

MECHANICS

Prof. Anjani Kumar Tiwari

Department of Physics

Indian Institute of Technology, Roorkee

Lecture 01

Course outline

Hello everyone, welcome to this course. I am Anjani Kumar Tiwari, faculty in the Department of Physics at IIT, Roorkee. This course is about mechanics and the following is the outline of this course. This course is going to run for 12 weeks. So, therefore, this has 12 modules.

Course Outline:

Module 1: Concepts of particles and rigid bodies, vectorial representation of forces and moments, couple moment, reduction of a force system to a force and a couple.

Module 2: Free body diagram, support reactions, equilibrium of rigid bodies in two and three dimensions.

Module 3: Distributed forces, beams, flexible cables.

Module 4: Truss, analysis of truss using method of joints and method of sections.

Module 5: Basics of constrained motion, degrees of freedom, generalized coordinates, principle of virtual work for a particle and rigid body, condition for equilibrium for a conservative system.

Module 6: Friction, static and kinetic friction, coefficient of friction, sliding friction, ladder friction, belt friction, rolling resistance.



In the first module, we will look at the concept of particles and rigid bodies. We will look at the vectorial representation of the forces and moment, couple moment, reduction of a force system to a force and a couple. In the second module, we will look at the free body diagram, support reactions, equilibrium of the rigid bodies in two and three dimensions. In the third module, we will look at the distributed forces, beams and flexible cables. The fourth module, we will look at truss, analysis of the truss using the method of joints and method of sections. In the fifth module, we will look at the basics of constrained motion,

degree of freedom, generalized coordinates, principle of virtual work for a particle and a rigid body, condition for equilibrium for a conservative systems. In the sixth module, we will discuss the friction, static and kinetic friction, coefficients of friction, sliding friction, ladder friction, belt friction and rolling resistance. So, this is all about the static part of this course.

In the second half, which is going to start from the seventh module, we will look at the

Module 7: Kinematics of particles, cartesian coordinates, planar polar coordinates, spherical coordinates, and cylindrical coordinates.

Module 8: Equation of motion in different coordinate systems.

Module 9: Work energy method, conservation of energy, impulse and momentum relation, conservation of momentum, impact, direct impact and coefficient of restitution, oblique central impact, variable mass problem.

Module 10: Moment of Inertia, parallel axis theorem, perpendicular axis theorem, radius of gyration, product of inertia of composite bodies, moment of inertia about any arbitrary axis, principal axis of inertia.

Module 11: Translation and rotation of rigid bodies, relative velocity and relative acceleration.

Module 12: Equation of motion, general plane motion and work energy relation, angular momentum, impulse-momentum equation, Euler's equation of motion and three-dimensional motion about a fixed point.



kinematic of particles. So, herein we will discuss the kinematic of particle, Cartesian coordinate, planar coordinates, spherical coordinate and cylindrical coordinate system. In the eighth module, we will look at the equation of motion in these different coordinate systems. The ninth module, we will look at the work energy method, conservation of energy, impulse and momentum relation, conservation of momentum, Impact, direct impact, and coefficient of restitution. Oblique central impact and variable mass problem. In the 10th module, we will look at the moment of inertia, parallel axis theorem, perpendicular axis theorem, radius of gyration, product of inertia of composite bodies, moment of inertia about any arbitrary axis and principal axis of inertia. In the 11th module, we will look at the translation and rotation of rigid bodies. relative velocities and relative accelerations. And finally, in the 12th module, we will look at the equation of motion,

general plane motion and work energy relation, angular momentum, impulse momentum, Euler equation of motion and three-dimensional motion about a fixed axis.

References:

1. J. L. Meriam and L.G. Kraige, "Engineering Mechanics: Statics", 6th Edition, John Willey and Son's (2012)
2. J. L. Meriam and L.G. Kraige, "Engineering Mechanics: Dynamics", 6th Edition, John Willey and Son's (2012)
3. F. P. Beer and E. R. Johnson, "Vector Mechanics for Engineers: Statics and Dynamics", 9th Edition, Tata McGraw-Hill Publishing Company (2010)
4. A. Pytel and J. Kiusalaas, "Engineering Mechanics: Statics", 3rd Edition, Cengage Learning (2010)
5. A. Pytel and J. Kiusalaas, "Engineering Mechanics: Dynamics", 3rd Edition Cengage Learning (2010)



In this course, we are going to follow these reference book. The first one is by Mariam and Kraige and the title is Engineering Mechanics Static and by the same author Engineering Mechanics Dynamics. We will also follow the book by Beer and Johnson, Vector Mechanics for Engineers and by Andrew Pytel, the book on Engineering Mechanics, Static and Dynamics. So, most of the part or most of the figure, most of the questions are taken from these reference book. Now, with this very brief discussion about the course content, now let us come to the topic.

So, in this course, as I already said, we are going to discuss about mechanics and mechanics deals with the effect of forces which are acting on the object. And it has broadly two parts. The first one is statics which concerns the equilibrium of bodies under the action of forces. And its second part is the dynamics which concerns with the study of forces and their effect on motion. So, well in both the topics, you will see that the forces are acting, but in one case, the forces are such that the body is in static equilibrium. In the second case, the forces are not balanced. Therefore, the body is in motion, but the point is in both the cases, the forces are acting on the body.

Mechanics \Rightarrow Deals with the effects of forces acting on the object.

└─ Statics \Rightarrow Concerns the equilibrium of bodies under the action of forces.
└─ Dynamics \Rightarrow Concerns with study of forces & their effect on motion.

Mass \Rightarrow Measure of the inertia of a body. (which is its resistance to change of velocity under the influence of force).

Force \Rightarrow Force is the action of one body on another. It tends to move the body in the direction of its action.

The action of a force is characterized by

\Rightarrow magnitude
 \Rightarrow direction of its action
 \Rightarrow Point of application.

} vector.

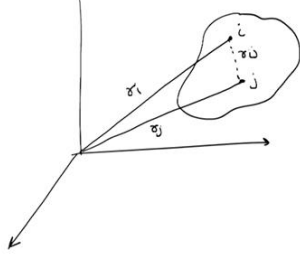


Now, to study this, we need some basic concepts like what is length, what is mass, what is rigid body, what is force, etc. Let us very quickly define two very important stuff which is mass and force. So, mass is the measure of the inertia of a body. So, when you apply the force, then the motion of the body is going to change, the velocity is going to change. So, basically it measures under the influence of force, how much the velocity is going to change.

So, therefore, this also defines or basically which is its resistance to change of velocity under the influence of force. So, now let us quickly define what is force. So, force is basically the action of one body on another. So, this is how you apply the force.

So, force is the action of one body on another, it tends to move the body in the direction of its action. So, that means you apply a force, then the body is going to move in the direction in which the force is applied. Now, this action it can be characterized by how much you are applying, in which direction you are applying and where you are applying. So, the action of a force is characterized by magnitude. It is classified by the direction of its action and also what is the point of application that means where you are at which point you are applying and of course force is a vector quantity. When we apply the force on the body, then depending upon whether the forces are balanced or not, the body will remain in the equilibrium or move under the influence of the forces.

Rigid body \Rightarrow




$$r_{ij} = |r_i - r_j| = c_{ij} \quad \forall i, j$$

\downarrow
const.

In a rigid body all particles of the body remain at fixed distance from each other irrespective of the forces that act on the body.

\Rightarrow The body does not deform under the action of forces.

* Concept of rigid body will be used in the study of statics & dynamics.



In real situation, however, this analysis is very complicated. Therefore, we require some idealization. One such idealization is the concept of rigid body. So let us see what is rigid body. So, let us say this is a body and we define a coordinate system and I take two points on the rigid body. So, let us say this is i^{th} point, this is j^{th} point and their position vector is r_i .

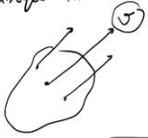
And r_j , then under the influence of force, the distance between i and j should not change. So, that means if this is r_{ij} , then this r_{ij} should not change. That means the body should not deform. So, note that r_i can change, r_j can change, r_i minus r_j can also change because that is a vector but r_i minus r_j modulus which is the spacing between them that should not change. So, r_{ij} which is r_i minus r_j mod this has to be constant for i and j actually for all i and j .

So, this is the definition. So, this is a constant and this would be constant. In a rigid body, all particles of the body remain at fixed distance from each other irrespective of the forces that act on the body. In another language, this means that the body does not deform under the action of forces. Now, this concept of rigid body we are going to use both in static and dynamics. So, the concept of rigid body will be used both in the study of statics and dynamics.

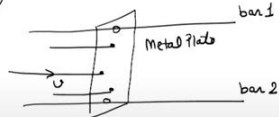
Vector \Rightarrow In mechanics, vector can be classified in three classes:

- ① free vector
- ② Sliding Vector
- ③ fixed vector

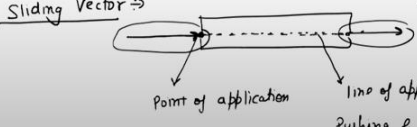
Free vector \Rightarrow A free vector is one which can be positioned anywhere in space & not confined or associated with a unique line.




If a body moves without rotation then the movement of every point in the body can be described by this vector V .



Sliding Vector \Rightarrow



Force can be applied at any point along its line of action without changing its effect on the body as a whole. Pushing & pulling produce the same effect.



Now, let us look at you know what is a vector because in this course like as I said we are going to deal with the forces et cetera and these are vector quantities. So, now let us look at the definition of different kind of vector that we are going to deal with. So, vectors in mechanics, vector can be classified in three classes. One is free vector, sliding vector and fixed vector. So, now let us look at the free vector. So, a free vector is one which can be positioned anywhere in the space and it is not confined or associated with a unique line. What do we mean by that? Let us understand it with an example.

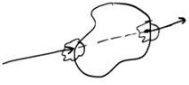
So, let us say I have a rigid body and this rigid body is moving without rotation. So, let us say it is moving with velocity v . Now, this velocity v is in principle the velocity of this point, but by the same vector, I can define the velocity of this point. Also, I can define the velocity of this point by the same vector. So, it does not matter where you put this vector because this body is moving without rotation. So, if a body moves without rotation, then the movement of every point in the body can be described by this vector V . And in principle, this vector V , I can place anywhere in the body.

Let us look at another example. Suppose I have a metal plate and this metal plate is moving between two bars. So, this is bar 1, this is bar 2 and this is the metal plate. Now, the velocity of this point I can define by V and by the same vector I can define the velocity of this point and the velocity of that point. So, therefore, it is in this sense that the velocity becomes a free vector you can place anywhere in the body. Now, let us look at the sliding vector. Suppose I have a rigid body and on this rigid body I am applying a force. So, this point wherein I am applying the force, this is called the point of application. And this line along

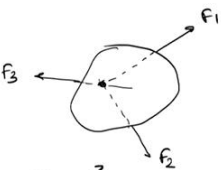
which I am applying the force is called the line of application. So when you apply this force, this force will have the same effect if you apply it at any points along its line of action. So the forces or the force can be applied at any point along its line of action. And this will not change its effect on the body as a whole. Okay, so this force that you are applying over here, this in principle you can apply anywhere. What does this means? That here you are pushing the body and here you are pulling the body. If you just translate this force at this point. So this means that pushing and pulling will produce the same effect. Okay.

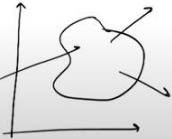
So this tells us a very important point that force is a sliding vector and it can slide along


* force is a sliding vector. slides along the line of action. [Principle of Transmissibility].


② fixed vector \Rightarrow  \Rightarrow has unique point of application.
Ex: forces on non rigid body.

Classification of forces \Rightarrow

① Concurrent force \Rightarrow  \Rightarrow if the line of action of all forces intersect at a common point \Rightarrow concurrent force.

② Coplanar force \Rightarrow  All forces lie in the same plane.

③ Parallel force \Rightarrow  Action lines of the forces are \parallel .

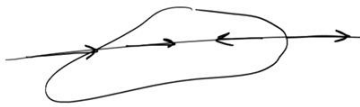


the line of action. This is also called the principle of transmissibility. Now, let us look at the fixed vector. So, this time let us suppose I have a body, but that body is not rigid and you want to know the dynamics of this. So, you apply a force. Now, this force because the body is not rigid, this is going to change the topology of the body at this point. Now, in this case you cannot say that let us apply the force along the line of action at this point because here the body is going to deform at this point. So, therefore, the body itself is going to deform depending upon where you are applying the force. So, in this case when the body is not rigid we cannot translate the force along its line of action and we have to exactly define where we are applying the force. So, that means this case this vector has unique point of application. So, as I said the examples are the forces on let us say non-rigid bodies.

Now, there are various situations in which the force can act on a rigid body. So, let us discuss those situations. So, let us call them the classification of forces. So, these are nothing but different situations in which the forces can act on a rigid body. So, the first one is the concurrent force.

So, let us say I have a rigid body and in this rigid body, the forces are acting on it. Now, if I look at the line of action of them and they meet at a single point, in this case, the forces, so these F_1 , F_2 and F_3 . F_1 , F_2 and F_3 , they become concurrent. So, that means, if the line of action of all forces, they intersect at a common point, then we call them the concurrent force.

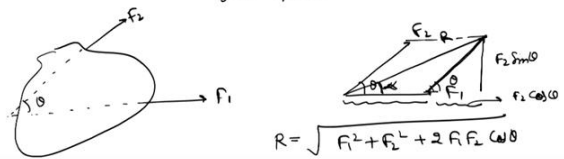
Collinear force →



All the forces have a common action line.

Composition or Resultant of forces →

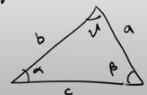
The process of reducing a force system to an equivalent & simplest force system.




$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta}$$

$$\tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

* The relationship b/w angles & length of sides (forces) can also be determined by the sine law.



$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$


Now, we can also have a situation wherein the forces are coplanar. So, in this case, so again we have a rigid body and in this rigid body the forces are acting, but these forces they will act in a plane, okay. So, here all forces they lie in the same plane. Now, there may be a situation where the forces are parallel.

So, again suppose I have a rigid body and in this the forces are acting, but all of them are parallel. So, here the action lines of the forces are parallel. We can also have a situation wherein the forces are collinear. So, in this case, let us say we have a rigid body, in this case all the forces will act in the same line.

So, here all the forces, they have a common x and y. Now, to analyze the forces, we will also need to reduce the forces or sometimes we have to divide the forces into different

components. So, now, let us look at the composition. or the resultant of forces. So, this is a process of reducing a force system to an equivalent and simplest force system.

Let us look at example. So, suppose I have this rigid body and on this rigid body two forces are acting and we want to reduce this to a single force system. So, let us draw F_1 and F_2 and its resultant will be R . So, this is F_2 this is F_1 and let us say they are making an angle θ between them. So, this angle that you see is θ and we want to determine what is R at what angle it is acting.

So, let us say this angle is α . So, this we already know that the resultant R will be $\sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos\theta}$ and the angle that resultant make from F_1 will be $\tan\alpha$ equal to $F_2 \sin\theta / F_1 + F_2 \cos\theta$, okay. Because this one is θ , this one is F_2 , so therefore, this will be $F_2 \sin\theta$. So, $\tan\alpha$ becomes $F_2 \sin\theta$ divided by this distance plus this distance and this one is $F_1 + F_2 \cos\theta$.

So, we get $\tan\alpha$ equal to $F_2 \sin\theta$ over $F_1 + F_2 \cos\theta$. Now, let us also recall the relationship between angles and length of sides. So, which are basically, you know, represented by the forces. They can also be determined by the sign law. So, this is something we know. Suppose, I have a triangle and the lengths are A , B and C and the angles are α , β and γ , then the sine rule is A over $\sin\alpha$ equal to B over $\sin\beta$ equal to C over $\sin\gamma$. So, let us stop this lecture here and let us continue the discussion in the next class. Thank you.