MECHANICS

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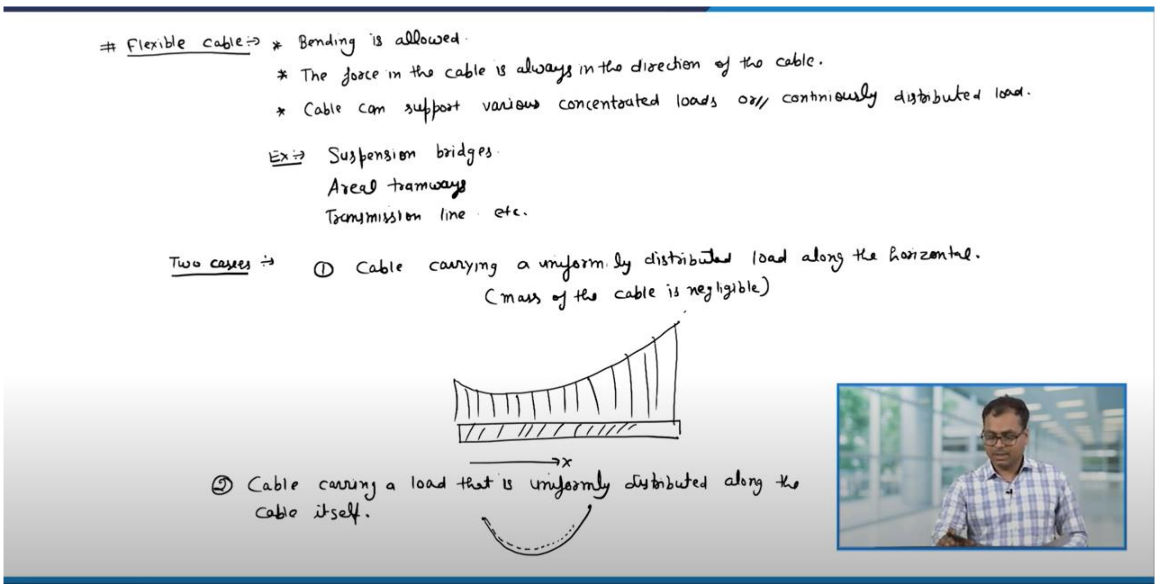
Lecture 11

Flexible Cable

Hello everyone, welcome to the lecture again. In the last lecture, we talked about the beam and distributed forces.

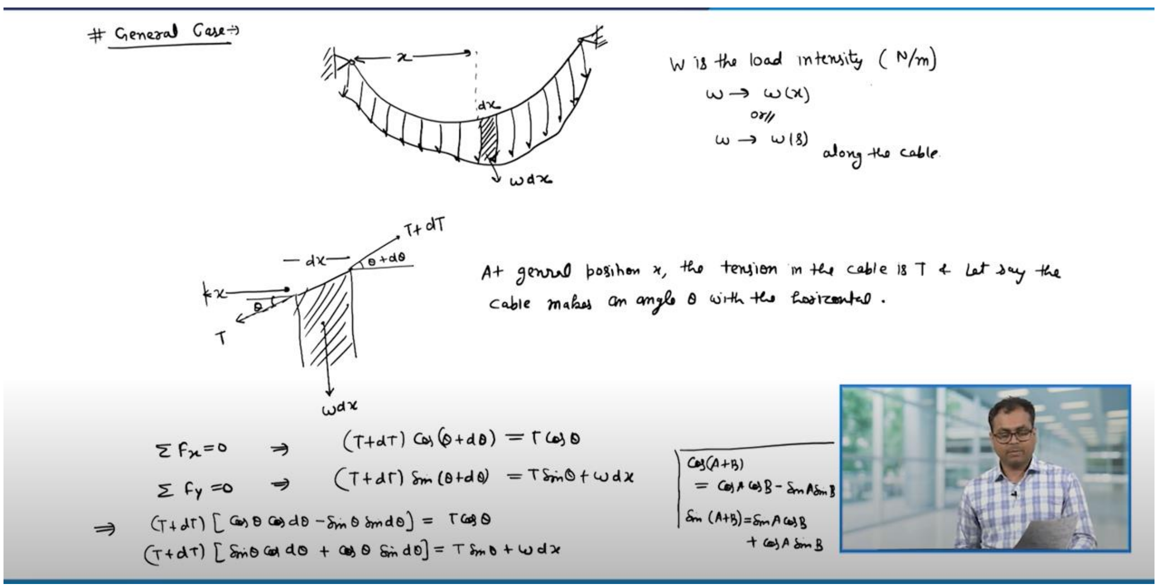


Today, we are going to talk about the flexible cable. Now, these cables can also support various kinds of loads. For example, it can be a distributed load or it can be a continuous load.

So, a cable is specifically flexible cable. In these cables, the bending is of course allowed and the force in the cable is always in the direction of the cable. So, as I said these cables can support various concentrated loads or continuously distributed For example, these cables are used in suspension bridges, in aerial tramways, in transmission line and so on. So, we will discuss two cases of this cable.

In the first case, the cable will carry a uniform distributed load along the horizontal axis. So, we have a cable carrying a uniformly distributed load along the horizontal line. And in this case, we will consider that the mass of the cable is negligible compared to the load that it is carrying.

So for example, we can have a bridge and this bridge is supported by this cable. Let us say the weight of these cables is negligible compared to the weight of this brace. In that case, if I move along the x direction, then this cable is supporting a uniform load. Now, the second case will be the cable carrying a load that is uniformly distributed along the cable itself. So, for example, we can have a cable which is hanging under its own weight. In this case, the weight is uniformly distributed along the cable.



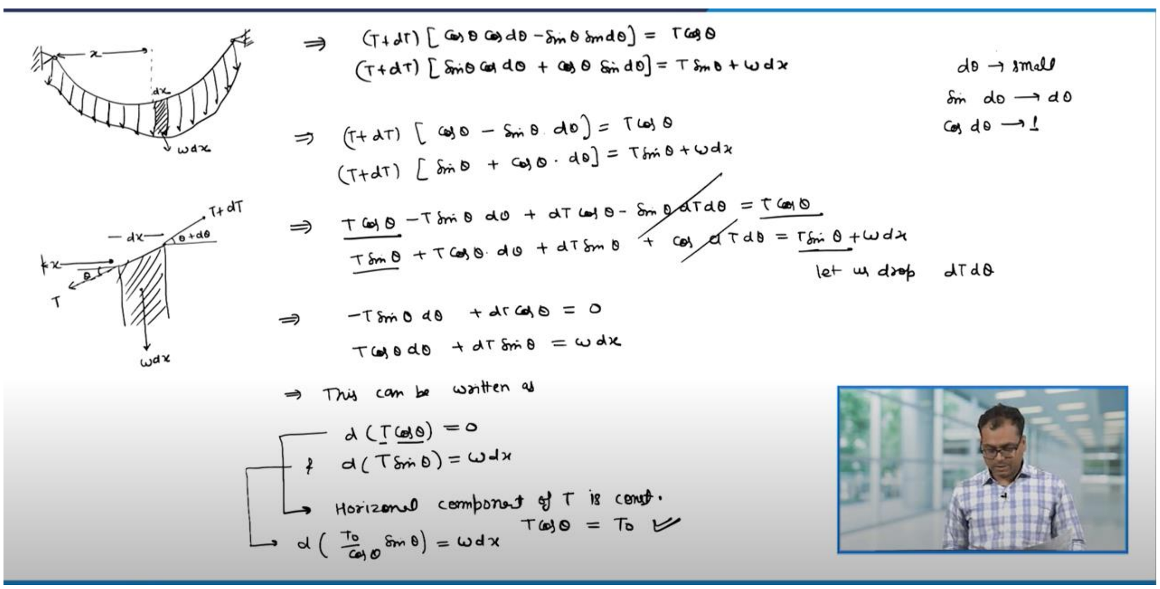
Now to analyze this, let us start with a general case. So let us say I have a cable and this cable is hanged between these two points and let us say on this cable there are some non-uniform load which is applied.

Let us say W is the load intensity So, it will be Newton per meter unit, and of course, this w is a function of x where x is the horizontal direction, or this w is a function of s where s is along the cable, okay. So, to analyze it, let us say we consider a differential element dx, which is at a distance x away. So, the weight of this differential element will be wdx. So, to analyze this cable, let us zoom this part.

So, we have this part; its weight is wdx, and from the origin, it is at a distance x, and let us say at this general position x, the tension in the cable is T. And let us say the cable makes an angle θ with the horizontal. So, therefore, over here, it will make an angle θ and at $x + dx$, let us say the angle is $\theta + d\theta$. Over here, the tension in the cable is T, and here, let us say the tension is $T + dT$.

Now, on this infinitesimal portion, we can apply the equilibrium condition. So, let us use the force balance equation. $\sum F_x = 0$. So, this will gives us $(T + dT)cos(\theta + d\theta) = Tcos\theta$.

And if we use the force balance in the y-direction, then $\sum F_y = 0$ will give you $(T + dT)\sin(\theta + d\theta) = Tsin\theta + Wdx$. Now, on the left-hand side, we have $cos\theta + d\theta$, which we can expand as $\cos(A + B) = cosAcosB - sinAsinB$, and in the second equation, we have $\sin(A + B) = sinAcosB + cosAsinB$. So, now let us expand this. So, we have $T + dTcos\theta cosd\theta - sin\theta sind\theta = Tcos\theta$ and we have $T + dTsin\theta cosd\theta + cos\theta sind\theta = Tsin\theta + wdx$. Now, under the condition that θ is small, $sinx$ can be approximated as x and $cosx$ can be approximated as 1.



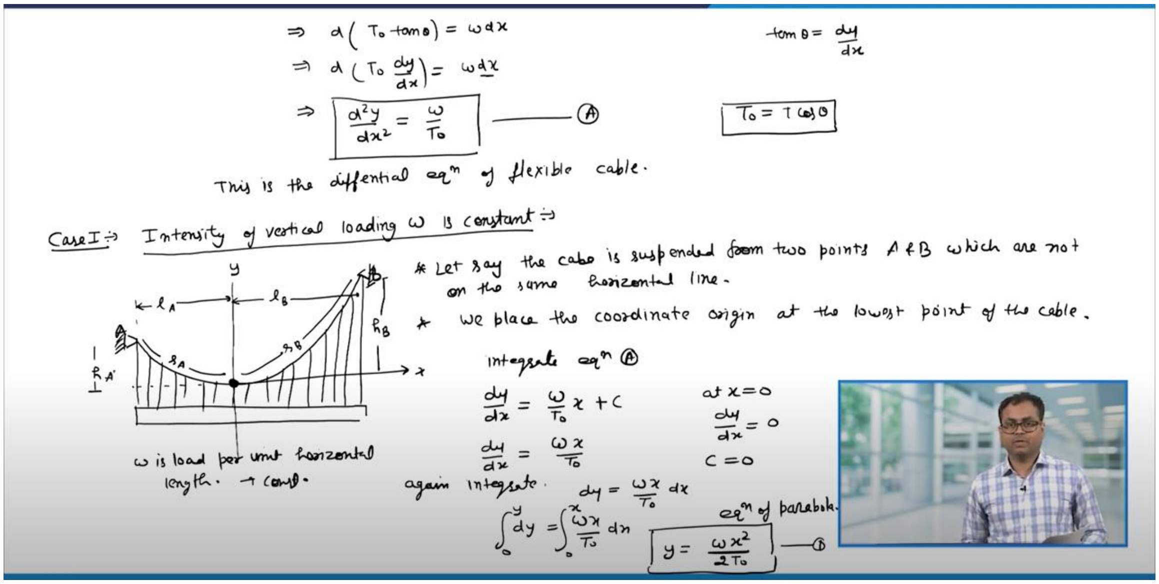
Now, under the approximation that $d\theta$ is small, we can approximate $sind\theta$ as $d\theta$ and $cosd\theta$ as 1. Let us put it back above. So, we have $T + dTcos\theta$, $cosd\theta$ becomes $1 - sin\theta$ and $sind\theta$ becomes $d\theta = Tcos\theta$ and we have $T + dTsin\theta cosd\theta$ becomes $1 + cos\theta d\theta = Tsin\theta + wdx$.

Now, let us multiply that. So, we have $Tcos\theta - Tsin\theta d\theta + dTcos\theta - sin\theta dTd\theta = Tcos\theta$ and $Tsin\theta + Tcos\theta d\theta + dTsin\theta + cos\theta dTd\theta = Tsin\theta + wdx$. Now, since $d\theta$ and dT, they are small.

So, therefore, let us drop $dTd\theta$ because that will be even smaller. So, this term we can neglect and this term also we can neglect. And here $Tcos\theta$ will get cancelled with this $Tcos\theta$ and $Tsin\theta$ will get cancelled with that $Tsin\theta$. So, we have $-Tsin\theta d\theta + dTcos\theta = 0$ and$Tcos\theta d\theta + dTsin\theta = wdx$. Now, these two equations can be written in the following manner.

So, this can be written as $dTcos\theta = 0$ and $dTsin\theta = wdx$. Here, you can use this T as first function and cos theta as second function and you can differentiate, you will get the above equation. Now, what is $Tcos\theta$? $Tcos\theta$, so let us look at this equation. $Tcos\theta$ is the horizontal component of T and if its differentiation is 0, that means it is constant.

So, the above equation tells me that the horizontal component of T is 0 constant. Let me repeat again, $Tcos\theta$ is the horizontal component and if its differentiation is 0 that means it is constant. So, therefore, $Tcos\theta$ is constant, let us call it T_0. Now, let us look at this equation. This equation tells me that D and T I can replace at $T_0/cos\theta$ from here and then we have $sin\theta$ over equal to wdx and $sin\theta/cos\theta$ is $tan\theta$.



So, therefore, I can write it down as $dT_0tan\theta = wdx$, but we know that $tan\theta$ is nothing but slope. So, this is dy/dx. So, I can write down as $d(\frac{T_0dy}{dx}) = wdx$. This dx I can put on the left-hand side and I will get $\frac{d^2y}{dx^2} = w/T_0$. Let us call it equation A and this is the differential equation of flexible cable. Note that here T_0 is the horizontal component of T and it was $Tcos\theta$. Now, let us discuss the first case where the intensity of the vertical loading is constant.

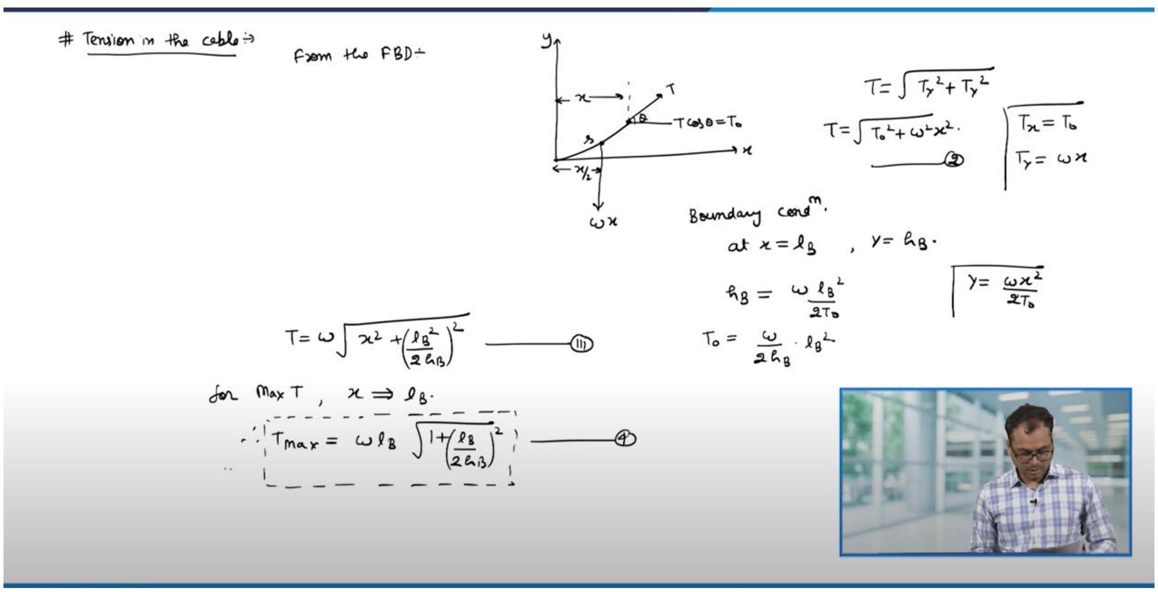
This one where the intensity of vertical load in w is constant. So, let us say you have some load w and this load w is held by this cable at two points A and B. And I have considered the general case wherein this A and B, they are not in the same horizontal line. So, let us say the cable is suspended from two points A and B which are not on the same horizontal line and also let us place our coordinate axis at the lowest point of the cable. So, let us say this is the lowest point I am going to put my coordinate axis over here. So, this is y axis and this is the x axis. So, we placed the coordinate origin at the lowest point of the cable.

Now, let us say the height of point A is h_A, and the height of point B is h_B. This distance is s_A and this distance is s_B. Similarly, let us say this length is l_A and this length is l_B. And as I said, w is the load per unit horizontal length and it is constant.

Now, here we have neglected the mass of the cable because the loading is much much larger than the weight of the cable. So, to analyze this problem, let us start with equation number A because that was the general equation and here W is constant. So, to find out the equation of the cable, let us integrate equation number A. So, integrate equation A. So, we have $\frac{dy}{dx} = \frac{w}{T_0}x + C$. Now, this constant C, we can determine from the boundary condition. For example, you can see that at $x = 0$, that means at this point dy/dx or the slope is 0.

So, if we put it back over here, then we immediately get $C = 0$. So, therefore, we have $\frac{dy}{dx} = wx/T_0$. Now, again integrate it. So, if we again integrate, then we have $dy = \frac{wx}{T_0} d\,x$.

So, integral d_y will be $\frac{wx}{T_0}dx$ and y is varying from let us say 0 to y and x is varying from 0 to x. So, we have $y = wx^2/2T_0$. And what is this? This is the equation of parabola. Let us call it equation number 1. So, under constant vertical loading, the equation of the cable will be the equation of parabola.



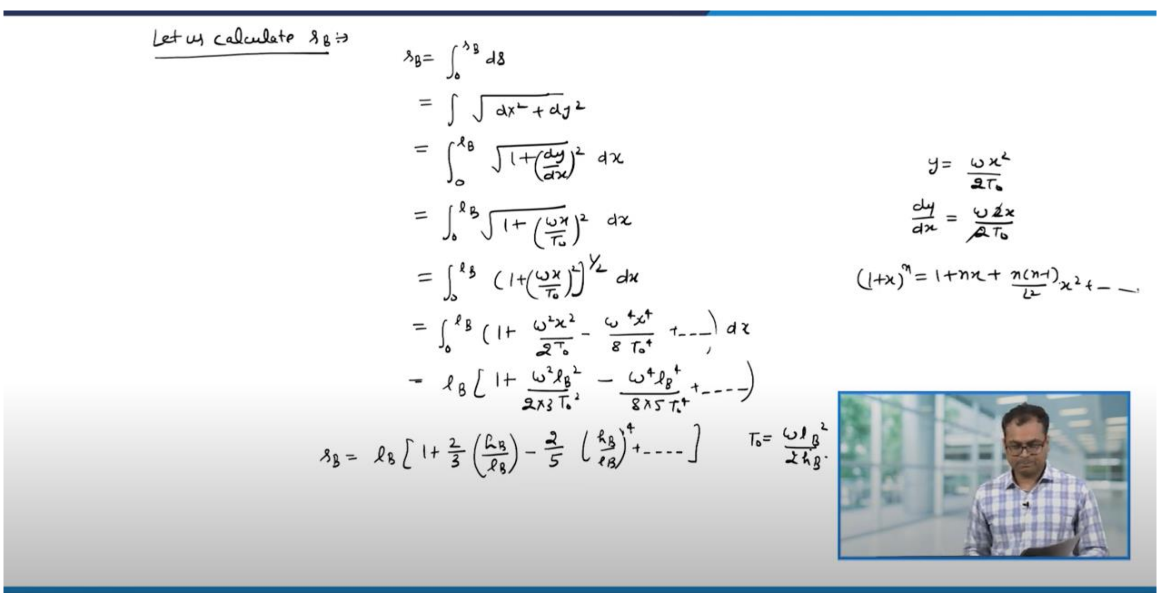
Now, let us calculate the tension in the cable. And for that, let us look at the free body diagram. So, we have the x-axis, we have the y-axis and take the portion of the cable which is up to x distance away from the origin and the tension T is going to act in the direction of the cable. So, let us say it is making an angle θ. So, therefore, the horizontal component will be $T cos\theta$ which is of course T_0 and it is constant and let us say the length of the cable is s. So, its weight will be wx and it is going to act at a distance of $x/2$. So, now let us calculate T, T will be $\sqrt{T_x^2 + T_y^2}$ and T_x is constant, it is T_0 and from the free body diagram, T_y will be w_x. Let us put it back. So, I have $T = T_0^2 + w^2 x^2$. Let us call it equation number 2.

Now, using the boundary condition, we can eliminate T_0. So, let us use the boundary condition. Let us use at $x = l_B$. So, let us look here at $x = l_B$, we have $y = h_B$. And we can also use equation number 1, which is the equation of parabola.

So, we have h_B so, let me just write down this equation. So, we have $y = wx^2/2T_0$. This was our equation number 1. Let us put $y = h_B$.

So, we have $h_B = wl_B^2/2T_0$ and from here, I can find out what is T_0. So, T_0 will be $w/2h_B l_B^2$. Let us put it in equation number 2. So, we have $T = wx^2 + l_B^2/2h_B^2$. Let us call it equation number 3. Now, you can see here for T_{max} , my x has to be maxima. And what is the maximum value of the x here? The maximum value of the x is l B.

So from here, I can also find out what is T_{max}. So for maximum T, my x should be l_B. So therefore, T_{max}will be $wl_B, 1 + l_B/2hB^2$. Let us call it equation number 4. So, this will be the maximum tension in the cable.



Now, let us calculate the length of the cable. For that, let us calculate s_B so, s_B will be 0 to s_B ds and ds will be $\sqrt{dx^2 + dy^2}$ and this can be written as 0 tol_B $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$. And dy/dx, I can calculate from the equation of the parabola.

Remember, $y = wx^2/2T_0$. So, therefore, dy/dx will be $w2x/2T_0$, 2 will get cancelled. So, we have 0 to $l_B \sqrt{1 + \left(\frac{wx}{T_0}\right)^2}\, d\, x$. And this I can just rewrite as $\left(1 + \left(\frac{wx}{T_0}\right)^2\right)^{\frac{1}{2}} dx$.

And under the condition that wx/T_0 is small, I can expand this $(1 + x)^n$ is $1 + nx + \frac{n(n-1)}{2!}x^2 +$ so on. So, we will have 0 to l_B $1 + \frac{w^2x^2}{2T_0^2} - \frac{w^4x^4}{8T_0^4} +$ so on dx and now we can very easily integrate this. So, it will be $l_B(1 + \frac{w^2 l_B^2}{2} \times 3T_0^2 - \frac{w^4 l_B^4}{8} \times 5T_0^4 + \cdots$ or S_B will be $l_B(1 + \frac{2}{3}\left(\frac{h_B}{l_B}\right) - \frac{2}{5}\left(\frac{h_B}{l_B}\right)^4$ and so on wherein we have used $T_0 = wl_B^2/2h_B$. So, in this class, we have discussed the case where the loading was uniform in the x direction or in the horizontal direction. In the next class, we will analyze the case where the loading will be uniform along the cable. With this, let me stop here. Thank you.