MECHANICS

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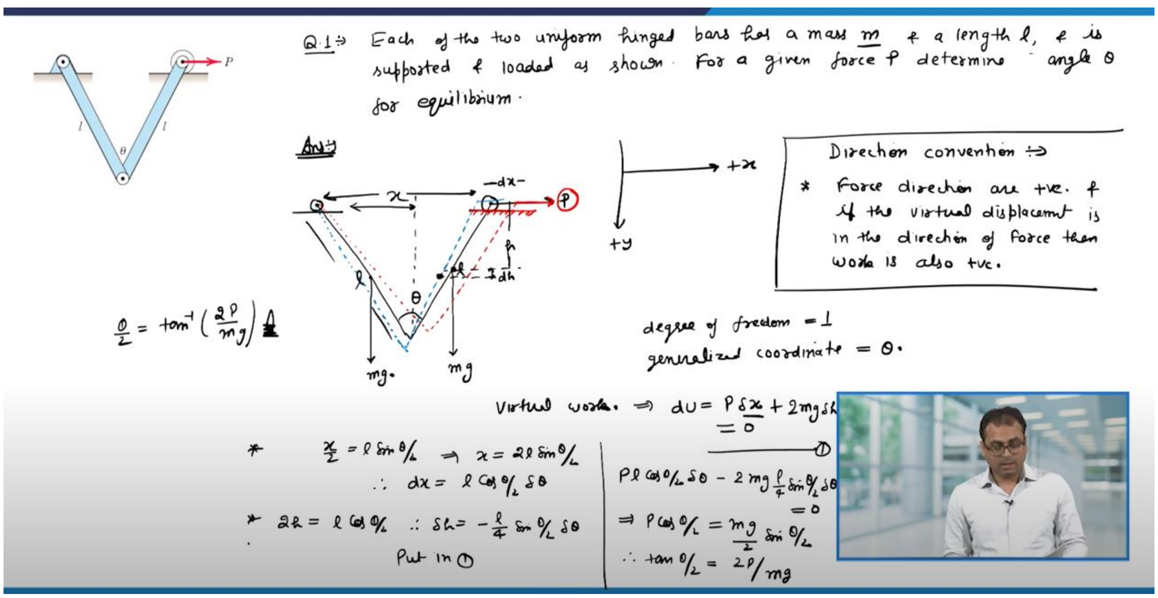
Lecture: 21

Principle of virtual work: examples

Hello everyone, welcome to the lecture again. In the last class, we looked at the concept of virtual displacement, virtual work and principle of virtual work.



Today, we are going to solve many examples so that we can get familiar with these concepts. Let us look at the first question statement. So, question number 1 and the problem statement is following.

Each of the two uniform hinged bars has a mass m and a length l and is supported and loaded as shown for a given force P determine the angle theta for equilibrium. Now, before we start this question, first let us follow some sign convention that we will use for the principle of virtual work. So, this is the direction convention.

The force direction I am going to take always positive. So, if the force is acting in the x direction, then that x will be positive. And if the virtual displacement is in the direction of the force, then work is also positive.

So with this, let us understand the problem. So here what is happening is you have these two bars and of course they have a mass m. Because of the mass m, they have a tendency to get close and because of the force P, we are balancing this tendency of the bar so that they remains in the equilibrium. And in the question statement, it has asked to determine the angle theta so that P balance the collapse of these bars. First of all, let me put the directions here.

So, I have taken plus x because the P is acting in that direction and I am taking this direction as positive y because the mass of these roads are acting downwards. So, let me draw the figure again. So, we have a pin support here and we have a roller support here. This is subjected to a force P and then the bars are like that.

They have a length of l, and in between, they make an angle of theta. Now, the mass of these bars are going to act at a distance of $l/2$, and it will be downwards. So, let us say this is mg, and that is also mg. Because of the P, the bars are going to go far. So, let us draw a virtual displacement.

This is the effect of P. And note that this is consistent with the constraint because we have taken the displacement along P. this surface, okay? And what will happen because of the weight? So, as I said, because of the weight, it have a tendency to do a motion like this. But note that everything is in equilibrium.

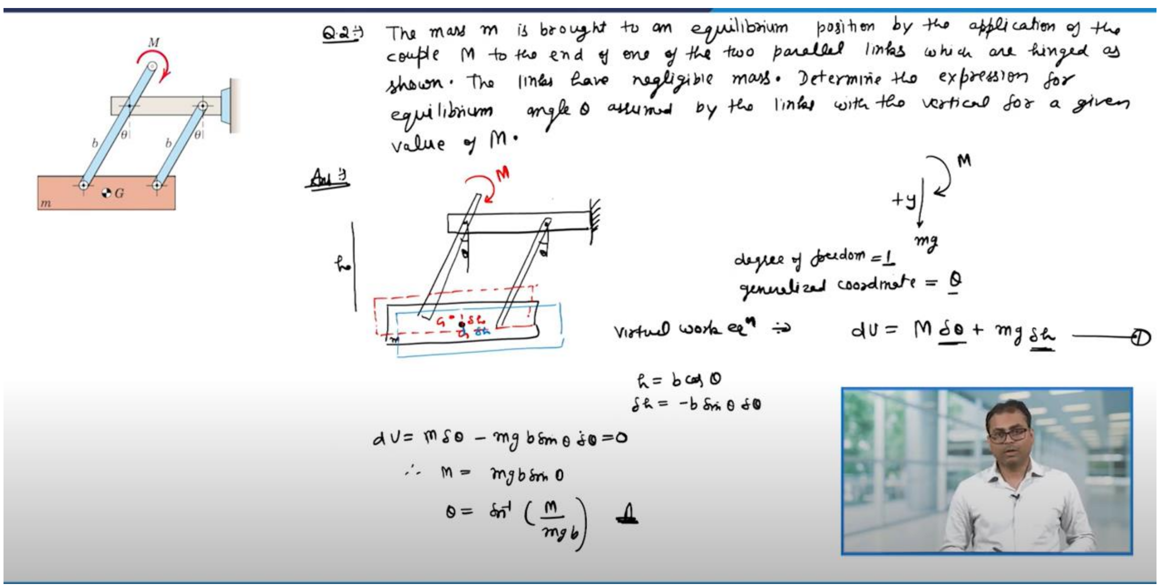
So let us find out the virtual work. So one more thing you have to appreciate here that the degree of freedom is 1 and the generalized coordinate is so, now let us look at the virtual work. Because of this force P, let us say this is x, because of this force P, this bar is going to displace by a length dx.

Therefore, virtual work dU will be Pdx because of that force P. Now, because of the mass, you can also see here that let us say this height is h. Because of the mass, these bars are going to come close. Therefore, their center of mass is is going to lower down, let us say by a height dh. So, earlier the center of mass was here, now it will shift to that point and let us say this is dh. So, therefore, because of the mass, the virtual work will be so there are $mg + mg$,$2mg$, into the virtual displacement which is dh, so δh.

And as per the principle of virtual work, this should be equal to 0. Let us call this equation number 1. Note that our generalized coordinates were θ. Therefore, this equation number 1, which is right now in δx and δh has to be converted to $\delta\theta$. So, let us find the relation between dx and θ or x and θ.

So, from the geometry, you can see that because this length is l, therefore, this length will be so, $x/2$ will be $lsin\left(\frac{\theta}{2}\right)$ or x will be $2lsin\left(\frac{\theta}{2}\right)$. Therefore, dx will be $lcos\left(\frac{\theta}{2}\right)\delta\theta$. So, we got this dx in terms of $d\theta$. Now, let us look at the relation between dh and θ.

So, again you can see here that $2h$ will be $lcos\left(\frac{\theta}{2}\right)$. Therefore, dh will be$-\frac{l}{4}\sin\left(\frac{\theta}{2}\right)\delta\theta$. Let us put this in equation number 1. So, we have $Plcos\left(\frac{\theta}{2}\right)\delta\theta - \frac{2mgl}{4}\sin\left(\frac{\theta}{2}\right)\delta\theta = 0$, and this gives us $Pcos\left(\frac{\theta}{2}\right) = \frac{mg}{2}\sin(\frac{\theta}{2})$ ortan $\left(\frac{\theta}{2}\right) = \frac{2P}{mg}$. And from here, you can say that $\theta/2$ is nothing but $tan^{-1}\left(\frac{2P}{mg}\right)$.



Now, let us look at another question. So, this is question number 2 let us say and the problem statement is following.

The mass m is brought to an equilibrium position by the application of the couple M to the end of one of the two parallel links. Which are hinged as shown. The link has negligible mass, and in the question it is asked you to determine the expression for equilibrium angle theta which is assumed by the links with the vertical for a given value of the moment M. So, again, because it has a weight mg, so therefore this direction is, let us say, plus y, and the moment is applied in that direction. So, let me take the positive moment in that direction. So, therefore this will be our sign convention. So, let me first make the diagram once again. So we have these bars here and with this we have two bar that is connected and from here we have the mass m and of course it is going to act at g. So, first of all, what will happen because of the moment?

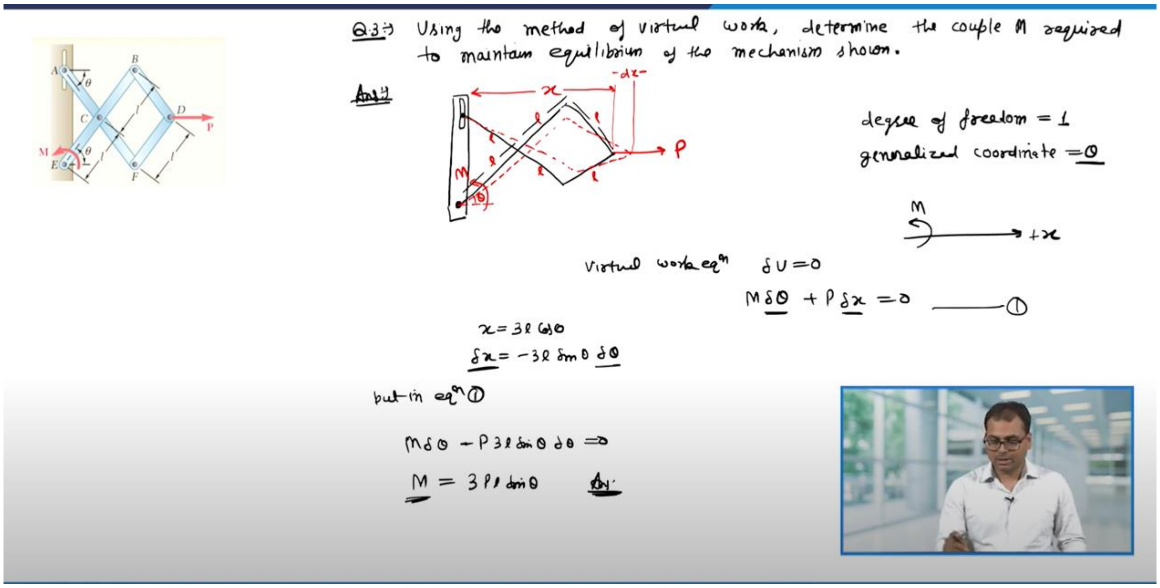
So, we can assume infinitesimal virtual displacement because of this m. So, you can think of because of this m, this mass is going to go up. So, earlier, let us say it is here and here. The G is going to go there and let us say this is delta h. And because of the mass m, you can assume the infinitesimal virtual displacement to go down. So, because of that, you can think of that this is going to go down.

So, this angle is given theta, this angle is also theta. So, first of all, the degree of freedom is 1, generalized coordinate is theta here because with this theta I can find out the position of the bar. Now, let us write down the equation of virtual work. So, virtual work equation is $dU = m$ is the moment and the virtual displacement which is $d\theta$.

Plus, it has a mass of mg, and because of this mass, the virtual displacement is dh. Now, let me write down this θ and dh in terms of θ. So, this $d\theta$ is already in terms of θ. The only thing that I have to do is I have to write down this dh in terms of θ because θ is our generalized coordinate. So, for that I have to find out a relation between h and θ.

So, you can clearly see here that if this height is h, then h is $bcos\theta$. Therefore, dh will be $-bsin\theta d\theta$. Let us put that in equation number 1. So, we get $dU = m\delta\theta dh - mgbsin\theta\delta\theta$, and this has to be equal to 0.

Therefore, M is $mgbsin\theta$ or $\theta = \sin^{-1}\frac{M}{mgb}$. This is the condition for the equilibrium. That means to maintain this at an angle θ, you have to apply you know a moment M and the relation is this.



Now, let us look at one more very important problem on this concept. So, let me again write down the problem statement and it is following.

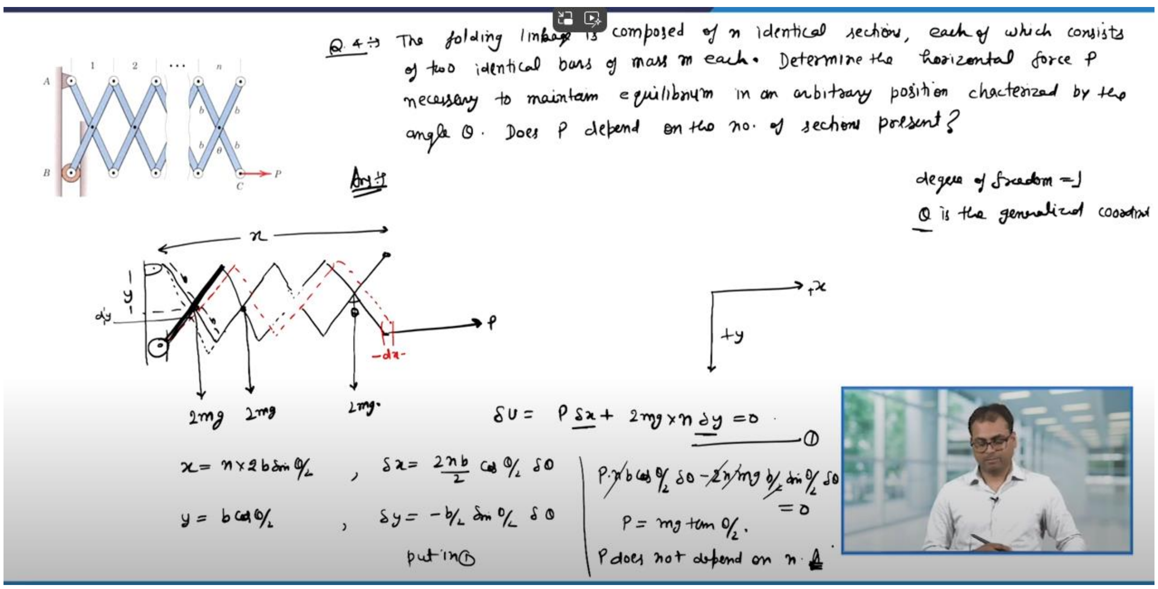
Using the method of virtual work, determine the couple M required to maintain equilibrium of the mechanism shown. So, you can see here that the degree of freedom is 1 and the generalized coordinate is θ because with this one θ I can find out the position of the link. Now again, the sign convention, the force P is applied in that direction. So, let me take this as positive x and the moment is applied in that direction.

So, let me take this direction as positive direction. Now, let me draw the system. So, we have a pin joint here and then here we have some kind of slot and we have the link like this and you apply a force P. This angle is given that it is θ and the moment M is applied to keep this thing in equilibrium.

Now, because of this force P, we can assume that the displacement is going to be like this. Let us say this distance is x and this is dx. It is already given that the length of each link is l. So, therefore, this is l, this is l, that is also l. And now, we are in a position to write down the equation for virtual work. So, the virtual work equation is $\delta U = 0$.

In this case, we have $M\delta\theta$ because of the moment and then we have P and the virtual displacement is $\delta x = 0$. Now, again because our generalized coordinate is θ, therefore we have to write down this equation only in terms of θ. So, this is already in $\delta\theta$; we have to write down this dx in terms of $d\theta$. For that, we have to find out the relation between x and theta. So, you can see here that x is $3lcos\theta$ because this length is l, this is also l, and this is also l. So, the x will be $3lcos\theta$.

From here, we can find out that dx will be$-3lsin\theta\delta\theta$. So, this gives me the relation between x and $d\theta$ and let us put that in equation number 1. So, we get $M\delta\theta - Pd3lsin\theta\delta\theta = 0$ and this gives me the relation between M and P so $M = 3Plsin\theta$. So, that means if I want to maintain an angle θ and on this system a force P is applied in that case I have to apply that much moment at point A.



Now, let us look at the generalization of this example and here the problem statement is following. The folding linkage is composed of n identical sections, each of which consists of two identical bars of mass m each. Determine the horizontal force P necessary to maintain equilibrium in an arbitrary position characterized by the angle θ. And it is also asked that does P depends on the number of sections present.

So, again here the degree of freedom is 1 and θ is the generalized coordinate. And let me draw the schematic once again. Here, you are applying a force P. This angle is given as theta. Now, since these links have a mass m, therefore, the mass of this part will be $2m$ and the weight will be $2mg$. Similarly, this will also be $2mg$ and that will also be $2mg$. Now, because the force P is acting in this direction, so let me take this as positive x and the force mg is acting in that direction. So, therefore, let me take positive y in this direction.

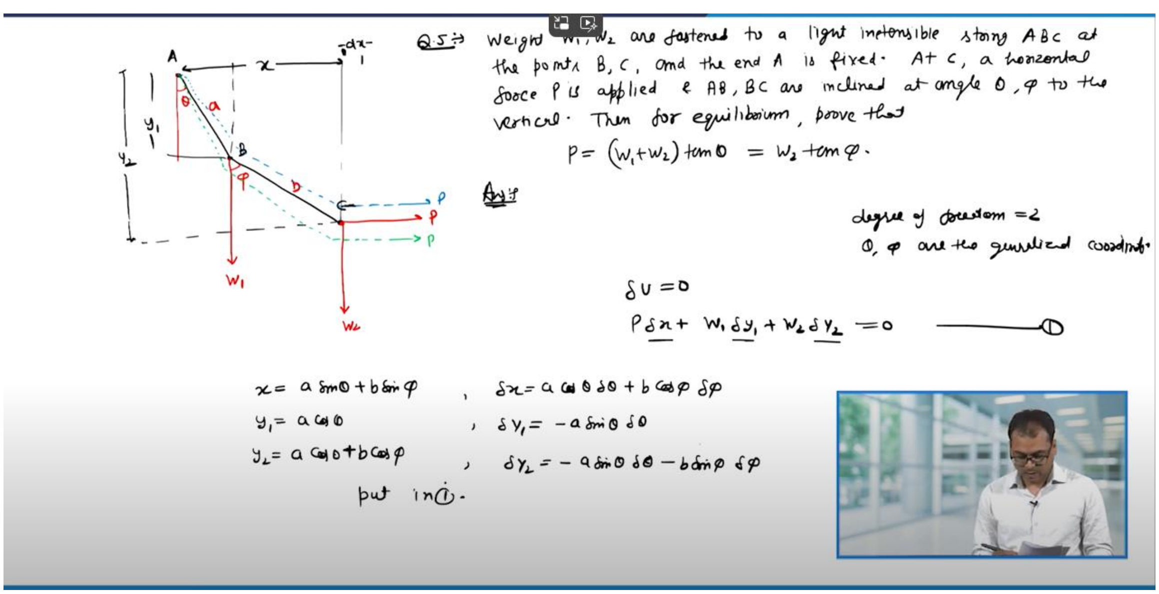
Now, let us write down the equation of virtual work. So, we have dU equal to, so let us say this whole length is x.

So, it will be Pdx wherein we assume a virtual displacement of δx because of this force P. So, let me just show you once again. So, this part is dx. So, dU will be Pdx plus because of this force. So, there are $2mg$ force, but there are, you know, n such, you know, sections. So therefore, the total weight will be $2mgn$, and because of that, its center of mass is going to lift.

So, let us say this is y and because of this mg force, we can assume the virtual displacement to be like that, and therefore, its center of mass is going to come down. So, it will be $2mgn\delta y = 0$. This is the virtual work equation. Now, again, we have to write down this δx and δy in terms of θ because θ is the generalized coordinate.

So, let us first see what is x. x is n because there are $n2bsin(\frac{\theta}{2})$ because this length is given that it is b. So, this part is b, this is also b. So, for one b, the projection in the x direction will be $bsin(\frac{\theta}{2})$. Therefore, for the n such bars, the total x will be $n2bsin(\frac{\theta}{2})$. Similarly, you can see here that y will be $bcos(\frac{\theta}{2})$.

And this equation helps us to write down dx in terms of $d\theta$. So, from here I can see that δx is $\frac{2nb}{2}\cos\left(\frac{\theta}{2}\right)\delta\theta$ and this gives me $\delta y = -\frac{b}{2}\sin\left(\frac{\theta}{2}\right)d\theta$. Let us put that in equation number 1. So, we get $Pnbcos\left(\frac{\theta}{2}\right)\delta\theta - \frac{2nmgb}{2}\sin\left(\frac{\theta}{2}\right)\delta\theta = 0$. So, 2 will get cancelled here. And we get $P = mgtan(\theta/2)$. And from here, you can also see that it is independent of n. So, P does not depend on n.

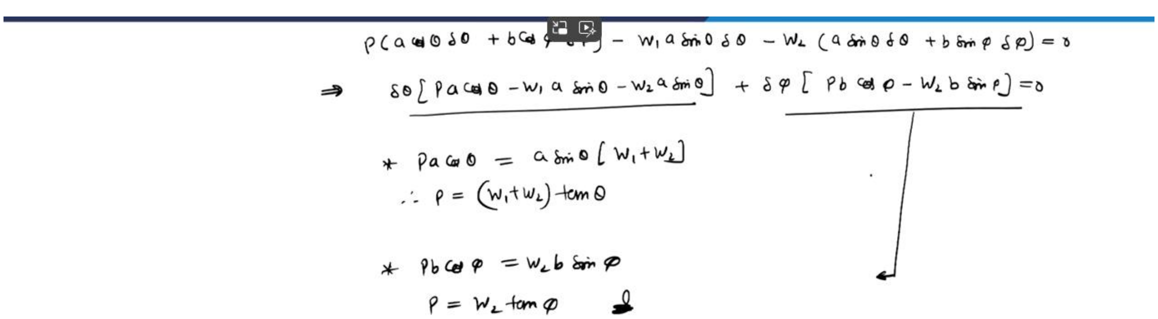


Now, let us look at another question. And here the problem statement is following. With W_1, W_2 are fastened to a light inextensible A, B, C at the points B, C and the end A is fixed. At C, A horizontal force P is applied and AB, BC are inclined at angle θ and ϕ to the vertical. Then for equilibrium prove that $P = W_1 + W_2 tan\theta$ which will be equal to $W_2 tan\phi$.

So we can think because of the weights W it is going to move like this and because of the force P we can think that the system is going to move like that. Now, you can also appreciate that here the degree of freedom is 2 and θ and ϕ are the generalized coordinate. Now, let us write down the virtual work equation, which is $\delta U = 0$.

Let us say this length is x, then because of this force P, it is going to move in the x direction. So, therefore, the virtual displacement is dx. Now, let us say this height is y_1 and this height is y_2. Therefore, δU will be $P\delta x$ plus because of W_1, let us say the virtual displacement is δy_1, and because of W_2, the virtual displacement is δy_2, and this has to be equal to 0.

Let us call it equation number 1. Now, again, note that this dx, dy_1, and dy_2, we have to write down in terms of the generalized coordinate. So, therefore, in terms of θ and ϕ. Now, from the geometry, you can see that $x = asin\theta + bsin\phi$ and y_1 is $acos\theta$ and y_2 is $acos\theta + bcos\phi$. Therefore, I can find out what is δx. δx Will be $acos\theta\delta\theta + bcos\phi\delta\phi$. Similarly, δy_1 will be minus $asin\theta\delta\theta$, and δy_2 will be$-asin\theta\delta\theta - bsin\phi\delta\phi$. Let us put these values of $\delta x, \delta y_1, \delta y_2$ in equation number 1.



So, we have $Pacos\theta\delta\theta + bcos\phi\delta\phi - W_1 asin\theta\delta\theta - -W_2 asin\theta\delta\theta + bsin\phi\delta\theta\delta\phi = 0$.

Let us collect the coefficient of $\delta\theta$ and $\delta\phi$. So, we have $\delta\theta Pacos\theta - W_1 asin\theta - W_2 asin\theta + \delta\phi Pbcos\phi - W_2 bsin\phi = 0$ and for this to be 0, both the coefficient of $\delta\theta$ and $\delta\phi$ has to be 0. So, let us first see this.

So, we have $Pacos\theta = asin\theta W_1 + W_2$ and this gives you $P = W_1 + W_2 tan\theta$. This is something that was asked in the question statement. Now, let us look at this equation. So, this gives you $Pbcos\phi = W_2 bsin\phi$ and this gives you $P = W_2 tan\phi$. This was also something that was asked in the question.

So, with this, let me stop here. See you in the next class. Thank you.