MECHANICS
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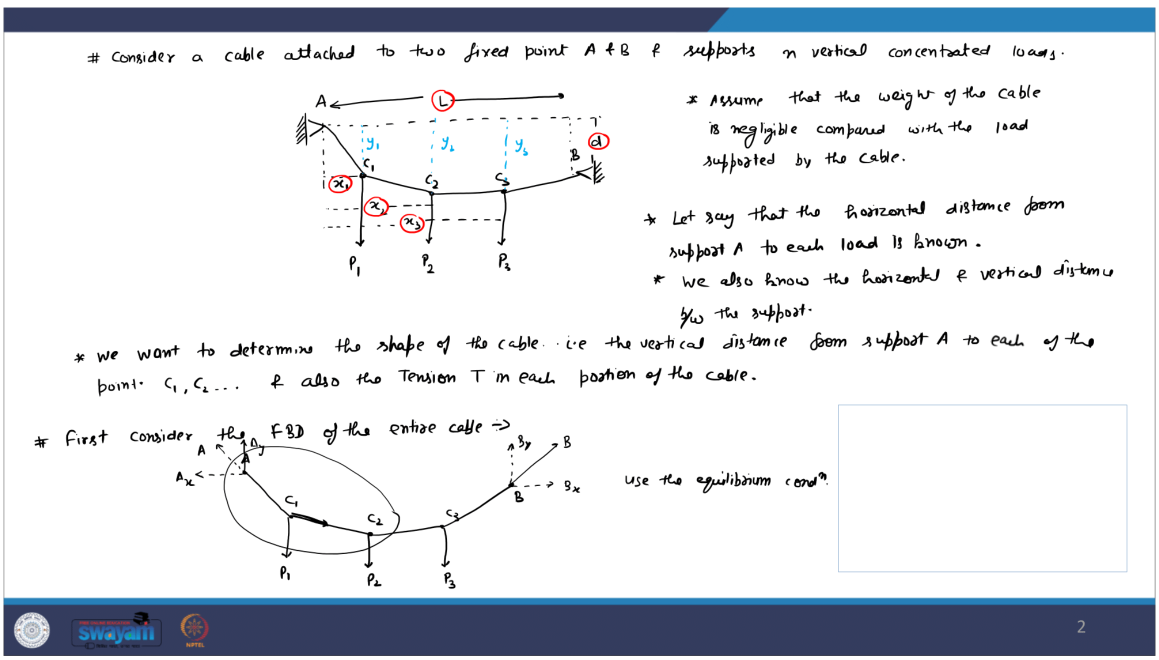
Lecture 14
Flexible Cable: Concentrated load

Hello everyone. Welcome to the lecture again. In the last couple of lecture, we looked at flexible cable which are hanging under its own weight or which has a uniform horizontal weight. Today, we are going to look at flexible cable which is loaded with concentrated load. So, the situation is following.



Let us consider a cable that is attached to let's say two fixed point, let us call them A and B and this cable supports let's say n vertical concentrated loads. And as we assumed previously, let's say this cable is flexible and also its weight is negligible compared to the weight of the load. So, we have let's say these two fixed points, let us call them A and let's say B and this cable is hanging between A and B and there are n number of concentrated loads.

So, these are the points from where the loads are hanging. So, let's say the magnitude is P_1, P_2 and P_3 and of course, there can be n. Now, as I said, you assume that the weight of the cable is negligible compared with the load that is supported by the cable. Now, in this kind of situation, let us assume that there are certain things we know. So, for example, let's say that the horizontal distance from, let's say, support A to each load is known. Okay. So, what do I mean by that? So, that means that from here, I know at what horizontal distance P_1, P_2 and P_3 are suspended. So, let's say this is x_1. So, x_1 is given to you. Similarly, this x_2 is also given to you and x_3 is also known. And let us also say that we also know the horizontal and vertical distance between the supports. So, what are supports here? Supports are A and B and let's say we know where they are located. So, that means that you know we know the distance from here to here. This is the horizontal distance.

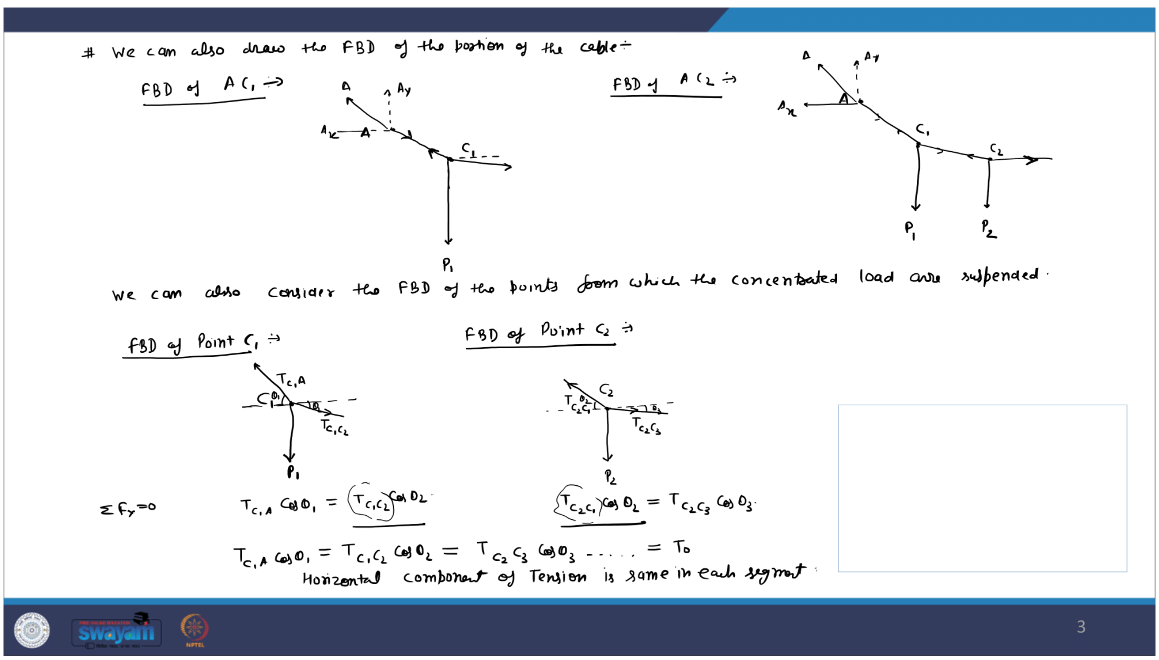
So, let's say this is L and let's say this L is also given in the problem statement. Similarly, its vertical distance from point A is also given. So, therefore, the knowns are L, x_1, x_2, x_3 and d. So, in the problem statement, these quantities are let's say given and what you have to find is, we want to determine the shape of the cable. And what is the meaning of the shape over here? That means that we want to determine the vertical distance from support A to each of the points which is let's say C_1, C_2, C_3. So, we want to know what is their horizontal distance C_1, C_2 and so on and also the tension T in each portion of the cable. So, we want to determine what is let's say this is y_1, this is y_2 and this is y_3 and also the tension in the cable. So, this is the problem statement. So, to solve this kind of problem, first you can consider the free body diagram of the entire cable. So, this is the starting point. So, we have to first consider the free body diagram of the entire cable. How do we make the free body diagram of the entire cable? So, when we make the free body diagram, then all these internal forces we will remove and you know also the supports we will remove and we will put by a you know reactive force at the support.

So, the situation was following, we have this point A, then we have point C_1, then point C_2, C_3 and point B. Now, at C_1, we have external loading P_1. Here, we have external loading P_2. And here I have external loading P_3. Now the tension at point C_2, I am not showing here because I am plotting the free body diagram of the entire body.

So, therefore, this internal tension at point C_2, C_1, C_3, I am not showing. Now at point B we will have the tension and it will be along the cable. So, this is the direction of the force at B. I am denoting it by force B. Now, this force, if you want, you can also resolve in B_x and B_y. So, if I need, I will resolve it in B_x and B_y.

Similarly, at point A, I will have a reaction force and that will be again along the cable. Let us call it A and again, I can resolve it in A_x and A_Y. Now, once I draw this kind of free body diagram, then I can use the equilibrium condition. So, I can use the equilibrium condition. So, $\sum F_x = 0$, $\sum F_y = 0$ and moment about, you know, let's say point A and point B should also be 0.

Okay. So, with this, I can determine, you know, what are the reaction forces at A and B. So, at least one quantity, I can, you know, determine using the free body diagram of the entire cable. Now, I can also consider the free body diagram of part of this cable. Okay. So, for example, because the whole cable, you know, that is in equilibrium.



So, therefore, its part is also in the equilibrium. So, what I can do is, let us consider the free body diagram. So, as I said, we can also draw the free body diagram of the portion of the cable. So, let's say that I want to draw the free body diagram of AC_1. So, I have to see what are the forces that are acting. So, let's say free body diagram of AC_1. So, this is point A and this is point C_1. So, at A, there will be force. So, look at this.

At A, I have a force along the cable. So, there will be a force like this. Let us call it force A. And as I said, you can write it down as A_y and A_x and at point C_1, I will have a force again along the cable. So, here, so, for example, this is the direction.

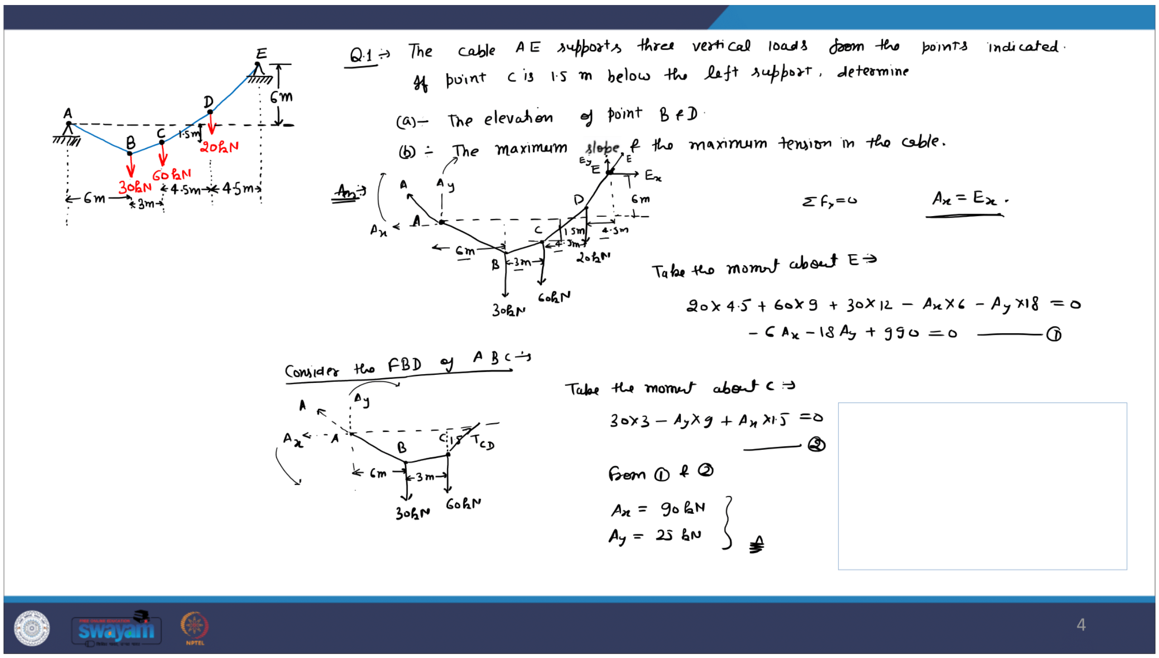
So, therefore, I will have a force along this line and again I can resolve it into perpendicular forces. Also, at C_1, you have external force P_1. Now, note that at this point, I will also have a force in this direction and that direction. But since they are equal, I do not need to show it over here because we are considering the free body diagram of AC_1. But you can also show them, and they will cancel together. Now, we can also look at the free body diagram of let's say AC_2. So, how do I make the free body diagram of AC_2? Now, I have to consider this portion of the cable.

So, let us do that. So, now I have point A over here, then you have point C_1, then you have point C_2, then and then there was point B. So, at A, as I said, there will be a force along the cable. I can divide it into A_y and A_x. At point C_1, I have external force P_1.

Now, I am not showing the forces you know, these one because, you know, we are looking at the free body diagram of the whole part AC_2, okay. So, at C_2, you have the force P_2 and then there will be another force which will act in the direction of C_2, C_3, okay. So, this is the free body diagram of AC_2. Now, these are the free body diagram of the portion of the cable.

What we can also do is we can also consider the free body diagram of the points from which the concentrated load are suspended. So, let us see. So, you can see here from point C_1, the load is suspended. From point C_2, the load is suspended.

Again, since the entire cable is under equilibrium, therefore, all these points are also in equilibrium. So, let us look at the free body diagram of let's say point C_1. So, at point C_1, you have external force P_1, you will have a tension along C_2. So, let me call it $T_{C_1C_2}$. Similarly, you will have a force along the direction A. So, let me call it T_{C_1A}. So, this is the free body diagram of this. So, you can also draw the horizontal line and you know you can say well let's say this angle is θ_1 , this angle is θ_2. So, this completes the free body diagram of point C_1. Now, let us look at the free body diagram of point C_2. So, let us consider point C_2. And at point C_2, we have an external force P_2 and then there will be a tension force along. So, point C_2 to C_3. So, it will be $T_{C_2C_3}$ and then you have a force along C_1. So, this will be $T_{C_1C_2}$. And again, we can say that let's say this angle is $\theta_2$and this angle is θ_3 to find out all the forces. So, this way we can construct the free body diagram either a portion of the cable or you know various points in the cable. Now, let us look at the equilibrium at point C_1. So, we have to balance the forces in the x direction and the forces in the y direction. Let me balance the forces in the x direction first. So, I am doing $\sum F_x = 0$ for point C_1. So, I have T_{C_1A}. So, this is the tension and its component along the x-axis will be $cos\theta_1$ and this will be equal to $T_{C_1C_2} cos\theta_2$. Similarly, for point C_2 , I will have $T_{C_1C_2} cos\theta_2 = T_{C_2C_3} cos\theta_3$. Now, note that tension C_1C_2 and C_2C_1 they are same. So, basically you have a cable and you know the force is acting in that direction and the force is acting in this direction. One case it is C_1C_2 and you know in the other case it is C_2C_1. It is the same cable. Therefore, $T_{C_1C_2}$ and $T_{C_2C_1}$ it will be equal. So, this C_1C_2 and C_2C_1. So, as I said over here, this force is equal to that force. So, either you take from C_1 to C_2 or C_2 to C_1. So, this and this are equal. Therefore, this and this are equal and therefore, I can say that $T_{C_1A}cos\theta_1 = T_{C_1C_2}cos\theta_2 = T_{C_2C_3}cos\theta_3$ and if there are n points, then it is so on. So, what are these? These are nothing but the horizontal component of the tension. So, let us denote it by T_0. What does this imply? This implies that the horizontal component of tension is same in each segment. So, with this, now let us look at couple of example.

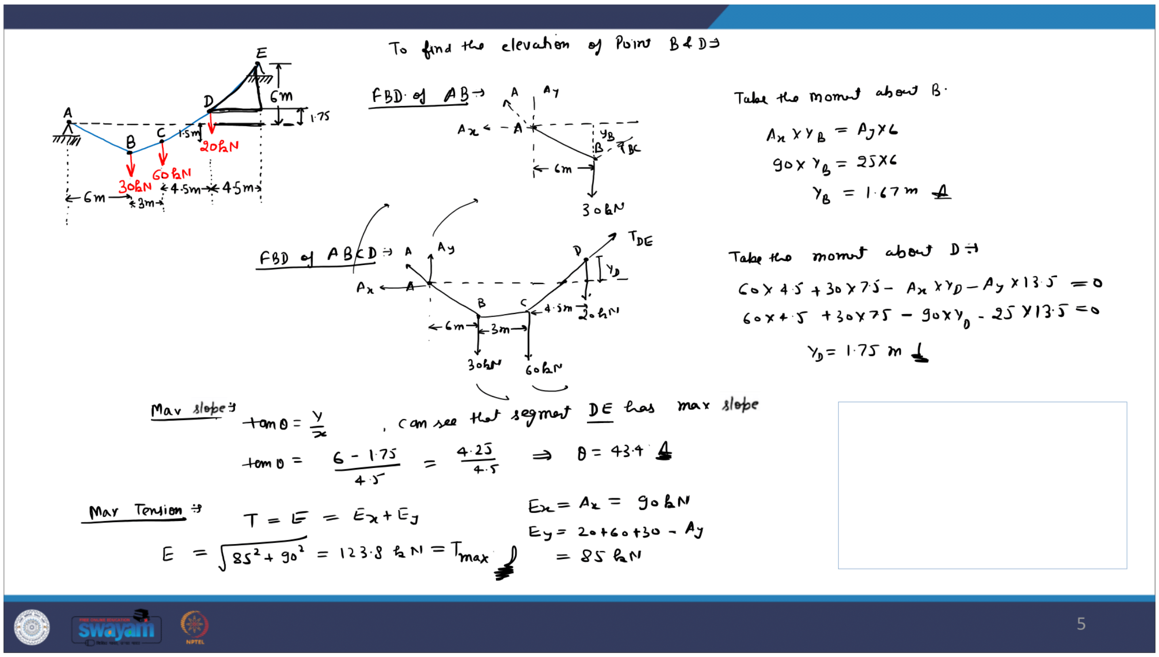


So, in this question number 1, the problem statement is following. The cable AE supports three vertical loads from the points indicated. If point C is 1.5 m below the left support, then in the question it is asked you to determine, (a) the elevation of point B and D and (b) the maximum slope and the maximum tension in the cable. So, first to solve it, let us look at the free body diagram of the entire body. So, we have point A, point B, C, D and then point E and at point B, we have external load of 30 kN. At C, you have 60 kN and at D, we have a force of 20 kN. Now, at point A, the tension is, so at point A, the support reaction is going to act along the cable. So, let's say this is A. Similarly, at point E, it will be along the cable. Let us call it E. Now, I can resolve this force A into A_y and A_x. Similarly, this force also I can resolve in E_x and E_y. Now, the dimensions of the cable is also given. So, for example, all this is given that this is 6 m, this is 3 m, this is 4.5 m and this distance is also 4.5 m. This height is 6 m and this height is also given, this is 1.5 m. Now, let us first look at the forces which are acting in the x direction. So, the forces that are acting in the x

direction are E_x and A_x. So, you can clearly see that from $\sum F_x = 0$, you get $A_x = E_x$. Now, to find out the other quantities, let us first take the moment about E, okay. So, I am going to take the moment about E. So, therefore, E_x and E_y will not contribute and I have a $20\ kN$ force. So, $20\ kN$ force into its distance from point E, the normal distance, it is $4.5\ m +\ 60 \times 4.5 + 4.5$, which is $9\ m + 30 \times 12$. Now, about point E, the force A_x has a tendency to rotate like that and A_y has a tendency to rotate like that. So, these are the clockwise direction. So, therefore, it will be negative. So, $-A_x \times 6\ m$, the perpendicular distance is $6m - A_y \times$ into the perpendicular distance. $20 \times 4.5 + 60 \times 9 + 30 \times 12 - A_x \times 6 - A_y \times 18 = 0$. So, from here, we get $-6A_x - 18A_y + 990 = 0$. Let us call it equation number 1. So, from considering the free body diagram of the entire cable, we got that $A_x = E_x$ and then we got this equation Now, there are two variables. So, we need one more equation. So, for that, let us consider the free body diagram of A, B, C. So, let us consider the free body diagram of A, B and C. So, let us look here.

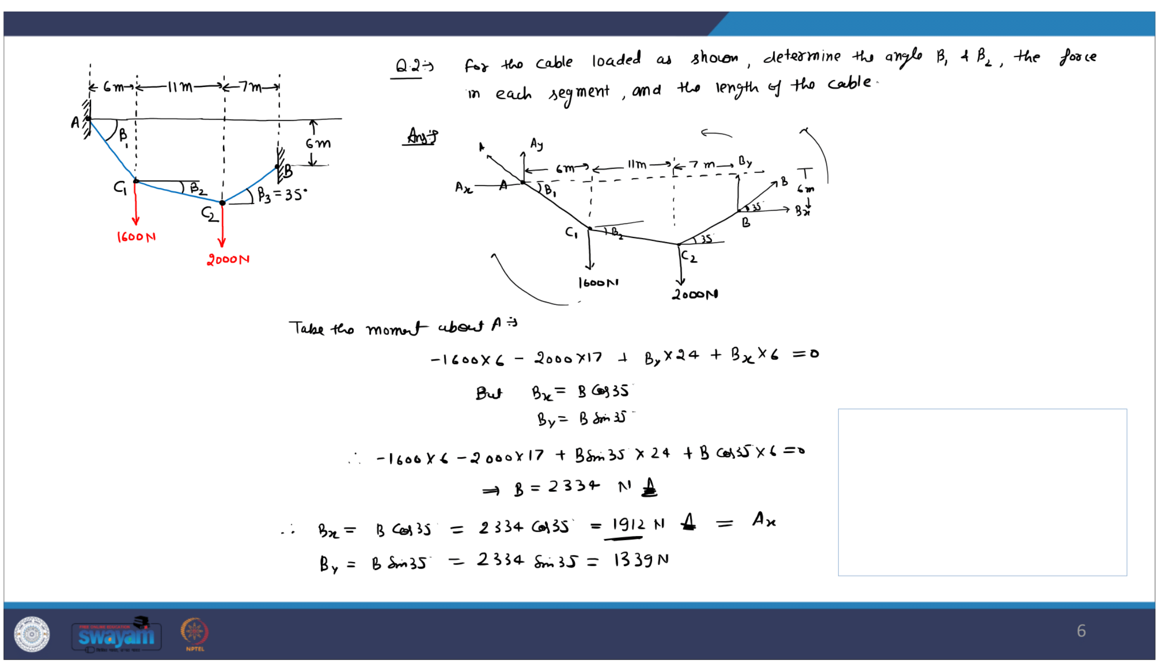
So, you have point A and we have point B, then we have point C, and then there will be a force which will act along the D direction. So, at point A, you have the reaction force. which I can divide into A_y and A_x. At B, you have $30\ kN$ force and at point C, we have a $60\ kN$ force and then there will be another tension T_{CD}. So, this makes the free body diagram. Now, again, various geometrical parameters are given. So, for example, this distance is $6\ m$, this is $3\ m$ and this distance is also given, it is $1.5\ m$. So, let us take the moment about C. So, we have. So, T_{CD} force will go away. So, we have a $30\ kN$ force multiplied by 3. Note that $60\ kN$ force will also not contribute because it is passing through C. So, $60\ kN$ and T_{CD} will go away. So, we have $30\ kN \times 3$ perpendicular distance $-A_y \times 9$ because this force has a tendency to rotate like that and this has a tendency to rotate like this. So, it will be $30 \times 3 - A_y \times 9 + A_x \times 1.5 = 0$. Let us call it equation number 2. Now, note that now we have two equations in A x and A y and therefore, we can solve them. So, you can solve equation number 1 and equation number 2 and this will give you $A_x = 90\ kN$ and $A_y = 25\ kN$.

Now, let us find out the elevation of point B and D which are asked in the question. So, to find the elevation of point B and D. So, let us look at the free body diagram of AB. So, here we have point A and then you have point B. At B, you have external loading of $30\ kN$. At point A, the reaction force is going to act along the cable, and this is what we are keep seeing. We can divide it into the vertical component A_y and A_x and at B, we will have a force T_{BC}. Now, this is given. This is $6\ m$ and we have to determine what is the elevation. So, let's say the elevation from point A is let's say y_B. Now, actually we have to find out what is y_B. For that, let us take the moment about B. So, if you take the moment about B, then 30 kN force and T_{BC} will not contribute. So, I will have $A_x \times y_B = A_y \times 6$. And just now we have find out what is the value of A_x and A_y. So, $90 \times y_B = 25 \times 6$.

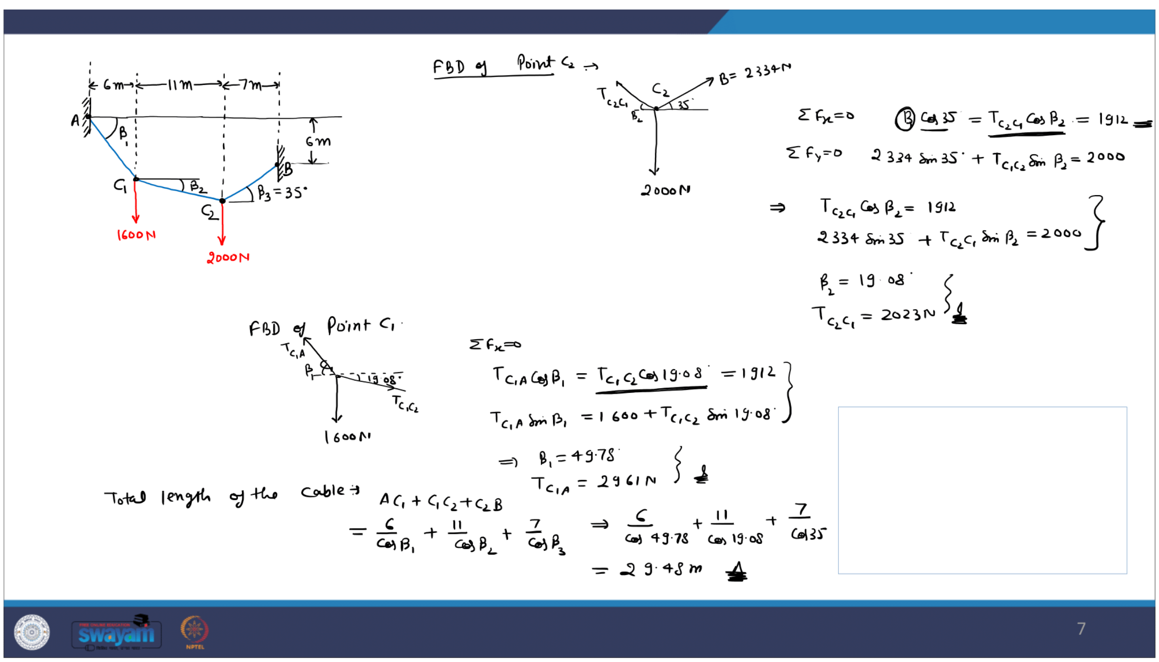


This immediately gives you what is $y_B = 1.67\ m$. So, this is the elevation of point B. Now, let us find out the elevation of point D. For that, let us consider the free body diagram of $ABCD$. So, this is point A, we have point B, point C, point D and then you know you have point E. So, at point A, the force will be along the cable. So, this will be A_y, this is A_x and at point B, I have 30 kN external load at point C, you have 60 kN external load and at point D, you have external load of 20 kN. Now, let us write down the other geometrical parameters. So, this is 6 m, this one is 3 m and this is 4.5 m. This was our horizontal direction, and, in this direction, the tension T_{DE} will act. Now, we have to find out what is its elevation.

So, therefore, from here to here, let us call it y_D and we have to find out what is y_D. For that, let me take the moment about D because I do not know what is T_{DE}. So, that will go away. So, let us take the moment about point D. So, I have 60 kN force multiplied by the normal distance which is $4.5 + 30 \times 7.5$ because this force has a tendency to rotate like that. So, they are in the clockwise direction. So, therefore, $-A_x \times y_D - A_y \times 13.5$ because $6 + 3 + 4.5 = 13.5$ and this is equal to $0.60 \times 4.5 + 30 \times 7.5 - A_x \times y_D - A_y \times 13.5 = 0$. So, again we know what is the value of A_x and A_y. So, therefore, the only unknown is y_D. So, let me just write down the values of A_x and A_y. $60 \times 4.5 + 30 \times 7.5 - 90 \times y_D - 25 \times 13.5 = 0$ and this will give you $y_D = 1.75\ m$. So, we have find out what is the elevation of this points now let us calculate the maximum slope. So, what is the slope? So, slope is nothing but $tan\theta = \frac{y}{x}$. Now you know what is the elevation you know what is its horizontal position. So, therefore, you can find out the slope of you know all the segment and you can see that segment DE has the maximum slope. So, let us find out that slope. So, $tan\theta = \frac{6-1.75}{4.5}$ because we are looking at the segment DE. So, let us look at this segment DE and this distance from here to here is 4.5 and this we have just now calculated it comes out to be 1.75. So, therefore, if you look at this triangle then this distance will be $6 - 1.75$ and this is of course 4.5. So, therefore, tan theta will be this and this comes out to be $\frac{4.25}{4.5}$ and this gives you $\theta = 43.4°$. Now, let us calculate the maximum tension. So, the maximum tension will be at the highest point and the highest point is E. So, therefore, at point E, we will have maximum tension. So, let us calculate that T is basically E and $E = E_x + E_y$. Now, when we consider the free body diagram of the entire cable, then we saw that E_x is nothing but A_x. So, $E_x = A_x = 90\ kN$ and E_y, now I can find out from the free body diagram of the entire cable because we have to see how much force is acting downward and how much force is acting upward. So, A_y I know. So, basically $A_y + E_y = 30 + 60 + 20$. So, $E_y = 30 + 60 + 20 - A_y$ and again $A_y = 25$. So, this will come out to be $E_y = 85\ kN$. Therefore, $T\ or\ E = \sqrt{85^2 + 90^2} = 123.8\ kN$ and this is nothing but T_{max}.



Now, let us look at the second problem and here the problem statement is following. For the cable loaded as shown, determine the angle β_1 and β_2, the force in each segment and the length of the cable. So, again to solve this problem, let us first look at the free body diagram of the entire cable. So, we have point A here, point C_1, C_2 and point B. At point C_1, you have external loading of $1600\ N$. At C_2, we have $2000\ N$ force. Now, at point B, you will have the force along the cable. So, let us call it B and of course, I can divide it into B_x and B_y. Similarly, at point A, we have a force along the cable, let us call it A and you can divide it into the horizontal and vertical component. So, this is A_y and this is A_x. Now, the geometrical parameters are given. So, this angle is β_1, this is β_2, this is So, therefore, this will also be $35°$ and this is $6\ m$, this is $11\ m$ and this is $7\ m$. Now, to find out the value of B, let us take the moment about A so that you know A_x and A_y will go away. So, let us take the moment about A. So, we have 1600 force. So, about A, this will go in the clockwise direction. So, -1600 perpendicular distance is 6, -2000 perpendicular distance is $6 + 11 = 17$, $+B_y$ because it has a tendency to rotate in the anticlockwise direction. So, I have taken it positive, into perpendicular distance is $6 + 11 + 7$=24. So, that will be $24 + B_x \times 6$ because this is $6\ m$ and this has to be equal to 0. $1600 \times 6 - 2000 \times 17 + B_y \times 24 + B_x \times 6 = 0$. But note that here this angle is known to you. So, I know what is B_x and B_y in terms of B. So, I can write down $B_x = B cos 35°$ and $B_y = B sin 35°$. So, let us put it back. So, therefore, $1600 \times 6 - 2000 \times 17 + B sin 35° \times 24 + B cos 35° \times 6 = 0$. And, you know, I can find out what is the value of sin 35 and cos 35. I can put it back and find out what is B. So, this gives you $B = 2334\ N$. Therefore, your



$B_x = Bcos35 = 2334cos35 = 1912\ N$. And note that from $\sum F_x = 0$, this is also the value of A_x. Now, $B_y = Bsin35 = 1339\ N$.

This we got by considering the free body diagram of the entire cable. Now, to find out what is β_1 and β_2, let us look at the free body diagram of let's say C_2 and C_1. So, first I am going to consider the free body diagram of point C_2. So, the free body diagram of point C_2. So, this is to find out what is β_2. So, I have point C_2. From point C_2, we have a force of $2000\ N$ acting downward and then there will be tension along point B. And I know the value of this tension. So, this is $2334\ N$ because just now I have found out and I also know that it is acting at $35°$. Now, at point C_2, I will also have a tension $T_{C_2C_1}$ and it is acting at β_2 angle. So, this completes the free body diagram of point C_2. Now, let us balance the forces. So, $\sum F_x = 0$, this gives you $Bcos35° = T_{C_2C_1}cos\beta_2$ and we have $\sum F_y = 0$. So, this gives you $2334sin35 + T_{C_1C_2}sin\beta_2 = 2000$. Now since the value of B is known and $cos35$ is also known, so, therefore, I know what is this. And you know, this was 1912. I can find out it is 1912 because anyway this is the force in the horizontal direction and we know that the force component along the horizontal direction is constant throughout the cable. So, you can use this value 1912 and you can also find out from, you know, B cos 35 because B is known, it comes out to be 1912. So, therefore, I have the following equation $T_{C_2C_1}cos\beta_2 = 1912$. This is this equation, and I have $2334sin35 + T_{C_2C_1}sin\beta_2 = 2000$. Now, in this there are two variables, and you know you have two equations. So, from here you can find out what is β_2 comes out to be $19.08°$ and $T_{C_2C_1} = 2023\ N$. Now, to find out β_1, let us consider the free body diagram of C_1. So, let us look at the free body diagram of point C_1. So, we have C_1 point, and the external force is $1600\ N$, then you have the force along C_2. So, $T_{C_1C_2}$ and I know at what direction it is acting because I find out what is β_2. This is $19.08°$ and then you have a force which is acting in A direction. So, it is from C_1 to A and this angle is β_1. So, this completes the free body diagram of point C_1. Now, again, let us use the force balance equation. So, $\sum F_x = 0$. So, I have T_{C_1A}. So, I have $T_{C_1A}cos\beta_1 = T_{C_1C_2}cos19.08°$ and this I already know because $T_{C_1C_2}$ is known and you know $cos19.08$ you can find out. So, this comes out to be 1912. Similarly, the force balance along the y direction will give you $T_{C_1A}sin\beta_1 = 1600 + T_{C_1C_2}sin19.08°$. Now, again we have two equation and two unknown. So, we can find out what is β_1. It comes out to be $49.78°$ and the tension comes out to be $2961\ N$. Now, let us find out the length of the cable. So, we have to find out what is total length of the cable. So, total length is $AC_1 + C_1C_2 + C_2B$. Now, the horizontal lengths are given, angle we have find out. So, we can find out what is AC_1. So, this will be $\frac{6}{cos\beta_1} + \frac{11}{cos\beta_2} + \frac{7}{cos\beta_3}$. And we can write down the values of this β_1, β_2, β_3. So, this is $\frac{6}{cos49.78} + \frac{11}{cos19.08} + \frac{7}{cos35} = 29.48\ m$. So, with this, let me stop here. Thank you.