MECHANICS

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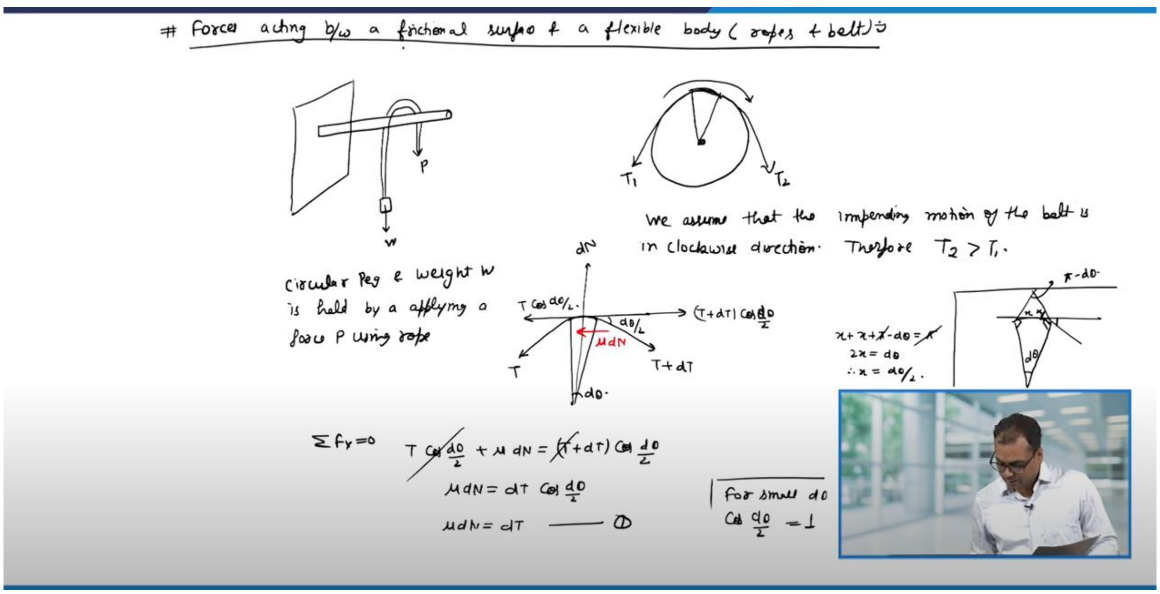
Lecture: 26

Rope and belt friction

Hello everyone, welcome to the lecture again. In the last class, we looked at various examples of the frictional forces.



Today, we are going to study the forces which are acting between the frictional surface and a flexible body, for example, the rope and belt. So, we are looking at the forces which are acting between a frictional surface and a flexible body.

For example, some ropes and belt. We can have a situation wherein there is a peg and by this peg a weight W is held by a rope and by applying a force P. So, we have a circular peg and weight W is held by applying a force P using some rope. And the second situation can be wherein you have some circular disc or circular drum, and on this, there is a belt, and by applying tension T_1 and T_2, there is an impending motion.

Let us say the impending motion of the belt is in the clockwise direction. So, therefore, T_2 will be larger than T_1. So, we assume here that the impending motion of the belt is in clockwise direction. Therefore, T_2 must be larger than T_1 and we are going to analyze this case under this assumption.

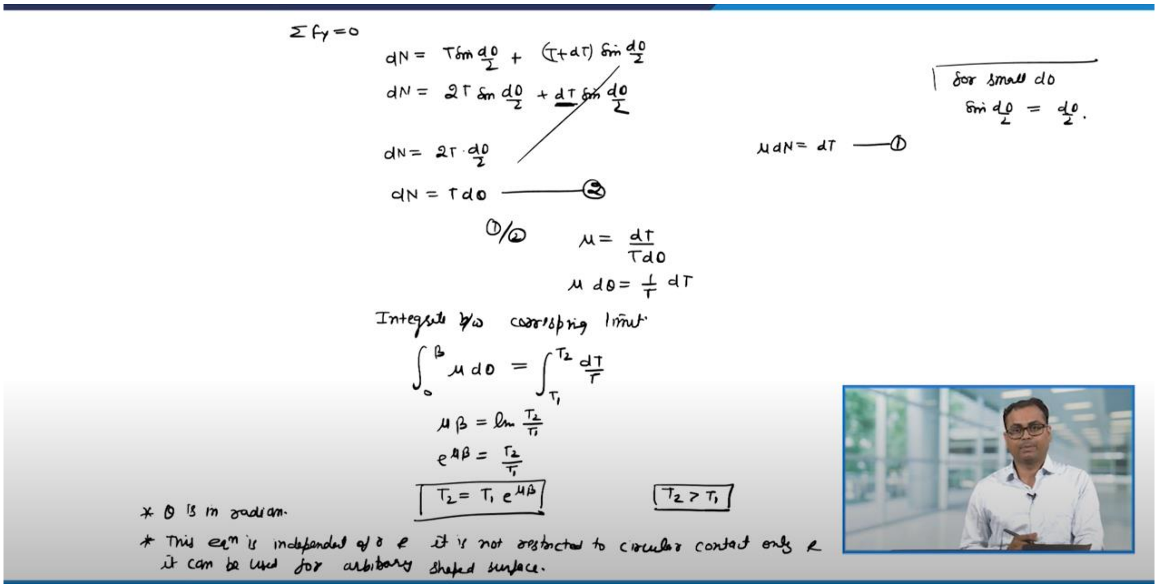
We can also have T_1 larger than T_2. In that case, the motion will be in the anticlockwise direction. To find out the equilibrium condition, let us look at a part of this belt. So, let us say this is the part of this. Let us say this angle is $d\theta$ and because of that, the normal force is N.

From the center is going to act outward. Since it is $d\theta$, let us call it dN. On this direction, let us say the tension is T, and in this direction, therefore, the tension will be $T + dT$ because we have assumed that T_2 is larger than T_1. So, therefore, it will be $T + dT$. Now, let us look at the angles. This angle will be $d\theta/2$. Let us see why we have this angle as $d\theta$. This is the belt. Let us draw a perpendicular.

Therefore, this angle will be $\pi - d\theta$ and we have to find out how much is this angle. This angle will be equal to that angle. Let us say this is x, this is x. So, we have $x + x + \pi - d\theta$ should be equal to 180^0 equal to π. So, this gives you π will get cancelled.

You get $2x = d\theta$. Therefore, x will be $d\theta/2$. Now, we can balance the forces in the x and y direction. So, the forces which are acting in the x direction will be $T + \frac{dTcosd\theta}{2}$, and in this direction, we will have $\frac{Tcosd\theta}{2}$. Note that this drum is rotating in the clockwise direction.

Therefore, there will be a frictional force in the opposite direction. So, there will be a force μdN which is going to act in the minus x direction. Now, let us balance the forces along the x and y direction. So, $\sum F_x = 0$. This will give you $\frac{Tcosd\theta}{2} + \mu dN = T + + \frac{dTcosd\theta}{2}$ and $\frac{cosd\theta}{2}T$ will get cancelled with this one. So, we have $\mu dN = \frac{dTcosd\theta}{2}$. Now, we also know that for small $\frac{d\theta cosd\theta}{2}$ or $d\theta$, that will be 1. Therefore, we have $\mu dN = dT$. Let us call it equation number 1. Now, let us balance the force in the y direction.



So, in the y direction, we have the $\sum F_y = 0$. In the y direction, we have dN force, and that will be equal to $\frac{Tsind\theta}{2} + T + \frac{dTsind\theta}{2}$. Therefore, we have $dN = \frac{2Tsind\theta}{2} + \frac{dTsind\theta}{2}$. Now, we also know that for a small $d\theta$, $sin\theta$ becomes θ. So, therefore, $\frac{sind\theta}{2}$ will be $\frac{d\theta}{2}$ and therefore, here I have $d\theta$ term and I have $d\theta$ term and both are very small. Therefore, its multiplication will be even smaller. Therefore, we can neglect this, and we have $dN = \frac{2Td\theta}{2}$ or $dN = Td\theta$. Let us call it equation number 2.

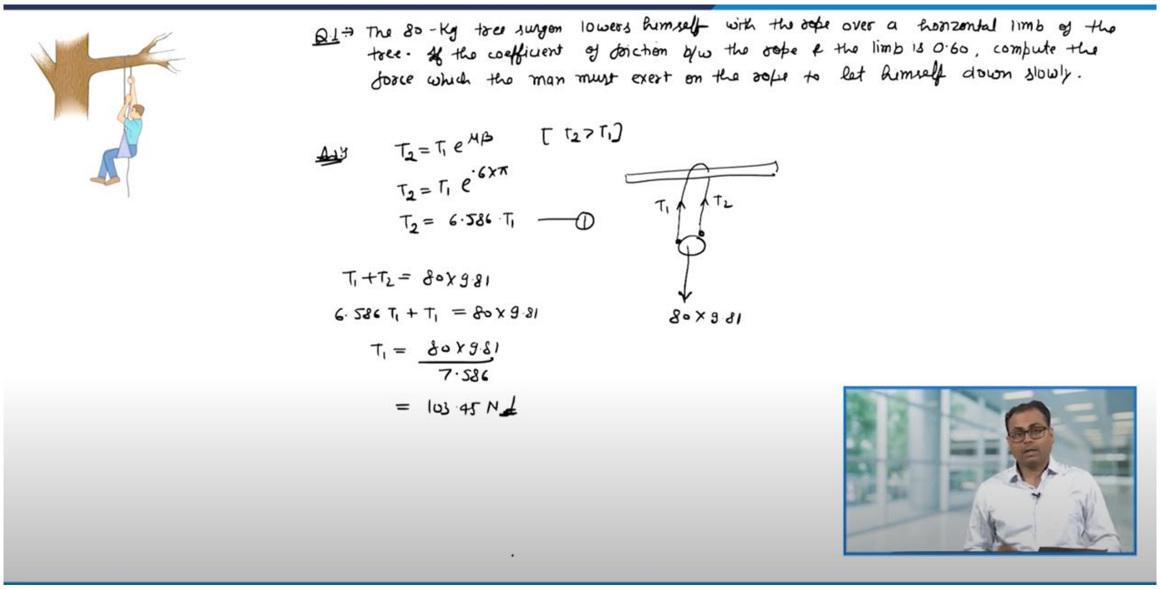
Note that equation number 1 was $\mu dN = dT$. So, let me just write it down $\mu dN = dT$. This was our equation number 1. From equation number 1 and 2, I can get rid of dN. So, let us divide 1 by 2.

So, we have $\mu = dT/Td\theta$ or $\mu d\theta = \frac{1}{T} dT$ and this equation I can integrate over the angle. So, let us integrate between the limit. So, let us see corresponding limit. So, we have integral. Let us say the belt makes an angle β.

So, let us say this angle is β. Therefore, the integral will be from 0 to $\beta \mu d\theta$ equal to we already know the limit on the tension. It is start from T_1 to T_2. So, T_1 to $\frac{T_2 dT}{T}$, this gives me μ and $d\theta$ here is β. The integral of $d\theta$ is $\beta = lnT_2/T_1$ or $e^{\mu\beta} = \frac{T_2}{T_1}$ or we get $T_2 = T_1 e^{\mu\beta}$.

Note that in this derivation, we have assumed that T_2 is larger than T_1. So, this is also very important point that this derivation is for T_2 larger than T_1. Now, herein, we have assumed that the motion is impending. If there is a motion in that case, instead of μ, you will assume the kinetic friction. So, therefore, it will be μ_k.

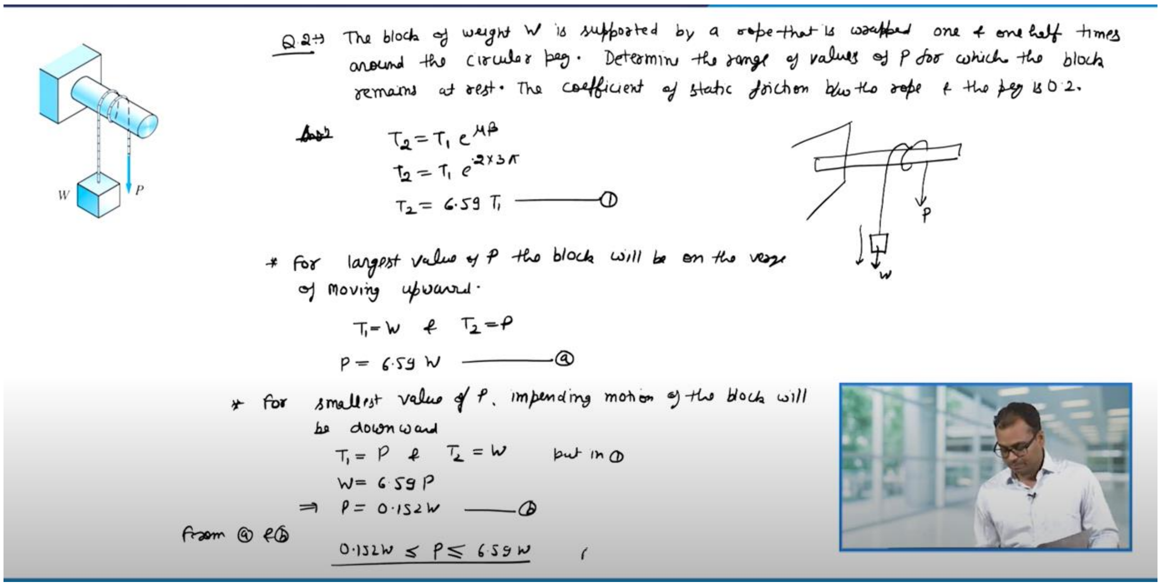
Now, let us look at two very important points about this formula. Number one, theta here is in radian and another important point is in this formula, the radius of the drum has not come into picture. Therefore, this equation is independent of R and therefore, it is not restricted to circular contact only and in fact, it can be used for arbitrary shape or arbitrary shaped surface.



With this very basic concept, now let us look at the examples. So, the first problem statement is following. The 80 kg pre-surgeon lowers himself with the rope over a horizontal limb of the tree. If the coefficient of friction between and the limb is 0.60, compute the force which the man must exert on the rope to let himself down slowly. So, the situation is following. So, this is the branch of the tree and there is this man. So, he applies the tension. So, let us say this is tension T_1, and this one is tension T_2 to lower himself down.

So, we can use the formula T_2 equal to $T_1 e^{\mu\beta}$ where μ is the frictional coefficient, and β is the angle that the rope is making with the branch in radian. So, again, let me emphasize here that T_2 is larger than T_1. So, T_2 will be T_1 e to the power μ is 0.6β. β Is the angle you can see from the figure, and the angle that it is making is π.

Therefore, T_2 will be $e^{0.65}$ we can find out it is $6.586T_1$. Let us call it equation number 1 and now let us balance the forces in the y direction. So, we have $T_1 + T_2$ equal to the weight of the object. So, here in person. So, its weight is 80 kg. So, 80 into g, 9.81. So, therefore, $T_1 + T_2$ will be 80×9.81, and we can put the value of T_2 from equation number 1. So, $6.586\, T_1 + T_1 = 80 \times 9.81$. This gives you $T_1 = 80 \times \frac{9.81}{7.586}$ and which will be $103.45\ N$.



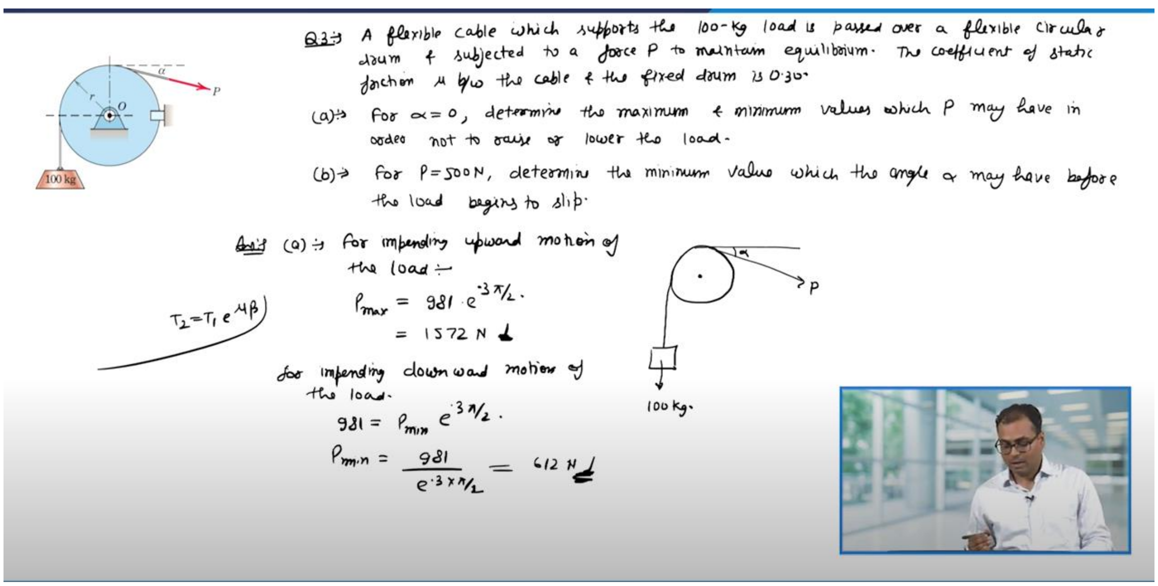
Now, let us look at another problem on the same concept and the problem statement is following. The block of weight W is supported by a rope that is wrapped one and one-half times around the circular peg. Determine the range of values of P for which the remains at rest and it is given that the coefficient of static friction between the rope and the peg is 0.2. So, again we have this peg and we have the weight W and we have the force P. So, let me write down $T_2 = T_1 e^{\mu\beta}$

So, T_2 will be T_1,e^{μ}, μ is given, it is 0.2, and β is also given, it is three π because it is rep 1 and 1 and half times. So, therefore, T_2 will be $6.59T_1$. Let us call it equation number 1. Now, let us first find out the largest value of P. So, if we are looking at the largest value of P, in that case, you can think that the impending motion of the block is upward because P is largest.

So, therefore, for largest value of P, the block will be on the verge of moving upward. Okay. So, therefore, whatever is the largest value that has to be T_2. So, in this case, T_1 will be W and T_2 will be P. Let us put that in equation number 1.

So, we have $P = 6.59\,W$. Let us call it equation number 8. Now, let us look at the smallest value of P. In that case, you can think as if the impending motion of the block is downward. So, for a smallest value of P, the impending motion of the block will be downward. So, therefore, in this case, now T_1 will become P and T_2 will become W. Let us put that in 1 again.

So, we get W equal to 6.59 P or $T = 0.152\,W$. Let us call it equation number B and from A and B, we get that P should be between 6.59 W and 0.152 W. Therefore, the block will be in rest if P is in this range.



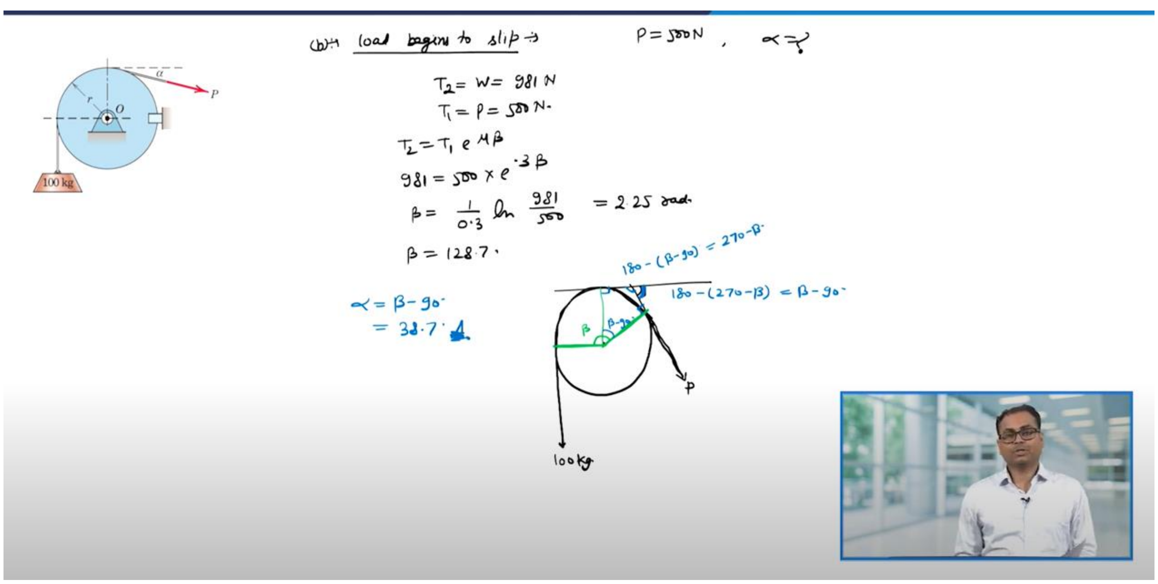
Now, let us look at one more problem on the same concept and the problem statement is following. A flexible cable which supports the 100 kg load is passed over a flexible circular drum and subjected to a force P to maintain equilibrium, the coefficient of static friction μ between the cable and the fixed drum is 0.30, then (a) for $\alpha = 0$, determine the maximum and minimum values which P may have in order not to raise or lower the load and (b) is for $P = 500\,N$, determine the minimum value which the angle α may have before the load begins to slip.

So, let us look at part A. So, we have a drum and with this 100 kg mass is suspended and it is held by a force P and this angle is α. So, initially it is given that $\alpha = 0$. To find out the maximum value of P, let us say that the motion of the block is upward. So, for impending upward motion, of the load.

So, we will use $T_2 = T_1 e^{\mu\beta}$, and here, my T_2 will be P. Let us say this is $P_{max} = T_1$. T_1 Is $100g$. So, that becomes $981e^{\mu}$ is given 0.3 and β because $\alpha = 0$, this becomes $\pi/2$. So, therefore, $P_{\max}$ becomes $1572\ N$. Now, if the impending motion of the block is downward, in that case, T_2 becomes 100 kg. So, for the impending downward motion of the load,

We have $T_2 = T_1 e^{\mu\beta}$. T_2 Will be 100 kg. So, 100×9.81 which becomes $981 = T_1$ becomes P. Let us say it is $P_{min} e^{\mu\beta}$. Therefore, we have $P_{min} = 981/e^{\frac{0.3\pi}{2}}$ and this comes out to be $612\ N$.

Now, in the second part, it is given that P is equal to $500\ N$ and we have to find out the angle α for which the load begins to slip. So, that means the load is larger than P. So, therefore, T_2 will be 100 kg.



So, let me write down part (b). The load begins to slip and it is given that $P = 500\ N$ and we have to find out what is α. So, T_2 will be W which is $981\ N$, and T_1 will be P which is $500N$.

So therefore, again $T_2 = T_1 e^{\mu\beta}$. From here, we have $981 = 500e^{\mu}$ is 0.3, and let us say the entire angle that it is making is β. Okay. So, we can find out what is β. β Will be $\frac{1}{0.3}\ln(\frac{981}{500})$ and this comes out to be $2.25\ radian$.

Therefore, in degree, β will come out to be 128.7^0. In the question statement, we have asked to find out the angle alpha and this we can find out from the simple geometry. So, we have this drum and the weight is hang like this, 100 kg. Force P is acting like that.

So, we have to find out this angle and this angle we can easily find out because this angle is β. Therefore, this angle will be $\beta - 90^0$ and since this angle and this angle are 90^0, therefore, this angle will be $180 - \beta - 90^0$ which will be 270 minus β minus Therefore, this angle over here will be $180 - 270 - \beta = \beta - 90^0$. And this is the angle that we want to find out. So, therefore, angle α will be $\beta - 90^0$ and therefore, it will be 38.7^0.

With this, let me stop here. See you in the next class. Thank you.