MECHANICS

Prof. Anjani Kumar Tiwari

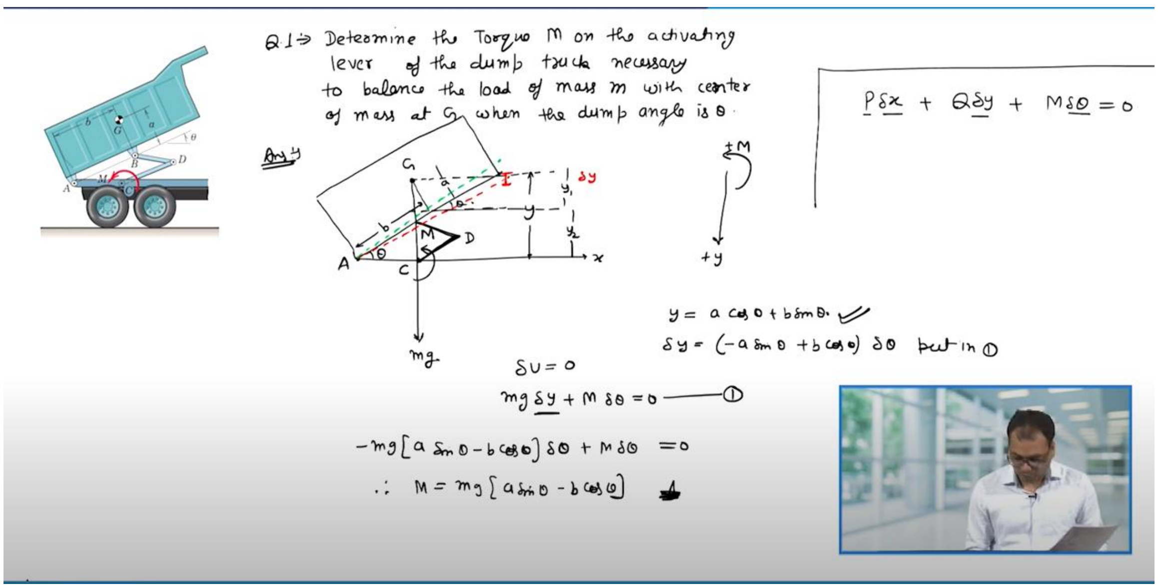
Department of Physics

Indian Institute of Technology, Roorkee

Lecture: 22

Principle of virtual work: examples - II

Hello everyone, welcome to the lecture again. In the last class, we solved some example based on the principle of virtual work.



Today, we are going to continue the discussion and we will solve some more example based on the same concept. So, before I write down the problem statement, let me just summarize what is the principle of virtual work. Suppose I have a rigid body and on this rigid body there are various external forces are acting.

So let us say P is one such force, Q is another force and let us say there is some moment that is also acting on the rigid body. Let us say corresponding to this force P if the virtual displacement is δx corresponding to this force Q there is a virtual displacement let us say δy and because of the moment let us say the virtual displacement is $\delta\theta$ then sum of them has to be equal to 0. Now, after that, there is one more step. You have to write down this

$\delta x, \delta y$ and $\delta\theta$ in terms of the generalized coordinate. So, you have to identify which one is the generalized coordinate.

You write down this equation in terms of that and we solve it to find out whatever is the unknown or whatever is the relation between them. Now, let me write down the first problem statement. And the problem statement is following.

Determine the torque M on the activating lever of the dump truck necessary to balance the load of mass m with centre of mass at g when the dump angle is θ. So, first let me make the free body diagram of this. So, let us say this is my x-axis and we have this term. This is making an angle θ with the horizontal because it is given that this angle is θ. And this length is b. So, its center of mass is at G, and this length is b, this height is a, and its mass is going to act downward.

So, let us say it is mg and over here, a moment M is applied, and this is kept into the equilibrium by this assembly. This point is given as D, and this is the free body diagram. You can see here that the generalized coordinate is θ because, with this θ, I can identify the location of, you know, all parts of this system. Now, let us follow the sign convention.

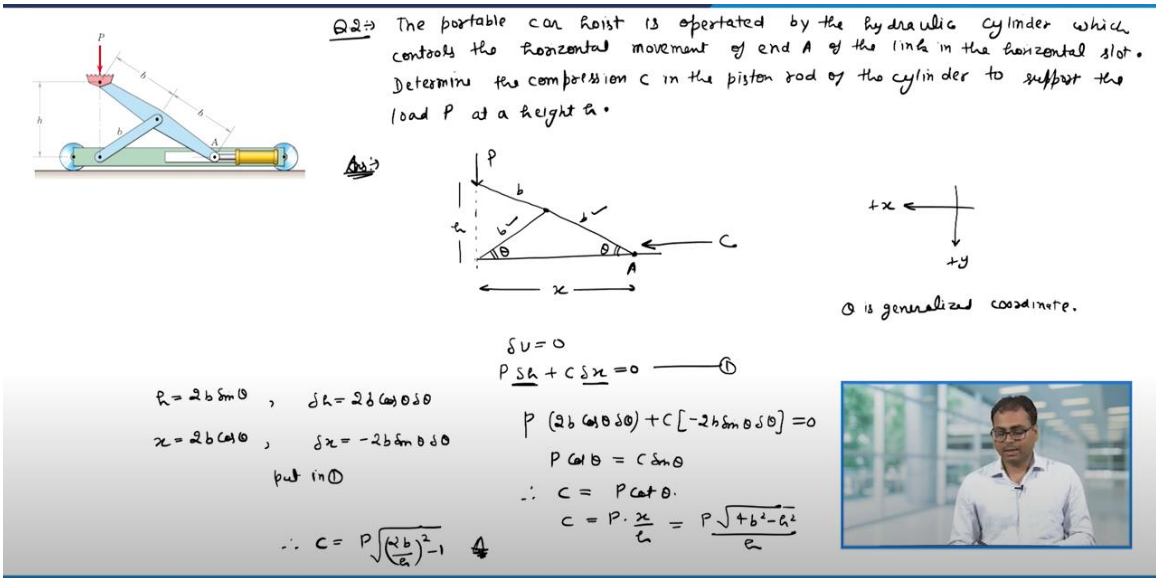
So, you can see here that the force mg is acting downwards. So, let me take this as positive y. This is in the same direction in which the force is acting and the moment is acting in that direction. So, let me take positive m in that direction. So, now, we have the sign convention. Let us say this height is y and the

This y, I can write down in terms of a's and b's. So, you can see here that this y is nothing but $y_1 + y_2$. And from the geometry, you can see that it will be $acos\theta + bsin\theta$. Now, let us write down the principle of virtual work equation.

So, we have $\delta U = 0$. So, we have force mg , so let us say $mg\delta y + m\delta\theta = 0$. Now, what we have to do is, we have, so as I said, here the generalized coordinate is θ. Therefore, this δy, I have to write down in terms of $\delta\theta$, but I know this relation. So, therefore, I can find out what is δy.

So, from here, my δy will be $-asin\theta + bcos\theta\delta\theta$. Let us put in equation number 1. So, we have$-mgasin\theta bcos\theta\delta\theta + M\delta\theta = 0$. Therefore, M comes out to be $mgasin\theta - bcos\theta$.

So, here in this example, I have not shown you the virtual displacement, how the virtual displacement will happen. But of course, I mean you can identify because of the force mg, this dump is going to come down. This is one virtual displacement and because of that your δy is going to change. And because of the moment M, you can assume that the virtual displacement will be like this and your θ in that case is going to increase. But since we have solved many problems, so we can you know think of it and we do not need to now show it here.



Now, let us look at one more problem statement. And the statement is following. The pull table car host is operated by the hydraulic cylinder which controls the horizontal movement of end A of the link in the horizontal slot.

Determine the compression C in the piston rod of the cylinder to support the load P at a height h. So, let me first make the free-body diagram of this. At A, because of the piston, there will be a horizontal force. So, since it is a cylinder, let me call this force as C and we have the force P acting at this point and then various geometrical parameters are given. So, for example, this is b, this is b and this is also b, this height is h. Let us say this distance is x.

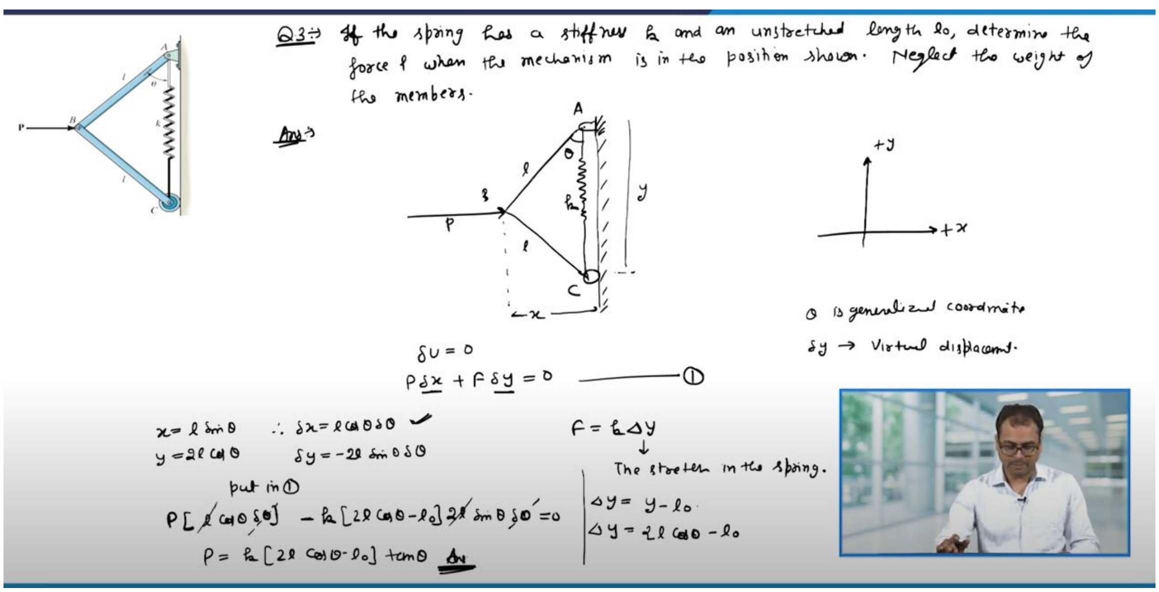
Now, you can see that the force is acting in that direction. So, therefore, let me take plus x like this and the force P is acting downwards. So, let me take plus y in that direction. So, I can write down the principle of virtual work equation. It is $\delta U = 0$.

So, which is $P\delta h + C\delta x = 0$. Now, Let us identify what is the generalized coordinate in this system. So, if I define my θ here, then with this θ, I can identify, you know, where the members of this systems are. So, therefore, this θ becomes the generalized coordinate.

Now, given the fact that this is b, this is b, this will also be θ. So therefore, since θ is the natural choice and it is the generalized coordinate. Therefore, we have to write down this δh and δx in terms of θ. So, let us see how the h and $\theta's$ are related.

So, you can see from the geometry that h will be $2bsin\theta$ and x will be $2bcos\theta$. Therefore, δh will be $2bcos\theta\delta\theta$ and δx will be$-2bsin\theta\delta\theta$. Let us put this in equation number 1. So, we have $P(2bcos\theta\delta\theta$ plus C into δx is$-2bsin\theta\delta\theta$ equal to 0 or we get $Pcos\theta = Csin\theta$.

Therefore, P is nothing but $Pcot\theta$. However, this θ was not mentioned in the question. So, therefore, let us eliminate this θ in terms of the given parameter that we have a, b and h. So, $cot\theta$ is nothing but x/h and x is $\sqrt{4b^2 - h^2}/h$. Therefore, C will be $P\sqrt{\left(\frac{2b}{h}\right)^2} - 1$. So, in equilibrium, the 4C and P should be related by this relation.



Now, let us look at, you know, another problem statement. This question is based on a spring. The problem statement is following. If the spring has a stiffness k and an unstretched length l_0, determine the force P when the mechanism is in the position shown.

Neglect the weight of the members. So, to solve this question, let us first make the free body diagram. So, we have a force P acting at point B. From here, we have members of length l, and they are connected by a spring of spring constant k. This angle is given as θ. And here you have a pin support and here you have a roller support.

This point is C, this point is A and that point was B. The spring constant is k. Now, since the force P is acting in that direction, so let me take this as positive x and in this direction, let me take positive y. Now, let us say this length is x, and this height is y. So, you can see here that the force P is balanced by the force which is provided by the spring and therefore, the system is in equilibrium. So, there are two forces P and let us say the spring force is F. So, we have equation $\delta U = 0$ and force $P\delta x$ plus spring force $F\delta y = 0$. Here, you can also appreciate that θ is the generalized coordinate.

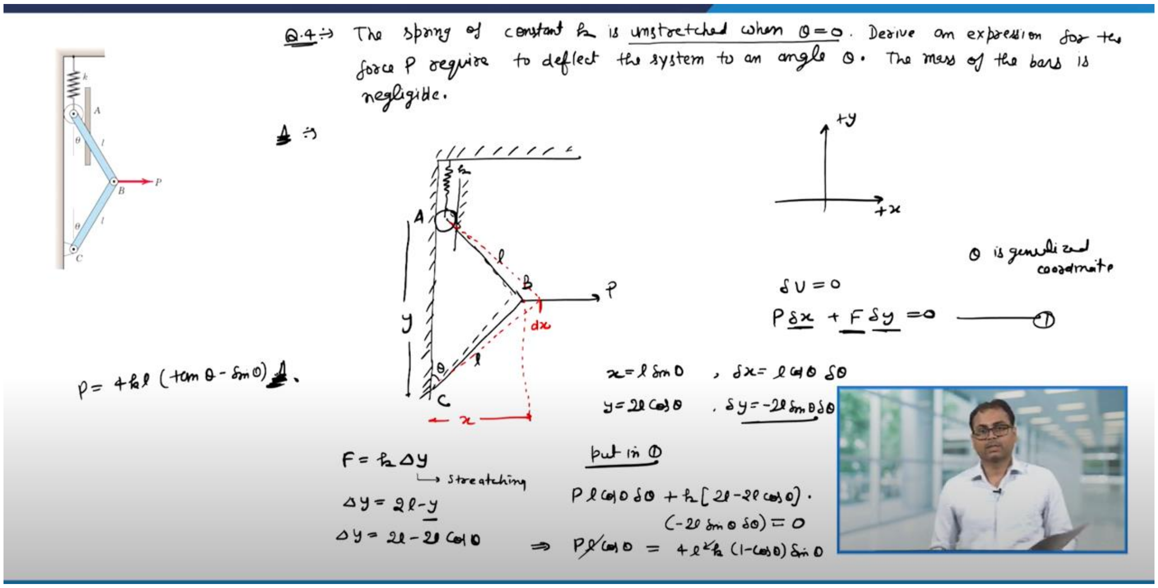
Therefore, this δx and δy, I have to write down in terms of θ. And also note that δy here is the virtual displacement. So, I would like to explicitly mention here that this δy is the virtual displacement. x is also virtual displacement in the x direction, δy is the virtual displacement in the y direction. Now, let us write down this $\delta x \delta y$ in terms of θ.

So, from the geometry, you can see that x is $lsin\,\theta$ and y, the way we have defined it, will be $2lcos\,\theta$. δx Is $lcos\theta\delta\theta$ and δy is$-2lsin\theta\delta\theta$. Now, in the question statement, P is mentioned. So, therefore, P is given. So, the spring force is not given.

So, let us write down what is the spring force here. So, $F = k\delta y$ where this δy, you know, it is the stretch in the spring, okay. So, we have to find out how much the spring has stretched in this particular case. It is given that the unstretched length of the spring is l_0.

So, therefore, δy will be $y - l_0$, okay. So therefore, again, this δy I can write down in terms of generalized coordinate because y is known. So, therefore, this will be $2lcos\theta - l_0$. Now, we are ready to rewrite equation number 1 in terms of the generalized coordinate. So, let us put everything in equation number 1.

So, we have $P\delta x$, δx is $lcos\theta\delta\theta + F$, F is $k\delta y$. So, $k\delta y$ is $2lcos\theta - l_0\delta y$. δy Is$-2lsin\theta\delta\theta$, and this will be equal to So, from here $\delta\theta$ can get cancelled, l can also get cancelled and we have $P = k2lcos\theta - l_0 tan\theta$. This is the relation between P and k for the system to be in equilibrium at angle θ.



Now, let us look at one more problem statement and the problem statement is following. The spring of spring constant k is unstretched when θ is equal to 0^0. Derive an expression for the force P require to deflect the system to an angle θ and it is given that the mass of the bar is negligible. So, again let us first make the free body diagram.

So, at A, we have a ruler which is connected by a spring of a spring constant k and then we have the bars. This length is l. It is given that this angle is θ, and here the force P is acting to keep everything in equilibrium. This point is given as C and this point is given as B. Let me show you the virtual displacement because of P, but we know like how it is going to look like. It is not required now, but anyway let me just show it.

So, let us say this length is x and then the virtual displacement will be dx and because of the spring, the whole system is, you know, will try to go up. So, therefore, that will be the other way around, something like that. Let us say this length is y and therefore, let me take this direction as positive x and this direction as positive y. Now, I can write down the equation for the virtual work.

So, this is $\delta U = 0$. We have force P acting in the x direction. So, virtual displacement is δx which is consistent with the constraint that we have plus $F\delta y$ equal to 0. Now, as usual, we have to identify what is the generalized coordinate and from here, it is obvious that θ

is the generalized coordinate. Therefore, this δx and δy, we have to write down in terms of θ.

So, for that, let us look at the relation between x and θ. So, we have x equal to, from the geometry, it will be $lsin\theta$ and y will be $2lcos\theta$. Therefore, δx will be $lcos\theta\delta\theta$ and δy will be minus $2lcos\theta sin\theta\delta\theta$. Let us find out this force F.

So, as we have done in the previous case, F will be $k\delta y$ where this δy is the stretch in the spring and δy will be $2l - y$ because it is given that the spring is unstretched when $\theta = 0$. So, in that case, my y will be $2l$. So, when the spring is unstretched, therefore, in this particular case, it will be $2l - y$. Therefore, δy I can write down. So, we have this y in terms of generalized coordinate, it will be $2l - 2lcos\theta$.

Now, we can put everything in equation number 1. So, let us put in 1. So, we have $P\delta x$; δx is $lcos\theta d\theta$, $lcos\theta\delta\theta + F$, F we have calculated, $F = k\delta y$. So,$k\delta y$, δy is $2l - 2lcos\theta$ into small δy and this $\delta y = -2lsin\theta d\theta = 0$.

So, from here, we can see that $Plcos\theta = 4l^2k(1 - cos\theta sin\theta)$, l will get cancelled, and$cos\theta$, we can take it on the right-hand side. We will get $P = 4kltan\theta - sin\theta$. With this, let me stop here. See you in the next class. Thank you.