Mechatronics
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Lecture - 28
Fluid System Model

I welcome you all to today's NPTEL online certification course lecture on Mechatronics. Today, we are going to talk about the modeling of fluid systems. So, here we will be dealing with how to model the fluid systems, which includes the modeling of the hydraulic system as well as modeling of a pneumatic system. So, first, we will be looking at the basic building blocks for each system. After the basic building block, I will be taking up examples of each type that is hydraulic and pneumatic. Then at the end, I will summarize both the systems.

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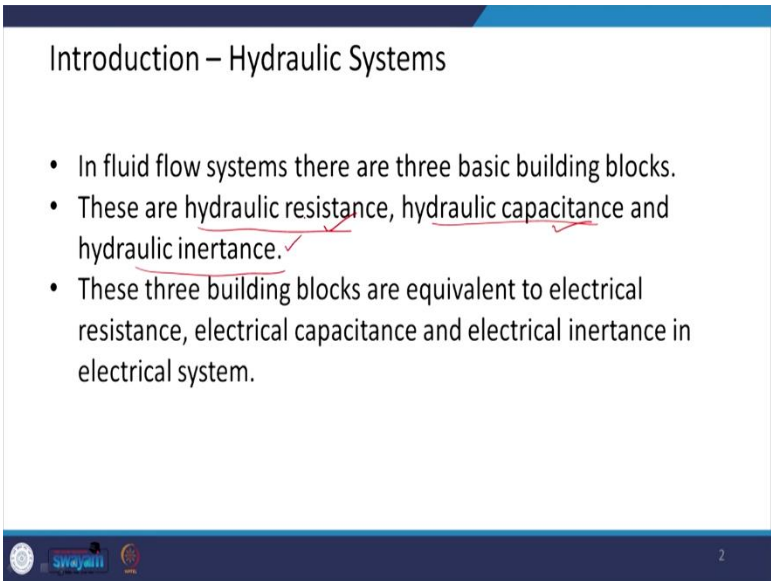

So, hydraulic systems, as we have seen in the case of electrical or mechanical systems, in the hydraulic system, there are also three basic building blocks. These basic building blocks are hydraulic resistance, hydraulic capacitance, and hydraulic inertance, and these basic building blocks are equivalent to electrical resistance, electrical capacitance, and electrical inertance in an electrical system.

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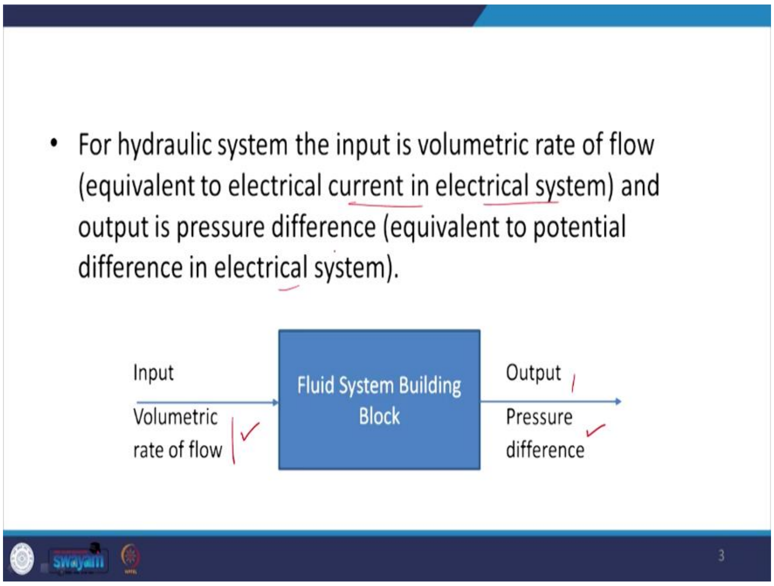

Now, for the hydraulic system, the input is usually the volumetric flow rate, and the output is the pressure difference. And this is equivalent to if we talk about the electrical circuit, so it's equivalent to the electrical current in the electrical system that is the input and output as the electrical potential difference in the electrical system.

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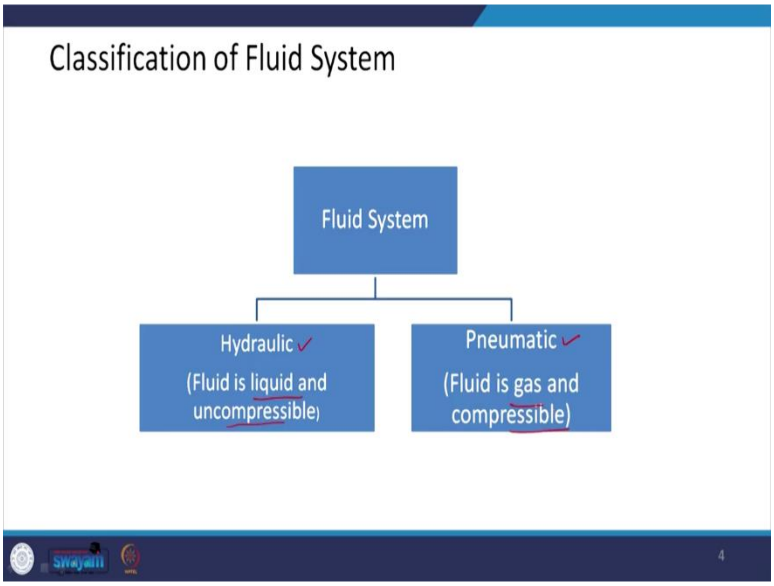

Now, coming to the classification of the fluid system, so as all of you are aware, the fluid system can be classified into two types that are hydraulic and pneumatic. In the case of hydraulic, the fluid is liquid, and it is incompressible, whereas in the case of pneumatic, the fluid is gas, and it is compressible. So, we will be looking at both these hydraulic and pneumatic systems today because, in a mechatronic system, a subsystem may consist of either hydraulic or pneumatic parts. So, it is very important for us to know how do we model this hydraulic and pneumatic part. So, first of all, let us take the hydraulic resistance.

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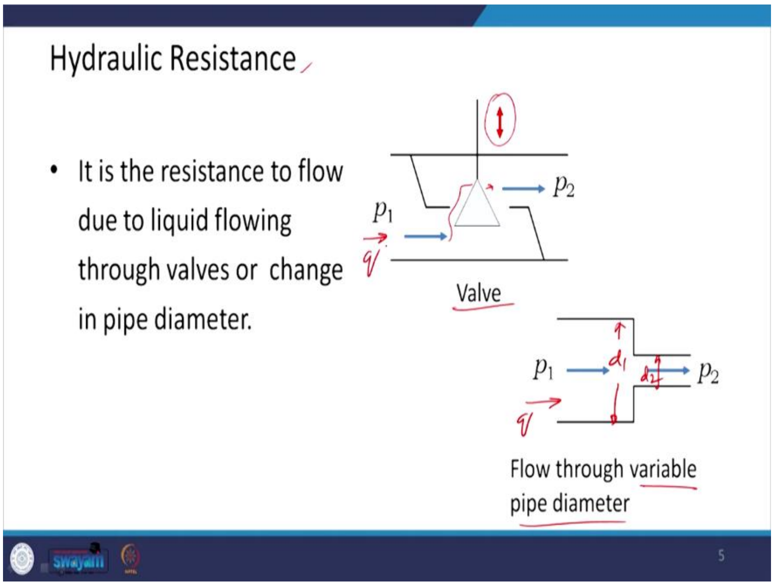

This hydraulic resistance is resistant to flow due to liquid flowing through a valve or through a variable pipe diameter. So, if you look at this valve, this is the direction through which your fluid flow takes place. This valve is operated by a lever to open or close. When the liquid flows through this valve, there is a drop in pressure. Similarly, if the liquid is flowing through a variable pipe diameter, here you can this is the pipe diameter d_1 here and d_2 here. When it is flowing through the wearable pipe diameter, you are going to have a drop in pressure.

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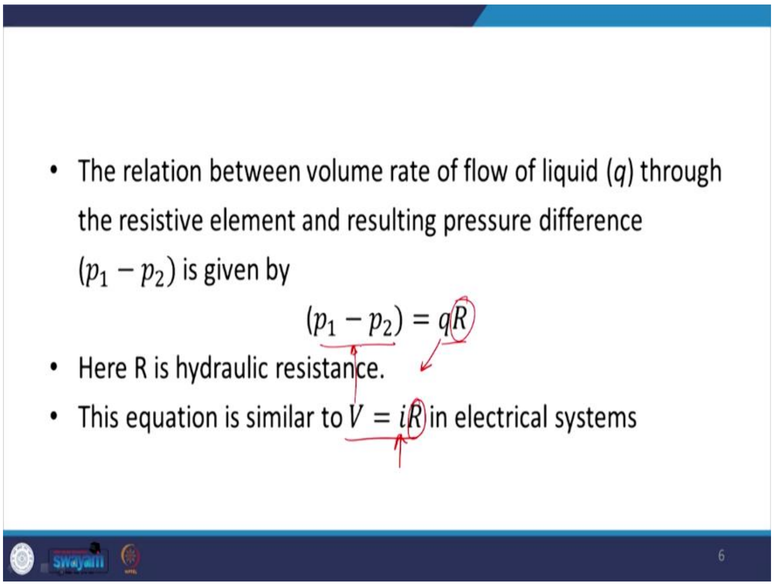

The relationship between the volume flow rate of liquid through the resistive element and resulting pressure difference is given by,

$$(p_1 - p_2) = qR$$

Here, q is the flow rate. And this is we can take an analogy with the electrical circuit where $V = iR$. Here R is the electrical resistance, and I am talking about R as the hydraulic resistance. As I said, current here is analogous to the volume rate of flow of liquid, and the voltage is analogous to the pressure difference at the two sides.

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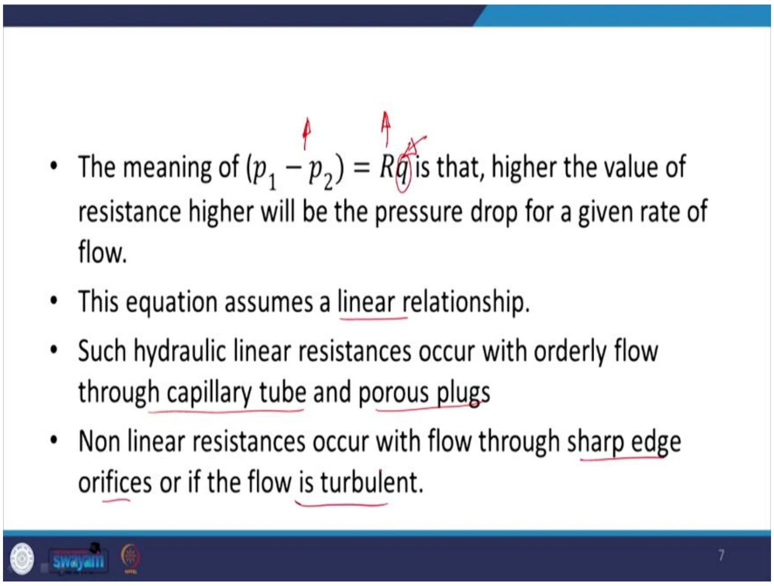

So, what does this relationship means? Relationship means that the higher the value of resistance higher is going to be the pressure drop for a given rate of flow. So, if here I keep this q as constant, so if R increases, $(p_1 - p_2)$ will also be increasing. So, what does that means? That means that if there is more resistance, more resistance hydraulic resistance, then you are going to have more pressure drop. This equation assumes a linear relationship. Such hydraulic linear resistances occur with orderly flow through a capillary tube or through the porous plug. There are situations where these resistances are non-linear resistances, and they occur, for example, flow-through sharp is orifice or if your flow is going to be turbulent.

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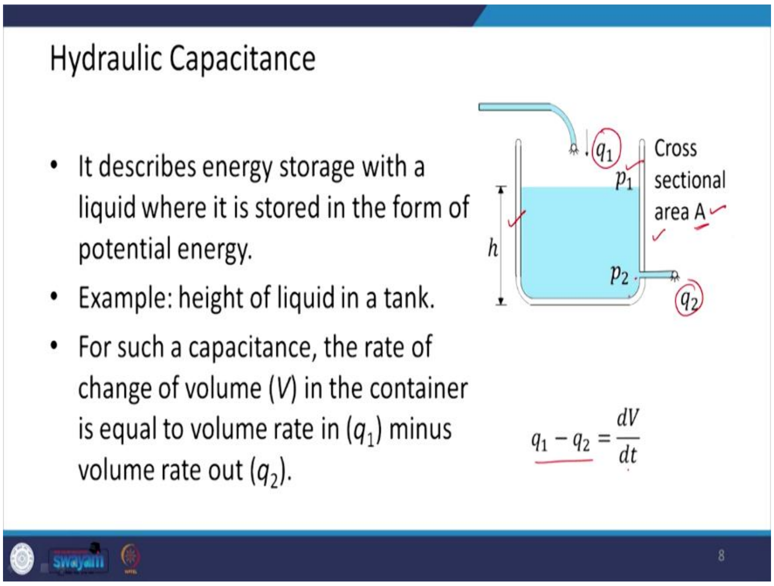

After the hydraulic resistance, let us look at another important building block which is hydraulic capacitance. It describes energy storage with a liquid where it is stored in the form of potential energy. We can take an example of liquid being stored inside a tank, so the height of a liquid in a tank. For such a case, the capacitance relates to the rate of change of the volume in the container, and that is,

$$q_1 - q_2 = \frac{dV}{dt}$$

So, hydraulic capacitance relates to that. So, this is we have a tank over here of cross-sectional area a, and there is q_1 flow in and q_2 flow out from it. And there is a liquid of height h, and it means that at the top, you have a pressure p_1, and at this point, at the bottom, you have pressure p_2. So, you see that this the net volume which is going to be inside that will be or the liquid inside is going to be equal to the volume flow rate, rate of change of volume.

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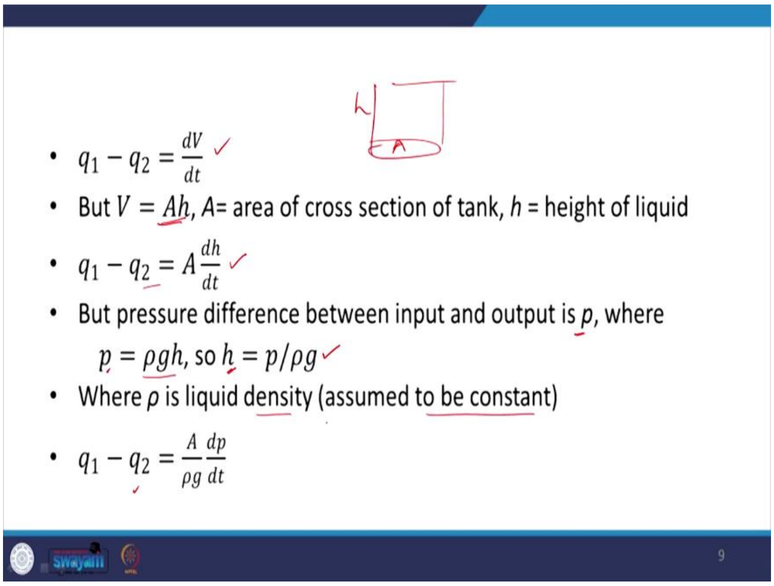

So, this,

$$q_1 - q_2 = \frac{dV}{dt}$$

And you see that we know $V = Ah$, where your A is your area of the cross-section. And h is the height. So, I can substitute that and I get,

$$q_1 - q_2 = A\frac{dh}{dt}$$

But you see, the pressure difference between input and output here is p, and this,

$$p = \rho g h$$

And so I can relate, $h = \frac{p}{\rho g}$

where ρ is the liquid density which we are assuming to be constant in the case of the hydraulic fluid flow, so,

$$q_1 - q_2 = \frac{A}{\rho g}\frac{dp}{dt}$$

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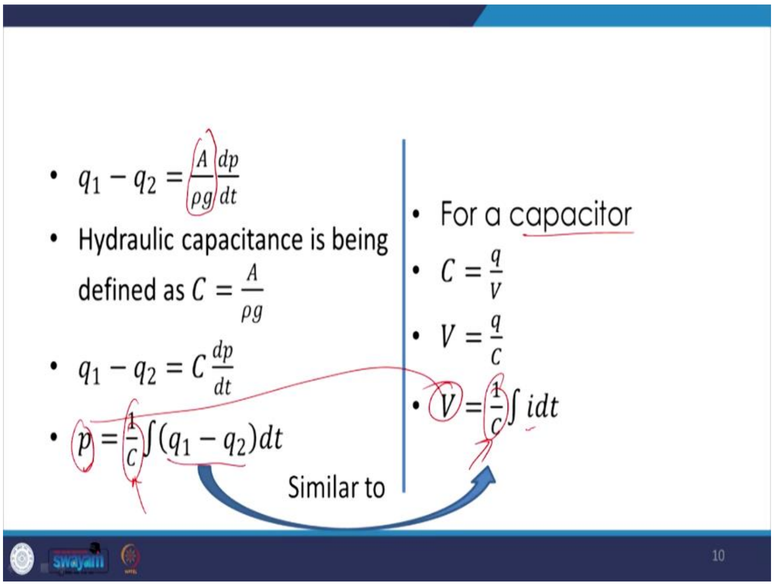

And I am defining this term as hydraulic capacitance. So, if I do that, then this,

$$q_1 - q_2 = C\frac{dp}{dt}$$

From here, I can write

$$p = \frac{1}{C}\int (q_1 - q_2)dt$$

And this you see that if I compare this with an electrical capacitor, so for that, you see,

$$C = \frac{q}{V}$$

So, and q_1 can write as integral of idt.

So, we can just compare this. This p is analogous to V. 1/C is where C is the hydraulic capacitance, here it is the electrical capacitance, and this is your analogous to current.

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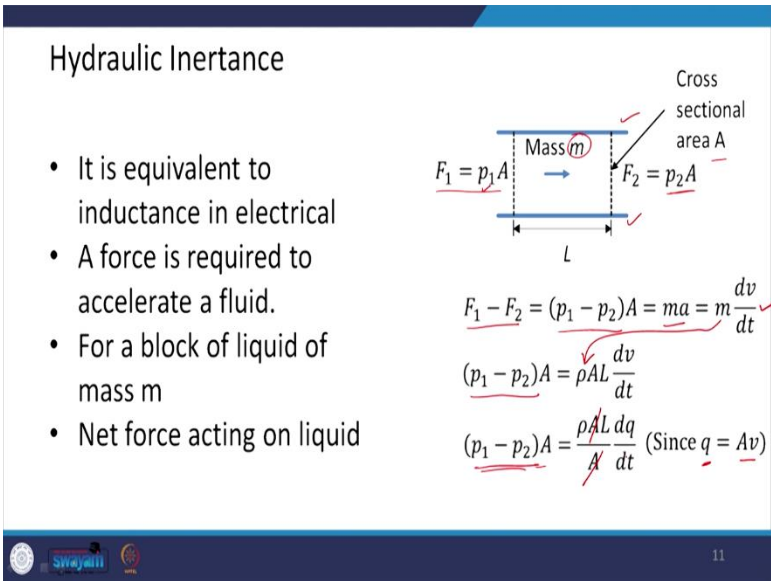

Next, let us look at hydraulic inertance. So, it is equivalent to the inductance in the electrical circuit. And so, let us assume that we have A cross-sectional area, A constant cross-sectional area duct or pipe through which a fluid is flowing. And at an instant in this length, I have got a mass m. And let us assume the pressure this side left-hand side as p_1. So, we have the force acting over here will be $p_1 A$. And at the right-hand side, it is pressure p_2, so force acting here will be $p_2 A$. And this is the force difference that is going to be responsible for the acceleration of the liquid. So, I am writing this,

$$F_1 - F_2 = (p_1 - p_2)A = ma = m\frac{dv}{dt}$$

And,

$$(p_1 - p_2)A = \rho AL\frac{dv}{dt}$$

So,

$$(p_1 - p_2)A = \frac{\rho AL}{A}\frac{dq}{dt}$$

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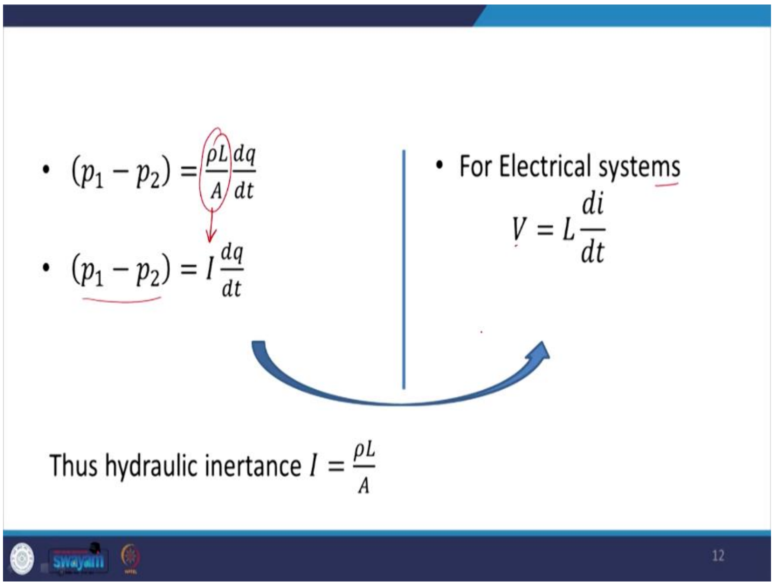

So,

$$(p_1 - p_2) = \frac{\rho L}{A}\frac{dq}{dt}$$

So, this is I can write inertance hydraulic this,

$$I = \frac{\rho L}{A}$$

$$(p_1 - p_2) = I\frac{dq}{dt}$$

And you see that for the electrical system, we define inductance as,

$$V = L\frac{di}{dt}$$

So, we have an analogous relationship.

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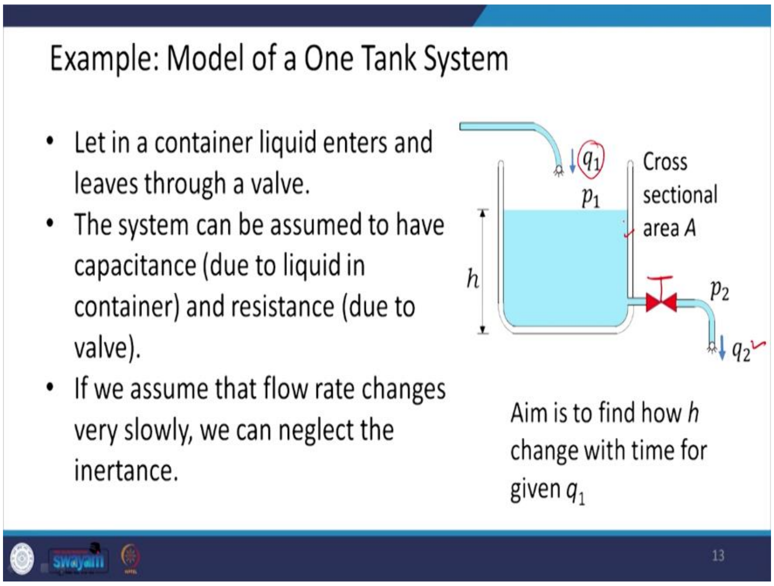

Let us take an example of a one-tank system. Now, here have in the one tank system, there is a tank here, and a liquid is flowing at the rate q_1 over here, and it is passing through a valve here. And through the valve, it is passing, and the discharge here is q_2. So, in this system, we can consider or this system we can identify the building blocks in this system. And these are the hydraulic resistance because of this valve and hydraulic capacitance because of the liquid of a certain height inside this tank. And we can assume that the liquid is that is the flow rate changes very slowly. So, we can neglect the effect of inertance. So, with this, let us take this example. So, we define capacitor as,

$$q_1 - q_2 = C\frac{dp}{dt}$$

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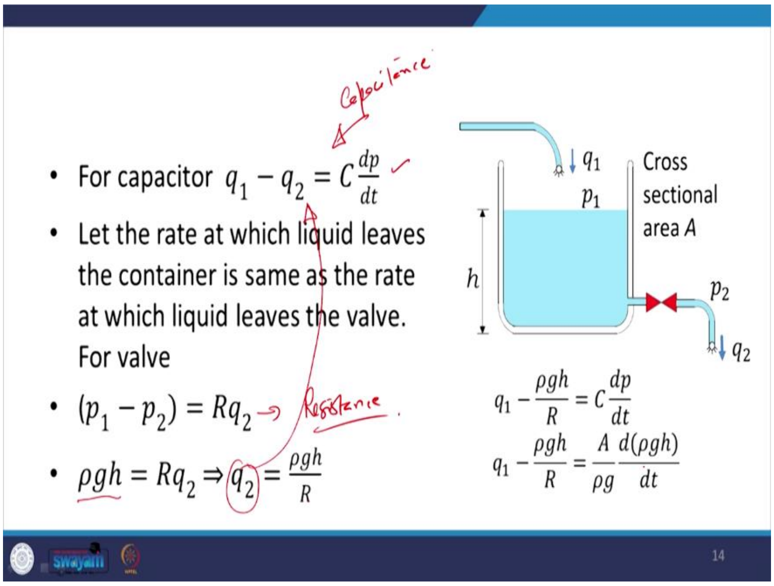

And let us assume that the rate at which liquid leaves the container is the same at the rate as with liquid leaving the valve. So, I can put,$(p_1 - p_2) = Rq_2$. So, this is the equation that I have got from the hydraulic resistance. And this is the equation that I have got from the hydraulic capacitance. So, this is there. And now next, what we do is we combine these two equations and get,

$$\rho gh = Rq_2$$

$$q_2 = \frac{\rho gh}{R}$$

And then, I can substitute this q_2 in this equation of the capacitance, so this is what I get,

$$q_1 - \frac{\rho gh}{R} = C\frac{dp}{dt}$$

And,

$$q_1 - \frac{\rho gh}{R} = \frac{A}{\rho g}\frac{d(\rho gh)}{dt}$$

So, I define this one, substitute over there.

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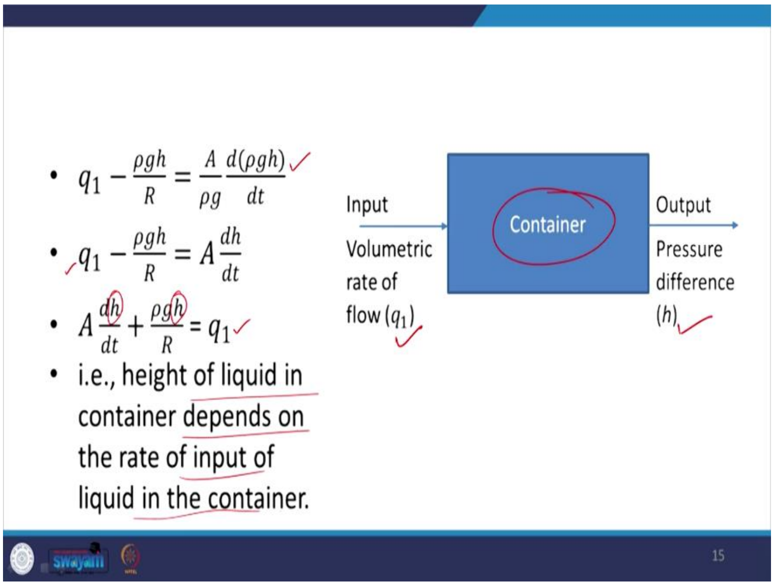

$$q_1 - \frac{\rho g h}{R} = A\frac{dh}{dt}$$

So, then we I take the parameter related with h one side, so I have,

$$A\frac{dh}{dt} + \frac{\rho g h}{R} = q_1$$

So, we can here it is a first-order differential equation in terms of h on the left-hand side and the right-hand side; we have the forcing function or what we call it as the input for this case which is the volume flow rate of the liquid in the tank. Here we can see that the height of liquid in the container depends on the rate of input of liquid in the container. So, this container as a physical system has input q_1 and output what we are measuring the pressure difference as h. So, this way here is what we have done in this example. I have tried to combine the two building blocks, which are hydraulic resistance and hydraulic capacitance, in order to get the system equation.

From here, we have got the system equation that relates the height of the liquid inside the tank and the input forcing function or the input, which is the volume flow rate into the tank.

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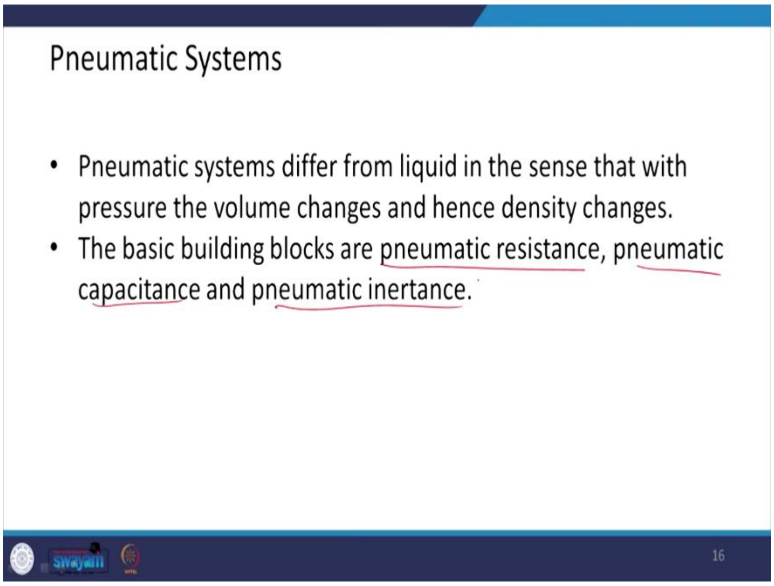

Now, let us look at the pneumatic system. The pneumatic system differs from the liquid in the sense that with pressure, the volume changes, and hence the density changes. Here, also we have the three basic building blocks that are the pneumatic resistance, pneumatic capacitance, and pneumatic inertance.

So, with the help of these three building blocks, we will try to take an example to illustrate how we derive the system equation of a pneumatic system.

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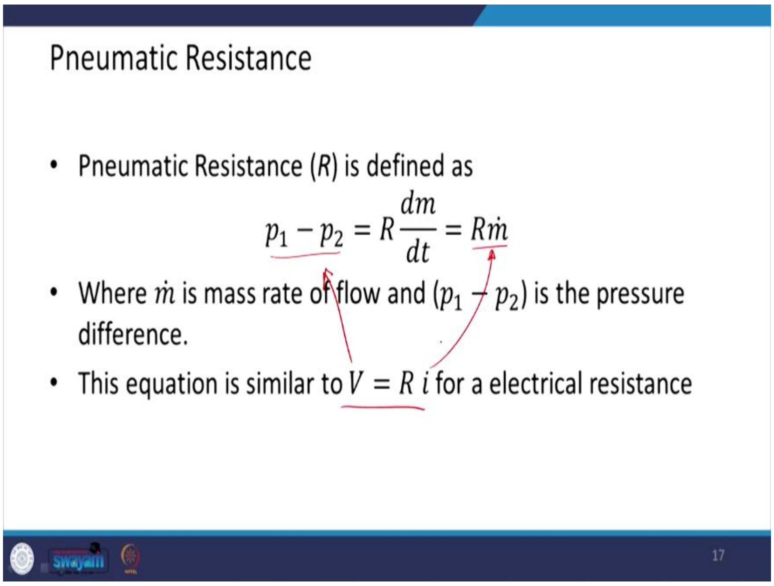

So, first of all, let us look at the pneumatic resistance. Pneumatic resistance is defined as,

$$p_1 - p_2 = R\frac{dm}{dt}$$

So, we can if we want to compare it with the electrical system, I can compare it as V = Ri. V is analogous to the pressure drop; i here is analogous to the mass flow rate $\frac{dm}{dt}$. In the case of the hydraulic system, if you can recall, we have just taken analogous of i as the volume flow rate, but in the pneumatic system, we take the mass flow rate.

The next element is the pneumatic capacitance. Pneumatic capacitance is due to the compressibility of the gases. And this is comparable to the compression of spring which stores energy. So, similar is the case with the pneumatic capacitance. So, to derive the expressional relationship for that, let there be a container of volume V, and let the mass flow rate entering the container be $\dot{m}_1$, and the mass flow rate leaving the container be $\dot{m}_2$.

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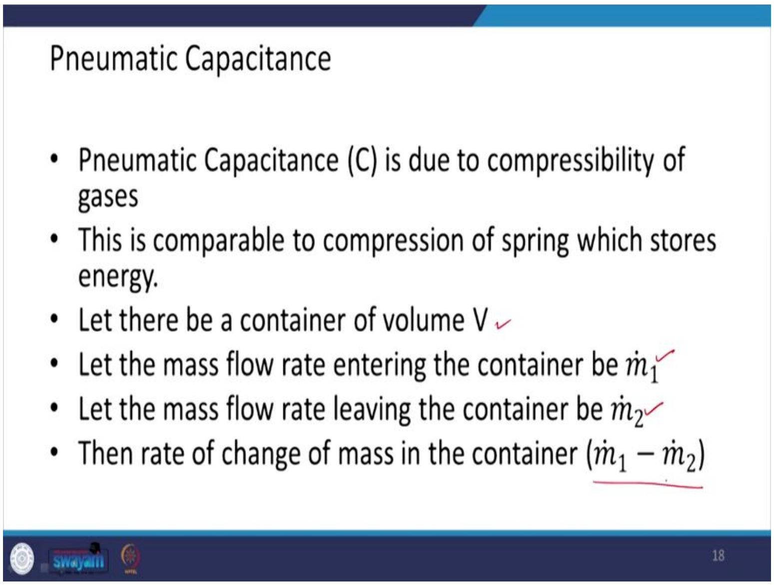

So, then the rate of change of mass in the container is, $\dot{m}_1 - \dot{m}_2$.

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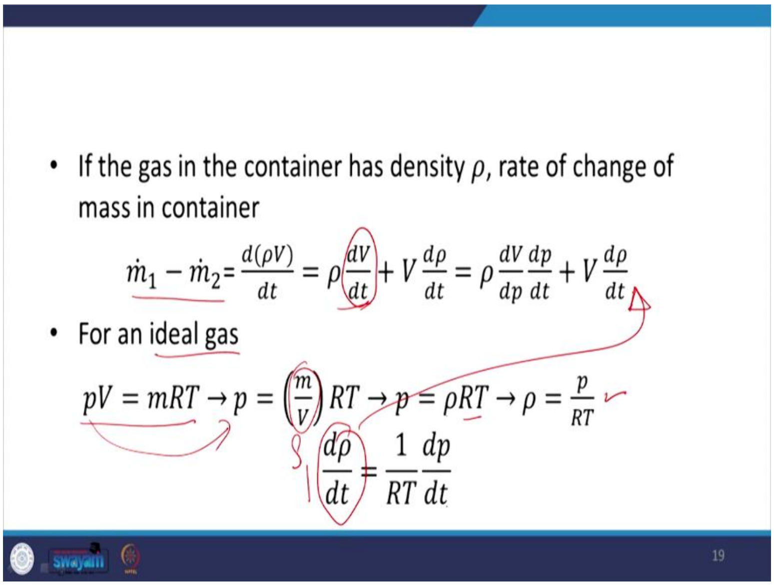

Now, this,

$$\dot{m}_1 - \dot{m}_2 = \frac{d(\rho V)}{dt} = \rho \frac{dV}{dt} + V \frac{d\rho}{dt} = \rho \frac{dV}{dp}\frac{dp}{dt} + V \frac{d\rho}{dt}$$

ρ is density and V is the volume so that I will be getting mass. So, I can expand here because my density and volume are both changing here. So, I can expand this and this term can be written as above.

Now, for an ideal gas, we know that the ideal gas equation is pV = mRT. So, from here, the p = (m/V) RT. And m/ V if I represent it by ρ that is the density, so p = ρRT; or I can write ρ is equal to p/ RT. So,

$$\frac{d\rho}{dt} = \frac{1}{RT}\frac{dp}{dt}$$

Now, I can substitute for this $\frac{d\rho}{dt}$ over here, that is $\frac{1}{RT}\frac{dp}{dt}$.

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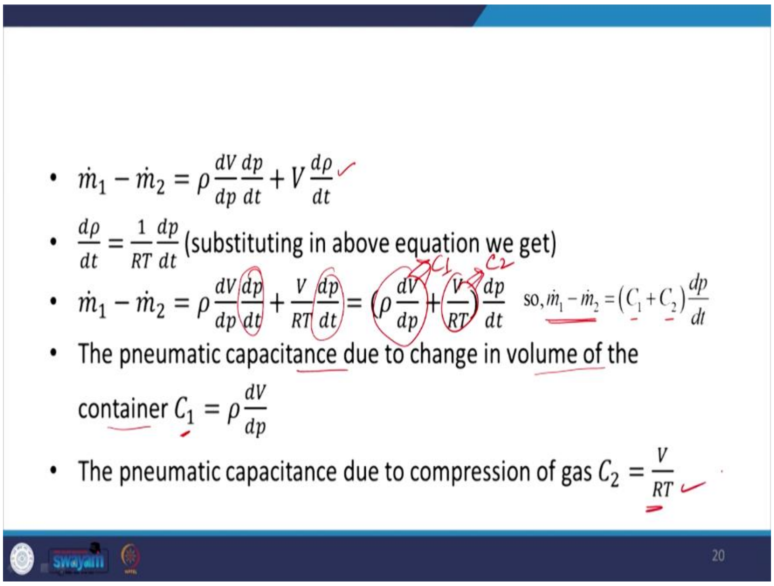

Now, I have

$$\dot{m}_1 - \dot{m}_2 = \rho \frac{dV}{dp}\frac{dp}{dt} + \frac{V}{RT}\frac{dp}{dt} = (\rho\frac{dV}{dp} + \frac{V}{RT})\frac{dp}{dt}$$

Or I can simplify this further as,

$$\dot{m}_1 - \dot{m}_2 = (C_1 + C_2)\frac{dp}{dt}$$

Now, here this C 1 is what this is the pneumatic capacitance due to the change in volume of the container. So, this $\rho\frac{dV}{dp}$. And the other one that is $\frac{V}{RT}$ is the pneumatic capacitance due to the compressibility of the gas.

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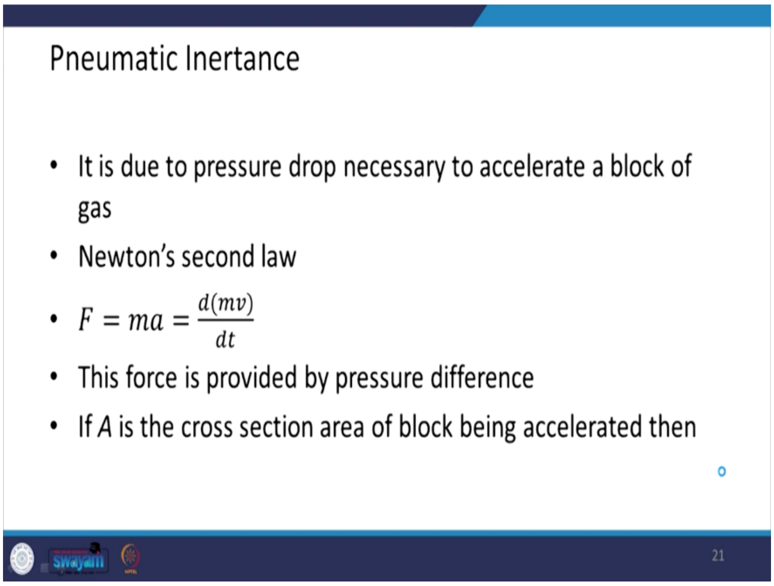

Now, the next element, let us take the pneumatic inertance. And you know it is due to the pressure drop necessary to accelerate a block of gas. So, we can derive the relationship for that using Newton's second law, which is,

$$F = ma = \frac{d(mv)}{dt}$$

And this force is provided here in this case by the pressure difference.

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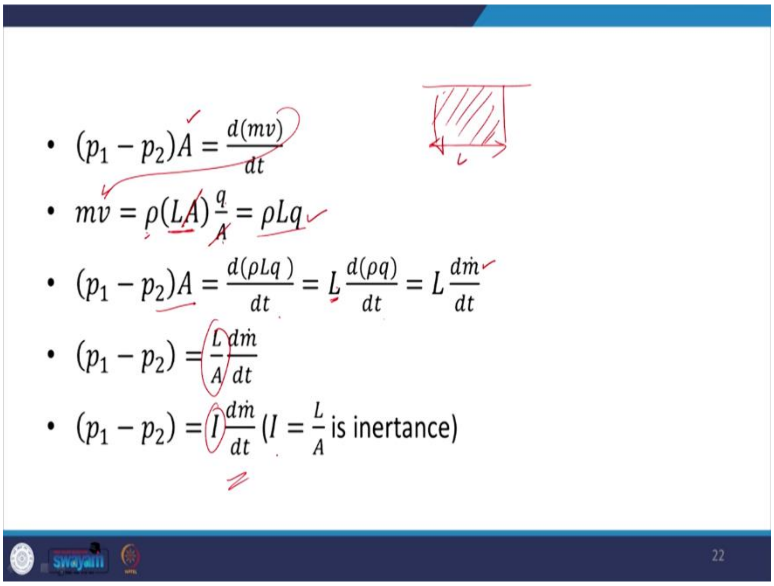

So, if A is the cross-section area of the block being accelerated, then I can write,

$$(p_1 - p_2)A = \frac{d(mv)}{dt}$$

This is the force. And what is this mv? This is ρ V that will be giving me the mass. And velocity, I can write as q/ A that is the discharge divided by the area of cross-section. So, I get, $\rho L q$.

So,

$$(p_1 - p_2)A = \frac{d(\rho L q)}{dt} = L\frac{d(\rho q)}{dt} = L\frac{\dot{m}_1}{dt}$$

Remember here L is the length of the section which we are considering here and whose, so that is there. So,

$$(p_1 - p_2) = \frac{L}{A}\frac{\dot{m}_1}{dt}$$

Or,

$$(p_1 - p_2) = I\frac{\dot{m}_1}{dt}$$

and this is L by A is the inertance. So, this way, I can define the pneumatic inertance.

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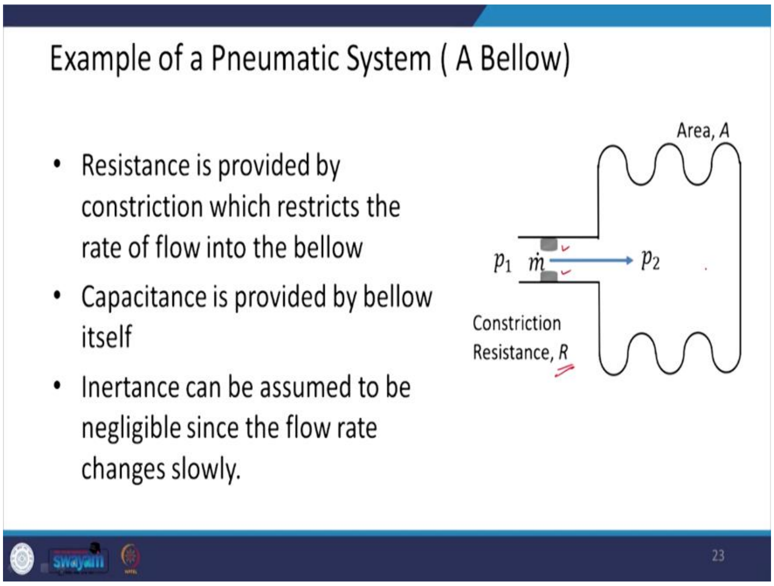

Next, let us take an example. After defining all the three pneumatic components, let us take an example bellow. So, suppose in this bellow, there is a constriction over here, and that constriction provides a resistance R., And there is a capacitance of the bellow we can take a capacitance of the bellow. And we can assume the inertance to be negligible since the flow rate changes very slowly.

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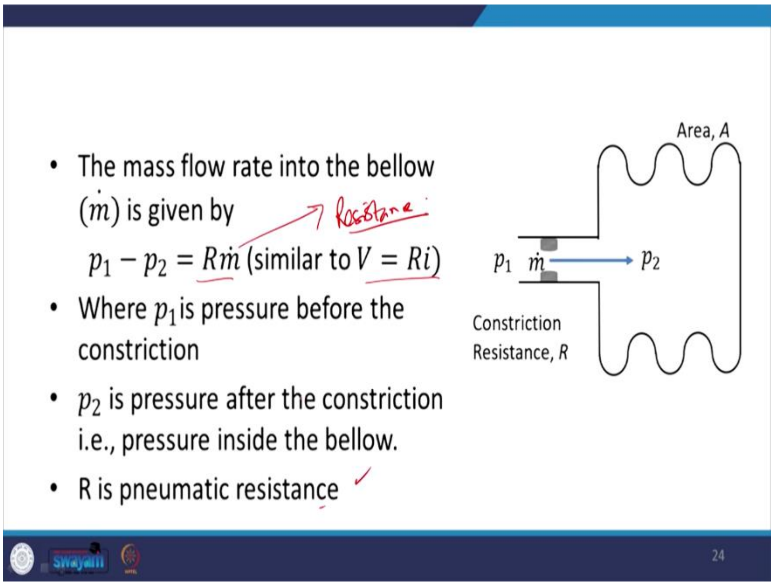

So, with that, the mass flow rate into the bellow can be given by,

$$(p_1 - p_2) = R\dot{m}$$

This is similar to V = R i or this we have given from the pneumatic resistance relationship, where p_1 is the pressure before the constriction, and p_2 is the pressure after the constriction, and R is the pneumatic resistance. So, this is the relationship.

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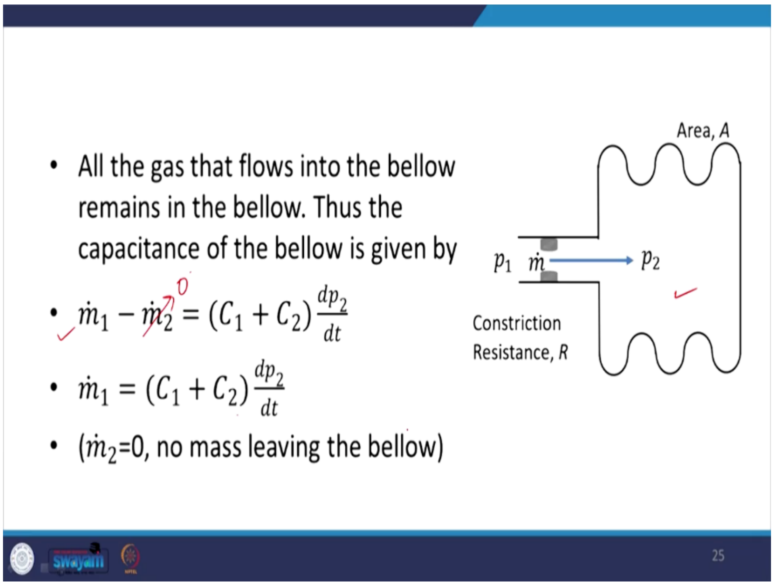

And all the gas that flows into this bellow, you can see that it remains in the bellow. So, thus the capacitance of the bellow, we can use the same equation,

$$\dot{m}_1 - \dot{m}_2 = (C_1 + C_2)\frac{dp}{dt}$$

because we are considering bellow pressure inside the bellow is p 2. So, I am using this term over here. And there is no mass leaving, so I can take this m_2 to be equal to 0. So,

$$\dot{m}_1 = (C_1 + C_2)\frac{dp}{dt}$$

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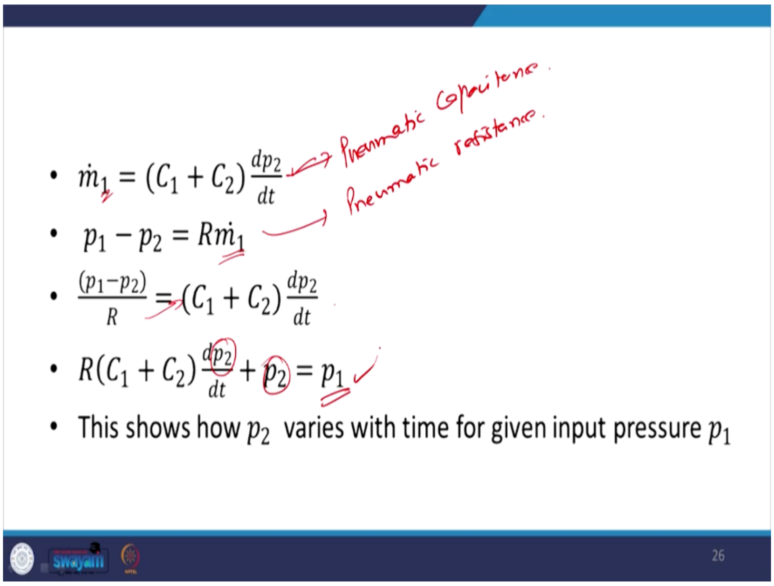

Now so this is our equation of what we call pneumatic capacitance, and this is the relationship of pneumatic resistance. Now, I can combine both of these. So,

$$\frac{(p_1 - p_2)}{R} = (C_1 + C_2)\frac{dp_2}{dt}$$

And then, I can simplify this equation. I multiply with R this side. So,

$$R(C_1 + C_2)\frac{dp_2}{dt} + p_2 = p_1$$

So, here we can see that on the left-hand side, we have the p 2 differential first-order differential equation in terms of p 2 is there, and which tells the how the pressure varies inside the bellow. And here we have got the forcing function or the input that is the pressure p_1.

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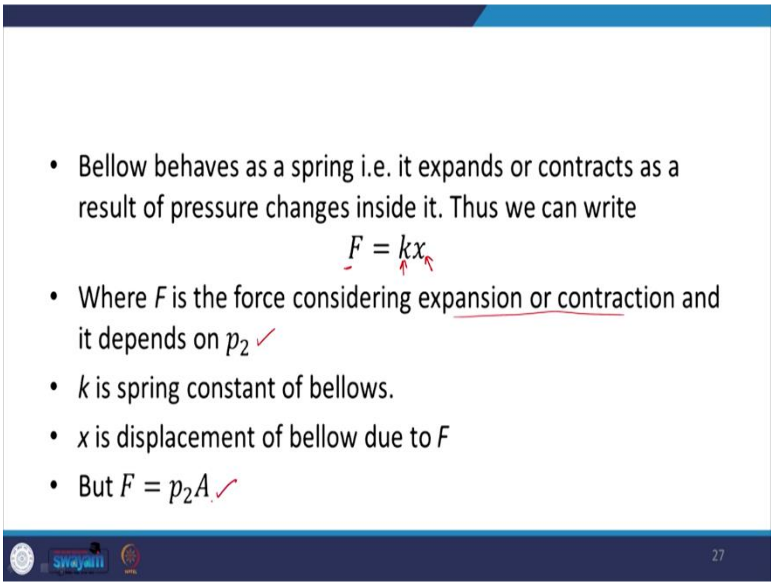

Now, I can further simplify this equation, or I can write this equation in terms of the displacement of the bellow. So, if I assume that my bellow works like a spring and the bellow has got a stiffness k, and it gets displaced by x, then we can write,

$$F = kx$$

And this force is the force considering the expansion or contraction, and it depends on the p_2, so that is there. So,

$$F = p_2 A$$

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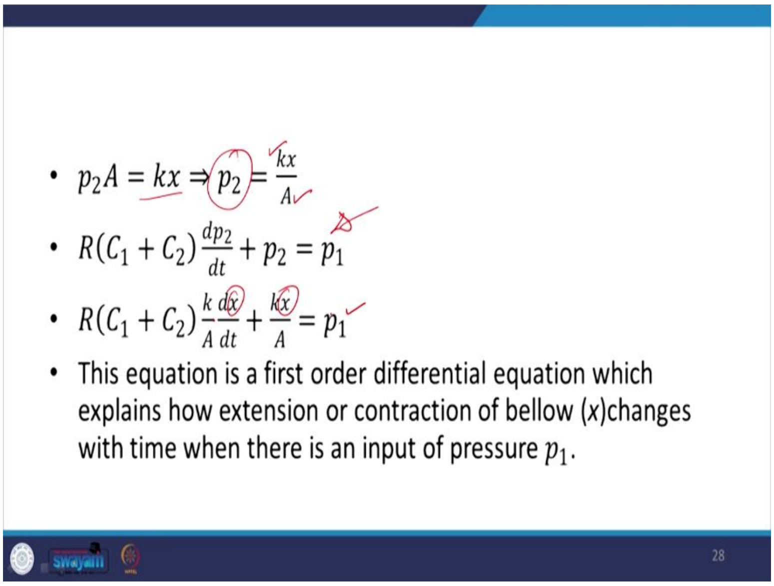

So, I can substitute over here. So, p_2A is kx. So, I get,

$$p_2 = \frac{kx}{A}$$

And now I can substitute for this p_2 in this equation which we just derived for the bellow. So, I can put it here $\frac{kx}{A}$ now here you see k, and A are constant. So, I can take them out. So, I get this differential equation in terms of x. So, here on the left-hand side, I have got the first-order differential equation in terms of the displacement of the bellow. And the right-hand side, we have the forcing function, which is the pressure of the air which is supplied into the bellow before the construction. So, this way, using the two building blocks that are the pneumatic resistance and pneumatic capacitance, we could derive the system equation for the bellow. And this equation tells us that if you are going to change the pressure p_1, how your displacement of the bellow is going to take place.

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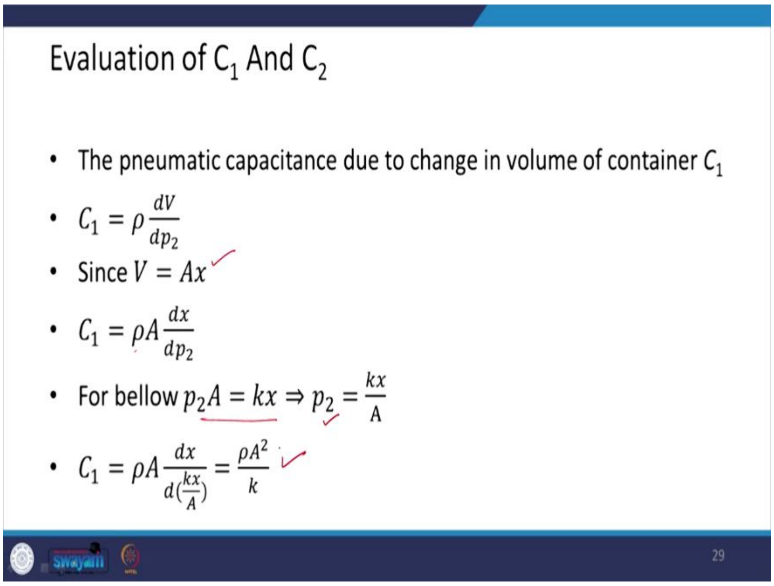

Here in this equation, the C_1 and C_2 for the bellow can be evaluated like this. You see,

$$C_1 = \rho \frac{dV}{dp_2}$$

and V is Ax . Area of the cross-section of the bellow and the displacement of the bellow, so

$$C_1 = \rho A \frac{dx}{dp_2}$$

And for the bellow p 2 A is kx we have already seen. So, p_2 I can write as $\frac{kx}{A}$. I substitute for that. So, this is what I get the value for C_1.

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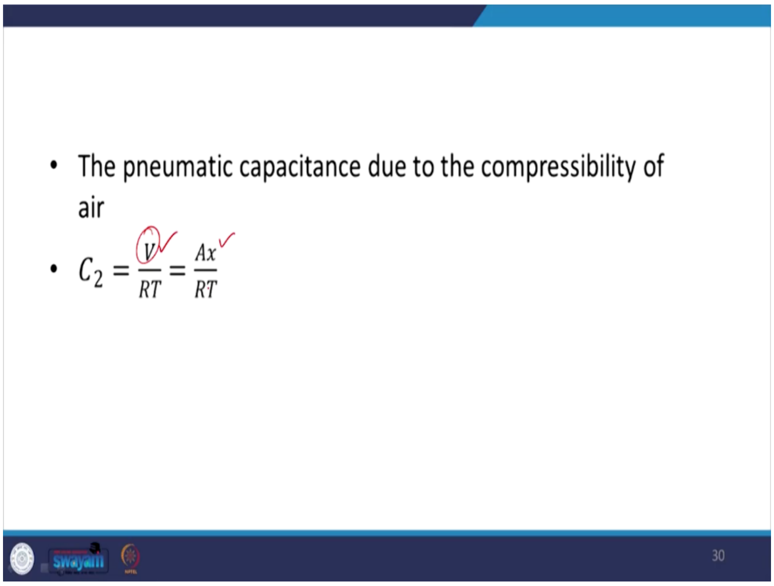

And similarly, for C_2 we know that it is the V/ RT, and this V is Ax. So, this is Ax/ RT. So, this way, we can evaluate C_1 and C_2 in terms of the bellow parameters and of the constants.

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SUMMARY

	Building block	Block equation	Energy stored/ power dissipated
Hydraulic	Inertance	$q = \frac{1}{I}\int (p_1 - p_2)dt$	$E = \frac{1}{2} I q^2$
	Capacitance	$q = C\frac{d(p_1 - p_2)}{dt}$	$E = \frac{1}{2} C (p_1 - p_2)^2$
	Resistance	$q = \frac{p_1 - p_2}{R}$	$P = \frac{1}{R}(p_1 - p_2)^2$
Pneumatic	Inertance	$\dot{m} = \frac{1}{I}\int (p_1 - p_2)\, dt$	$E = \frac{1}{2} I \dot{m}^2$
	Capacitance	$\dot{m} = C\frac{d(p_1 - p_2)}{dt}$	$E = \frac{1}{2} C (p_1 - p_2)^2$
	Resistance	$\dot{m} = \frac{p_1 - p_2}{R}$	$P = \frac{1}{R}(p_1 - p_2)^2$

So, here I provide a summary of hydraulic and pneumatic building blocks. So, we have seen the inertance, capacitance, and resistance, inertance, capacitance, resistance for hydraulic as well as pneumatic, and these are the equations. So, in the case of hydraulics, you can see that we have the volume flow rate, and here relations are in terms of volume flow rate. We are getting evaluating volume flow rate, whereas, in the case of pneumatic, we are evaluating the mass flow rate. And on the right-hand side, we have all the same terms.

Similarly, we can evaluate the energy stored in the inductor and capacitor as well as the energy dissipated by the resistor. Here I have got $\dot{m}^2$ in place ofqe^2. The rest of the things are the same.

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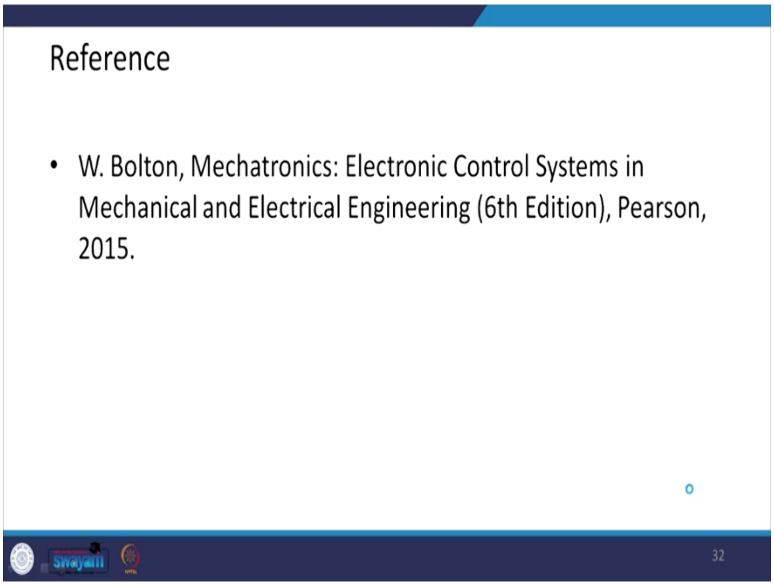

So, here are the references. It is you can refer to Bolton, Mechatronics. If you want to read it, do further exercise on the modeling of hydraulic and pneumatic systems further.

Thank you.