MECHANICS

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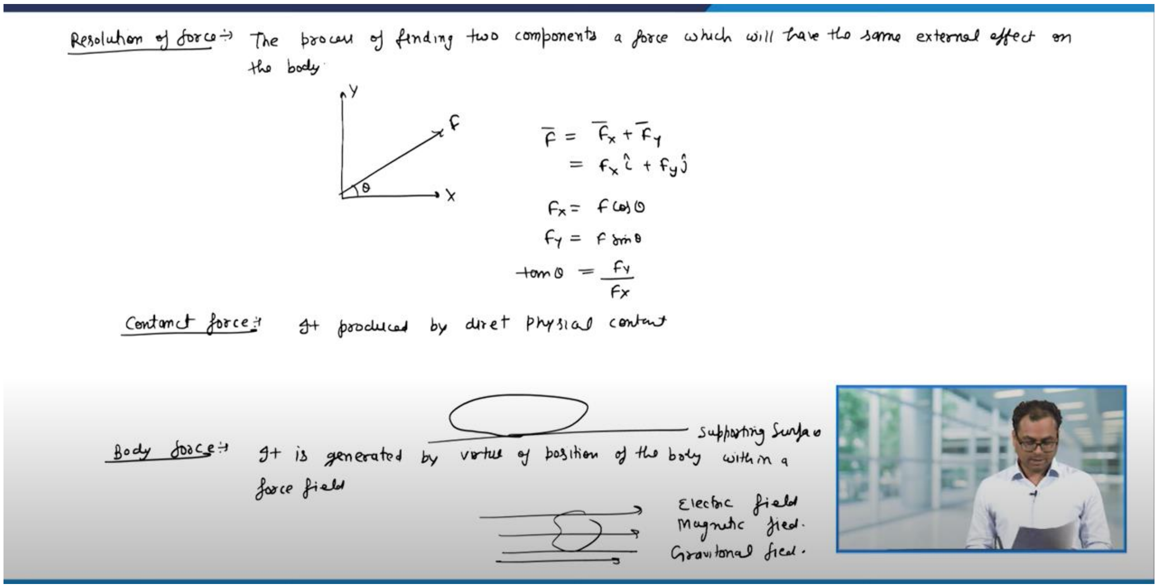
Lecture 02

Vectorial representation of forces and moments

I welcome you for the second lecture. So, in the last lecture, we look at how can we find out the resultant of you know various forces.



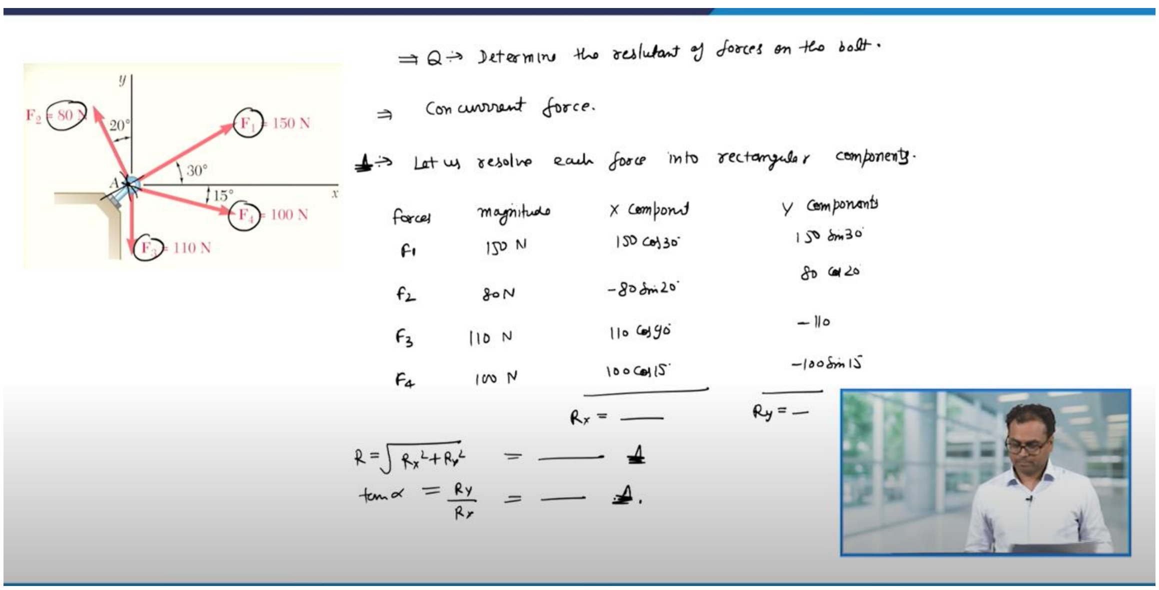
Now, let us look at the reverse process of that, that means the resolution of a forces. So, basically this is the reverse process of what we have done for the resultant of the forces, wherein we have various forces and then we find out a single force which has the same effect of all these forces, this is the reverse process of that. So, here this is the process of finding two component of a force.

Which will have the same effect, same external effect, on the body. So, let us say I have a force F which is acting on a rigid body and it is making an angle theta let us say from the x and y axis then this is the process wherein we can break this force into you know two component x and y. So, this force F can be written as F_x plus F_y. If i and j are the unit vector along x and y, then they can be written as x x i plus F y z and the value of F_x is nothing but $Fcos\theta$, the projection of F along x direction and F_y. is $Fsin\theta$ and there is a relation between theta and F_y and F_x. So, $tan\theta$ will be F_y divided by F_x.

Now, we will see that the body will be experiencing different kind of forces. So, let us very quickly discuss the definition of the contact force and body force. So, let us first discuss what is a contact force. A contact force is produced by direct physical contact.

So, for example, let us say I have a supporting surface and you put a body on it. So, here in the body will experience contact force because that is in contact with the supporting surface. We can also have a situation wherein we have body force and this body force is generated by virtue of positioning of the body within a force field. So, for example, you put a body in electromagnetic field, then the body will experience a force.

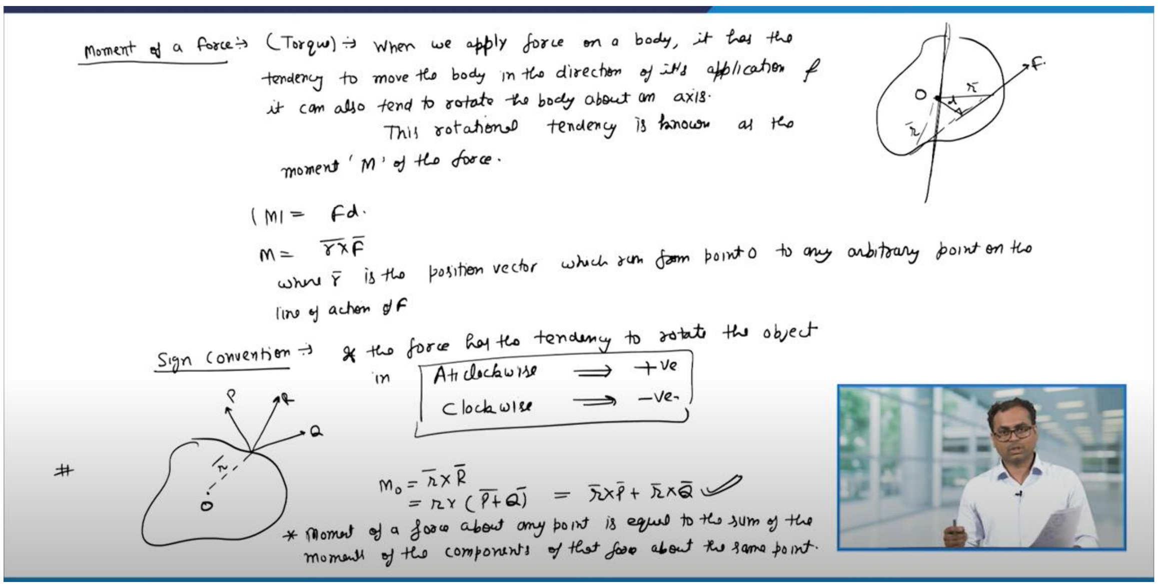
And, you know, in this case, it will be a body force. So, you can have electric field, magnetic field or or the gravitational field. So, the forces which are generated by the field and you place a body, then it will experience the body force. Now, let us look at, you know, the concept that we have learned by a simple example.



So, herein, there are four forces which are acting on a bolt. And the question statement is, determine the resultant of the forces on the book. So, you can see here there are four forces, their magnitudes and direction is given and the question is can we reduce this force system into a single force. So, first thing we have to see here that the forces are concurrent because their line of action they meet at a single point.

So, these are the example of concurrent force. Now, to determine the resultant let us resolve each forces into rectangular component. So, because here the x and y axis are given, so let us resolve them along the x and y direction. So, we have four forces F_1, F_2, F_3 and F_4 and their magnitudes are $150\ N, 80\ N, 110\ N$, and $100\ N$ their x component will be. So, for example, F_1 it will be $F_1 cos30^o$. So, this will be $150cos30^o$, $-80sin20^o$, $110cos90^o$, $100cos15^o$ and similarly the y components will be $150sin30^o$, $80cos20^o$, -110 , and $-100sin15^o$. So, we can sum them up and find out what is R_x.

We can sum them up can find what is R_y and then capital R will be $\sqrt{R_x^2 + R_y^2}$ and $tan\alpha$ wherein α is the angle that the resultant is making from the x-axis will be $\frac{R_y}{R_x}$.



Now, let us look at what is the moment of a force. This is also called torque. So suppose I have a rigid body and on this rigid body you apply a force.

Then this force has a tendency to translate this object and also this has a tendency to rotate this object. And this rotational tendency, it is called the torque or the moment of the body. So, when we apply force on a body, it has the tendency to move the body in the direction of its application and it can also tend to rotate the body about an axis.

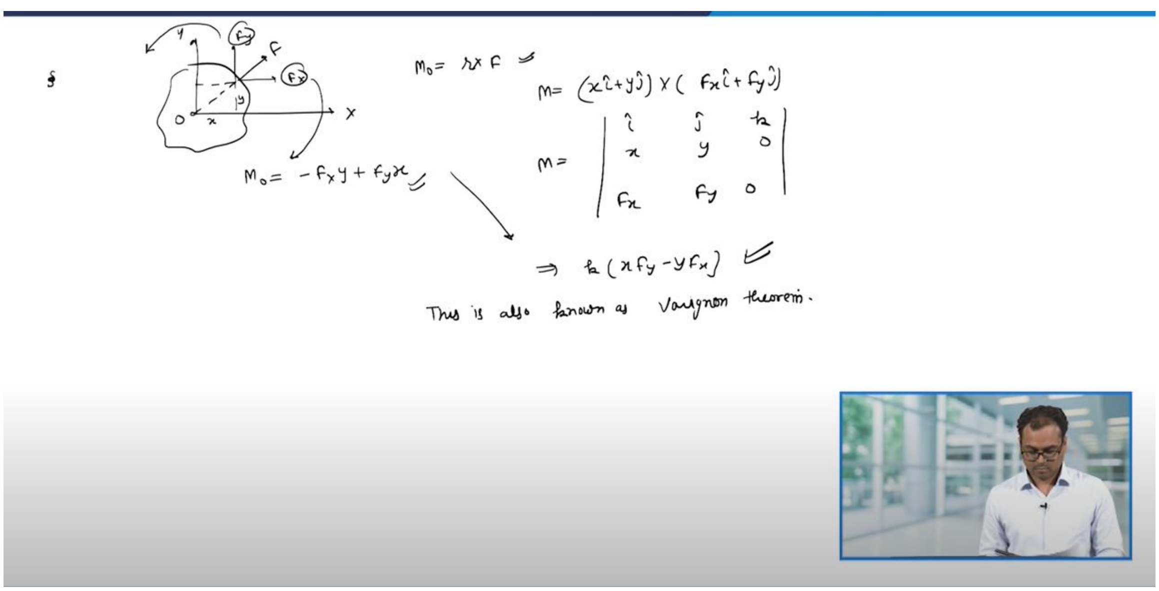
So, this rotational tendency is known as the moment M of the force. So, for example, here this is the axis and the force is applied. So, let us say it is passing through this O, then the moment M will be Fd, where d is the perpendicular distance from this force to the point

O. Now, the moment can also be find out using $r \times F$ where this r is the position vector which run from point O to any arbitrary point.

On the line of action of F. So, for example, you can take this as r or you can take that as r. So, you know, any position vector which runs from point O to any arbitrary point on the line of action of force. Now, in this course, we will follow the following sign convention. So, if the force has the tendency to rotate the object in anticlockwise, then we will take its moment positive and if it is clockwise, then we will take the moment negative. This is the convention that, you know, we will follow.

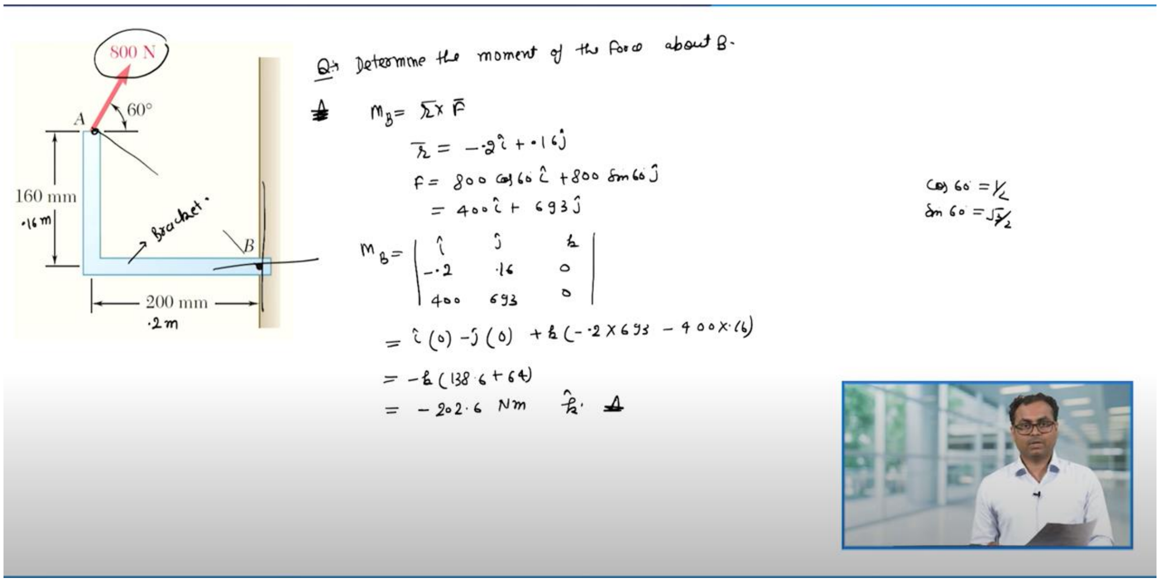
Now, let us say that I have a rigid body and on this rigid body, a force R is acting on it. Now, this force R, I can divide into two components. Let us say they are P and Q. then the moment about O will be $r \times R$. But since R, I can write down in terms of P and Q. So, this R can be written as $P + Q$ and this will be $r \times P + r \times Q$. What does this mean?

This means that the moment of a force about any point is equal to the sum of the moments of the components of that force about the same point. So, this is the statement that you know we have just shown that you take a force you break it into two force then the moment of that force about the point will be equal to the moment of those forces that we have broke you know about the same points.



So, let us look at the same concept once again. So, suppose I have object or rigid body and let us say these are the x-axis and the y-axis and let us say the force F is acting on it. So, this force F, I can break into F_x and F_y then the moment about O was $r \times F$, but now this F, I have broke into two, F_x and F_y. Now, about O, this F_x has a tendency to rotate the object in this direction and F_y has the tendency to rotate the object in that direction. Now, this F_x has a tendency to rotate it in the clockwise direction.

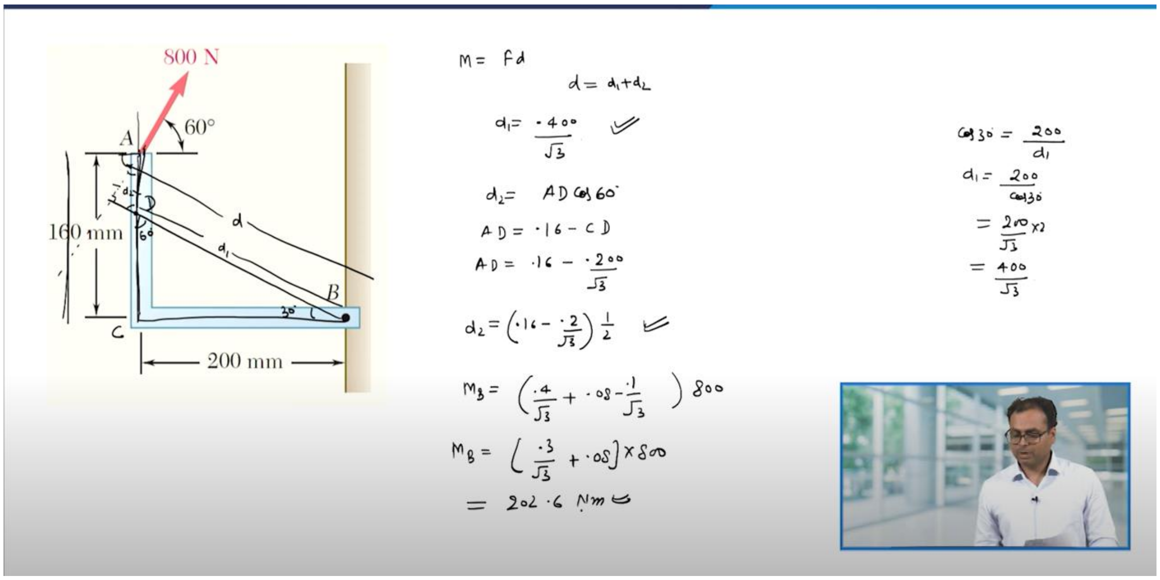
So, therefore, it will be negative. So, therefore, the moment will be minus F_x into the perpendicular distance which is y and this one is x. So, it will be minus $F_y + F_{yx}$. Now, let us see the same point using you know this equation. So, moment M will be r is $xi + y\,j$ cross F is $F_{xi} + F_{yj}$. So, this is just a vector product. So, this will be i j k x y 0 and F_x F_y 0. So, this will be $k(xF_y - yF_x)$, which is the same. Sometimes this is also called the Varignan theorem.



So, now let us look the concept of moment by an example. So, let us say a force of 800 N is acting on a bracket. So, this is a bracket. And let us say we were asked to determine the moment of the force about point B. So, this question we can solve you know by different way. So, let us say we use the equation of moment about B equal to $r \times F$. So, in the Cartesian coordinate system, let us fix our axis here. So, r is any point on the force.

So, let us say this is my r. So, r is minus point to I plus 0.16 j, wherein I have converted this distance in meter. And the force F is 800 cos 60 degree I plus 800 sin 60 degree j, okay. So, this is $400i + 693j$ because $cos60^o$, $1/2$ and $sin60^o$, $\sqrt{3}/2$.

So, therefore, the moment will be r cross F I j k minus 0.2, $0.160, 400, 693$ and 0 and this will be $i(0) - j(0) + k(0.2 \times 693 - 400 \times 0.16)$. And this will be $-k(138.6 + 64)$ equal to $-202.6\ Nm$ and its direction is $\hat{k}$. So, this is one way of calculating the moment about point B. We can also find out the moment by using the formula Fd where, d is the perpendicular distance from point B to the force.

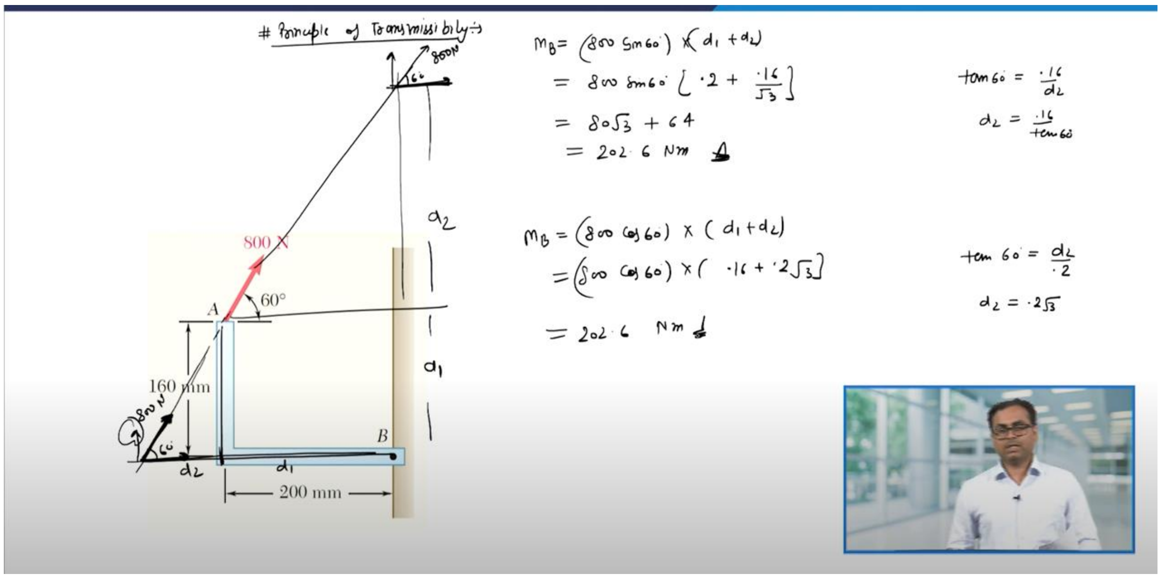


So, let us use that method and let us see if the answer is same using $M = Fd$ where this d is the perpendicular distance from B to the force. Now, force is a sliding vector. So, therefore, we can slide it and we can draw a perpendicular. So, this is d and this i can write down as d_1 and this i can write down as d_2. So, d becomes $d_1 + d_2$.

Now, d_1 is very easy to determine because this is 60^o. So, therefore, this will be 60^o, this will be 30^o and since it is perpendicular, this will be 60^o. So, therefore, this is 60^o and this one is 30^o. Now, cos 30 is $200mm/d_1$. So, therefore, d_1 will be $200/cos30$.

So, this will be 200 divided by square root 3 by 2, which is 400 divided by square root 3. So, d_1 is $0.4/\sqrt{3}$, wherein i have changed this distance into meter. Now, to determine d_2, let us say that this point is C, this point is D, then d_2 will be $ADcos60^o$ and A D will be $0.16 - CD$ because this is 0.16. So, AD will be this distance minus that distance and CD we can very easy find out. So, my AD will be $0.6 - 0.2/\sqrt{3}$. So, therefore, d_2 will be $ADcos\theta$ it will be $0.16 - 0.2/\sqrt{3}$ multiplied by $cos60^o$ which is $1/2$. So, this is my d_2, d_1 i already know. So, therefore, the moment will be the force multiplied by the distance, distance is d_1 plus d_2.

So, d_1 is $\frac{0.4}{\sqrt{3}}$ plus d_2 is $0.08 - 0.1/\sqrt{3}$. So, therefore, the moment will be let us simplify this $\sqrt{3}$ plus 0.08 multiplied by 800 and this comes out to be $202.6\ Nm$ which is same you know as we have obtained earlier.



We can also calculate the moment of this force using the principle of transmissibility. So, let us slide it along its line of action. So, we have slided it on the horizontal axis. So, this is the force $800\ N$ and this angle is 60^o. Now, the moment about B will be this force we can break into the x and y component. So, the x component will not contribute because its moment about B will be 0. So, the only moment that is going to contribute is the y component of the force.

So, the moment will be $800sin60^o(d_1 + d_2)$. Now, d_1 is $0.2m$ and d_2 we have to find. So, d_2 we can find using $1060 = 0.16/d_2$. So, therefore, d_2 will be $0.16/1060^o$.

So, this is the force multiplied by d_2, d_1 is 0.2 plus d_2 is $0.16/\sqrt{3}$. So, therefore, this will be $80\sqrt{3} + 64$ and this comes out to be $202.6\ N/m$. Now, we can also translate this force on the y axis. So, this is the force that we have $800\ N$, we can again break it into x and y components.

So, in this case, the moment will be $800cos60^o(d_1 + d_2)$ wherein d_1 and d_2 are this distance. So, again let us see what is d_2, then 60^o will be $\frac{d_2}{0.2}$. So, d_2 will be $0.2\sqrt{3}$. So, it will be 800.

$Cos60^o$ Multiplied by d_1 is 0.16 plus d_2 is $0.2\sqrt{3}$ and this again comes out to be $202.6\frac{N}{m}$. So, we have seen using various principles like the principle of transmissibility moment equal to $R \times F$ or you know FD, we get the same result. So, there are various ways that you can you know find out the moment and with this let me stop here and let us see you in the next class. Thank you.