

**Mechatronics**  
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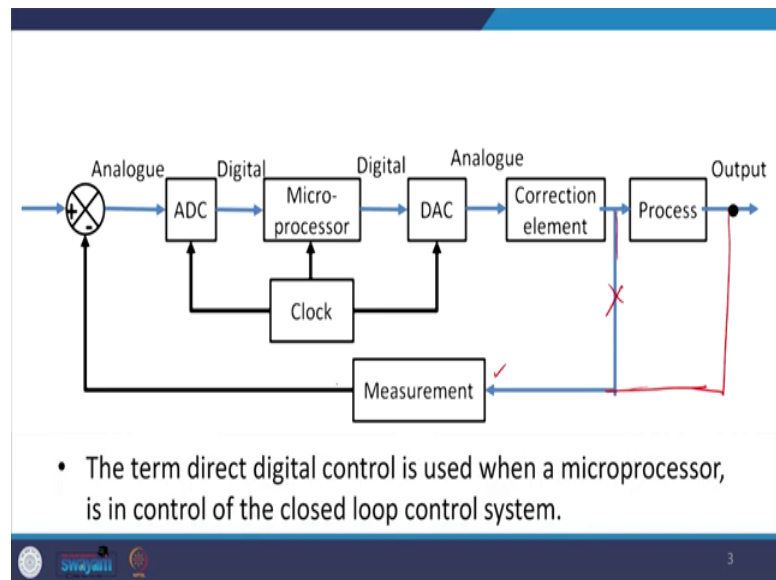
**Lecture - 32**  
**Digital Controllers**

I welcome you all to NPTEL online certification course on Mechatronics. Today we are going to talk about the Digital Controller. In the last lecture, we have seen the analog PID controller different types of PID controllers, and we have seen how its opamp implementation can also be done. So, in this lecture, I am going to discuss the digital controller how digitally we can implement the controller.

In the analog control system, all the signals are analog signals, and digital signals, as we have discussed earlier, are nothing, but they are a sequence of pulses or on-off signals with the value of the quantity being represented by the sequence of on-off signals. Now, most of the signals being controlled are analog, and thus it is necessary with a digital control system to convert these analog signals into a digital signal.

So, these signals can be processed by the microcontroller, and as the processed signal has to be given to the implementing device or rather actuating device, such types of devices are analog devices. The digital signals which are emanating from the microcontroller or microprocessor need to be converted back into the analog signal. So, we need analog to digital converter and digital to analog converter for these types of situations, and I have already discussed dedicated one lecture each for the analog to digital converter and the digital to analog converter earlier. So, I hope that concept will be helpful here in understanding this lecture.

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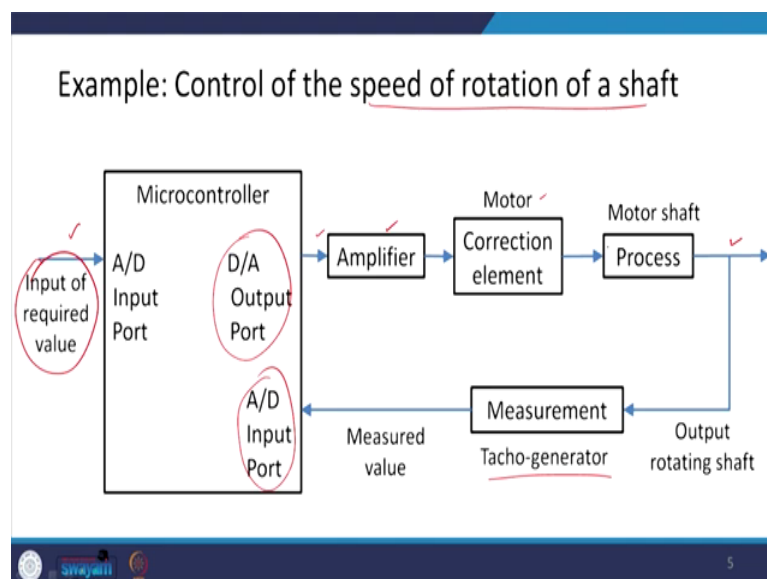
The term digital control, as I said, is used when a microprocessor is in control of the closed-loop control system. So, as you can see in this figure, suppose I have got some output from here, out of the correction element, and we take the measurement, and then this measurement is fed to the comparator from where you have the analog signal. This measured signal is also an analog signal as well a reference signal is also an analog signal. So, you need to have an analog to digital converter over here in order to get the digital signal. Then this digital signal is processed in the microprocessor, and as I said, the microprocessor gives the digital signal, and this again digital signal is converted back to the analog signal by a digital to analog converter, and then this is given to the correction element. Then this correction element does the correction in the process, and you have the output, and here this analog to digital conversion digital to analog conversion and microprocessor activities are clocked. So, this is how this process, or rather the digital control, is implemented.

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- Microprocessors as controllers have the advantage over analog controllers that the form of the controlling action, e.g. proportional can be altered by purely a change in the computer software.
- Here a clock supplies a pulse at regular time intervals and dictates when samples of the controlled variable are taken by the ADC.

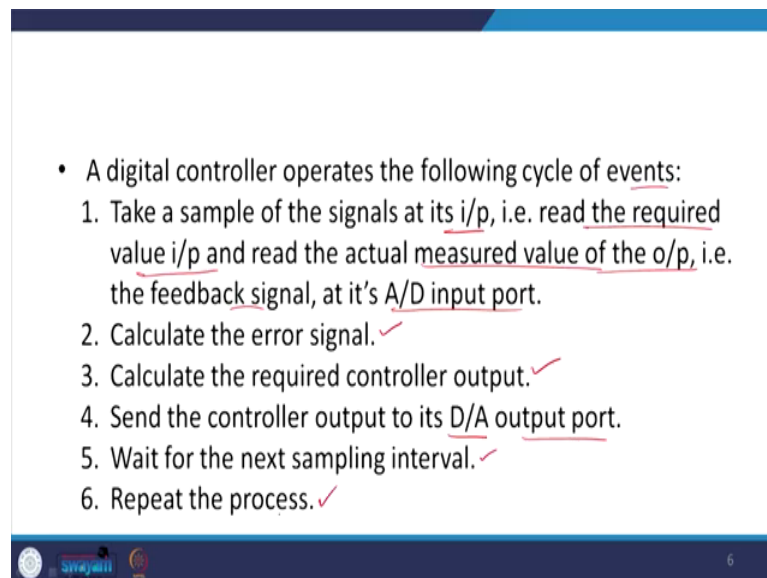
The microprocessor as the controller has the advantage over the analog controller that the form of the controlling action can be purely done with the help of simply changing the expressions in the computer software. One need not change or play with the hardware in order to implement a particular control action, and here clock supplies a pulse at regular time intervals and dictates when a sample of the control variables are taken by the analog to digital converter.

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If I am asked to give an example of that type of digital control system, I can take the case of speed control of the speed of a rotation of a shaft. So, from the motor shaft, I get a certain space or rather let me start from here, suppose I input the required value from here, and it is an analog value. So, in the case of the microcontroller, the microcontroller has in a bit the analog to digital as well as the digital to analog converter in it. There is an analog to the digital input port. So, the port accepts analog signal and convert them into a digital signal. This is the reference value, and from the output of the shaft, the motor shaft, I get the output of the rotating shaft, and with the help of a Tacho-generator, I measure that speed, and that measure speed is again and an analog signal. So, I require an analog to digital converter here. So, that analog signal is fed through the analog to digital input port over here, then you in the microcontroller you find out the error between the two signals and then implement a control scheme, and that control signal is again a digital signal. So, you require a digital to analog converter over here, and this digital to analog converter can give a signal. So, you can take out that signal from here, amplify that particular signal and then give it to the correction unit, which is the motor here, and then the motor will be the shaft of the motor will be rotating, and the cycle will go on. So, this is how the control of the speed of a rotation of the shaft can be done digitally.

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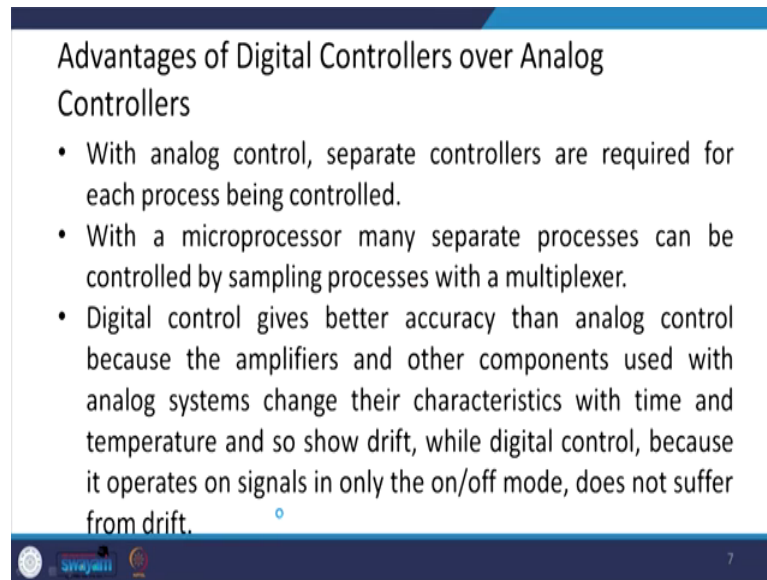


- A digital controller operates the following cycle of events:
  1. Take a sample of the signals at its i/p, i.e. read the required value i/p and read the actual measured value of the o/p, i.e. the feedback signal, at its A/D input port.
  2. Calculate the error signal. ✓
  3. Calculate the required controller output. ✓
  4. Send the controller output to its D/A output port.
  5. Wait for the next sampling interval. ✓
  6. Repeat the process. ✓

A digital controller operates in the following cycle of events. As I said, we take a sample of the signal at its input. That is, I read the required value of the input and read the actual measured value of the output. That is the feedback signal at its analog to the digital input

port and then calculate the error signal and then calculate the required controller output and send the controller output to its digital to the analog output port as I said, wait for the next sampling interval and then repeat the process. So, this is how it can be carried out.

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The slide is titled "Advantages of Digital Controllers over Analog Controllers". It contains three bullet points:

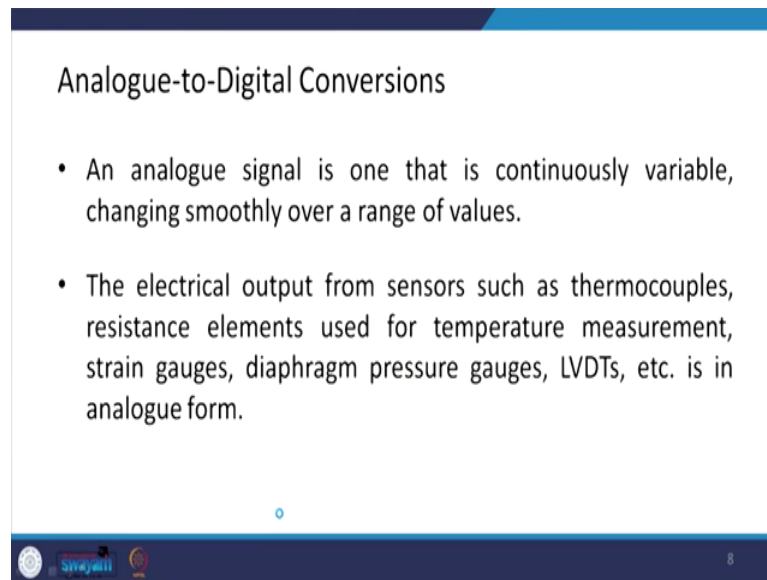
- With analog control, separate controllers are required for each process being controlled.
- With a microprocessor many separate processes can be controlled by sampling processes with a multiplexer.
- Digital control gives better accuracy than analog control because the amplifiers and other components used with analog systems change their characteristics with time and temperature and so show drift, while digital control, because it operates on signals in only the on/off mode, does not suffer from drift.

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The advantage of the digital controller over analog controllers is many. With analog control, separate controllers are required for each process being controlled, but in the case of microprocessor control, many separate processes can be controlled by sampling process with a multiplexer, and you see that multiplexer is a type of switching device.

The digital control gives better accuracy than the analog control because the amplifier and other components used with the analog system change their characteristic with time and temperature and show drift. While with the digital control, because it operates on the signal only in the on-off mode, it does not suffer from the drift type of behavior.

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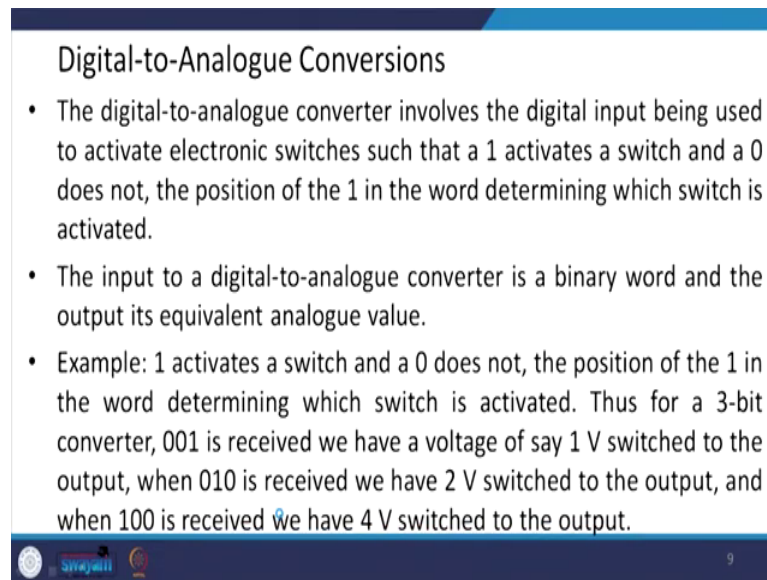
### Analogue-to-Digital Conversions

- An analogue signal is one that is continuously variable, changing smoothly over a range of values.
- The electrical output from sensors such as thermocouples, resistance elements used for temperature measurement, strain gauges, diaphragm pressure gauges, LVDTs, etc. is in analogue form.

Then as I said, we require the analog to digital conversion and the digital to analog conversion for implementing the digital controller. So, I have already devoted one lecture each on how these things are performed, but here I am just incorporating a slide just to for the sake of completeness of all the components.

The analog to digital convertor, as I said, is used to convert the analog signal into a digital signal. Analog signals are usually continuous only variable signals, and the change is smoothly over a range of values, and there are many devices, in fact, many of the sensors are analog sensors and the electrical output from such type of devices such as thermocouple, resistance element used in temperature measurement, strain gauges, diaphragm pressure gauges, LVDT etcetera is in the analog form.

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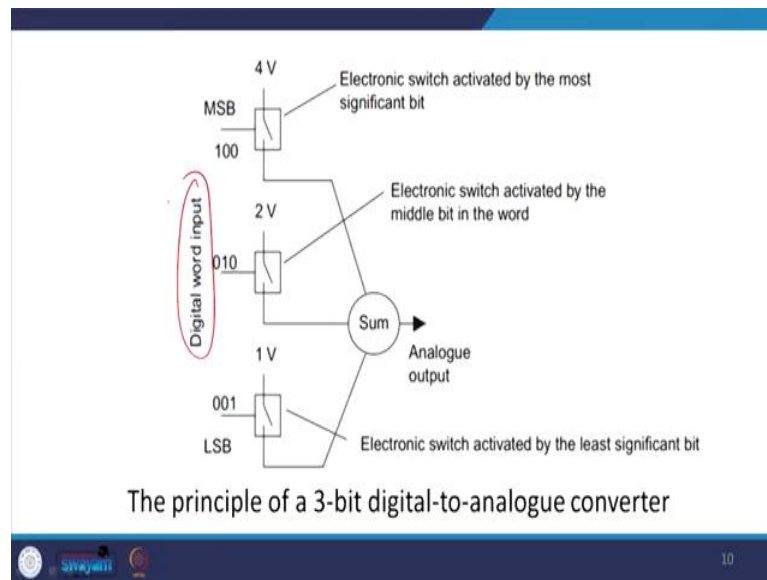


**Digital-to-Analogue Conversions**

- The digital-to-analogue converter involves the digital input being used to activate electronic switches such that a 1 activates a switch and a 0 does not, the position of the 1 in the word determining which switch is activated.
- The input to a digital-to-analogue converter is a binary word and the output its equivalent analogue value.
- Example: 1 activates a switch and a 0 does not, the position of the 1 in the word determining which switch is activated. Thus for a 3-bit converter, 001 is received we have a voltage of say 1 V switched to the output, when 010 is received we have 2 V switched to the output, and when 100 is received we have 4 V switched to the output.

Now, coming to the digital to analog conversion, as I said, digital to analog conversion involves the digital input being used to activate the electronic switches such that 1 activates a switch and 0 does not, and the position of one in the word is determined which switch to be activated. So, again as I said, we have already seen the digital to analog conversion how it is done earlier. The input to a digital to analog converter is a binary word, and the output is an equivalent analog value. So, I can give an example here 1 activates a switch, and 0 does not. As I said, the position of one in the word determined which switch to be activated. So, for a three-bit converter, 001 is if it is received we that a voltage of 1-volt switch to the output that is to be there and if it is 010 is received if we have a 2-volt switch to the output and 100 is received then we have 4-volt switch to the output.

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So, something like this, you can have the digital word input. So, 001 is the least significant bit, and 100 is the most significant bit. What are these least significant and most significant bits? We have already seen these things in the digital circuit lecture. Suppose if your 001 is there. So, it means that the switch corresponding to 1 volt has to be activated and 010 means this corresponding to 2 volts has to be activated and 1 double 0 corresponding to 4 volts has to be activated, and the analog output is the summation of all these. Summation means in-between values. So, this is the principle of 3-bit digital to analog converter. Next, let us look at how do we implement these control modes.

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### Implementing Control Modes

- To produce a digital controller which will give a particular mode of control it is necessary to produce a suitable program for the controller.
- The program has to indicate how the digital error signal at a particular instant is to be processed in order to arrive at the required output for the following correction element.
- The processing can involve the present input together with previous inputs and previous outputs.

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So, to produce a digital controller which gives a particular mode of control, it is necessary to program or produce a suitable program for the controller. The program has to indicate how the digital error signal at a particular instant is to be processed in order to arrive at the required output for the following correction elements all right the processing can involve the present input together with the previous inputs and the previous outputs.

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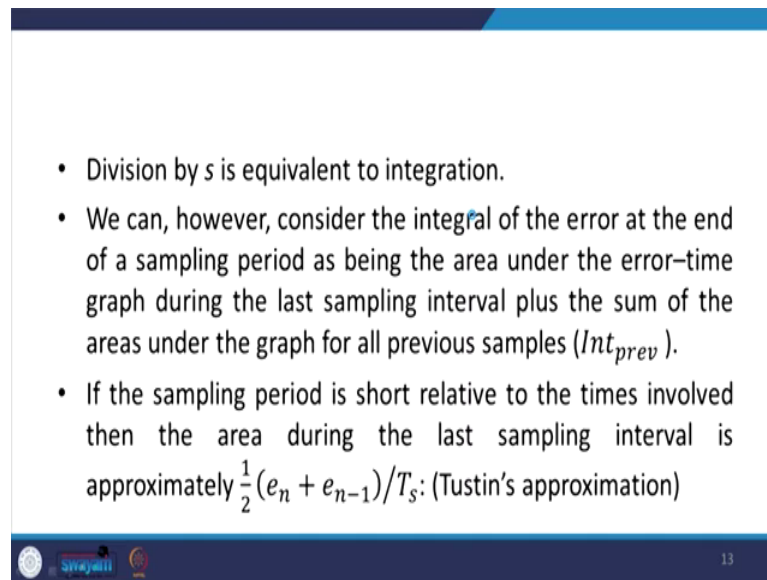
- The transfer function for a PID analogue controller is

$$\text{transfer function} = K_p + \frac{1}{s}K_I + sK_D$$

- Multiplication by  $s$  is equivalent to differentiation.
- We can, however, consider the gradient of the time response for the error signal at the present instant of time (latest sample of the error  $e_n$  minus the last sample of the error  $e_{n-1}$ )/(sampling interval  $T_s$ ).

So, the transfer function for a PID analog controller is what we have seen. If we look at the differentiation, then we can consider the gradient of the time response for the error signal at the present instant of time that is the (latest sample of error  $e_n$ ) – (the last sample of error that is  $e_{n-1}$ )/(the sampling time interval  $T_s$ ).

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- Division by  $s$  is equivalent to integration.
- We can, however, consider the integral of the error at the end of a sampling period as being the area under the error-time graph during the last sampling interval plus the sum of the areas under the graph for all previous samples ( $Int_{prev}$ ).
- If the sampling period is short relative to the times involved then the area during the last sampling interval is approximately  $\frac{1}{2}(e_n + e_{n-1})/T_s$ : (Tustin's approximation)

Now, and the division by  $s$ , as I said, is equivalent to the integration, we can, however, consider the integral of the error at the end of a sampling period as being area under the error time graph during the last sampling period.

Last sampling interval plus the sum of the area under the graph for all previous samples integration of previous ( $Int_{prev}$ ) and if the sampling period is short relative to the time involved, then the area under the last sampling period is approximately  $\frac{\frac{1}{2}(e_n + e_{n-1})}{T_s}$ . This is also called as Tustin's approximation.

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- Thus the controller output  $x_n$  at a particular instant the equivalent of the transfer function as

$$x_n = K_p e_n + K_I \left( \frac{(e_n + e_{n-1})T_s}{2} + Int_{prev} \right) + K_D \frac{(e_n - e_{n-1})}{T_s}$$

$$x_n = A e_n + B e_{n-1} + C (Int_{prev})$$

Where  $A = K_p + 0.5K_I T_s + K_D/T_s$ ;  
 $B = 0.5K_I T_s - K_D/T_s$  and  $C = K_I$

So, the controller output action at a particular instant the equivalent of the transfer function can be written as this.  $\frac{1}{2} \frac{(e_n + e_{n-1})}{T_s} + Int_{prev}$  corresponds to the proportional control and corresponds to the integration of the errors, and  $((e_n) - (e_{n-1})) / (T_s)$  corresponds to the derivative of the error it's equivalent.

Now, I can simplify this,

$$x_n = A e_n + B e_{n-1} + C (Int_{prev})$$

The constant A, B, and C can be defined like this.

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• The program for PID control thus becomes:

1. Set the values of  $K_p$ ,  $K_i$  and  $K_d$ .
2. Set the initial values of  $e_{n-1}$ ,  $Int_{prev}$  and the sample time  $T_s$ .
3. Reset the sample interval timer.
4. Input the error  $e_n$  and Calculate  $x_n$ .
5. Update, ready for the next calculation, the value of the previous area to  $Int_{prev} + 0.5(e_n + e_{n-1})T_s$ .

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So, the program of PID control thus becomes we set the value of  $K_p$ ,  $K_i$ , and  $K_d$  and then set the initial values of  $e_{n-1}$ ,  $Int_{prev}$  and  $T_s$ . Then reset the sample interval timer, input the error and calculate the action. So, this is done and updated, ready for the next calculation. The values of the previous area are this one  $Int_{prev} + 0.5((e_n) - (e_{n-1}))/ (T_s)$ .

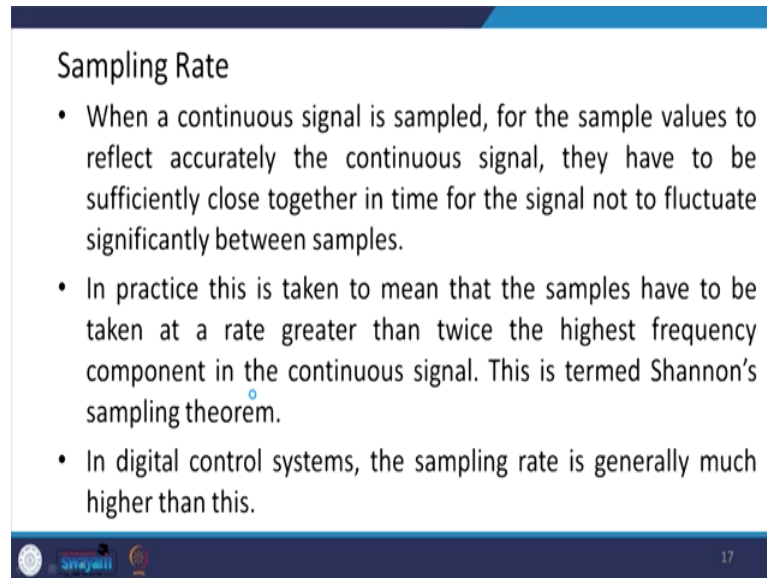
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6. Update, ready for the next calculation, the value of the error by setting  $e_{n-1}$  equal to  $e_n$ .
7. Wait for the sampling interval to elapse.
8. Go to step 3 and repeat the loop.

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So, then update ready for the next calculation the value of the error by setting  $(e_n) = (e_{n-1})$ , and then we go for the next one. So, wait for the sampling interval to elapse and then go to step 3 and repeat the loop. So, this is how the PID control can be implemented.

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**Sampling Rate**

- When a continuous signal is sampled, for the sample values to reflect accurately the continuous signal, they have to be sufficiently close together in time for the signal not to fluctuate significantly between samples.
- In practice this is taken to mean that the samples have to be taken at a rate greater than twice the highest frequency component in the continuous signal. This is termed Shannon's sampling theorem.
- In digital control systems, the sampling rate is generally much higher than this.

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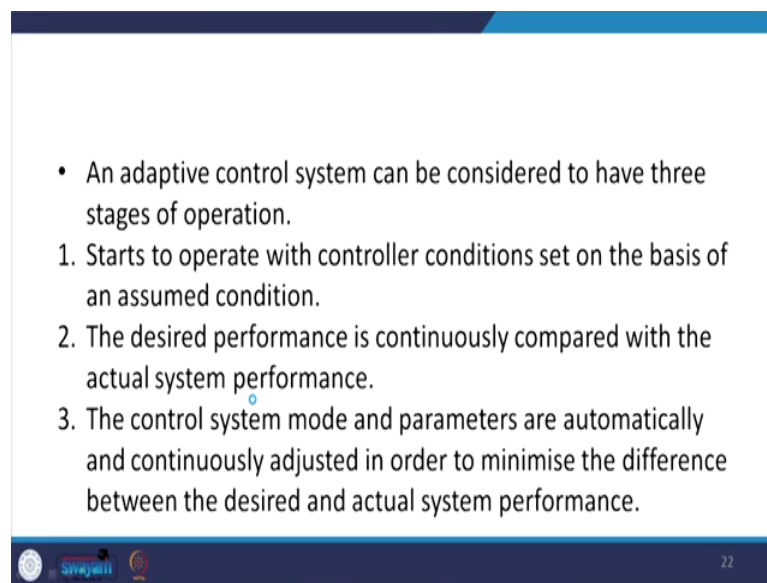
Now sampling rate what at what rate the sampling has to be carried out we have already seen Shannon's sampling theorem that whatever be the highest frequency in the signal, you have to take the twice of that highest frequency you for the sampling. So, when a continuous signal is sampled for the sample values to reflect the continuous signal accurately, they have to be sufficiently close together in time for the signal not to fluctuate significantly between the samples, and in practice, this is taken to mean that the sample has the taken at a rate greater than the twice of the highest frequency component in the continuous signal. In a digital control system, the sampling rate is generally much higher than this one. Now next, let us look at the computer control system. So, in the case of a computer control system, the setpoint and the control parameter are entered from the keyboard. So, the computer control system can be explained using the diagram for the digital control system, which we have seen at the beginning of this lecture. The same diagram can be explained for that the only thing is here we set the values with the help of keyboard the software for use with the system provides the program of instructions needed. For example, for the computer to implement the PID control, they provide the operator with display, recognize the process, recognize and process the instruction inputted by the operator, provide information about the system, provide startup and shutdown instruction,

and supply clock/calendar information. So, all these are there in a computer control system, and an operator display is likely to show that is such information as what is the said point value or what we call as the reference value, what is the actual measurement value, what is the sampling interval, what is the error and the controller setting and state of the correction elements. So, all these displays are going to be there in the case of the computer control system. This display is likely to be updated every few seconds. Then the question comes that how do we get these parameters that are  $K_P$ ,  $K_I$ , and  $K_D$ . So, how do we select these parameters? They are proportional gain, derivative gain, and integral gain, and this is what is usually called as controller tuning. Many researchers what they do is that they do manual tuning. That is, they look at the system performance by changing these gain values. And then select the gain values corresponding to the best performance of the system and what the system performance has to be that we have already seen in case of the performance of performance parameter for the system such as a steady-state error (Refer Time: 22:16) settling time, maximum overshoot and all. With a proportional controller, it means that we need to select the value of  $K_P$  with a PID controller, we need to select the value of proportional gain  $K_P$ , derivative gain  $K_D$  and integral gain  $K_I$ . And there is a number of methods for doing this. So, the method by Ziegler and Nichols assumed that when the control system is in an open-loop, a reasonable approximation to its behavior is a first-order system with a built-in time delay, and based on this, they derived the parameters for the optimum performance. Then there could be a situation when we are talking about the control of a mechatronic system, where these gain parameters keep on changing. For example, I take a situation where the parameter of a plant changes with time. Like a robot manipulator is used for moving a load and when the load changes.

Suppose a robot is having a glass of water and the glass of water is a the from the glass of water the water is being poured. So, what is happening here? The load at the end-effector is changing. So, that type of situation is there. So, in such type of situation, these gain parameters need to be changed corresponding to the change situation. So, if your load has changed, the gain parameter has to be changed. But the continuous changing of these gain parameters is not easy. As I said, usually we go with the estimation of, or rather we select the gain parameter based on the certain estimation, and it is assumed that this gain parameter is kept constant. So, what happens? As the parameter has to be changed because of the load change, such a system needs to adapt to these changes, and it has to fit the circumstances prevailing. So, the adaptive control system is based on the use of a

microprocessor as the controller, and such devices enable the control mode and the control parameter used to be adopted to fit the circumstances and modify them as the circumstances changes and of. So, an adaptive control system can be considered to have three stages of operation. First, the start to operate with the controller condition, set on the basis of the assumed condition for which we have the parameter, the desired performance is continuously compared with the actual system performance, and the control system mode and parameter are automatically and continuously updated or adjusted in order to minimize the difference between the desired and the actual system performance.

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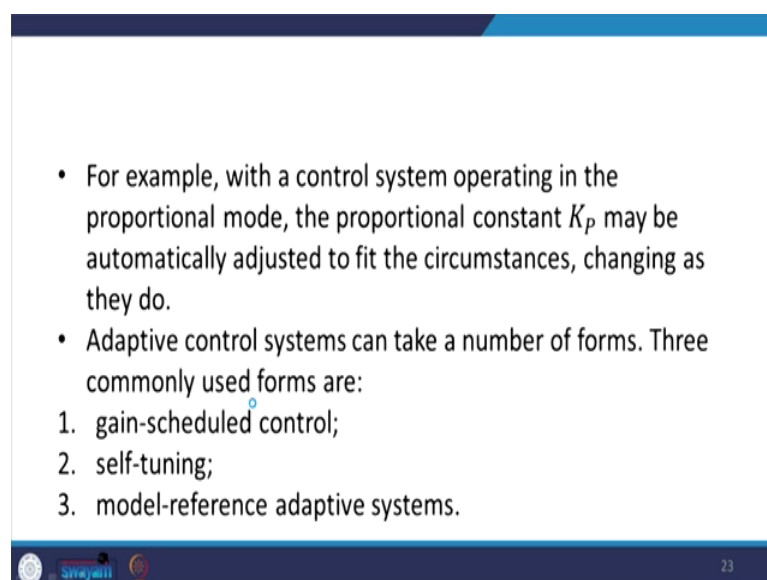


Slide 22 contains a bulleted list describing the three stages of an adaptive control system. The text is as follows:

- An adaptive control system can be considered to have three stages of operation.
  1. Starts to operate with controller conditions set on the basis of an assumed condition.
  2. The desired performance is continuously compared with the actual system performance.
  3. The control system mode and parameters are automatically and continuously adjusted in order to minimise the difference between the desired and actual system performance.

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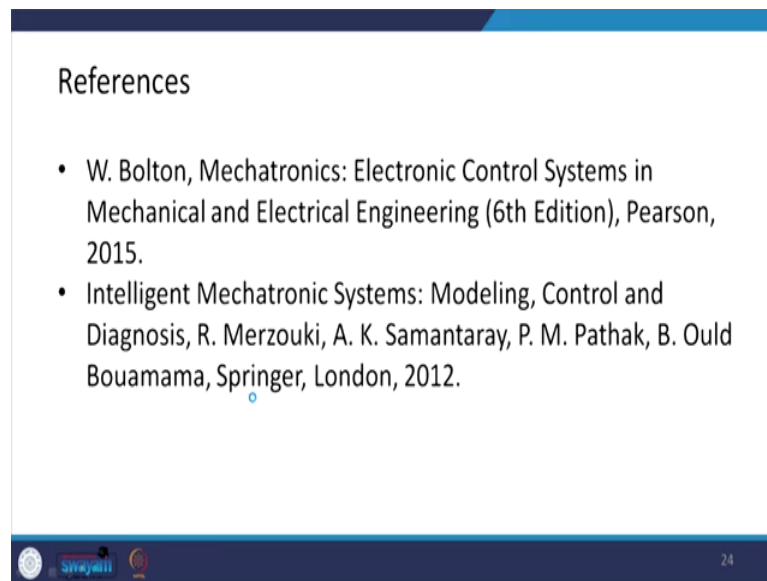
Slide 23 contains a bulleted list describing the forms of adaptive control systems. The text is as follows:

- For example, with a control system operating in the proportional mode, the proportional constant  $K_p$  may be automatically adjusted to fit the circumstances, changing as they do.
- Adaptive control systems can take a number of forms. Three commonly used forms are:
  1. gain-scheduled control;
  2. self-tuning;
  3. model-reference adaptive systems.

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For example, if there is a control system that is operating in a proportional control mode, we may keep on changing the  $K_P$  value so that it fits the circumstances, and as the changes are taking place, there could be a number of forms for such adaptive control. The three common use forms are gain scheduling control, self-tuning, and model reference adaptive system. If you want to see further detail of these methods for adaptive control, you can refer to the book by Bolton.

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So, these are the references by Bolton Mechatronics. You can also refer to our book for some of the material discussed in this lecture.

Thank you.