So, for A and C, let me write down the combined way. So, it is 0.2β is $\pi + \pi$, which is 2π. And then we have e to the power μ_2 is 0.5 and β_2 is π.

So, this gives you $T_2 = 2e^{0.9\pi}$ or T_2 will be 33.8 kg. Now, for the impending motion towards let us say 2 kg. So, in this case, we have again $T_2 = T_1 e^{\mu_1\beta_1 + \mu_2\beta_2}$, but now this T_2 will be 2 kg. So, 2 into equal to $T_1 e^{0.9\pi}$ or T_1 will be $\frac{2}{e^{0.9\pi}} = 0.118\ kg$. Therefore, the range of masses will now be less than equal to 33.8 kg and it should be greater than equal to 0.118 kg. Now, let us look at couple of example of the belt friction. So, let me write down the first problem statement and it is following.

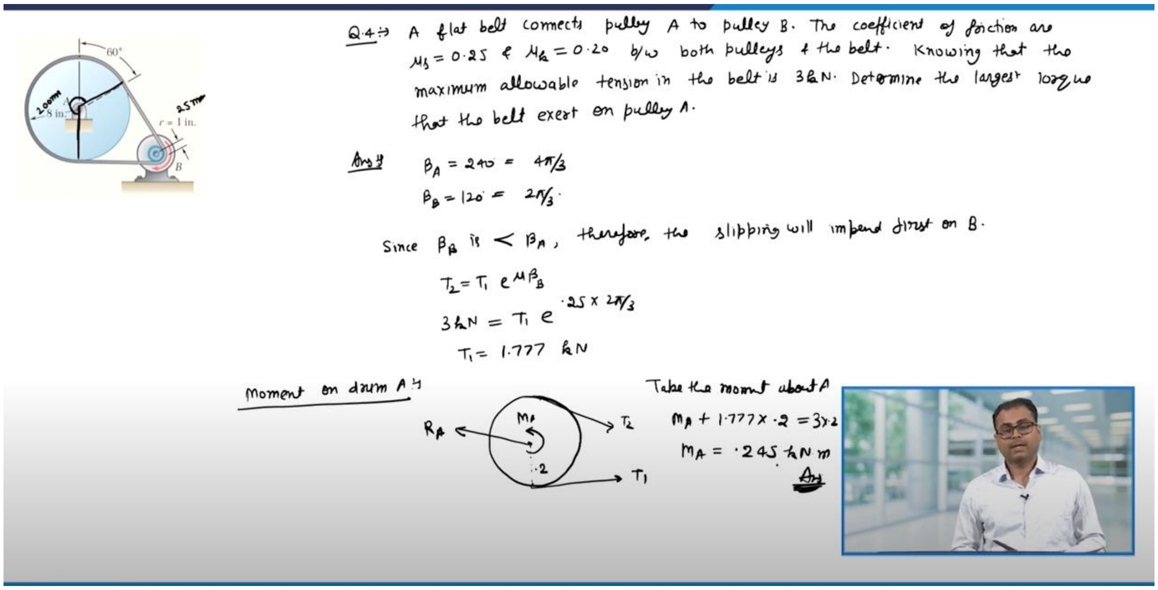
A flat belt is used to transmit a couple from drum B to drum A knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is $450\ N$. Determine the largest couple that can be exerted on drum A. So, first let us see how much angle the belt is making with drum A and drum B. Let us call it β_A. So, β_A is the angle that the belt is making with drum A. So, you can see here it will be $180^0 + 15^0 + 15^0$. So, which is $\frac{7\pi}{6}$ and the angle that the belt is making with drum B will be $180^0 + 15^0 + 15^0$ that will be $\frac{5\pi}{6}$.

So, here since the β_B is small than β_A, therefore the slippage or the slipping will happen first on drum B. Therefore, let us calculate the tension in drum B. So, we use $T_2 = T_1 e^{\mu \beta_B}$ and T_2 is given it is $450 = T_1 e^{\mu}$ which is $0.4\beta_B = \frac{5\pi}{6}$. This gives you $T_1 = 157.9\ N$. Now, in the problem statement, we have been asked to find out what is the couple on drum A. So, let us look at the free body diagram of drum A. So, we have this drum, this point is A and we have tension T_1 and tension T_2.

And since there is a connection over here, so therefore, this will also apply a force, let us say A_y in the y direction and there will be a resistive moment. So, let us call it M_A. This is the free body diagram of the drum and to find out what is the value of M_A, we can take the moment about A. So, let us take the moment about A. In this case, A_y will not contribute. So, we have M_A plus the tendency of T_1 about point A is to rotate it in the anticlockwise direction.

So, it will be T_1 into the perpendicular distance which is the radius and it is given it is 120 mm. So, it will be $0.12 - T_2 0.12$ and this would be equal to 0. Therefore, you get $M_A = 0.12, 450 - 157.9$ because T_1 is 157.9 and T_2 is $450\ N$. Therefore, M_A will be $35\ Nm$.

Now, let us look at one more problem on the similar concept. So, here the problem statement is following. A flat belt connects pulley A to pulley B. The coefficient of frictions are $\mu_s = 0.25$ and $\mu_k = 0.20$ between both pulleys and the belt. Knowing that, the maximum allowable tension in the belt is 3 kN determine the largest torque that the belt exerts on pulley A. So, first let us see how much angle the flat belt is making with pulley A and pulley B. So, with pulley A, let us say the angle is β_A and you can see that this angle is $180 + 60^0$ which is 240^0 and this I can write down as $\frac{4\pi}{3}$. Similarly, β_B will be 120^0 and this is $\frac{2\pi}{3}$.



Now, the radius of the pulley A is 8 inch which can be written as $200\ mm$ and the radius of pulley B is 1 inch which is 25 mm. So, again you can use the same concept since β_B is less than β_A. Therefore, the slippage or the slipping will impend first on B.

Therefore, we have to analyze pulley B first and find out the corresponding tension. So, we have $T_2 = T_1 e^{\mu \beta_B}$ and T_2 is given it is 3 kN. So, let me just write down 3 kN itself equal to $T_1 e^{\mu}$ is $0.25 \times \frac{2\pi}{3}$. And this gives you $T_1 = 1.777\ kN$. Now, since we know what is T_1 and T_2, let us find out the moment on drum A and for that let us look at the free body diagram of drum A.

So, we have this drum there is T_2 and T_1 force and from pin A, there will be a reaction force which will be in the direction which connects A and B. So, let us say it is like this R_A and there will be a moment to oppose the rotation. Let us say it is some M_A. So, now, to find out what is the value of M_A, let us take the moment about A. So we have M_A and R_A will not contribute plus 1.777 into the perpendicular distance.

So, it is given that this is 200 mm. So, therefore, it is $0.2\ m$ into 0.2 equal to 3 kN into 0.2 . And this gives you $M_A = 0.245\ kNm$. So, with this, let me stop here.

See you in the next class. Thank you.