MECHANICS

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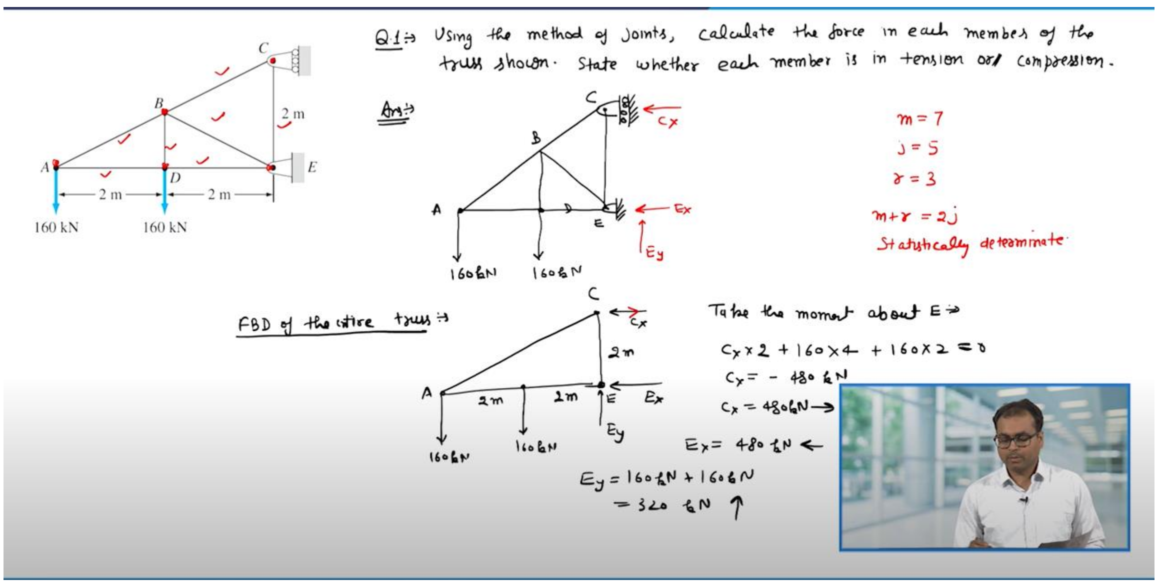
Lecture: 16

Analysis of trusses: Method of joints

Hello, everyone; welcome to the lecture again. In the last lecture, we look at you know how do we analyze the structure, particularly we look at the planar truss and we look at method of joint.



Today, we are going to solve two more examples on the method of joint. So, let us look at the first problem statement and the problem statement is following. Using the method of joints calculate the force in each member of the truss shown and you have to state whether each member is in tension or compression. So, to analyze the truss, first we have to make the free body diagram of the entire structure. You can see here that the, so let me first make the diagram.

So, you have at point A, I have 160 kN force that is acting and then at point D, there is Again, 160 kN of force that is acting and then at C, you have a roller support and at point

E, you have a pin support. So, let us look at the reaction forces first. So, since at C, you have roller support. So, therefore, it will not support the vertical forces.

So, you will have a force in the x-direction. Let us call it C_x. And at E, you will have a pin support. So, therefore, it will support both the x-component. So, E_x and E_y.

Now, let us see first whether this truss is statistically determinate or this is indeterminate. So, you can see that m which is the member in the truss. So, let us count the member 1, 2, 3, 4, 5, 6 and 7. So, there are 7 members. Now, let us count how many joints are there.

So, you have 1, 2, 3, 4, 5. So, there are 5 joints. And now, let us see the reaction forces. So, we have 1, 2 and 3. So, there are 3 reaction forces.

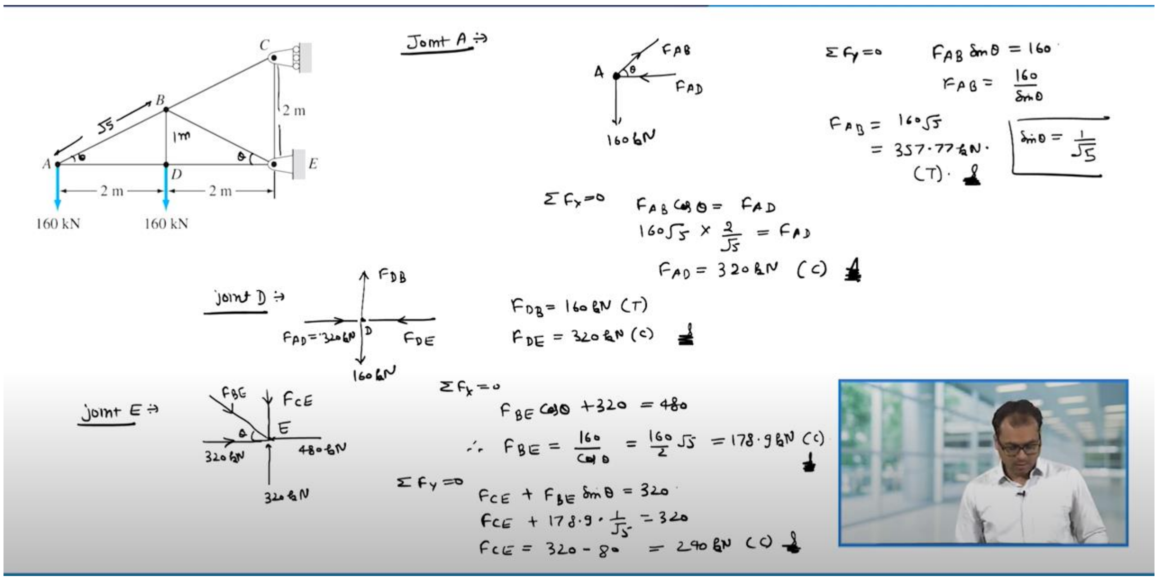
So, you can see that $m + r = 2j$. Therefore, this stress is statistically determinate. That means we can calculate all the forces or all the internal forces that are acting on the member of this truss. Now, to analyze the truss, let us first look at the free body diagram of the entire truss. So, free body diagram of the entire truss.

So, we have this truss, the entire truss, internal members we will not consider because we are looking at, you know, the whole truss. So, you have $160\ kN$ force acting at this point and you have $160\ kN$ force which is acting over here and here you have E_y, E_x and then we have a force C_x. So, this is the free-body diagram of the entire truss. Now, to calculate various forces, let us first take the moment about E. So, this point is E, this point is A and this point is C. So, let me take the moment about E.

So, E_x and E_y will not contribute in the moment because they are passing through E. So, we have $C_x \times 2$ plus because this is $2m$ and 160×4 the first force plus 160×2 equal to 0. So, this is the moment equation and from here I can see that C_x is $-480\ kN$. So, this means that I have to change the direction of C_x. So, the C_x should act in that direction.

So, C_x is $480\ kN$ and it should act in this direction. Now, summation F_x gives you $E_x = C_x$. So, this is $480\ kN$ and of course, that should act in the opposite direction of the C_x. Now, E_y also I can calculate from $\sum F_y = 0$ equation. And E_y will be $160\ kN + 160\ kN$ which is nothing but $320\ kN$ and just to tell you it's direction is upward.

Now, all these parameters of you know C_x, E_x, E_y we got by considering the entire truss. Now, let us calculate you know other forces, other internal forces by using you know joint by joint.



So, let us first look at joint A, the free-body diagram of joint A. So, at A, You have a force of 160 kN which is acting downward.

There will be a force along AB and since I have to balance the horizontal force, let me put the direction like this. So, this force will be F_{AD}. Now, let us say this angle is θ and this θ, of course, I can find out from the geometry. Now, let me use the force balance equation along the y-direction. So, $\sum F_y = 0$.

So, this gives you $F_{AB} sin\theta = 160$. So, therefore, F_{AB} is $160/sin\theta$, and this θ is that θ. And $sin\theta$ you can find out because this is $2m$. So, therefore, this will be $1m$ and therefore, this length will be $\sqrt{5}$. So, therefore, $sin\theta$ will be $1/\sqrt{5}$.

So, let us put here. So, you get $F_{AB} = 160\sqrt{5}$. At this, you can calculate this is $357.77\ kN$. And since its direction is away from point A or joint A, so therefore, this will be a tensile force.

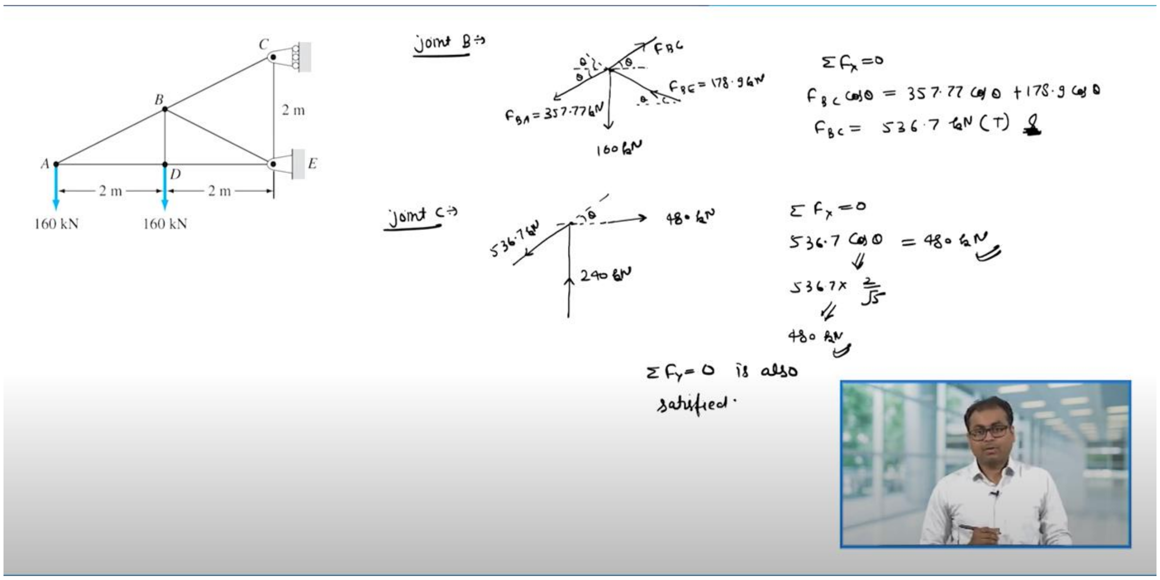
So, for tensile, let me just write down T. Okay. So, this T stands for tensile. Now, let us use another equation, $\sum F_x = 0$. So, you have $F_{AB} cos\theta = F_{AD}$, and F_{AB} is $160\sqrt{5}\ cos\theta$ is $\frac{2}{\sqrt{5}} = F_{AD}$ and this gives you $F_{AD} = 320kN$.

Now, this force is acting towards A. So, therefore, this will be compressive force. So, therefore, let me write down C. Now, let us use joint D. So, the free body diagram of joint D is like this, you have a $160 kN$ force that is acting downward and then you have a force which is acting upward, F_{DB}. Then you have the force F_{DE} and you have the force of AD which we have just calculated. So, F_{AD} this is nothing but $320\ kN$ and this is point D. So, you can again use $\sum F_x = 0, \sum F_y = 0$. So, immediately we get $F_{DB} = 160\ kN$ and since it is away from the joint, so therefore, this is tensile, and F_{DE} is $320\ kN$ and since it is acting towards point D, so therefore, this will be the compressive force. So, now let us look at joint E. At joint E, we have the E_x and E_y forces so, this E_x and E_y forces which we have already calculated.

So, this is $480\ kN$ and then we have $320\ kN$. Now, there will be a force from D to E which we have calculated. So, this was $320\ kN$ DE force, and we have a force which from C to E. So, let us call it F_{CE} and we have a force from BE. So, let me put it like this F_{BE}.

Let us say this angle is θ. So, what we are saying is this angle is θ. So, therefore, I can find out what is the value of $cos\theta$ and $sin\theta$, etc. Now, let us balance the equation along the x-direction. So, $\sum F_x = 0$.

So, we have $F_{BE}cos\theta + 320 = 480$. So, this gives you $F_{BE} = 160/cos\theta$. This is $160/2\sqrt{5}$ and this comes out to be 178.9 kN and from the direction, it is clear that it should be a compressive force. Now, let us use $\sum F_y = 0$. So, you get $F_{CE} + F_{BE}sin\theta = 320$. So, we have $F_{CE} + 178.9sin\theta$ is $1/\sqrt{5}$ equal to So, therefore, F_{CE} is equal to, so you can simplify it, this will be $(320 - 178.9)/\sqrt{5}$, this comes out to be 80 and this will give you 240 kN and again it will be the compressive force. Now, to analyze further, let



us look at joint B. So, we are going, you know, joint by joint. So, first we analyze point A, then point D, then point E.

Now, let us look at point B. So, joint B, let us make the free body diagram of this joint. So, at B, you have 160 kN force. which is acting downwards. Then there is a force of F_{BA}, which we have calculated. So, F_{BA} is 357.77 kN.

Then we have the force F_{BC} and you have a force F_{BE}, which again we have calculated. So,F_{BE} it was 178.9 kN. And from here, we can find out what is the value of BC. So, let us look at the geometry.

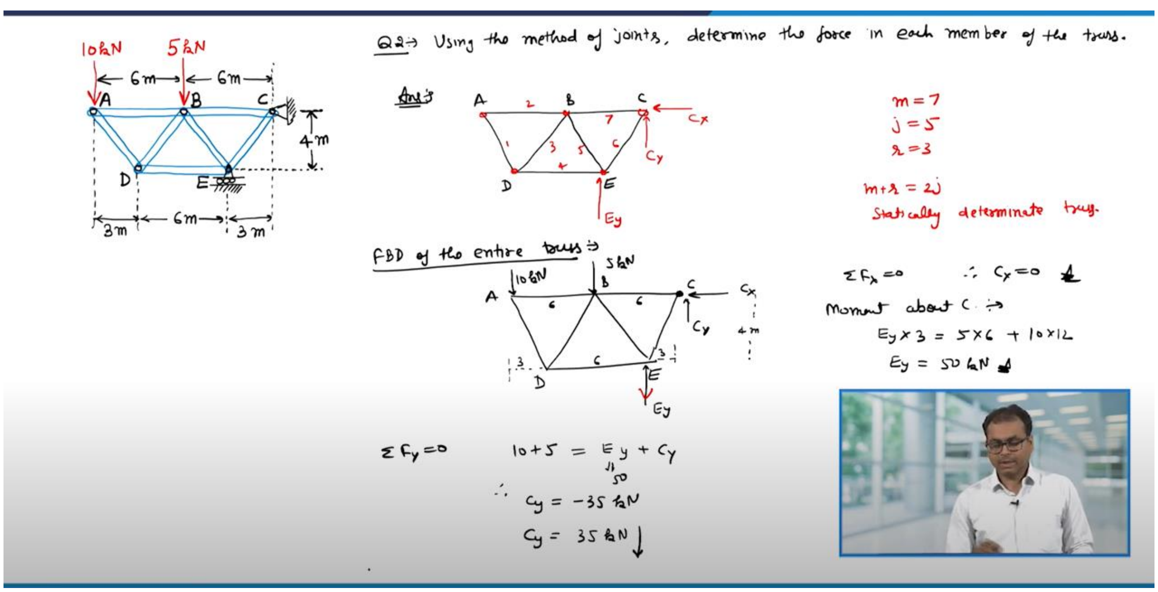
So, you know, this angle we have find out that this angle is theta. So, if this angle is theta therefore, this angle will also be theta and again from the symmetry this will be theta and that will be theta. So, let us balance the force along the x-direction $\sum F_x = 0$. So, we have $F_{BC}cos\theta = 357.77cos\theta + 178.9cos\theta$.

Now, $cos\theta$ you can you know already you can find out. So, F_{BC} will be $536.7\ kN$ and it will be the tensile force. So, now let us look at the last joint which is joint C. So, at C, we have the force which will be along CE. So, this force we have already calculated.

This is $240\ kN$ and just now we calculate the force BC. So, this will be along this direction and its value will be $536.7\ kN$ and from the horizontal let us say it is making an angle theta in this direction we have a $480\ kN$ of the force. So, this will be C_x force which we already know that $480\ kN$ so, now let us use the force balance along the x-direction.

So, here you know we do not need to find anything because all the parameters are given. So, let us see whether you know $\sum F_x$ and $\sum F_y$ equation is satisfied for joint C or not. So, let us use $\sum F_x = 0$. So, we have $536.7 cos\theta = 480\ kN$ and this $cos\theta$ we have already find out this is $2/\sqrt{5}$ and you calculate that it comes out to be $480\ kN$.

So, therefore, this and this is satisfied. Similarly, you can also see that $\sum F_y = 0$ is also so, with this, we have calculated all the forces that are acting on the joint. Now, let us look at the second example.



So, this is another truss and here the problem statement is following using the method of joints. Determine the force in each member of the truss. So, first of all, since you have asked to find out the force in each member, so therefore, anyway we have to use the method of joint. So, let us see here. So, here you have this truss. this point is A, B, C and D and E. Now, at point E, you have a roller support.

So, therefore, it will support the force only in the y-direction, only E_y. At point C, you have a pin support. So, therefore, there will be a force along x-direction. So, let us call it C_x, and then there will be a force in the y-direction.

So, let me call it C_y. Now, you can see that the total number of members are 1, 2, 3, 4, 5, 6, and 7. So, there are 7 members and there are 1, 2, 3, 4, 5. So, there are 5 joints and total reaction forces are 3.

So, again we see that $m + r = 2j$. That means, this stress is again statistically determinate, statically determinate. determinate truss. Now, to analyze as we did earlier, let us consider the free body diagram of the entire truss which is given here just that I have to put the forces. So, let me put the forces. So, free body diagram of the entire truss.

Again, we have this truss This point is A, B, C, D and E. At E, you have E_y force. At C, we have C_x and C_y force. Now, at point A, I have a force of $10\ kN$. Here, I have $5\ kN$ force, and this is $6m$; this is also $6m$, and this distance is $3m$; this is $6m$, and this is also $3m$, and this one is $4m$.

To analyze this truss, let us take the force balance equation. So, from here, you can see that C_x is not balanced. So, therefore, $\sum F_x$ gives you $C_x = 0$. So, $\sum F_x = 0$. So, there is nothing to you know compensate for C_x.

So, therefore, C_x will be 0. Now, let us take the moment about C to find out what is the value of A_y. So, if I take the moment about C, then C_x and C_y will go away and I will have E_y into the perpendicular distance is $3m$ equal to5 into perpendicular distance is $6 + 10\ kN$ force multiplied by the perpendicular distance, which is $12m$ and this gives you $E_y = 50\ kN$.

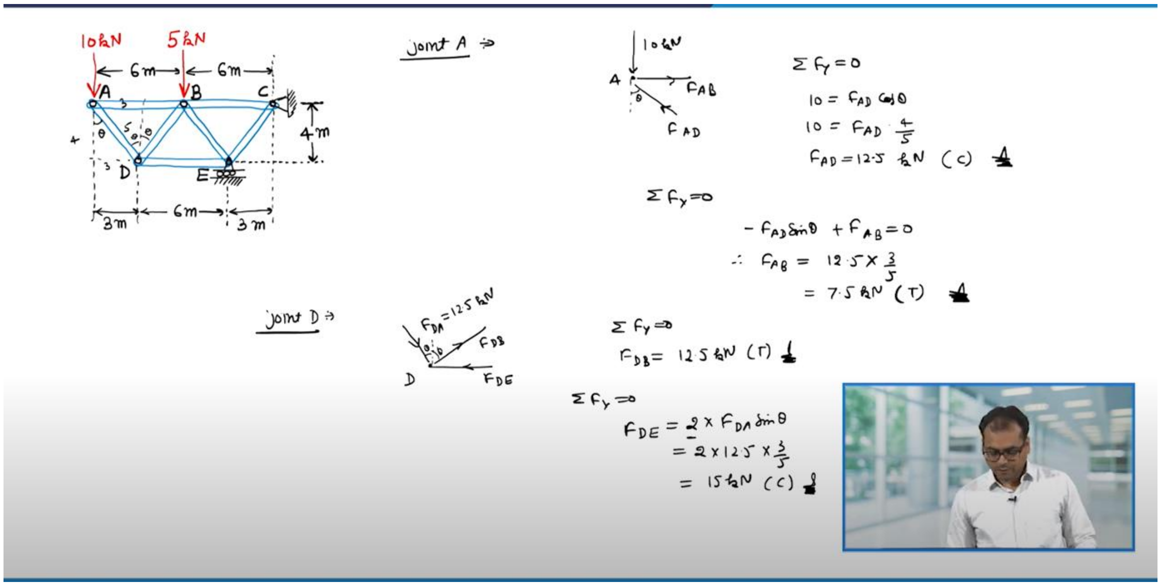
Now, let us balance the force equation in the y-direction also. $\sum F_y = 0$. So, this gives you $10\ kN + 5\ kN = E_y + C_y$ and E_y we have already calculated it is 50. So, therefore, E_y is 50. Therefore, C_y will be $-35\ kN$ or C_y will be $35\ kN$, but we have to change the direction.

So, let us change this direction downwards. So, now to further calculate, let us look at each point. So, let us look at joint by joint. So, let us look at joint A. The free body diagram of joint A. So, we have point A. On this point, we have an external force of 10 kN, then there will be force along AB and there will be a force along AD.

So, since I have to balance that 10 kN force downward, so let me put the force direction like that and this will be F_{AD}. Let us say this is making an angle of θ. So, what I am saying is this angle is θ. So, you can find out what is the value of $cos\theta$ and $sin\theta$ because this is 4, this is 3. So, therefore, this will be 5 and $cos\theta$ will be $4/5$.

So, let us balance the force along the y-direction $\sum F_y = 0$. So, we get $10 = F_{AD} cos\theta$, $10 = F_{AD}$ and $cos\theta$ is $4/5$. This gives you $F_{AD} = 12.5\ kN$ force. And you can see that this force is compressive because it is towards the pin. Now, let us balance the force along the x-direction.

So, $\sum F_x = 0$. So, we get $-F_{AD} sin\theta + F_{AB} = 0$. So, this gives you $F_{AB} = 12.5 sin\theta$. $Sin\theta$ Is $3/5$, and this comes out to be 7.5 and this is the tensile force because it is acting away from point A.



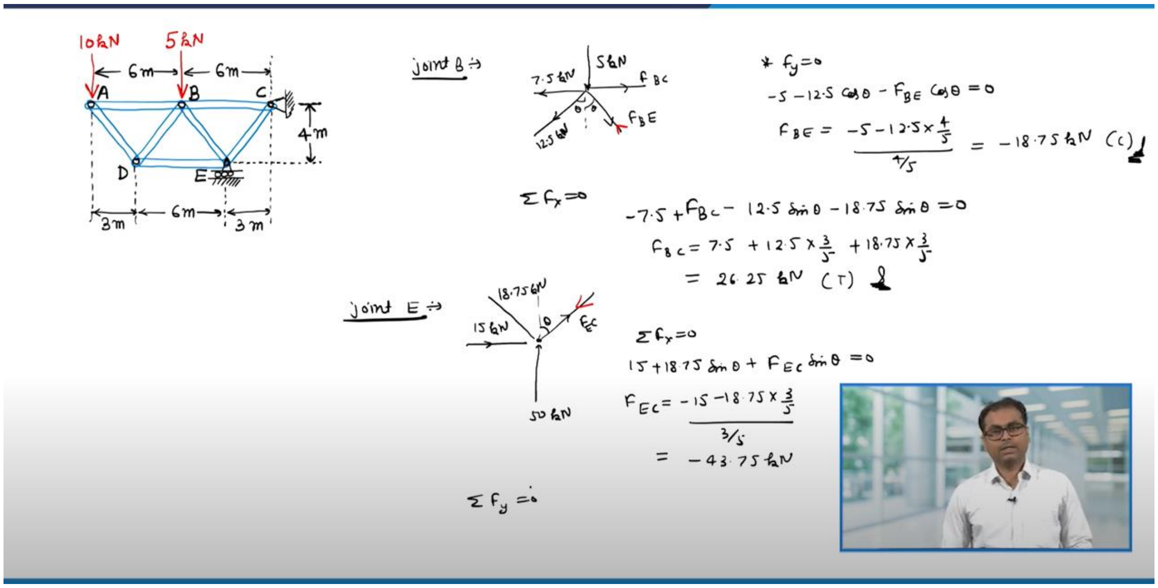
Now, let us look at joint D. So, let us make the free body diagram of joint D. So, we have point D, and we have a force that is acting

along AD. So, we have F_{DA} and its value we have already calculated, it is $12.5\ kN$. So, let us put the other force F_{DB} and F_{DA}. And let us say, these angles are θ.

So, again you can find out what is θ. So, what we are saying is this angle is θ and this angle is also θ and $cos\theta$ and $sin\theta$ I can find out because this is 3 and the other length is 4. So, let us balance the force along the y-direction. So, $\sum F_y = 0$. And from the symmetry, you can see that F_{DB} will also be12.5 kN, and it is tensile.

Now, let us balance the force along the x-direction. So, you have $F_{DE} = 2F_{DA}sin\theta$. So, this is $2 \times 12.5 \times \frac{3}{5}$, this comes out to be 15 kN and this is compressive, okay. I have taken two here because F_{DA} and F_{DB} is already, you know, they are equal.

So, either you write it down, you know, again or just multiply it by 2. So, with this, we have solved joint D.



So, now, let us look at the next joint, joint B. So, at B, we have an external force of 5 kN, and then you have F_{BC} and F_{BA}, which we have already calculated. This is 7.5 kN, and then there are two forces. F_{BD}, which we have calculated, this is 12.5 kN, and then F_{BE}. Now, to calculate it, let us again say that these angles are θ and θ we know. So, let us use the force balance along the y-direction.

$\sum F_y = 0$. So, this gives you $-5 - 12.5cos\theta - F_{BE}cos\theta = 0$. This gives you $F_{BE} = -5 - 12.5cos\theta$ is $4/5$ and divide by $cos\theta$ which is again $4/5$. So, this comes out to be-18.75 kN.

So, that means we have to change the direction of BE. So, this force should be in that direction and therefore, this is a compressive force. Now, let us balance the force along the x-direction, $\sum F_x = 0$. So, we have$-7.5 + F_{BC} - 12.5sin\theta - 18.75sin\theta = 0$.

Now, we know you know all the $sin\theta$. So, from here F_{BC} comes out to be $7.5 + 12.5 \times \frac{3}{5} + 18.75 \times \frac{3}{5}$ and this is 26.25 kN and you can see that this is a tensile force. Now, let us look at the last joint which is joint E. So, at E, you can see there is a force of 50 kN. This is the reaction from E towards y-direction.

And we have a force along DE, which we have already calculated. So, this is 15 kN. We have also calculated the force along BE. It is 18.75 kN and then the force along C. So, this is F_{EC}. Now, let us say this angle is θ.

So, let us balance the force along the x-direction. So, $\sum F_x = 0$. So, you have $15 + 18.75sin\theta + F_{EC}sin\theta = 0$. So, this gives you $F_{EC} = -15 - 18.75 \times \frac{3}{5}$ divided by $sin\theta$ which is $3/5$.

This gives you $-43.75\ kN$. This means that we have to change the direction of EC. So, with this, we have calculated all the forces, and you can check by yourself that $\sum F_y = 0$ equation is already satisfied, and this confirms that all the forces that we have calculated is correct. So, with this, let me stop here.

In the next class, we will look at the method of sections. Thank you.