**MECHANICS**

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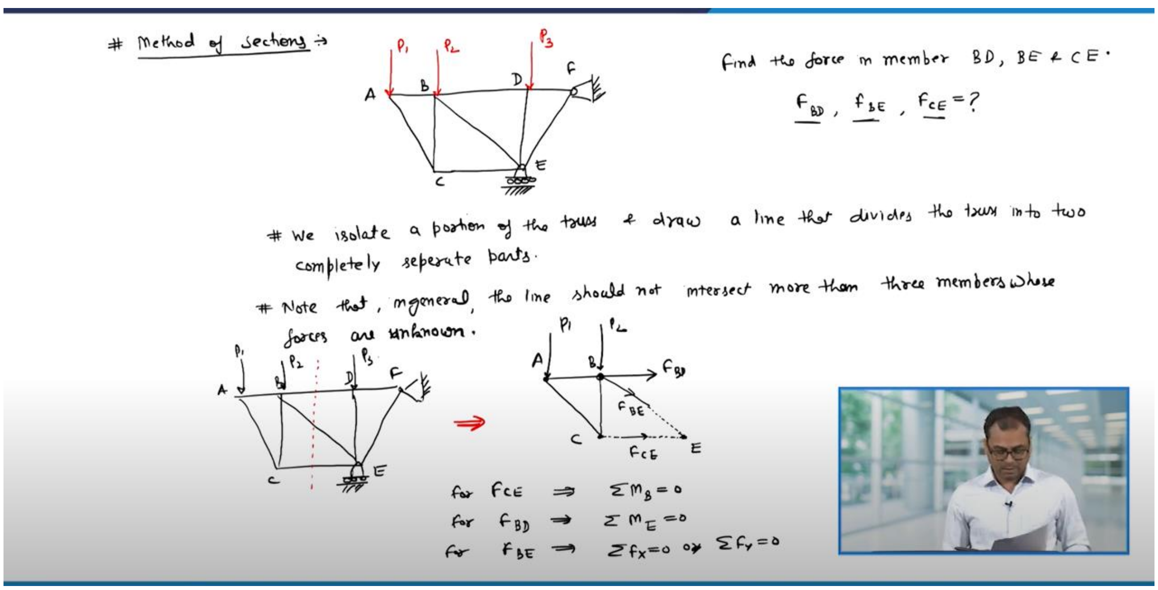
**Lecture: 17**

**Analysis of trusses: Method of sections**

Hello everyone, welcome to the lecture again. In the last few lectures, we were analyzing the structure and we said that there are two methods. One is method of joint and another one is method of section. We also discussed that method of joint is very effective when you want to calculate the force in all the members and method of section is useful when you want to calculate the forces in some members.




So, today we are going to discuss the method of sections. Let me tell you the scheme of method of sections. So, let us say I have a truss. So, let us say the truss is of this type and this truss is held by the supports.

So, let us say you have a ruler support over here and you have a pin support over here and let us say there are various forces, various external forces that are acting on the truss. So, let us say this point is A, this is B, this is C, D, E and F and the external forces that are acting on this truss are, let us say at A, you have external force $P_1$, at B, you have external force $P_2$ and at D, you have some external force $P_3$. And let us say I want to find out what is the force.

So, let us say I want to find out the force in member BD, BE and CE. So, now in this example, you can clearly see that I am not interested in calculating all the forces in all the members, but the forces only in certain members. So, what I want is force in BD, force in BE and force in CE. So, the idea of method of section is following.

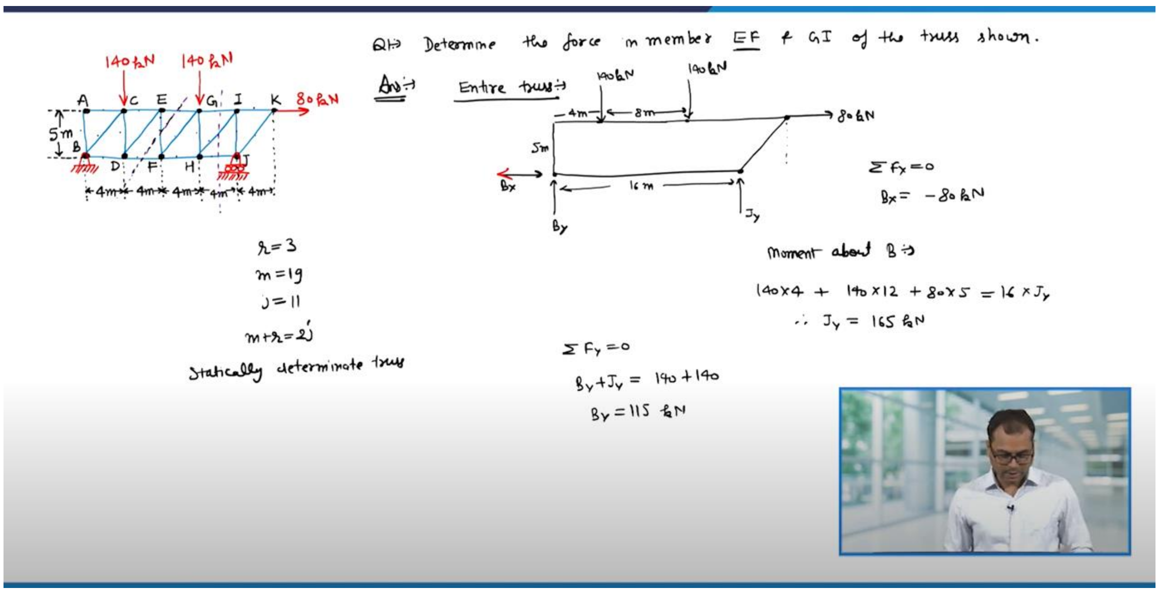
So, first we isolate a portion of this truss and draw a line that divides the truss into two completely separate Now, when we divide the truss into two completely, you know, separate part, we have to note that we have three equations, right? We have two force equation because it is a planar truss. So, therefore, we have $F_x$ and $F_y$ equal to 0 and we have a moment equation. Therefore, when we divide the truss into two different part, we have to, you know, preferably, we have to make sure that this division, it cut this truss into not more than 3 parts. So, note that in general, the line that is cutting the truss should not intersect more than three members and the reason is clear because we have three independent equation whose forces are unknown. So, let us understand it by you know example once again. So, we have this truss

So, here you have roller support and here you have pin support. Now, of course, you have $P_1, P_2$ and $P_3$. Now, since we are interested in finding out the force BD, BE and CE, so therefore, let us cut the truss like this. Okay, and then you analyze the part of the truss which we have got. So, here I have point A, point B, point C and then we have a force along BD, we have a force along CE and then we have a force along BE. So, this is the free body diagram of the part that we have separated. And of course, you have $P_1$ and you have $P_2$. Now, how do I find out all these forces?

So, let us say I am interested in first the force in the CE direction. So, for the force in the CE direction, what you can do is you can take the moment about B. So, if I take the moment about B, then $F_{BD}$ and $F_{BE}$ will not contribute and I have, you know, $P_1$ and $P_2$ are known. So, therefore, I can find out $F_{CE}$. So, for that, what I will do is I will take the moment about B and put it equal to 0 and this will give me the force in the member CE.

Now, if you want to find out what is the force in BD, then you can take the moment about, so let me just extend it. So, this arm BE and CE, they are meeting at point E. So, if I take the moment about E, In that case, force BE and force CE will not contribute because that is passing through E. So, therefore, it will be 0. And again, I have only one unknown parameter $F_{BD}$. So, what I can do is in this case, I can take the moment about E and put it equal to 0.

So, I will get the force in the BD member. Now, for the force, In BE, you can either use the equation $F_x = 0$ or $\sum F_y = 0$ because now I know what is CE, force CE and what is force BD. Now, let us understand this by few examples.




So, let us look at this problem statement. So, again this is a planar truss and the problem statement is following. Determine the force in member EF and GI of the truss shown. So, we have to find out the force in member EF and GI. So, let us first divide this truss into two parts making sure that we are not going to cut more than three members of this truss.

So, for EF, I can cut the truss like this. And for the force in GI, I can cut the truss like this. Okay. Now, before I analyze these, you know, individual, you know, cutted truss, let me first look at the entire truss and find out the support reactions that are there. So, first, let us look at the entire truss.

The free body diagram of the entire truss. So, we have this truss and then over here $80\ kN$ force is acting. At this point, we have $140\ kN$ of force. Over here, I have $140\ kN$ of force. This is $5m$, this is $4m$ and this one is $4 + 4 = 8m$.

At point B, we have a pin support. So, therefore, it will support two kind of, you know, reaction forces. So, since this point is B, let me write down the x force as $B_x$ and the y force as $B_y$. At point J, we have a roller support. So, therefore, it will only support the force in the y direction.

So, let us call it $J_y$. So, with this, we are ready with the free body diagram. Also, note that here total number of reaction forces are equal to 3 and if you count there are 19 members. So, let us count it 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19.

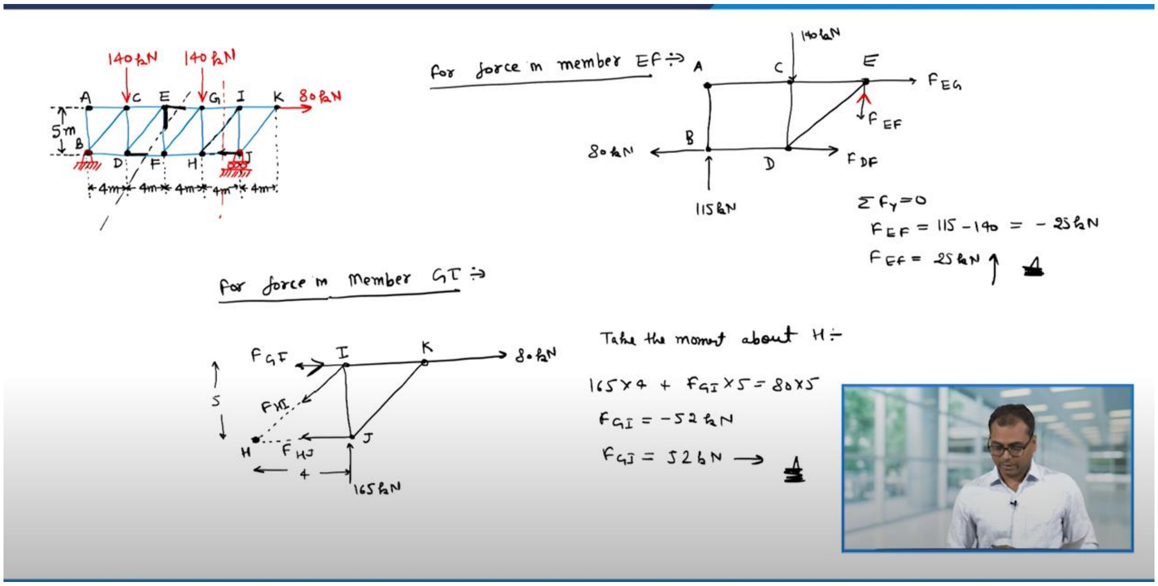
So, there are 19 members. So, $M = 19$ and you can count that there are 11 joints. So, therefore, you have$m + r = 2j$. So, that means this truss is statistically determinate or statically determinate truss. So, that means I can find out all the forces that are asked.

Now, for the entire truss to find out these reaction forces, first let us do $\sum F_x = 0$. So, if I do $\sum F_x = 0$, I get $B_x = -80\ kN$. So, that means I have to change the direction of this force. Now, let us take the moment about B to find out what is J because in that case $B_x$ and $B_y$ will go away. So, let us take the moment about B.

So, I have $140\ kN$ of force, its distance or the perpendicular distance from B is $4m + 140 \times 12$. So, $8 + 4$ is $12 + 80 \times 5$. So, this is the vertical distance of this force from point B which is $5m$ equal to 16 because this distance is $16J_y$. So, this gives you $J_y = 165\ kN$.

Now, to find out $B_y$, I can balance the force in the y direction. So, I get $B_y + J_y = 140 + 140$. This gives me $B_y = 115\ kN$ because $J_y$ is known, it is 165. So, therefore, I have calculated the reactions in the, you know, the external reaction in the truss.

Now, to find out EF, as I said, let us divide the truss like this and look at the free body diagram of this part of the truss.




So, let me do it once again. We have divided like this and let us look at the free body diagram. So, for the force in member EF, so we have this truss so, here it is point A, point B, C, D and E. Now, we have already calculated the reaction forces the $B_x$ and $B_y$.

So, this was 80 $kN$ and $B_y$ was 115 $kN$. At point C, you have an external force of 140 $kN$ and from E, we will have a force along EG. We will have a force in this direction. So, this will be $F_{EF}$ and from D, we will have a force along this direction.

So, let us call it $F_{DF}$. So, this is the free body diagram of the isolated part of this truss. Now, to find out what is EF, we can balance the force in the y direction. So, let us do $\sum F_y = 0$. So, we get $F_{EF} = 115 - 140\ kN$ and this gives you $-2500\ kN$.

So, therefore, $F_{EF} = 25\ kN$ and we have to change its direction. So, therefore, this force should be in that direction. Now, to find out force GI, let us look at the second isolated part.

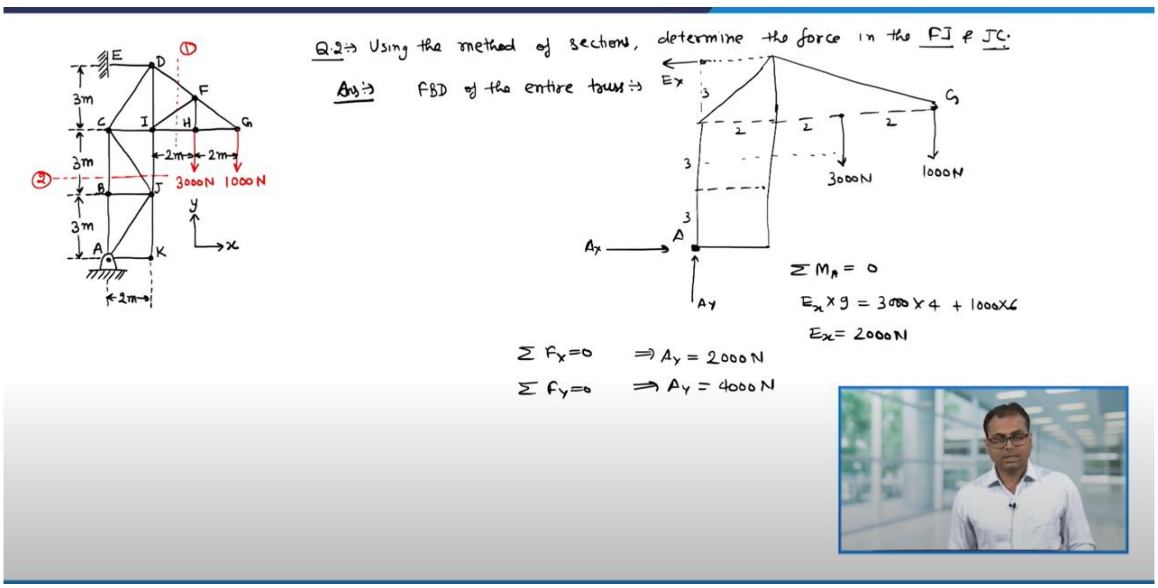
So, which was like this. So, for that we have cut the truss like this and let us look at the free body diagram. So, this is for force in member GI. So, in this case we have the free body diagram like this.

And here we have external force of 80 $kN$. This point is $K$. This is I. This is J. At J, we have the support reaction in the y direction, which we have calculated. It was 165 $kN$. And from J, we have the force in this direction.

Let us call it force $F_{HJ}$. We have a force in this direction. So, that will be $F_{GI}$ and we have a force in this direction that will be $F_{HI}$. Let us extend this and we know from the free body diagram that they meet at point H. So, because this and this force, they meet at point

H. So, therefore, this point is H. Now, to find out GI, what I can do is I can take the moment about H because in that case, $F_{HI}$ and $F_{HJ}$ will not contribute and we have only one unknown GI which we can easily calculate. So, let us take the moment about H. So, we have $165\ kN$ force, its perpendicular distance is 4 because this is $4m$ plus we have $F_{GI}$ into its perpendicular distance is $5m = 80 \times 5$. So, this gives me $F_{GI} = -52\ kN$. That means that $F_{GI}$ is $52\ kN$ and we have to change its direction. So, GI, so we have to change this direction. So, it will be in this direction.

So, with this, we are able to calculate the forces in EF and GI. So, we do not need to calculate all the forces in all the members, but this technique is very effective when you want to calculate the forces in some of the member. Note that the truss is very long. So, therefore, if you go to calculate the forces joint by joint, then it will take a lot of time.

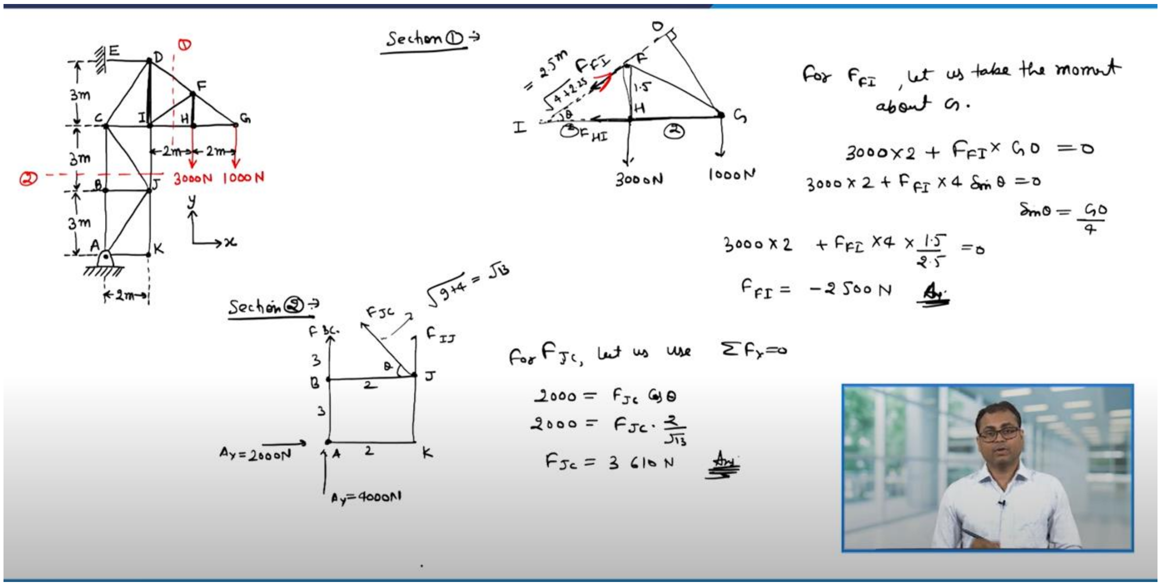



Now, let us look at another example. So, again, this is a big truss and the question statement is using the method of sections, determine the force in the FI and JC member. To find out these forces, what we can do is we can again, you know, divide this truss. So, for that, let me divide the truss like this. So, when I divide, I make sure that I am cutting no more than three members.

So, let us say this is first section. This is useful when I am going to find out the force in the FI member. For JC, I can divide the truss like this and before I look at these part or these sections, let us first look at the free body diagram of the entire truss to find out the support reactions. So, let us look at the free body diagram of the entire truss. So, we have this truss and this point is, of course, A. Here, I have a pin support.

So, therefore, two forces are going to act, $A_y$ and $A_x$. At point E, I have another support. So, in this case, the force is going to act in the x direction. Let us call it $E_x$.

And at point G, you have external loading of $1000\ N$ and at H, we have external loading of $3000\ N$. Let me put the dimensions. So, this is $3m$, this is also $3m$ and this is also $3m$, this one is $2m$, this is $2m$. Now, to calculate what is $E_x$, I can take the moment about A because in that case, this $E_x$ and $A_y$ will go away.



So, to find out $E_x$, let us take the moment about A. So $\sum M_A = 0$. So, we have $E_x$ into perpendicular distance from A. This is $3 + 3 + 3$, which is $9m$ equal to $3000$ into because this is also $2m$. So, the perpendicular distance of this from A will be $2 + 2 = 4 + 1000\ N$ force multiplied by 6 and this gives you $E_x = 2000\ N$. Now, to find out $A_x$, let us balance the force in the x direction.

So, we have $\sum F_x = 0$. This gives you $A_x = E_x$ which is $2000\ N$ and the force balance along the y directions gives you $A_y = 3000 + 1000$ which is equal to $4000\ N$.

To find out the force FI and JC, let us now look at the sections. So, let me again draw the sections that we have considered. So, this was first section and this was second section. So, let us first consider section 1. And let us look at the free body diagram of this section.

So, we have point G, point F and point H. There is an external loading at G which is $1000\ N$. At H, you have external loading of $3000\ N$.

Now, the force at point G will also be along the HI direction. So, this is a force at H, it will be in HI direction. So, let me say that this point is I and this force is F HI and there will be a force in the FI direction. So, there will be force F in the I direction.

Now, the dimensions are given. Since this height is $3m$, so therefore, this height from the symmetry will be $1.5\ m$ and this is $2m$. This is also $2m$. Therefore, I can find out how much is this length. It will be $\sqrt{4 + 2.25}$, which is nothing but $2.25m$.

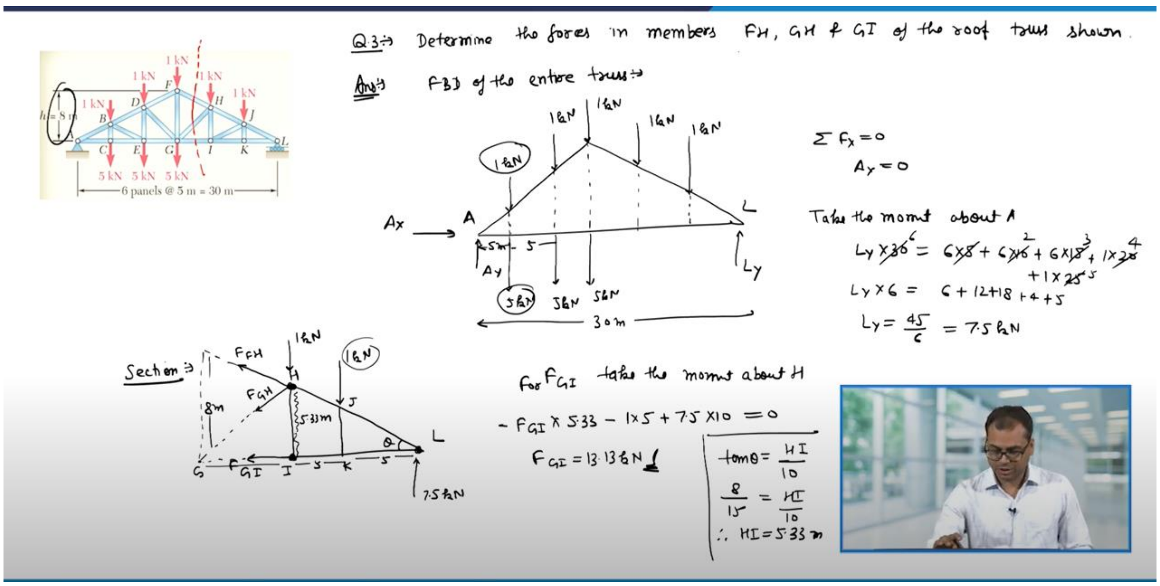
Let us call this angle as θ. So, to calculate the force along the FI direction, let us take the moment about G. So, for FI, let us take the moment about G. In that case, the contribution of $F_{HI}$ will not be there because it is passing through G. So, we have $3000\ N$ force multiplied by the perpendicular distance which is $2 + F_{FI}$ into the perpendicular distance. So, I have to draw a perpendicular on this FI from G. So, let us call it some point O. So, it will be this force multiplied by GO and this would be equal to 0. So, I have to find out what is GO. So, let us look here. You have from the geometry; you have $sin\theta = \frac{GO}{4}$ because this is 2 and this is also 2. So, therefore, it will be, you know, $\frac{GO}{4} = sin\theta$. So, therefore, GO will be $4\ sin\theta$.

So, I have $3000 \times 2 + F_{FI} \times 4sin\theta = 0$. Now, $sin\theta$ I can find out from this triangle. So, $sin\theta$ is $1.5/2.5$. So, I have $3000 \times 2 + F_{FI} \times 4 \times sin\theta$ is $\frac{1.5}{2.5} = 0$ and this gives me $F_{FI} = -2500N$. So, that means I have to change the direction of $F_{FI}$. So, this force will be in that direction. Now, to find out force JC, let us look at the second section. So, let me first draw the free body diagram of this.

So, we have point A, B, this point is K, this point is J, and at A, we have two support reactions in the $A_x$ direction, which we have calculated. This is $2000\ N$. We have a force in the y-direction, which is $4000\ N$. And at J, we will have the force in the IG direction.

And we also have a force in in the JC direction. At point B, we will have a force in the BC direction. So, this is the free body diagram of this section and of course, this is $2m$ and this length is $3m$. Now, for force JC, let us use the force balance along the x-direction.

So, $\sum F_x = 0$. So, you can see here that $2000 = F_{JC}$ and its component along the x-direction which is let us say this angle is $\theta$. So, this becomes $cos\theta$. Now, $cos\theta$ I can find out from the geometry. So, this is $2000 = F_{JC}$ and since this is $2m$, this is $3m$. So, therefore, this length will be $\sqrt{9+4} = \sqrt{13}$. Therefore, $cos\theta$ will be $2/\sqrt{13}$. So, therefore, $F_{JC}$ will be $3610\ N$.




Now, let us look at one more example on the method of section and the problem statement is following.

Determine the forces in members FH, GH and GI of the roof truss which is shown here. So, since we have asked to find out the forces FH, GH and GI, what we can do is we can divide this truss like this. So, before we isolate this part of the truss, let us look at the free body diagram of the entire truss to find out the support reactions. So, we have the free body diagram of the entire truss and this point is A, this point is L. So, look at A. At A, you have a pin support.

Therefore, two forces $A_y$ and $A_x$ will act. At L, you have a roller support. So, therefore, it will be only one force in the y direction. And then we have external loading.

So, at B, we have 1 $kN$ of force and then we have 5 $kN$ Here you have 1 $kN$ and we have 5 $kN$ and then we have 1 $kN$ and 1 $kN$ of force. Now, it is given that this entire truss is

$30m$ long. So, AL is $30m$ and the length between these bars are $5m$. So, to find out what is $A_x$, let us first balance the force in the x direction.

So, automatically you get $\sum F_x = 0$ gives you $A_x = 0$. Now, to find out what is the force in the y direction $L_y$, let us take the moment about A. So, let us take the moment about A. So, $A_y$ will not contribute in this case. So, I have $L_y$ into 30 equal to.

So, I have $5\ kN$ force and $1\ kN$ force and they are acting in the same direction. So, therefore, this will be $5 + 1 = 6\ kN$ of force and its perpendicular distance from A is $5\ m$. Similarly, I have $6\ kN$ of force $5 + 1$ acting at a distance of $5 + 5$ which is $10m$. Similarly, $6 \times 15$ equal plus I have $1\ kN$ of force acting at a distance of 20 and then another $1\ kN$ of force acting at a distance of $25m$.

So, let us divide the whole thing by 5. So, I get this. So, I got $L_y 6 = 6 + 12 + 18 + 4 + 5$ and this gives you $L_y = 45/6$ which is nothing but $7.5\ kN$. Now, to find out the asked forces, let us look at the section now.

So, let me isolate the section. So, we have point L, this is K, this is Z, this is H and this one is I. At L, we have a force in the y direction, $L_y$ which we have calculated. It is $7.5\ kN$. This is $5m$.

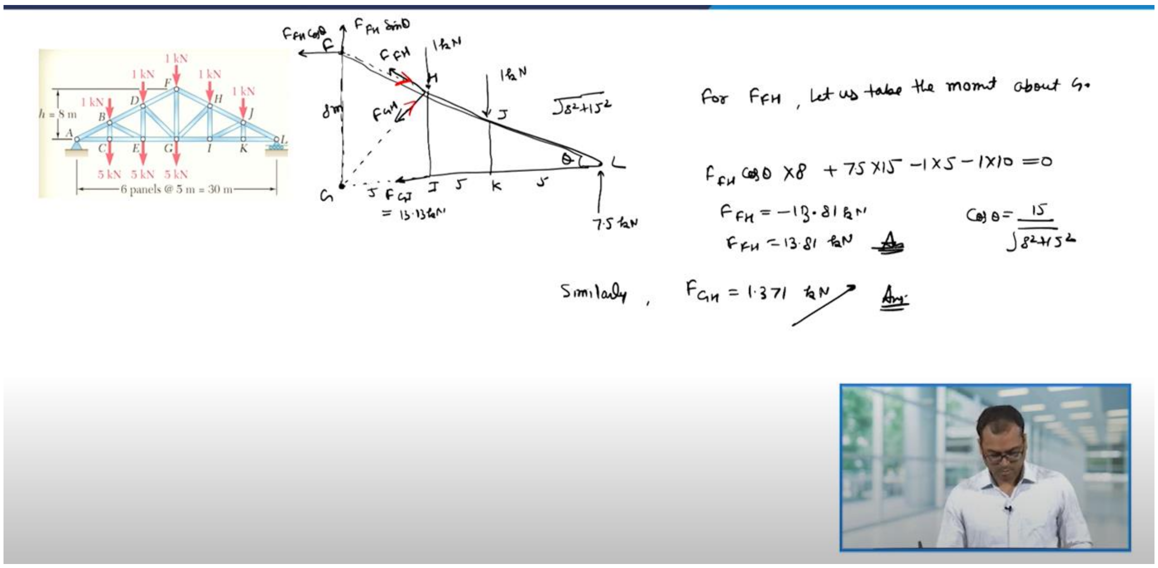
This is also $5m$. And at J, you have an external loading of $1\ kN$. Also at H, there is external loading of $1\ kN$. Now, at i, we will have the force in the G direction.

So, we have a force $F_{GI}$. At H, you will have a force in the GH direction. So, $F_{GH}$ and also there will be a force in the FH direction. Now, this GH force and GI force, they intersect at point G and this height is also given in the question, this is $8m$.

So, you can see here, this is $8m$. This is the free body diagram of the section. Now, for GI force, let us take the moment about H. So, if you take the moment about H, in that case GH, force GH and force FH will not contribute. So, we have minus $F_{GI}$ into the perpendicular distance, which is this distance.

So, HI, let us find out how much is HI. So, this angle is let us say $\theta$. So, I know that tan theta will be $HI/10$. Now, $tan\theta$ from the bigger triangle, it is $8/15$ because this is $8m$ and this is also $5m + 5m + 5m$ will give you$15m = HI/10$. So, therefore, HI from here will be $5.33m$.

So, this is $5.33m$. So, now the moment about H will be force $F_{GI}$ into the perpendicular distance $5.33m - 1\ kN$ of force, its perpendicular distance from H is $5m$. So, we are talking about this force plus $7.5\ kN$ force, its perpendicular distance is $10m$ equal to 0. So, this gives you $F_{GI} = 13.13\ kN$.




Now, to find out the force FH and GH member, let us again look at the free body diagram of the part that we have isolated. So, we have L, K, J, this is I, this one is H, and we have the force in the GH direction. We have the force in the GI direction, which we have calculated. So, this is $13.13\ kN$.

And at L, we have the support reaction of $7.5\ kN$. At J, we have an external loading of $1\ kN$ and also at H, you have external loading of $1\ kN$. Now, in this direction, we have a force $F_{FH}$ and it is given that this is $8m$. So, this point is F. Now, to find out what is FH, let us take the moment about G. So, in this case, $F_{GI}$ and $F_{GH}$ will not contribute and this force $F_{FH}$, I can divide into two parts.

So, let us say this is $F_{FH} sin\theta$ and this one is $F_{FH} cos\theta$. Now, about G, $F_{FH} sin\theta$ will also not contribute because this force is passing through G. So, let us take the moment and let us say this angle is $\theta$. So, I have $F_{FH} cos\theta \times 8$ and plus $7.5 \times 15 - 1 \times 5 - 1 \times 10 = 0$. And here $cos\theta$ is nothing but $15 / \sqrt{8^2 + 15^2}$.

Because this is $5m$, this is $5m$, this is $5m$. So, therefore, this will be $\sqrt{8^2 + 15^2}$. So, from here, I can find out what is $F_{FH}$. So, that comes out to be $-13.81\ kN$. Or $F_{FH} = 13.81\ kN$ and we have to change the direction of the force.

So, its direction should be like this. Similarly, you can also find out the force in the GH. So, this I am not doing, but the same exercise can be done to find out what is $F_{GH}$ and it comes out to be $1.3714\ kN$ and its direction will be the opposite. So, with this, let me stop here.

Thank you.