Mechatronics
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Lecture - 31
Controllers

I welcome you all to NPTEL online certificate course on Mechatronics in this lecture on Controllers. So, today we are going to talk about controllers. Specifically, I am going to talk about the proportional controller, derivative controller, and PID controller. Before we go into the controller, let us have a look at the concept of open-loop versus closed-loop control.

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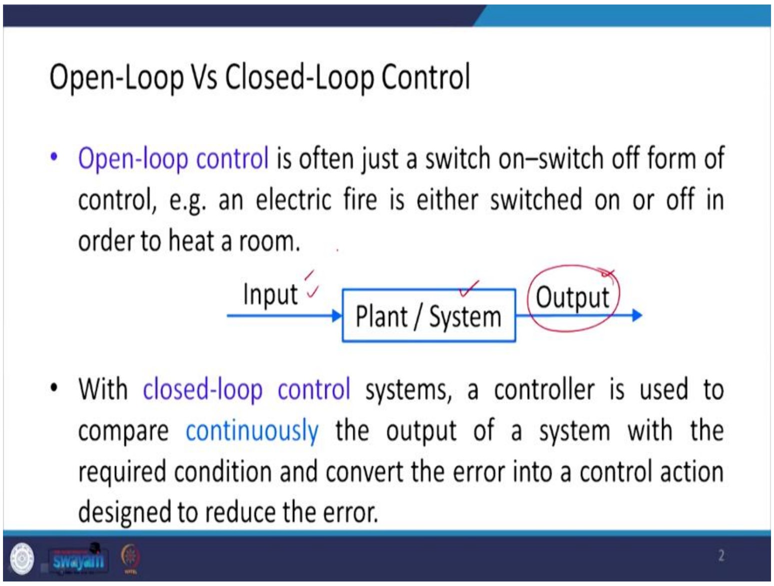

Open-loop control is often just a switch on-off switch form of control; for example, if there is a room and you want to control the temperature of the room, you may switch on or switch off the heater of the room. For such a system, you have a plant or a system is there, and you have an output, the desired output, the actual temperature, and input is the desired temperature in this case. Now, this desired temperature may be a function of how many heaters you have to switch on, what wattage capacity, and so on. The other type of control is the close loop control, where a controller is used to compare the output of a system continuously with the required condition and convert the error into a control action designed to reduce the error.

So, in the open loops control system, we can see that there is no role of the actual output in deciding the input over here. There is no continuous feedback, whereas, in the case of a closed-loop controller, there is a provision of continuous feedback.

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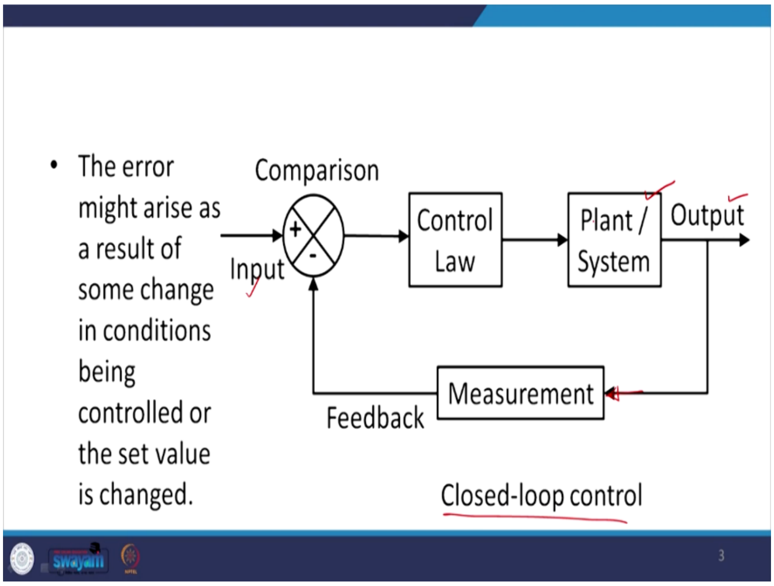

This figure explains the concept of the closed-loop control, where we have a plant or system; there is certain output from that plant and system, and there is a measurement device that measures this output. And this the out measured output is compared with the reference input in a comparator, and then it is given to the control unit, where the control law is there. The error is fed to that control unit, and the control unit gives the control action to the plant. So, these types of systems are called the closed-loop system or called the error-driven system. So, it is the error that drives the system, and the intent of such a control system is to minimize that error. Now, this error might arise as a result of some change in the condition being controlled or the set value is changed. The term direct digital control is used when the computer is there in the feedback loop. So, I will be devoting one complete lecture on the digital control that is the next lecture here.

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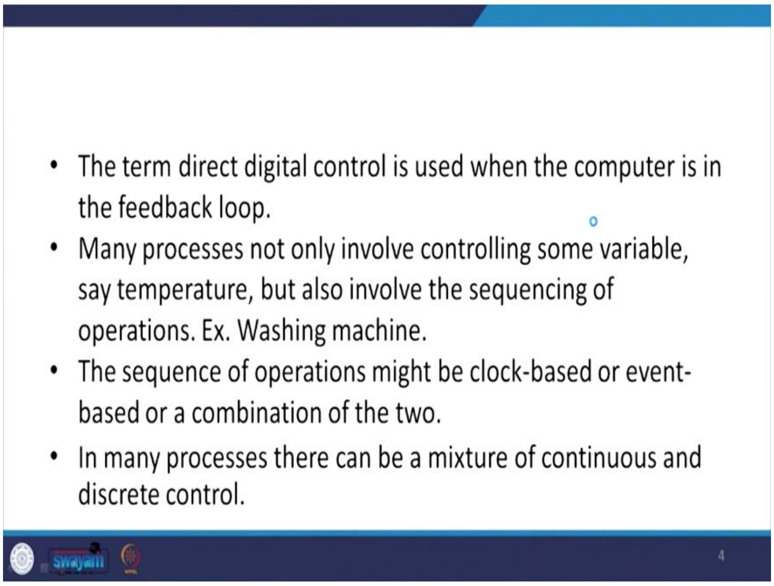

Now, there are many processes not only involve controlling some variable's temperature; but also involve in the sequencing of the operations, for example, washing machine, where one needs to follow a definite sequence because after cleaning, only your drawing operation could be there. Here you need to have a certain sequencing also. Now, the sequence of operation might be clock-based, event-based, or a combination of two. The clock-based means it is dependent on time and, event-based means it is dependent on an event to happen; that is the next task will be completed only if some earlier task has been completed. This is what we mean by event-based. Or you could have a combination of both types. In many processes, there can be a mixture of continuous and discrete control.

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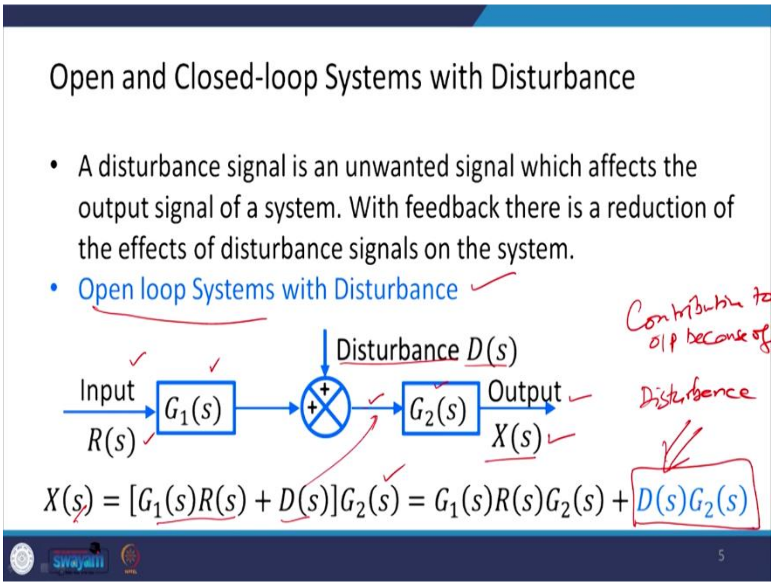

Now, let us look at what happens if you have an open-loop and closed-loop system and there is a certain disturbance in it. A disturbance signal is an unwanted signal which affects the output signal of a system, and with feedback, there is a reduction of the effect of the disturbance signal on the system. Now, let us first look at an open-loop system with the disturbance, open-loop system.

So, suppose you have an input given by Rs over here. There is the block with the transfer function $G_1(s)$; you have output here given by X(s), and here there is another block $G_2(s)$, whose transfer function is given by $G_2(s)$. And here, in between both these, there is a disturbance D(s). So, this is being summed up. This disturbance is being added up in the signal, which is coming from $G_1(s)$.

So,

$$X(s) = [G_1(s)R(s) + D(s)]G_2(s) = G_1(s)R(s)G_2(s) + D(s)G_2(s)$$

So, you can see that this is the term that is contributing because of the disturbance. So, this is the contribution, contribution to output because of disturbance.

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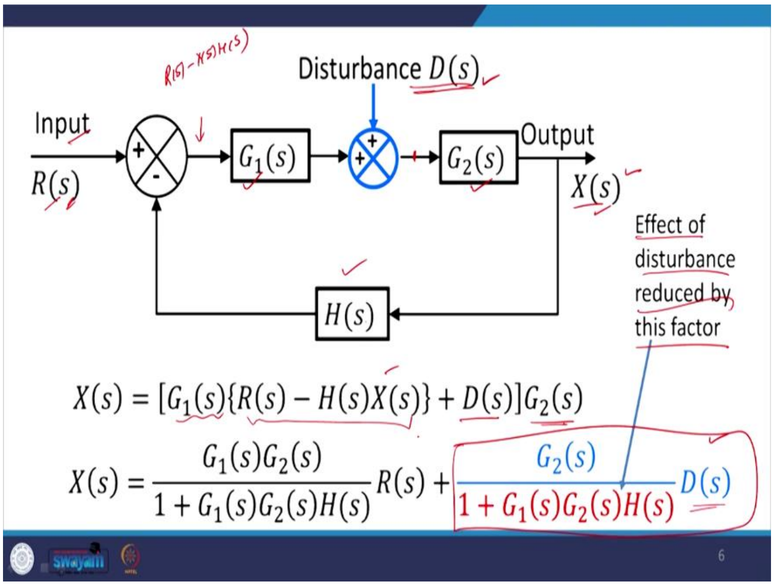

Now, let us look at a closed-loop system with a disturbance. So, again here, I have got the first block with transfer function$G_1(s)$, the second block with transfer function$G_2(s)$, and in between, I have got the disturbance D(s) over here. Here is the X(s), and this is the feedback, and there is a transfer function in the feedback path, that is, H(s), and then this is sent for comparing with the reference inputs R(s).

So,

$$X(s) = [G_1(s)\{R(s) - H(s)X(s)\} + D(s)]G_2(s)$$

$$X(s) = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}R(s) + \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}D(s)$$

If I compare this with that of the open-loop case. So, in open-loop it was $D(s)G_2(s)$, and here you see that this is $\frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}D(s)$. So, the effect of disturbance is reduced by this factor in the case of the closed-loop control system. So, we can see that the closed-loop control system is able to reduce the disturbance.

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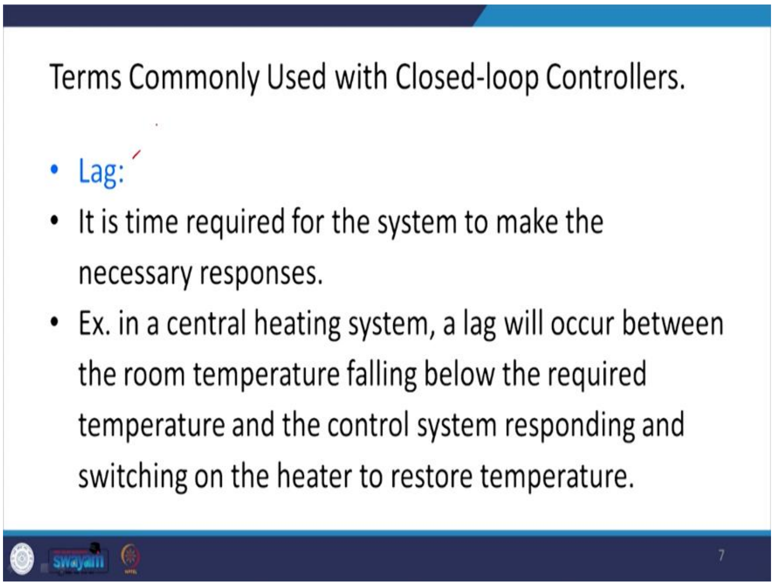

There is the term commonly used with closed-loop controllers is a lag; lag is a very common term which is used, and it is the time required for the system to make the necessary responses. So, for example, in a central heating system, a lag will occur between the room temperature falling below the required temperature and the control system responding and switching on the heater to restore the room temperature. This process does not take place immediately, so there is going to be a certain amount of lag in it.

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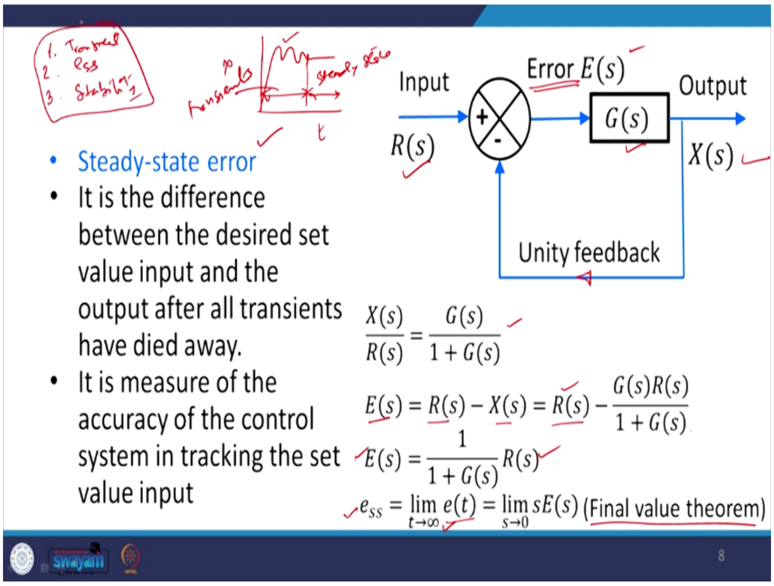


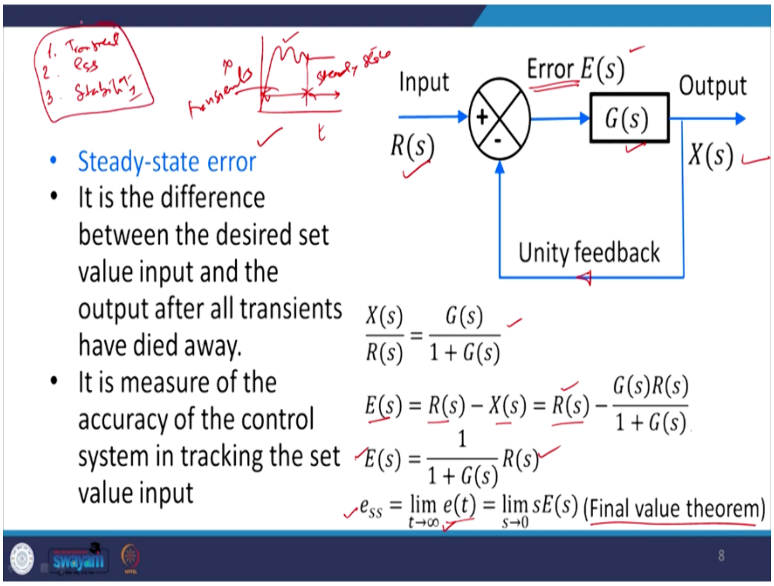
The next thing is letting us see the steady-state error. As the name indicates, the error occurs when there is a steady state. So, it is the difference between the desired set value input and the output after all transients have died away.

This is about a steady state when you plot output versus time. So, if all your transients have died away, then this state is what we call the transient state, and this state is steady, and this is the transient. When all the transients have died away, that state is what we call the steady-state, and it is the measure of the accuracy of the control system in tracking the set value input. So, suppose I have got a control system here with the reference input R(s) and output X(s). It is a unity feedback control system.

So, you have unity feedback here, and the transfer function of the feed forward-path is a G(s) over here, and here is the error. So, the error will be the (input – output). So, for this system, as we have seen earlier, the transfer function is,

$$\frac{X(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

The general expression is, $\frac{G(s)}{1+G(s)} H(s)$, where Hs is the feedback, but we are talking about the unity feedback here.



Now, if I look at what is the value of the error,

$$E(s) = R(s) - X(s) = R(s) - \frac{G(s)R(s)}{1 + G(s)}$$

Now, if we want to find out the steady-state error in the time domain, using final value theorem the steady-state error is going to be,

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

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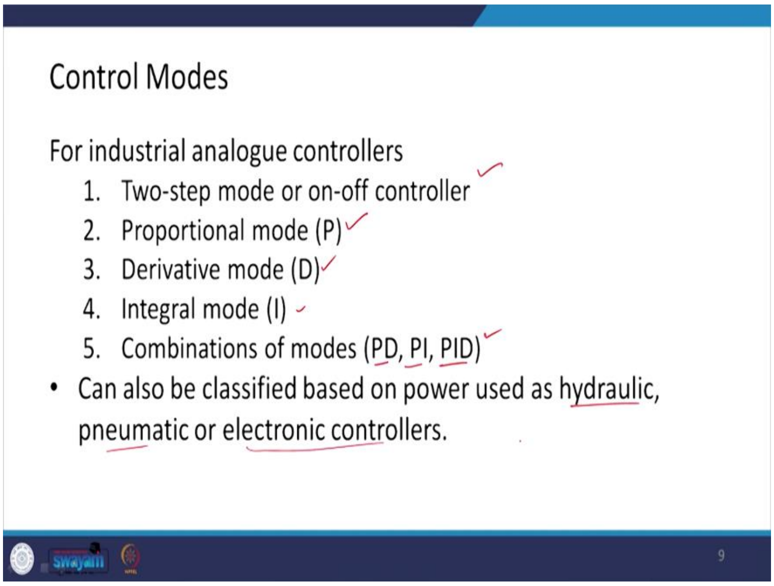

Next, let us look at the various type of control modes. So, now, here, the use of these control modes is to work for the three things; that is, any control system is designed to look after the three aspects. One is to improve the transient behavior, the second thing is to reduce the steady-state error, and the third thing is about the stability to make the system stable. So, in any control system, we look at improving upon these three factors. The control modes for industrial analog controllers various possibilities are the two-step mode or what we call the on-off controller; then we have the proportional mode, proportional controller derivative mode, derivative controller integral mode, or integral controller and combination of modes, that is we could also have the combination of this. That is, the proportional derivative, proportional integral as well as proportional integral derivative. So, these could be the three combinations possible. We can these can also be classified based on the power used as a hydraulic controller, pneumatic controller, or electronic controller.

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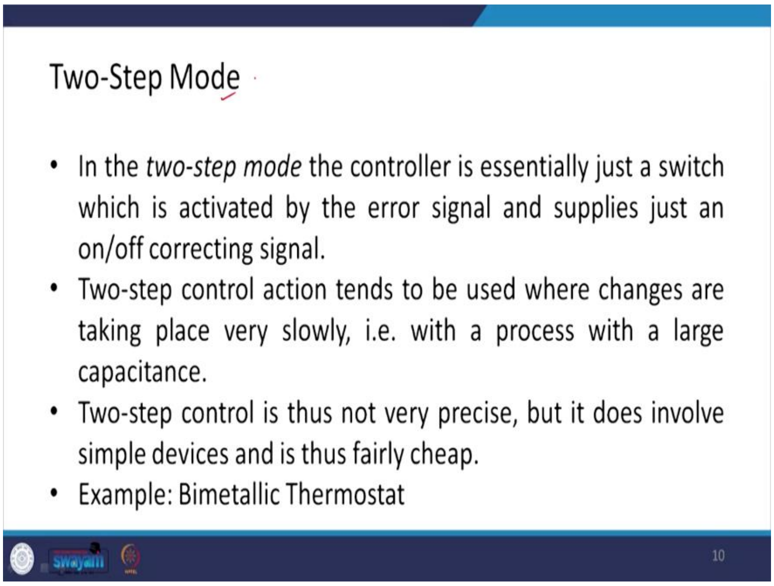

Now, let us look at, first of all, the two-step mode. In the two-step mode, the controller is essentially just a switch, which is activated by the error signal and supplies just an on-off correcting signal. The two-step control action tends to be used, where changes are taking place very slowly; that is, with a process with a large capacitance. The two-step control is thus not very precise, but it does involve simple devices and thus fairly cheap, for example, our bimetallic thermostat.

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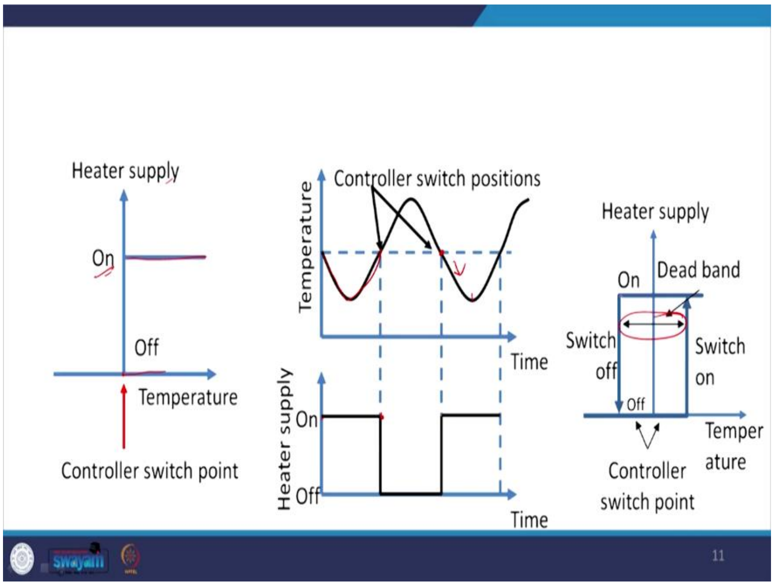

So, here you can see that we have the heater supply. It's on condition is this one, and this one is the off condition, and the controller switch point is this one. So, you can see that here the controller could be on, at this position, if your temperature is falling, so if your temperature is falling, as you can see over here.

So, you could switch on the heater supply then temperature, and then after some time, the temperature starts increasing. So, you can switch off the heater supply over here. And again at this position when the temperature starts falling, then again you may like to switch on the heater, so that again the after some time, the temperature starts increasing. This is what is this on-off control is. And here, as I was telling you, there is a dead band; that is, after switching on, it takes some time, it takes much time for the action to take place.

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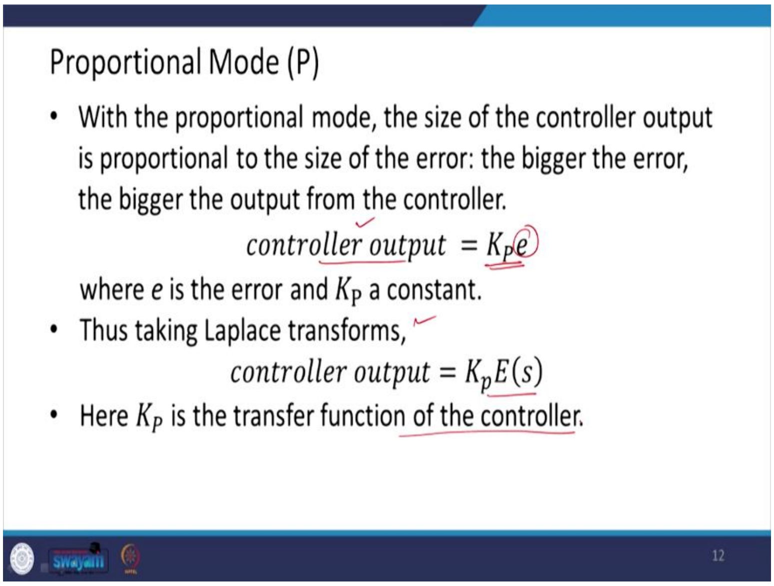

Next is the proportional mode; with the proportional mode, the size of the controller output is proportional to the size of the error. So, the bigger the error, the bigger the output from the controller is. So, in this case, the controller output is $K_p e$. And so, if your error is more, your controller output is more. So, that is there. And if you want to reduce the error, as you can see from this equation, the K_p-value has to be increased. But you cannot go for increasing the K p-value to any limit; because a very high value of K p makes your system unstable, and that is also not good for a system. As I discussed with you, the three essential criteria which we are looking for in any control system are transient behavior, steady-state error, and stability.

So, naturally, we do not like to go for a very high value of K_p. It is a constant called proportionality or proportional gain rather. So, if I take the Laplace to transform for this, this is going to be K_p times$E(s)$, and the K_p is the transfer function of the controller.

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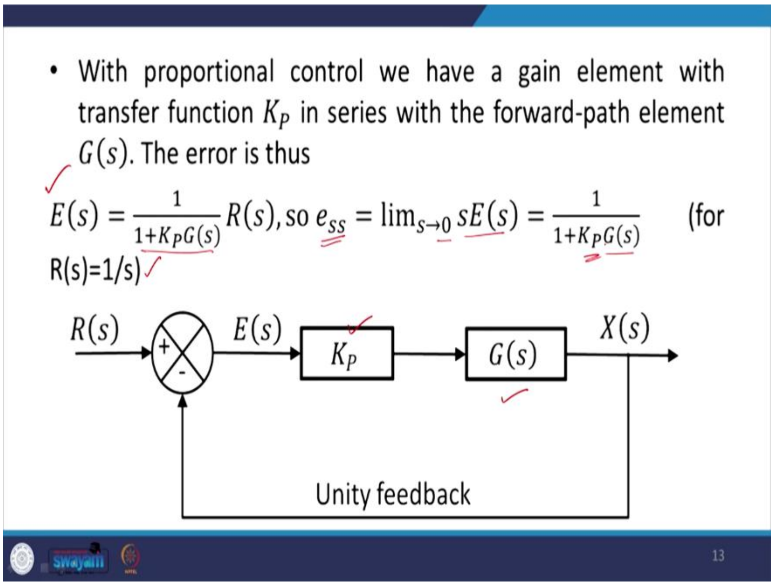

With the proportional control, we have a gain element with transfer function K_p in series with a forward path element G(s). So, as you can see here, so, this is the proportional controller over here. So, in this case, we have, we can see that my error is,

$$E(s) = \frac{1}{1 + K_p G(s)} R(s)$$

So, if I am looking for the steady-state error, this is going to be,

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{1}{1 + K_p G(s)}$$

If I am taking R(s) = 1 /s as the input, that is, I am taking the step function as the input. So, this is going to be the steady-state error. So, what does this means? This means that in your system, which is proportional control, you are always going to have a steady-state error. With the increase in value of K_pYou can reduce this steady-state error to some extent, but you cannot eliminate it to 0. For eliminating it to 0, we need to put up or combine this controller along with another controller that is the integral control, which I will be discussing with you in the coming slides.

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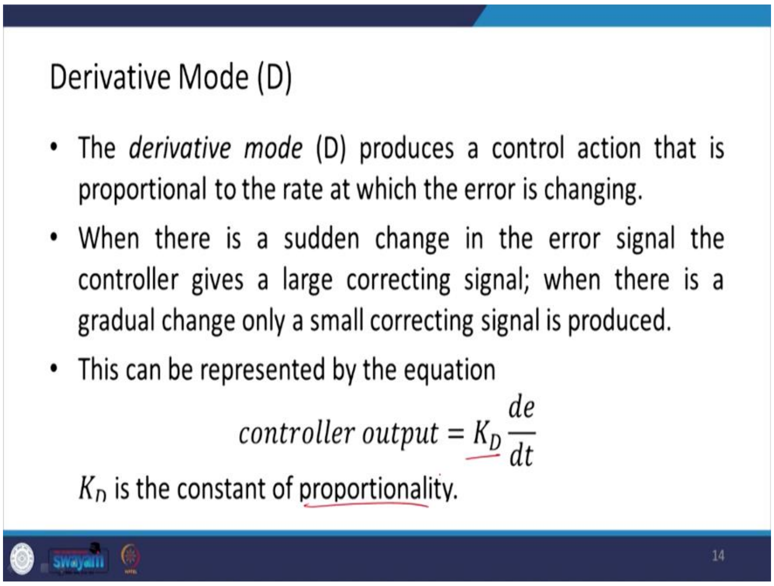

Now, next, let us look at the derivative mode. The derivative mode produces a control action that is proportional to the rate at which the error is changing. That is,

$$\text{Controller output} = K_D \frac{de}{dt}$$

It is proportional to $\frac{de}{dt}$, and here K_D is the constant of proportionality. And here, so, when there is a sudden change in the error signal, the controller gives a larger correcting signal. When there is a gradual change, only a small correcting signal is produced. This is what is meant by derivative control.

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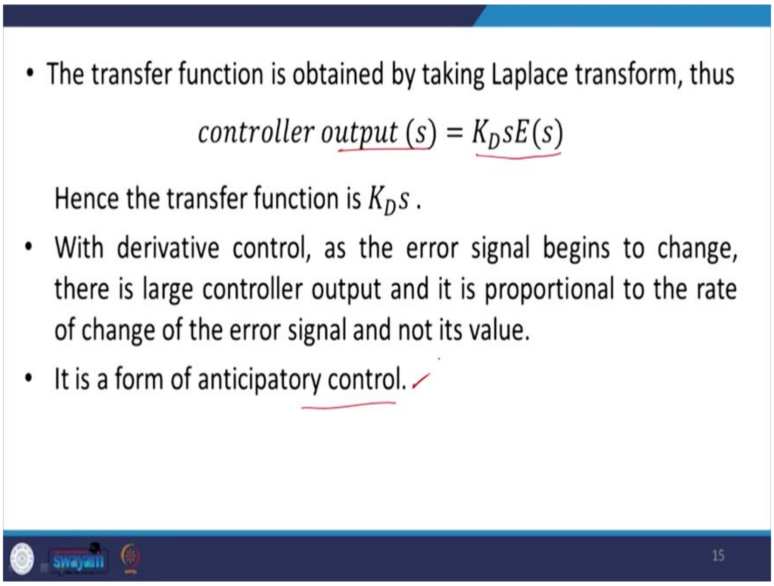

So, the transfer function is going to be,

$$\text{Controller output (s)} = K_D sE(s)$$

Now, with derivative control, as the error signal begins to change, there is a large controller output, and it is proportional to the rate of change of the error signal and not its value. And it is some form of what we call the anticipatory control, that is one that anticipates what is going to happen there in the future.

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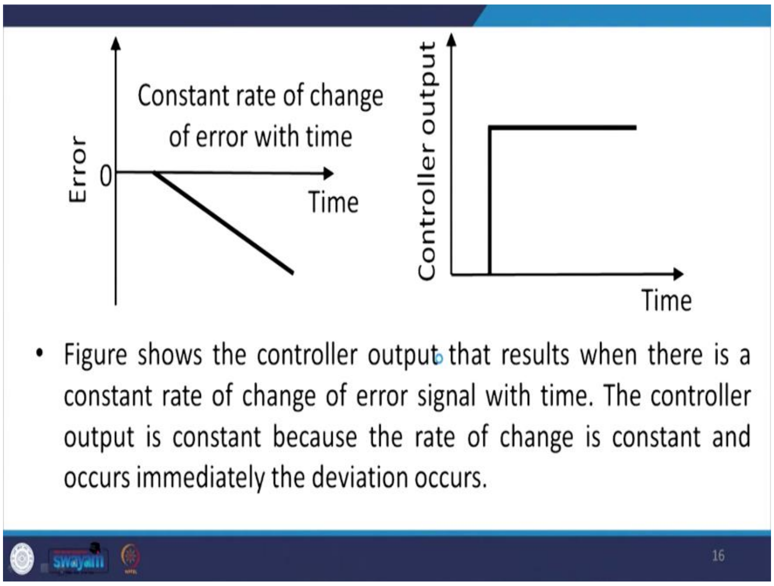

Here you can see that if there is a constant rate of change of error with time, as you can see by the constant slope for this one. So, you are going to have a controller action like this. So, the, as the controller output that results when there is a constant rate of change of error signal with time and the controller output is constant, because the rate of change is constant and that occurs immediately, the deviation occurs over here.

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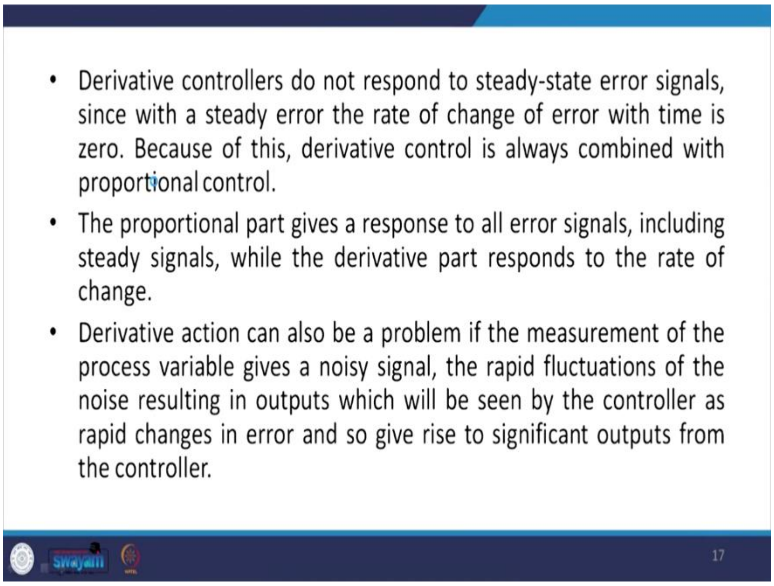

Now, the derivative controller does not respond to the steady-state error signal. Why? Because you see, if a steady-state error signal that value is going to be there, then $\frac{de}{dt}$ will be zero because your error has become constant in the steady-state. So, in that case, you are not going to have any control action. So, with the steady-state error, the rate of change of error with time is zero, and because of this, the derivative control is always combined with the proportional control. So that when the steady-state error is reached, the proportional control takes over. So, usually, the role of the derivative control is to improve the transient behavior, and for the steady-state portion, the proportional control usually takes over. So, the proportional part gives a response to all error signals, including the steady signal, while the derivative part responds to the rate of change, and that is what is happening in the transient zone.

Now, this derivative action can also be a problem if the measurement of the process variable gives a noisy signal. In a mechatronic system, as I said, we have sensors to measure the signal values. And if there is a noisy signal, then the derivative action can give you a problem. The rapid fluctuation of noise results in the output, which will be seen by the controller as a rapid change in the error and so give rise to the significant output from the controller.

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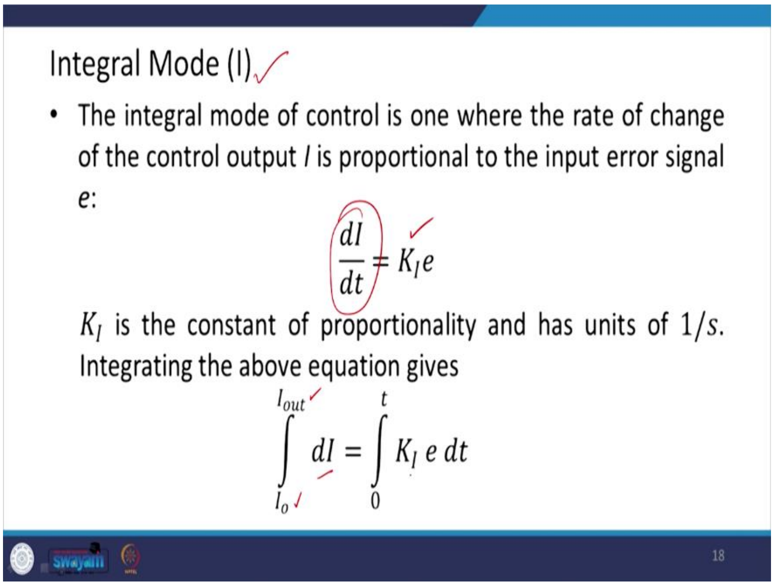

Next, let us look at the integral mode. Now, the integral mode is of control is one, where the rate of change of control output is proportional to the input error signal. So, is this is,

$$\frac{dI}{dt} = K_I e$$

This K I is the constant of proportionality and has the unit of 1/ s. Integrating the above equation gives you,

$$\int_{l_o}^{l_{out}} dI = \int_0^t K_I e\, dt$$

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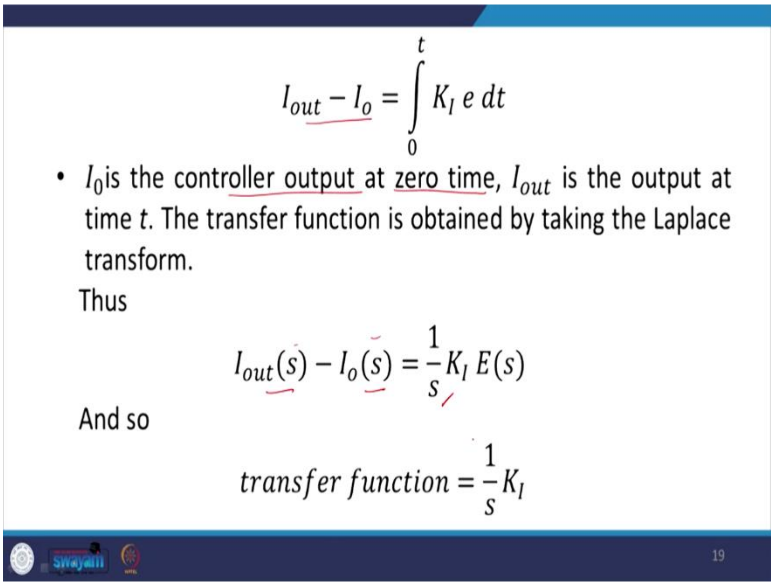

So, you have,

$$I_{out} - I_o = \int_0^t K_I e dt$$

Here I_o is the controller output at zero time, and I_{out} is the output at time t. And the transfer function is obtained by taking the Laplace of this. So,

$$I_{out}(s) - I_o(s) = \frac{1}{s} K_I E(s)$$

So, the transfer function for this controller = $\frac{1}{s} K_I$.

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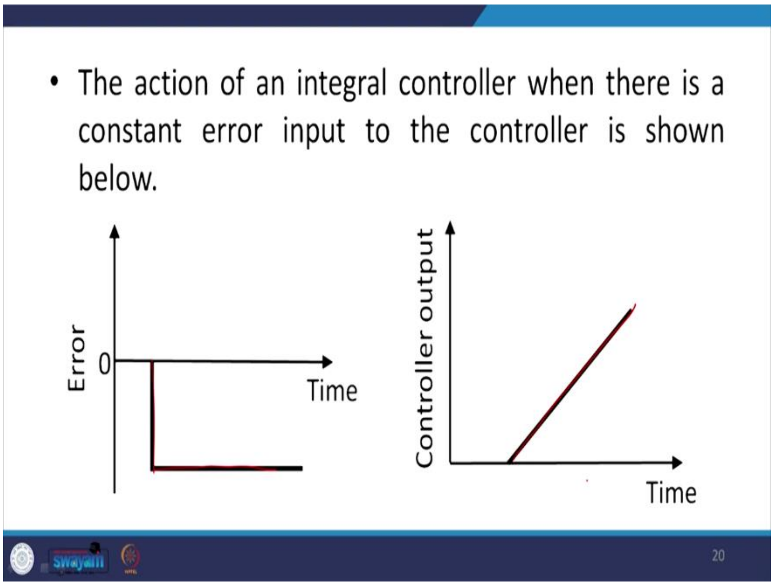

The action of an integral control when there is a constant error input to the controller is like this here. So, if you have a constant input error here, then the controller output is going to be like this. So, this control is very much useful, as I indicated to you earlier; if there is a steady-state error or a constant error, then the controller, this type of controller is used. Then we could have. So, we have seen the characteristic of the proportional control; we have seen the characteristic of the derivative control. We have seen the characteristic of integral control.

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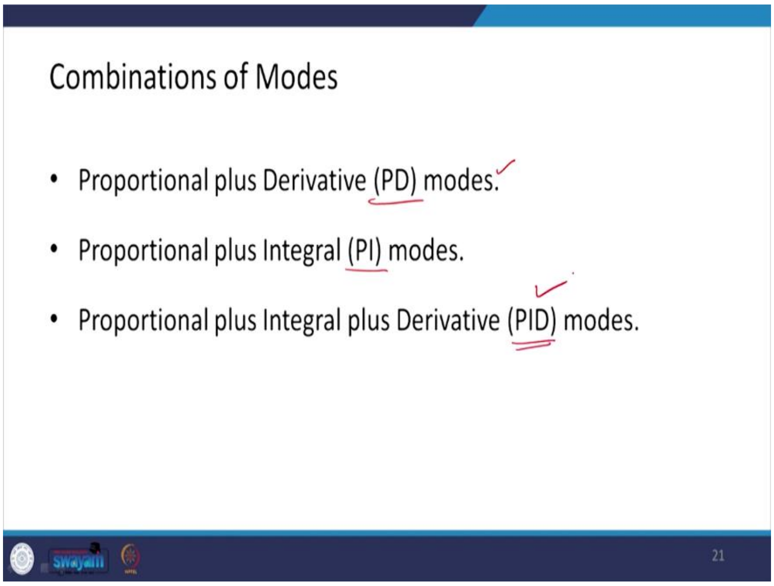

So, by combining this, we could get a combined characteristic of this controller. So, there is various combination of modes possible. So, this one is proportional plus derivative mode, that is what we call the PD controller; then we are proportional plus integral control or what we call the PI controller, and we have proportional plus integral plus derivative or what we call the PID controller.

Now, let us look at the first one, proportional and derivative control. So, the derivative control is never used alone, as I have explained earlier also. Why? Because it is not capable of giving an output when there is a steady-state error signal is there. And the control action will be zero in case of steady-state error. So, that is why it is usually used in combination with proportional control.

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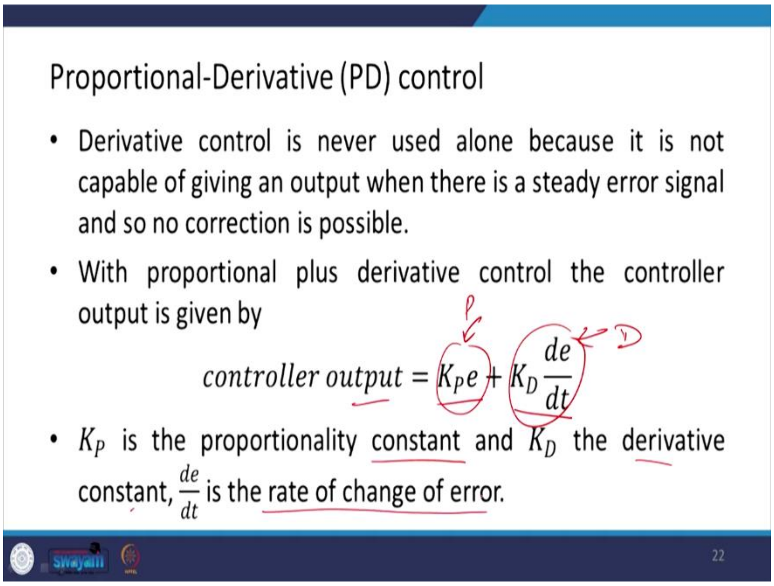

So,

the controller output $K_p e + K_D \frac{de}{dt}$

So, this part is, this part is your derivative control, and this part is your proportional control; that is why this is called the PD controller. So, K_p is the proportionality constant, K_D is the derivative constant, and $\frac{de}{dt}$ is the rate of change of error.

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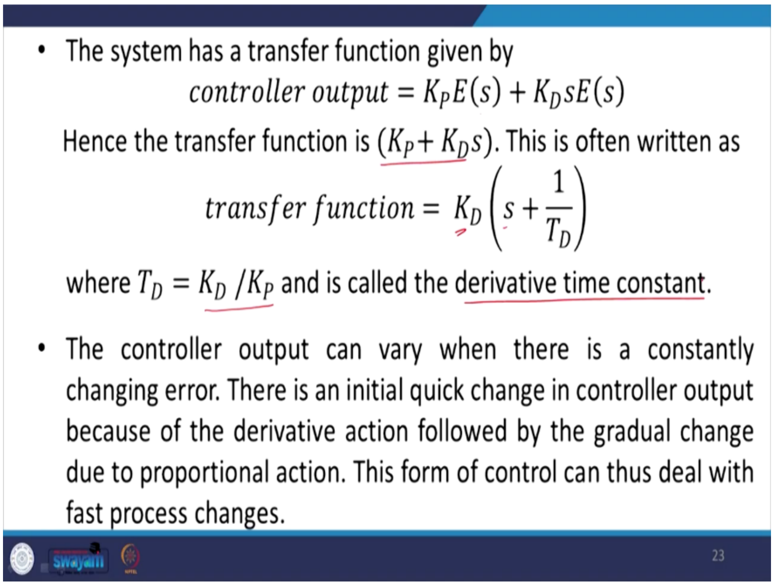

So, the system as a transfer function we can write in terms of the s domain. So,

Controller output (s) = $K_p E(s) + K_D s E(s)$

And so, the transfer function for this controller is $K_p + K_D s$. And this is often written in another form.

$$\text{Transfer function} = K_D(s + \frac{1}{T_D})$$

$T_D = K_D/K_P$ is called the derivative time constant.

The controller output can vary when there is a constantly changing error. And there is an initial quick change in the controller output because of the derivative action followed by gradual change due to the proportional action. So, this form of control can thus deal with fast process changes.

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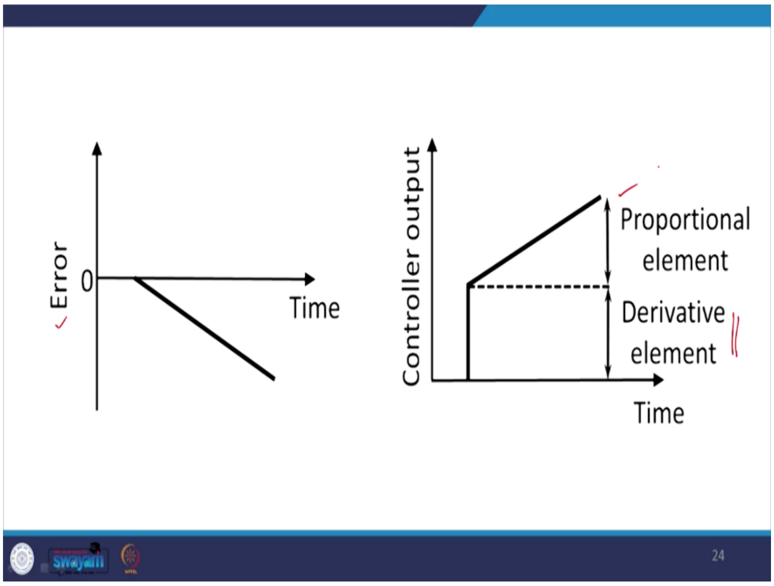

So, this is how it is. So, you have the error over here, and here is the constant over error over here. So, you have the derivative element in this zone, and you have the proportional element over here. Now, let us look at the proportional-integral control. The integral mode of control is not usually used alone, but it is frequently used in conjunction with the proportional mode. And as I said, it is usually used to reduce the steady-state error.

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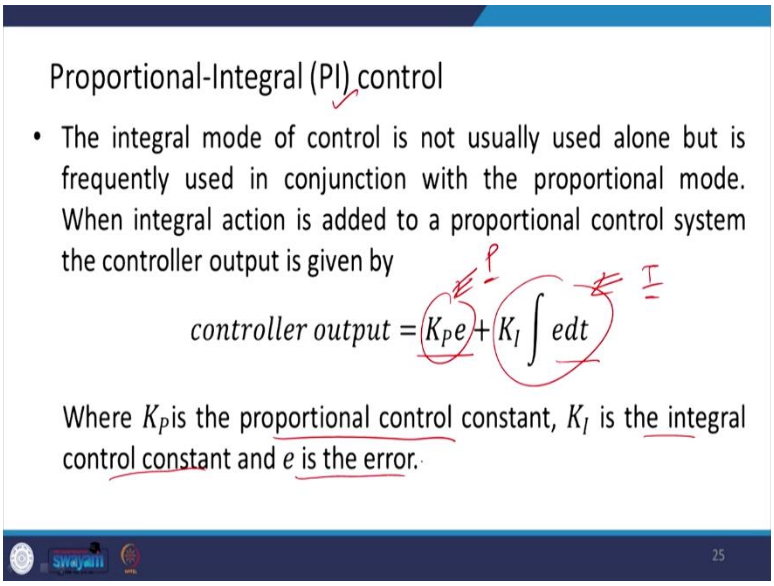

When integral action is added to a proportional control system,

$$\text{Controller output} = K_P e + K_I \int e dt$$

This is called the P I control over here. So, the K_P is the proportional constant control, K_I is the integral constant control constant, and e is the error.

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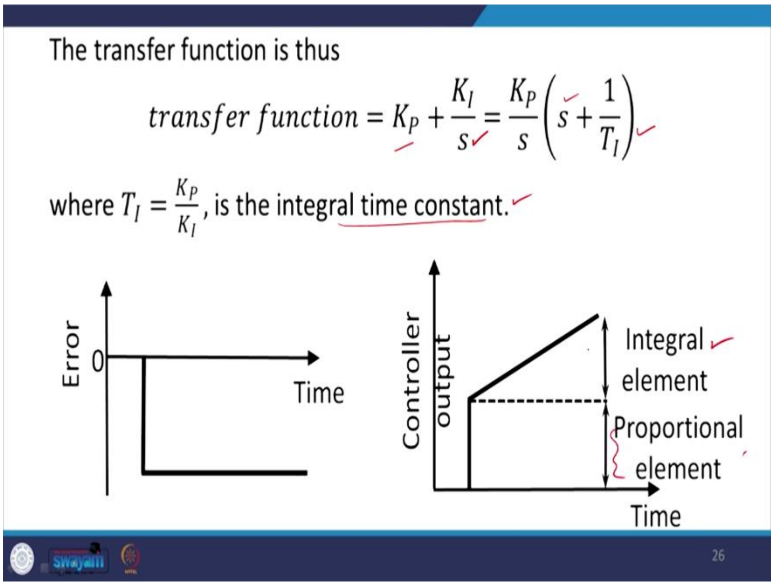

Now, the transfer function = $K_p + \frac{K_I}{s} = \frac{K_p}{s}\left(s + \frac{1}{T_l}\right)$

Here, $T_l = \frac{K_p}{K_l}$. So, here you can see your error if it is a constant over an error over here. So, this constant error is taken care of by the integral control action over here. So, this one portion corresponds to the proportional element, and this one corresponds to the integral element.

Next is the combination of all three, which is proportional, integral, and derivative controller, which is a very popular controller and is mostly used in the industry.

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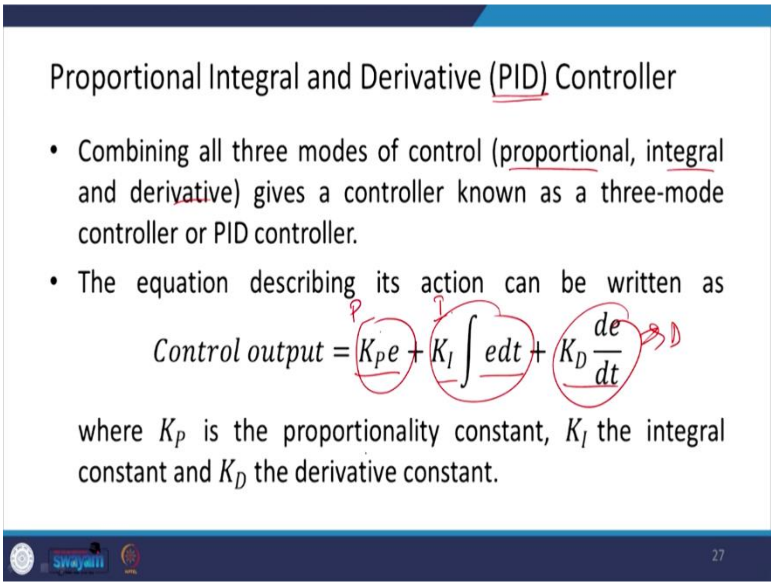

So, the PID controller is the combination of all the three modes of control; that is, the proportional, integral, and derivative control. And it is also known as the three-mode controller or the PID controller.

$$\text{Control output} = K_P e + K_I \int edt + K_D \frac{de}{dt}$$

These terms or have they have the usual meaning, which I have discussed earlier.

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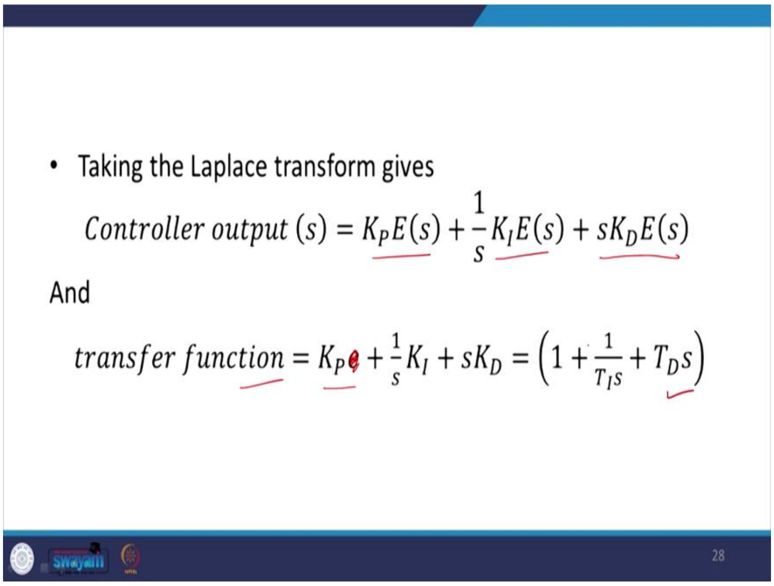

So, if I take the Laplace to transform for that, the controller output is this one for the proportional one, this is for the integral one, and this is for the derivative one. So, the transfer function for the PID control controller is,

$$\text{Controller output (s)} = K_p E(s) + K_D sE(s) + \frac{K_I}{s} E(s)$$

$$\text{Transfer function} = K_p + \frac{K_I}{s} + sK_D$$

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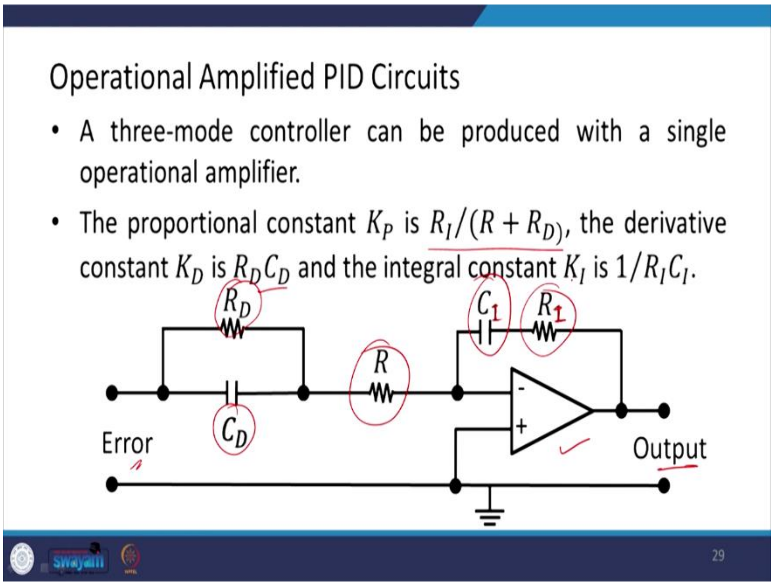

Now, how this these controller could be implemented in a discrete circuit? So, we can realize, physically realize these types of controllers with the help of an operational amplifier. So, I am just discussing this for the PID circuit; you can refer to standard text for the other type of implementation for the other type of circuit other types of control actions.

So, the three-mode controller can be produced with a single op-amp or operational amplifier. So, here you can see that this is the op-amp, and here in the feedback path of the op-amp, I have a resistor, and I have a capacitor here. And here I have a resistor over here, and here in the inverted input one, I have a resistor, and I have a capacitor over here. And the non-inverting terminal is grounded over here; I get output here, and this is the error one.

Now, I am not deriving this, we have already seen the op-amp, and it is working. An op-amp has the signal conditioner we have already seen. And for this case,

the proportional constant = $\frac{R_1}{R+R_D}$,the derivative constant is $R_D C_D$, and the integral constant is one by $R_1 C_1$. So, this way, we can physically implement these controllers.

These are the references by Bolton, and you can refer to our book on the intelligent mechatronic system also.

Thank you.