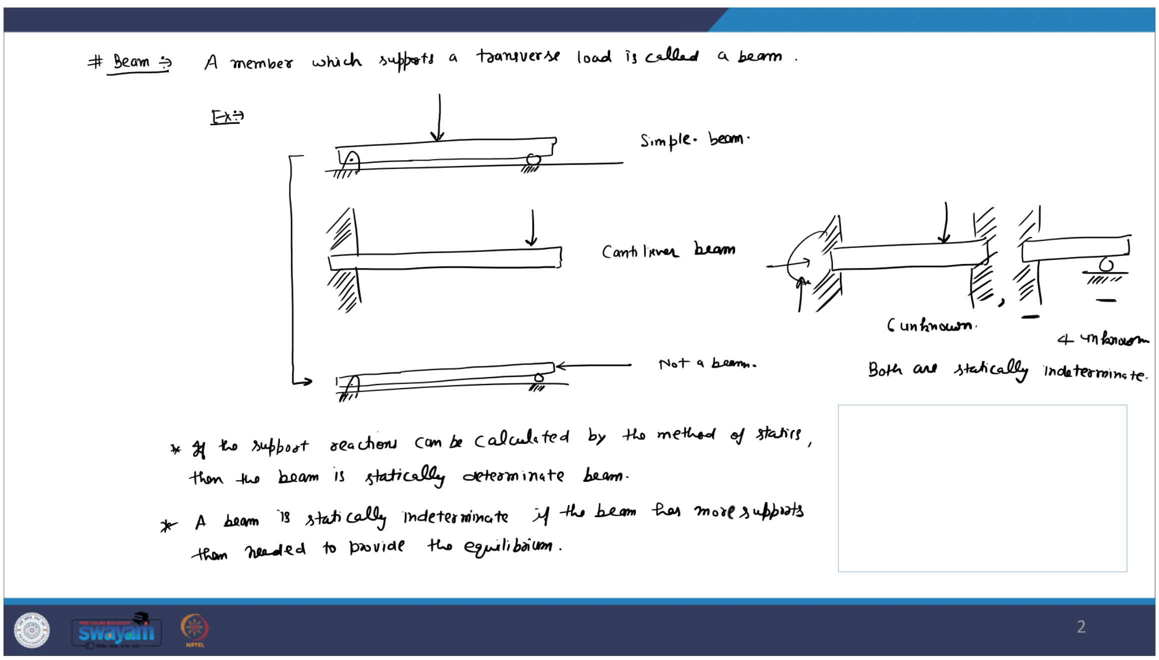
MECHANICS
Prof. Anjani Kumar Tiwari
Department of Physics
Indian Institute of Technology, Roorkee

Lecture 10
Examples: Beams and distributed loads

Hello everyone, welcome to the lecture again. In the previous classes, we have calculated the moment etc about the forces that were concentrated. So, basically they were applied at particular points, but there are many cases wherein the load or the force can be distributed across the members. So, today we are going to discuss about the beams and the forces which are distributed.



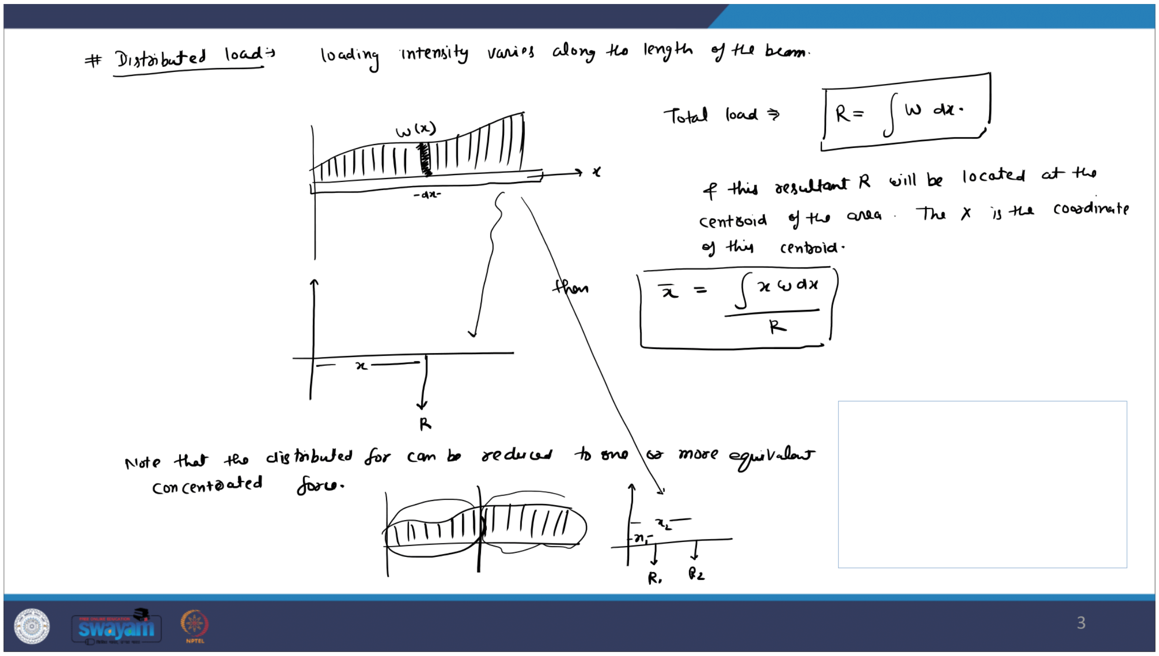
So, first let us see what is beam. So, we are going to use the following definition of the beam. This is a member which supports a transverse load. So, let us look at the example. So let's say I have a pin and I have a roller support and on this I put a beam and I apply a vertical force. Now, this vertical force or the transverse force that is supported by the beam. So, therefore, this qualify to be called a beam and this is the example of a simple beam. Similarly, we can have a configuration like this wherein inside the wall, we insert the beam and you apply a force.

So, this is the example. This is again a beam because it is supporting a vertical force. This is the example of a cantilever beam. Now, think of the following situation. So, again, we have a pin support here and you have a roller support there and you have this beam, but now you are applying the force in this way. So, this is the same example, but the forces are applied differently. Here, the force is transverse and in this direction and in this case, the force is horizontal. So, therefore, this is not a beam. Now, you will find various cases wherein all the support reactions that are acting, it cannot be calculated and there will be some cases where the support reactions can be calculated.

So, if the support reaction can be calculated by the method of statics. By the method of statics, I mean using the equilibrium condition, then the beam is statically determinate beam. And as I said, you will also see various cases wherein there are various support reactions. So, in that case, if I cannot calculate all the support reactions, then it will be statistically indeterminate beam.

So, a beam is statically indeterminate if the beam has more supports than needed to provide the equilibrium. And in this case, you will not be able to calculate all the support reactions. Let me give you an example. So, for example, let us take this beam and this beam is held like that and you apply a force on it. Here, there are six unknowns. You have the x and y reaction and you will have a moment. So, there are three unknown over here. Similarly, three unknown on the other side. So, there are six unknown. And you know all the unknowns cannot be calculated given this information. So, therefore, this is statically indeterminate beam. Similarly, let us look at this example. So, in this example, there are four unknown, three unknown over here and one vertical reaction over here. So, there are four unknown and you will see in the examples that, you know, such kind of cases, all the reaction forces cannot be calculated. So, therefore, both are statically indeterminate beams.

Now, let us look at the cases wherein the load is not uniformly distributed. So, we are looking at the distributed load. So, this means that the loading intensity it varies along the length of the beam. So, for example, you have a beam and if we look at the loading, so it has, you know, some variation. So, let's say this axis is x-axis, then the loading will be a function of x. So, to calculate the total loading, what you have to do is you have to find out the total load. So, let us say this is dx, then total load can be find out by taking the integral of this w as a function of x.

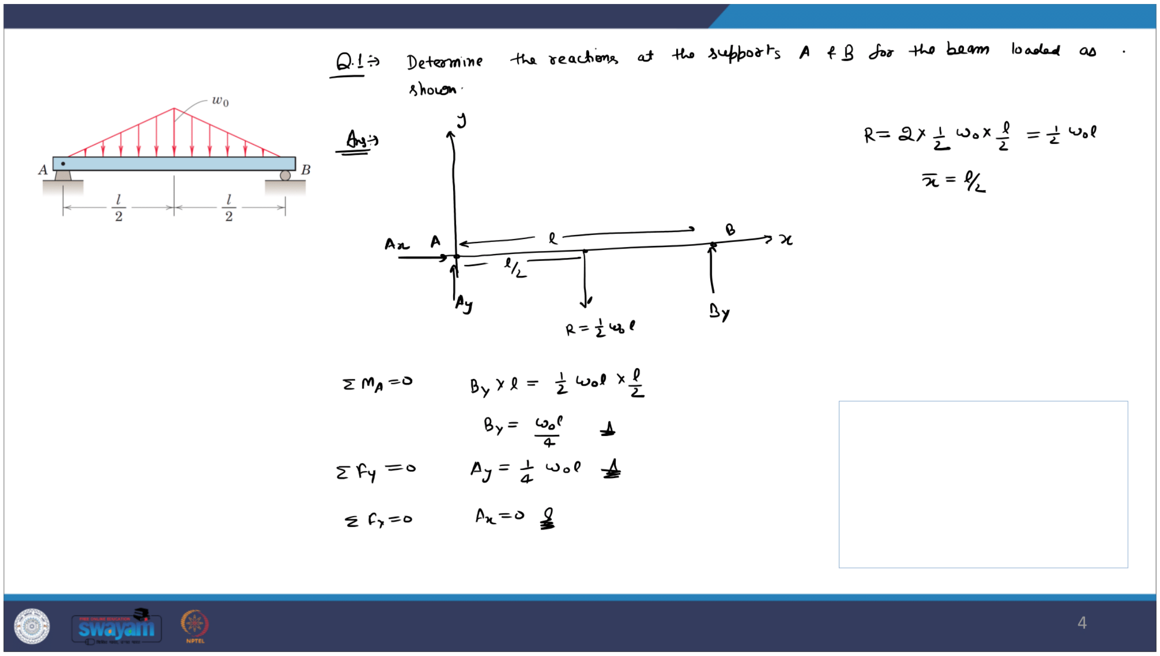


So, total load is let us say $R = \int w\, dx$. And where this R is going to act? Well, it is going to act at the center of mass of this system. And this resultant R, it will be located at the centroid of this area that we have.

Let's say x is the coordinate of this centroid. Then I can find out this $\bar{x} = \frac{\int xwdx}{R}$. So, therefore, this distributed force that we have, I can replace it by a concentrated force and it will act at a distance of $\bar{x}$ and its value will be R. So, this whole system I can replace by this.

Now, note that the distributed force can be reduced to one or more equivalent forces or equivalent concentrated forces. What do I mean by that? So, what I can do is in the above example, I can take this part and calculate the total load and at what distance it is going to act. So let's say it is going to act at a distance x_1 and then I can take this part and calculate at what distance it is going to act. So let us say it is r_2 and it is going to act at a distance of x_2. So the same system either I can replace by this or I can replace by that.

Let us look at the first problem and the question statement is following. Determine the reactions at the supports A and B for the beam loaded as shown. So, to find out the support reactions, first let us look at the free body diagram of this.

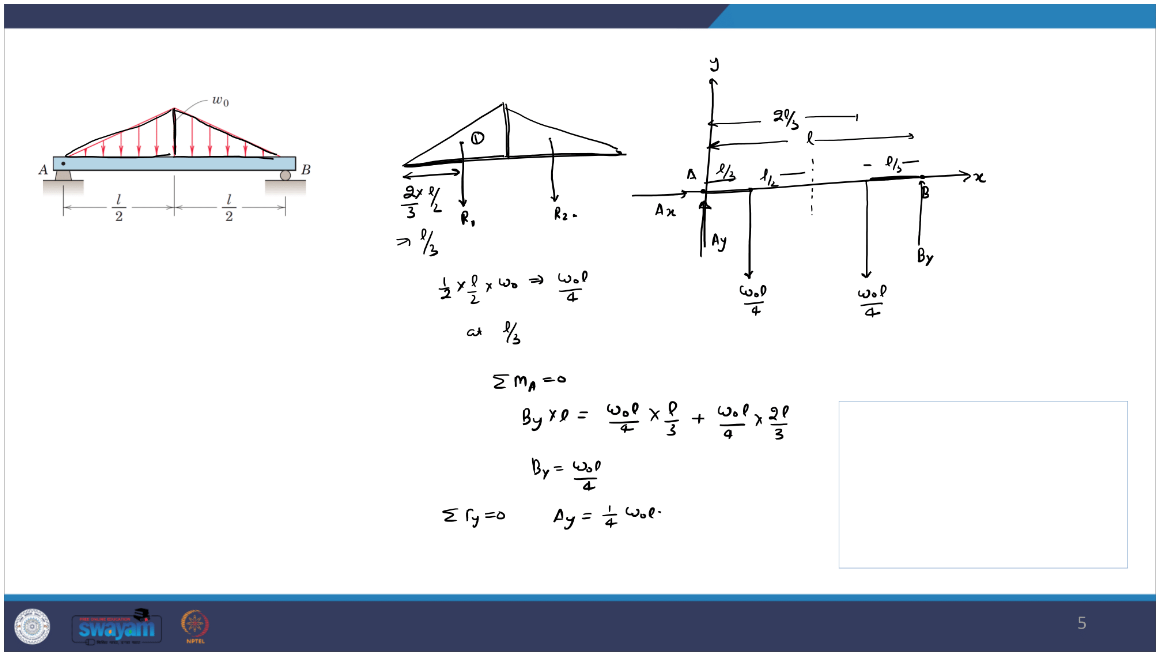


Let's say this is x-axis, this one is y-axis. Now, first let us calculate the total resultant of these loads. So, total resultant R will be, so there are two triangles which are equivalent. So, therefore, its area will be $2 \times \frac{1}{2} \times w_0 \times \frac{l}{2} = \frac{1}{2} w_0 l$.

So, that much force is going to act. Now, where it is going to act? Again, from the inspection, you can see that it is going to act at $\frac{1}{2}$. So, $\bar{x} = \frac{l}{2}$. So, therefore, at point A, first of all, it is a pin support. So, therefore, it cannot support the moment. It can only support the A_y and A_x forces. At point B, we have a roller support. So, therefore, force B_y is going to act.

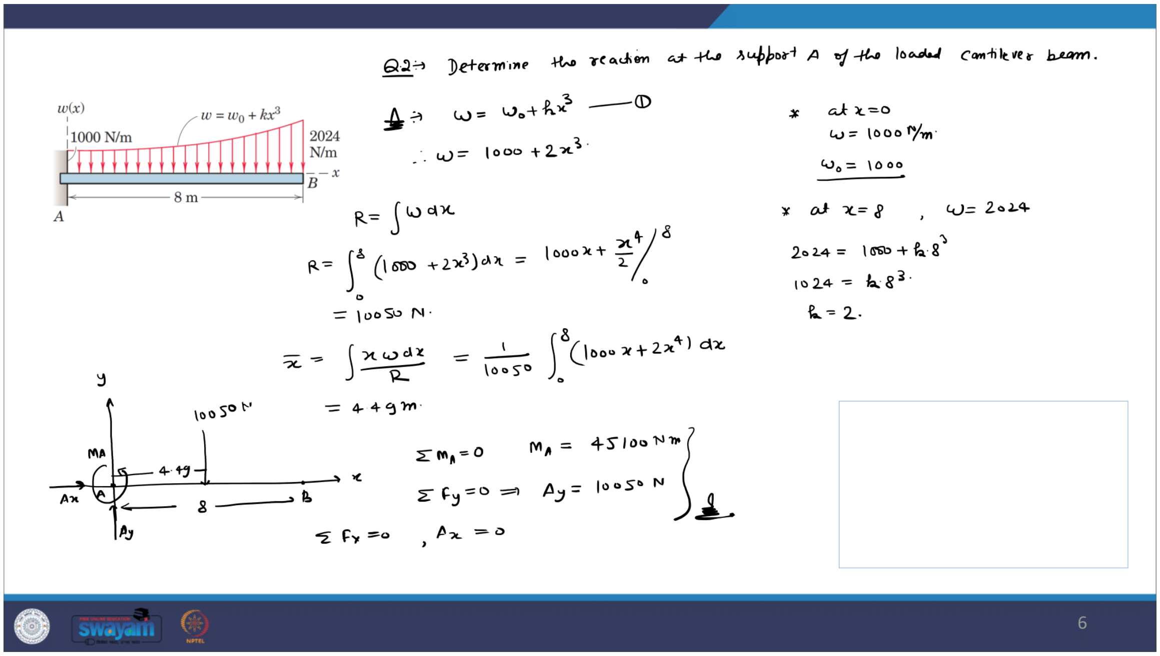
Now, R is going to act at $\frac{1}{2}$. This length is of course l and this is $R = \frac{1}{2} w_0 l$. So, this makes the free body diagram of this. Now, to find out A_y and B_y, let us take the moment about A first. So, if you take the moment about A, then A_x and A_y will go away.

So, we will have $B_y \times l = \frac{1}{2} w_0 \times \frac{l}{2}$ and this gives you $B_y = \frac{w0l}{4}$ and I can also have $\sum F_y = 0$. This gives you $A_y = \frac{1}{4} w_0 l$. And since there are no forces acting in the x direction, so therefore, $\sum F_x = 0$. We have only one force A_x and it has to be balanced. So, therefore, $A_x = 0$. So, here we have replaced the entire distributed load by a single force. We can also solve this problem by considering two triangles and we can replace this system by two concentric loads. So, let us do this problem in the following manner.



Let us consider this triangle and then let us consider that triangle and let us replace it by two concentric load, let us say R_1 and R_2. So, for that, let me make the free body diagram. So, we have the x-axis and the y-axis. Now, this was point A, this is point B. At B, we have a roller support. So, therefore, force B_y is going to act. At A, there will be a force A_x and A_y. Now, this is the center at a distance of $\frac{1}{2}$. Now, the area of the first triangle is $\frac{1}{2}\left(\frac{l}{2}\right) w_0 = \frac{1}{4} w_0 l$. So, $\frac{w_0 l}{4}$ force is going to act at a distance of $\frac{l}{3}$ because this is 2 times or $\frac{2}{3}$ multiplied by $\frac{l}{2}$. So, this will be $\frac{l}{3}$. So, at a distance of $\frac{l}{3}$, I have the first load $\frac{w_0 l}{4}$. Similarly, from here at a distance of $\frac{l}{3}$, we have the second concentrated load.

So, this is the free body diagram. Now, to calculate B_y, let us take the moment about A. $\sum M_A = 0$. So, we have $B_y \times l = \frac{w_0 l}{4} \times \frac{l}{3} + \frac{w_0 l}{4} \times \frac{2l}{3}$. So, from here we get $B_y = \frac{w_0 l}{4}$ and I can have $\sum F_y = 0$. This gives me $A_y = \frac{w_0 l}{4}$. So, alternatively we have solved this problem by considering two concentrated load instead of one.



Now, let us look few more questions on this concept. So, here the problem statement is following. Determine the reaction at the support A of the loaded cantilever beam. So, let us look at this problem here. This $w = w_0 + kx^3$. Here, w_0 and k are some constant which can be determined by the boundary conditions that we have. So, for example, if you look at $x = 0, w = 1000\ N/m$.

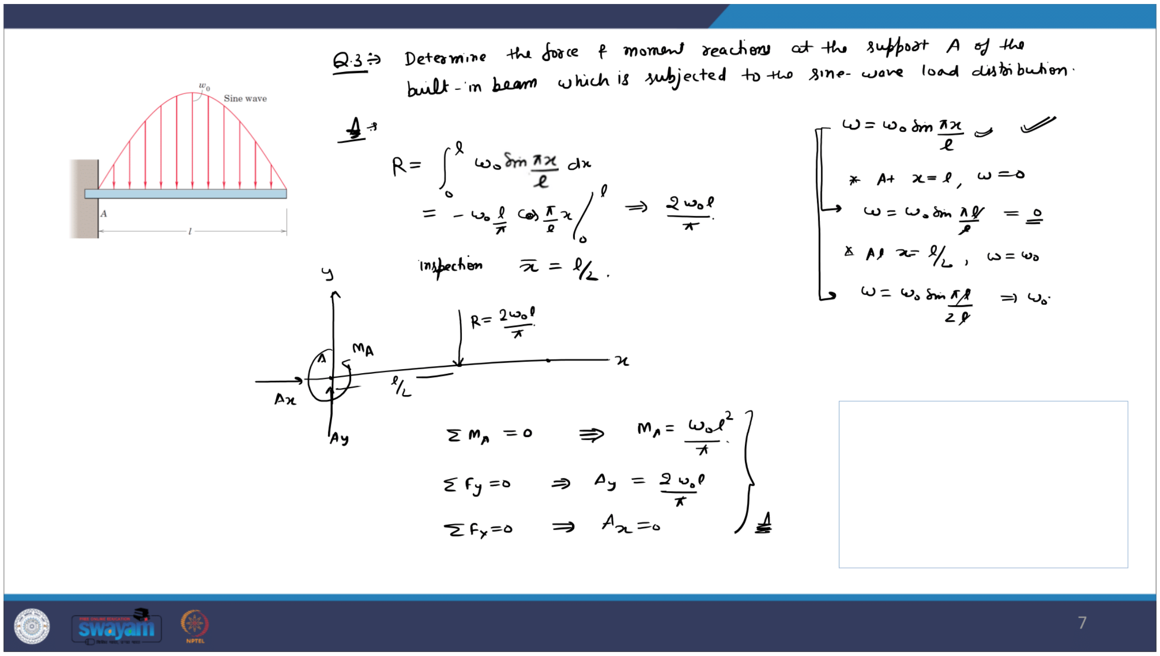
So, let us put that in the above equation. You will immediately see w is nothing but 1000. Now, at $x = 8,\ w = 2024$. Let us put that in equation number 1. So, we have $2024 = 1000 + k(8)^3$. So, we have $1024 = k(8)^3$. This gives us $k = 2$. Let us put in equation number 1. Therefore, we have $w = 1000 + 2x^3$.

So, this is how the load is varying. Now, to find out how much the load is, So, $R = \int w\, dx$. So, R will be integral and the x varies from 0 to 8. $R = \int_0^8 (1000 + 2x^3) dx$.

So, this will be $1000x + \frac{x^4}{2}|_0^8 = 10050\ N$. So, this much concentrated load is going to act, but where it is going to act? So, for that, let us find out the centroid of this. So, $\bar{x} = \int \frac{xwdx}{R} = \frac{1}{10050}\int_0^8 (x + 2x^4) dx = 4.49\ m$. Now, with this let us make the free body diagram. So, we have the x-axis, the y-axis. At point A, this beam is inserted inside the wall. So, therefore, it will also support the moment.

So, let us take the moment in the anticlockwise and then, you know, its sign will determine whether it is correct or not. We have A_x, we have A_y. Now, R is going to act at a distance of 4.49 from point A and its value is $10050\ N$. So, this is the free body diagram of that and this is of course, 8 meter.

Now, to calculate M_A, let us take the moment about A. So, if you take the moment about A, Then $M_A = 45100\, Nm$ because you multiplied 10050×4.45 and it is 45100 Newton meter. Now, let us do the force balance in the y direction and this gives you $A_y = 10050\, N$. And since there are no force to balance A_x, so therefore, $A_x = 0$.

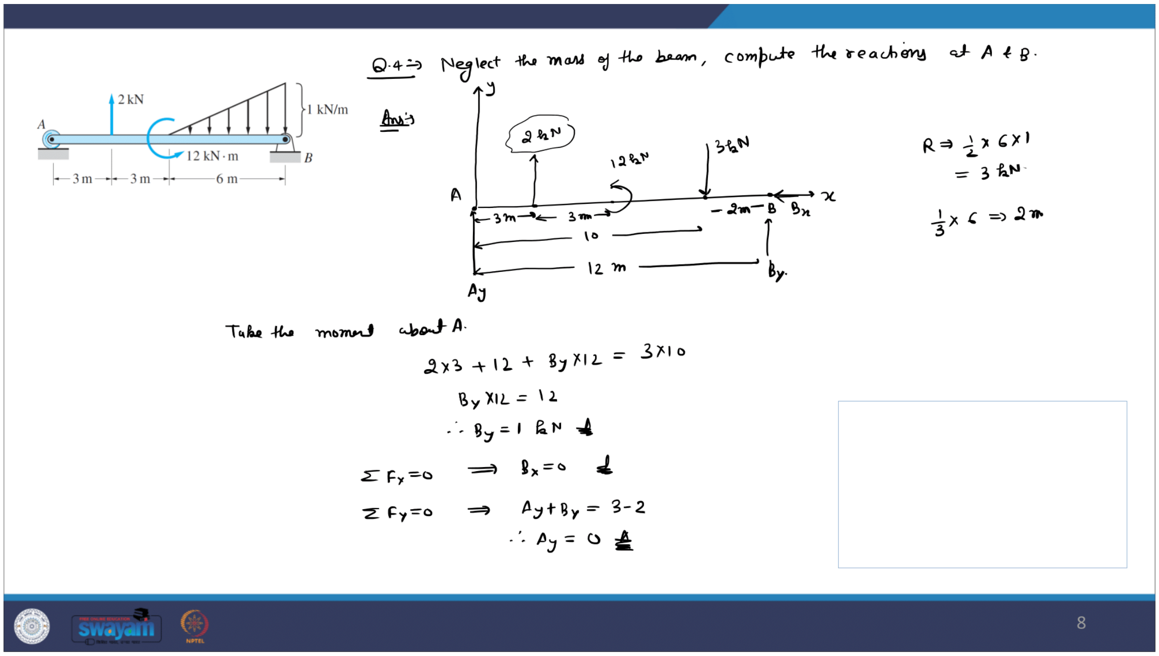


Now, let us look at this problem statement. Determine the force and moment reaction at the support A of the built-in beam which is subjected to the sine wave load distribution. So, since over here the distribution is in the form of sine wave, let us see if this is the correct function to represent the loading. Let us first check at the boundaries if this is correct. So, at $x = l$, we know that w=0. Let us see if this function correctly represent this. Now, over here if I put $x = l$, then I have $sin\pi$ and $sin\pi = 0$. So, therefore, this is correct.

Now, at $x = \frac{l}{2}$ from the figure, I know that the loading has to be maximum w_0. Now, in this formula, again if I put $x = \frac{l}{2}, w = w_0$. So, therefore, this is the correct representation for the weight distribution.

Now, let us calculate the total weight. So, total $R = \int_0^l w_0 \sin\left(\frac{\pi x}{l}\right) dx$. The integral of sin is $-cos$. So, this will be $-\frac{w_0 l}{\pi}\cos\frac{\pi}{l}x|_0^l = \frac{2w_0 l}{\pi}$.

So, this much load will be there. Now, let us find out where it will be. Now, from the inspection only we can find out that it will be at $\bar{x} = \frac{l}{2}$ is just by inspection or you can calculate the $\bar{x}$ by the formula that we have discussed earlier. So, therefore, the free body diagram of this is we have the x-axis, the y-axis and at point A, since it is a built-in support, there will be A_x, A_y and it will also support a moment, let us call it M_A and at $x = \frac{l}{2}$, we have the force $R = \frac{2w_0 l}{\pi}$. Now, to calculate M_A and A_y, let us take the moment about A. So, $\sum M_A = 0$. This gives you $M_A = \frac{w_0 l^2}{\pi}$and let us take $\sum F_y = 0$. So, this gives you $A_y = \frac{2w_0 l}{\pi}$ and $\sum F_x = 0$ obviously gives you $A_x = 0$. Now, let us look at one more problem and the problem statement is following.



Neglect the mass of the beam compute the reactions at A and B. So, to solve this problem, let us first make the free body diagram of this. So, let's say this is the x-axis, this one is the y-axis at a distance of 3 meter, we have a 2 kilo Newton force and at a distance of 3 meter from this 2 kilo Newton force, we have a couple. This is 12 kilo Newton and then we have this distributed force.

So, let us first calculate how much is the distributed force. So, $R = \frac{1}{2} \times 6 \times 1 = 3\ kN$. So, this force 3 kilo Newton is going to act as at one third of $\frac{1}{3} \times 6 = 2$ meter which is 2 meter from point B. So, let us put that 3 kilo Newton force over here. Now, let us look at the reaction forces. At point A, we have a roller support. So, therefore, a force A_y is going to act in the y direction and at B, we have a pin support. So, therefore, forces will be B_x

and B_y and this whole distance is 12 meter. So, this completes the free body diagram. Now, to find out B_y, let us first take the moment about A. So, if I take the moment about A, then $B_x = 0$, $A_y = 0$ and couple is a free vector. So, therefore, it will be $2 \times 3 + 12 + B_y \times 12 = 3 \times 10$.

So, this gives us $B_y \times 12 = 12$. Therefore, $B_y = 1\ kN$. Now, obviously, $B_x = 0$ because $\sum F_x = 0$ and this gives you $B_x = 0$. Now, $\sum F_y = 0$ will give you $A_y + B_y = 3 - 2 = 1$. So, therefore, $A_y = 0$.

With this let me stop here. See you in the next class. Thank you.