Mechatronics
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Lecture - 27
Electrical System Model

I welcome you all to this NPTEL online certification course on Mechatronics. Today, we are going to discuss the Electrical System Models. First, we will be looking at the various building blocks for modeling any electrical system, and then, we will see the various combinations of these building blocks in modeling of an electrical system, and then, we will take up an example of an electrical system such as a motor, and we will see how can we model either the armature control motor or the field control motor. So, this is what I have planned for you in this lecture.

The basic building blocks of the electrical system are the inductor, capacitor, and resistor. So, first of all, let us look at the inductor.

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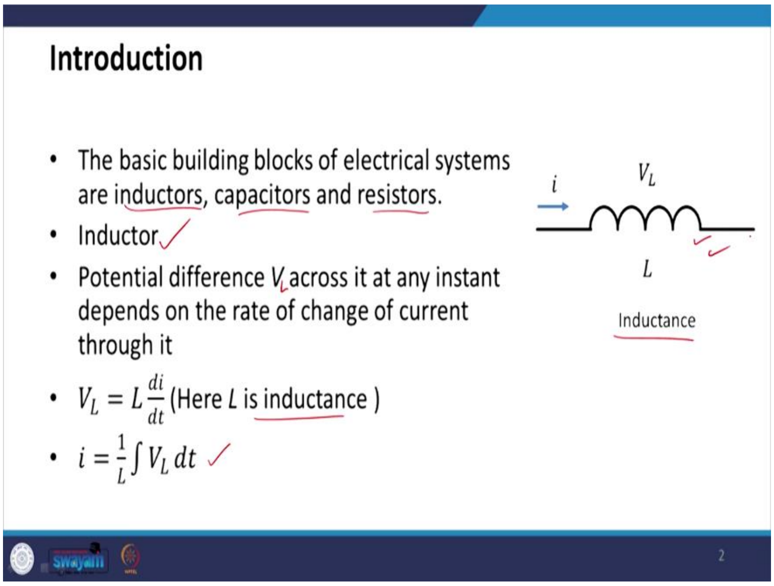

We see that this is the symbol for an inductor. So, the potential difference V_L across it at any instant depends on the rate of change of current through it. So,

$$V_L = L\frac{di}{dt}$$

where L is inductance, or I can write it in terms of current as,

$$i = \frac{1}{L}\int V_L dt$$

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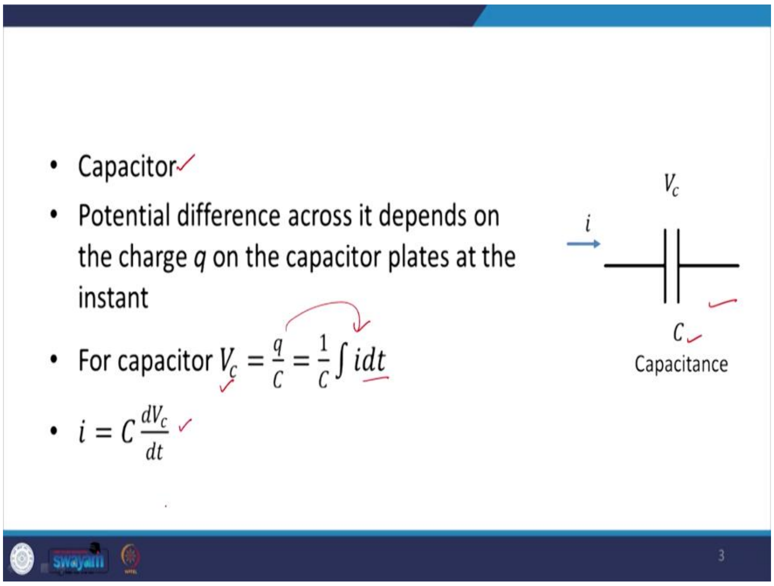

The next building block is the capacitor over here. So, see, there is a capacitor of capacitance C; the current through is i, and the V_C is the voltage across the plates over here. So, the potential difference across it depends on the charge on the capacitor plates at that instant. So, for here,

$$V_C = \frac{q}{C} = \frac{1}{C}\int idt$$

I can write,

$$i = \frac{CdV_c}{dt}$$

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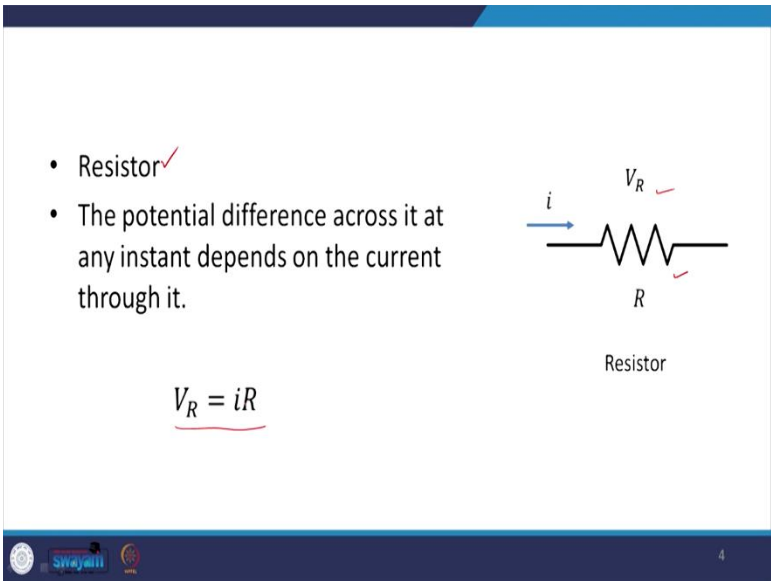

The next building block is the resistor which comes from the definition of the ohm, the definition for the resistor given by ohm's law. So, here the voltage drop across the ends of the resistor,

$$V_R = iR$$

where i is the current through the resistor and R is the resistance of the resistor.

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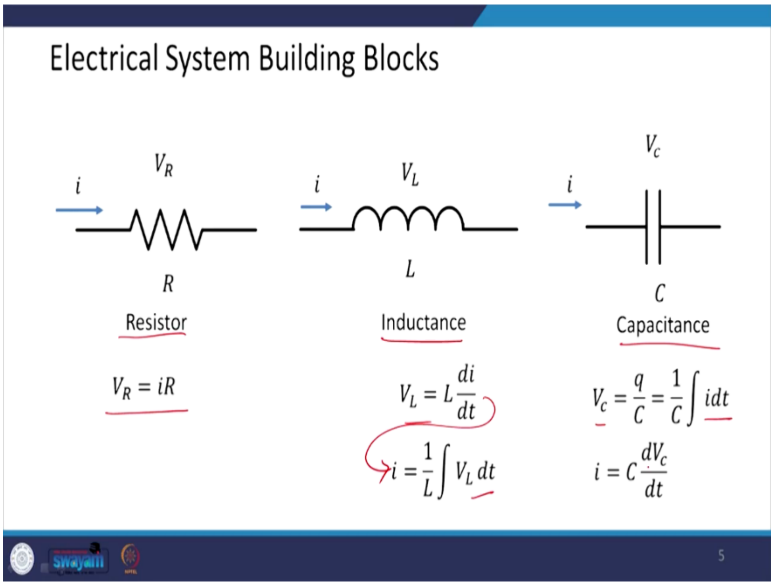

So, here are the three basic electrical system building blocks resistor, inductance, and capacitance.

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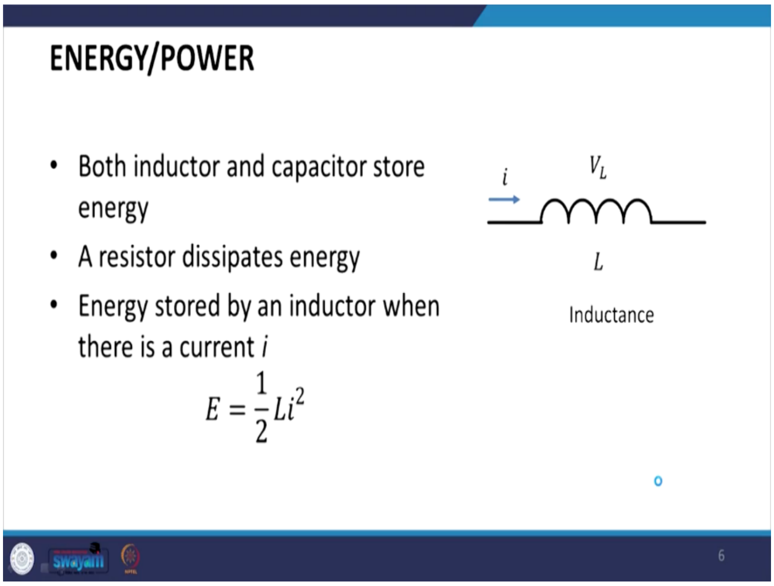

Now, what about energy and power? So, both the inductor and capacitor store energy, and a resistor dissipates energy. So, energy is stored by an inductor when there is a current *I*,

$$E = \frac{1}{2} L i^2$$

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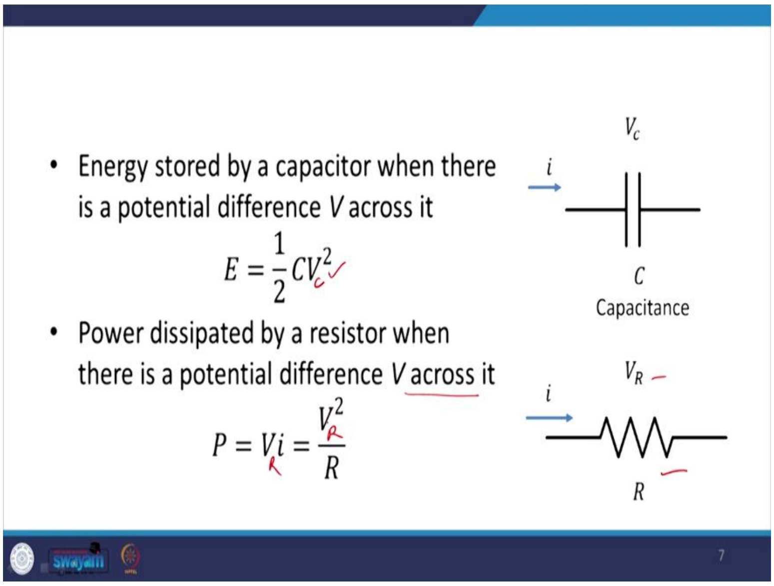

And energy stored in a capacitor,

$$E = \frac{1}{2}CV^2$$

I am not going into the derivation of this because you must have studied these things in your earlier classes. And the power is dissipated by a resistor, and when there is a potential drop V across it here,

$$P = Vi = \frac{V_R^2}{R}$$

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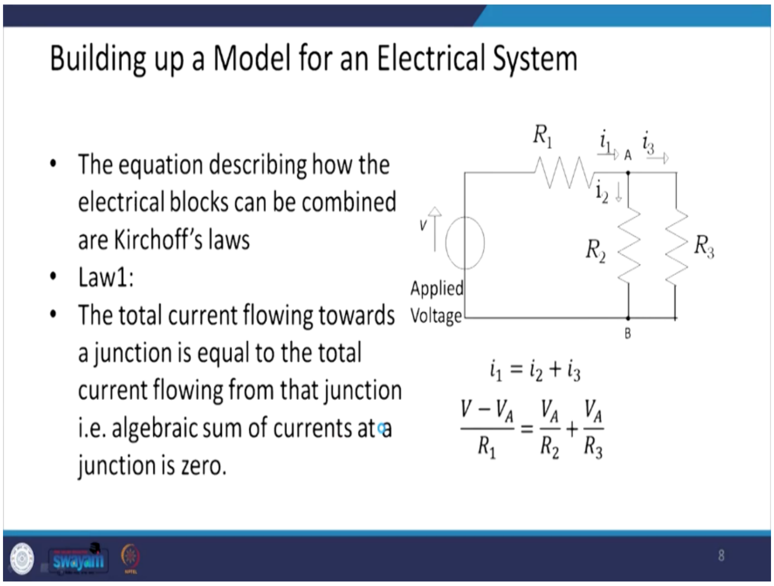

Now, building up a model for an electrical system; so, after defining these three basic elements, we can build up a model for the electrical system using Kirchhoff's current law and Kirchhoff's voltage law. I have discussed talked about these two Kirchhoff laws in my very introductory lecture in the first week of it. I am going to refer to those laws here again, which will help us in preparing the model for an electrical system.

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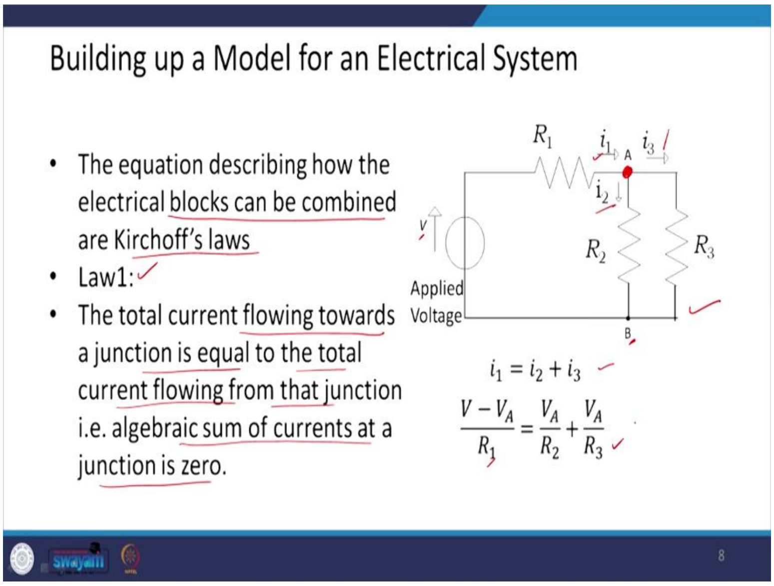

So, the equation describing how the electrical blocks can be combined is Kirchhoff's law, the combination of these blocks. So, the 1st law is the total current flowing towards a junction is equal to the total current flowing from that junction. So, the current in has to be equal to current out at a junction, or what we can is that the algebraic sum of current as it at a junction is 0.

So, if I look at this electrical circuit over here, so if I take this junction here, so here you can see the current in is i_1 and current out is i_2 and i_3. So,

$$i_1 = i_2 + i_3$$

I can write as if this voltage is V, and here,

$$\frac{V - V_A}{R_1} = \frac{V_A}{R_2} + \frac{V_A}{R_3}$$

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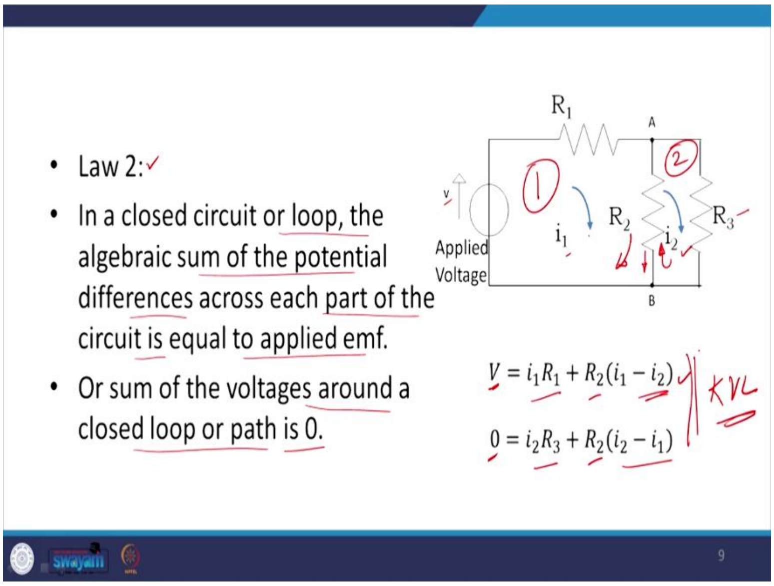

Then, there is the 2nd law by Kirchhoff, and it states that in a closed circuit or a loop, the algebraic sum of potential difference across each part of the circuit is equal to the applied emf or what we can that some of the voltages across a closed-loop or path is going to be 0. So, this law can be explained with the help of this very example over here.

So, it is the same figure which I have shown you in the previous slide. So, here suppose, I assume a current direction in this direction current i_1 and in this direction current i_2. Then, if I look at there are two loops; this is your loop 1, and this is your loop 2, so, if I write the expression for loop 1, its applied voltage is V.

So, this is Kirchhoff's voltage law for the first loop.

$$V = i_1 R_1 + R_2(i_1 - i_2)$$

Likewise, I can apply it for the second loop. So, for the second loop, there is no source over here.

$$0 = i_2 R_3 + R_2(i_2 - i_1)$$

So, this way, I can write Kirchhoff's voltage law for the two loops.

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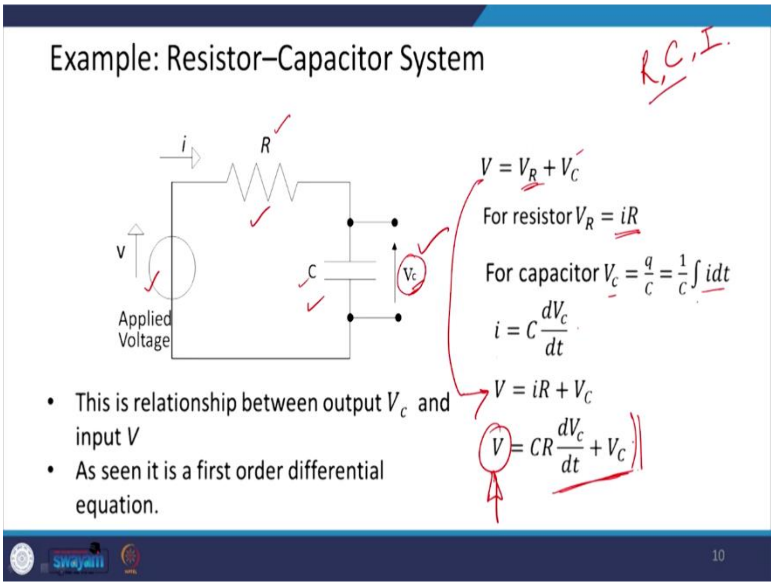

So, these are the building blocks for us if we want to model an electrical system. So, we have seen a resistor, we have seen a capacitor, we have seen an inductor. So, now, I am going to take the various combinations of these which exist in a circuit. So, here suppose I have a combination of resistor and capacitor.

So, there is a circuit, and you have an applied voltage source, a resistor, and a capacitor is there. So, this is my input and output, and I want to take across the capacitor that is V_C . So, I can apply Kirchhoff's voltage law. So, this,

$$V = V_R + V_C$$

That is the voltage across the resistor and voltage across the capacitor. For,

$$V_R = iR$$

Because it is a resistor and for capacitor,

$$V_c = \frac{q}{C} = \frac{1}{C}\int idt$$

So, I get,

$$i = C\frac{dV_C}{dt}$$

Now, I can substitute here. So, this,

$$V = iR + V_C$$

This *i* can substitute over here. So, this is,

$$V = CR\frac{dV_C}{dt} + V_C$$

So, as I said, my aim is what is my input here? Input is this voltage, and what output am I looking for? The output I am looking for is V_C. So, the forcing function here is the voltage supplied and the right-hand side. You can see that it is a differential equation of the first order. So, this way, I can derive the system equation for this system which consists of a resistor and a capacitor.

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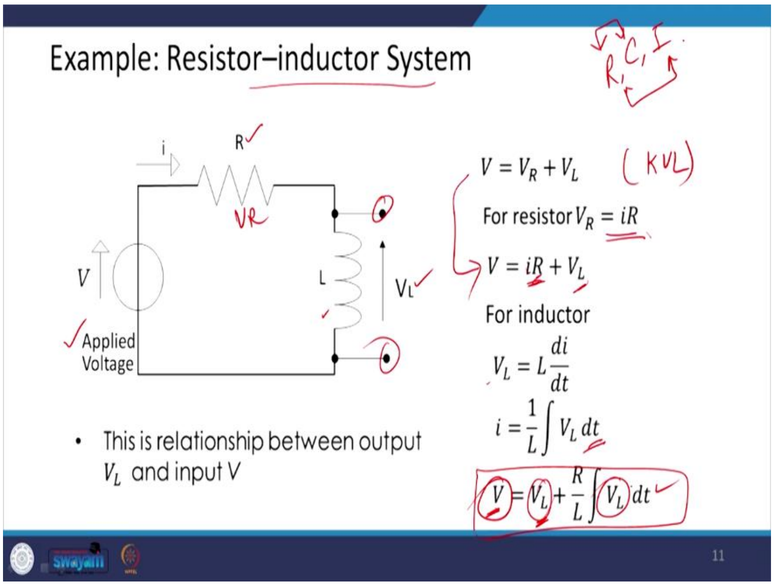

Next, let us take the example of the resistor and inductor. We have seen resistor, capacitor, and inductor. So, in the last slide, I have shown you the example of resistor and capacitor. Next, let us see an example of having the combination of resistor and inductor. So, you have a resistor, and you have an inductor over here. So, again my input is applied voltage V and output suppose I want to take across the two terminals of the inductor that is my output.

So, again I can apply Kirchhoff's voltage law over here. So,

$$V = V_R + V_L$$

$$V_R = iR$$

So, for V_L,

$$V_L = L\frac{di}{dt}$$

Or,

$$i = \frac{1}{L}\int V_L dt$$

Why am I writing it in this way? Because I want things to be expressed in terms of the output parameter that is V_L.

So, my expression V is,

$$V = V_L + \frac{R}{L}\int V_L dt$$

So, this is my equation. So, now, here also you can see that this is my left-hand side here, so this is my forcing function which is the input voltage, and on the right-hand side, I have got the V_L as the parameter, so, this is the voltage across the inductor.

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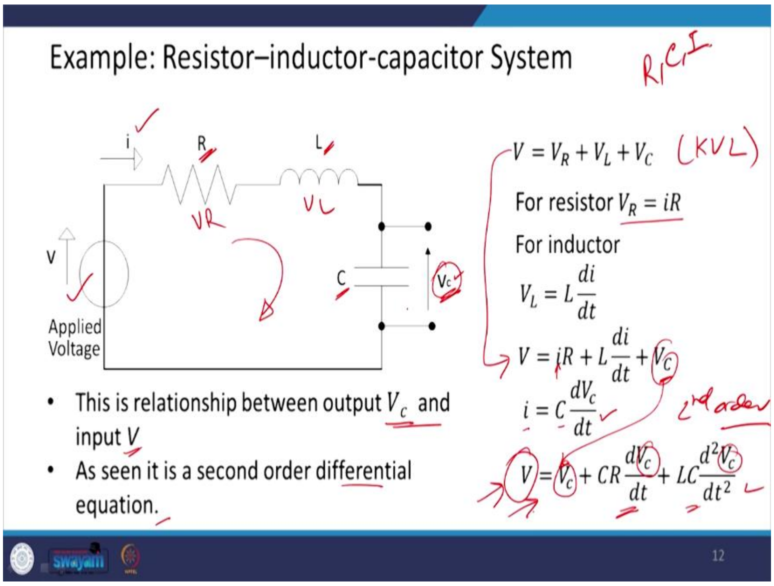

Now, next example, let us take where we have a combination of R C and I. So, I have got a resistor, I have got a capacitor, and I have got an inductor, and suppose my output, I am interested in seeing across the capacitor. So, this is my applied voltage or the voltage source, and here i is the current through the circuit.

So, what is my intention? My intention is to develop a relationship between V_C and V, and I have seen it is a second-order differential equation. So, this is how it looks like; but how it comes, let us see that. So, if I apply Kirchhoff's voltage law over here, so

$$V = V_R + V_L + V_C$$

$$V_R = iR$$

$$V_L = L\frac{di}{dt}$$

$$V = iR + L\frac{di}{dt} + V_C$$

$$i = C\frac{dV_C}{dt}$$

So, this is what I get. So, we see this V_C is already there, this *i* in this one, and if I substitute for i here, then I get this one.

$$V = V_C + CR\frac{dV_C}{dt} + LC\frac{d^2V_C}{dt^2}$$

So, it is a left-hand side, and you can see the forcing function is the voltage supplied, and on the right-hand side, we have got the V_C . So, it is a second-order differential equation for the RLC circuit.

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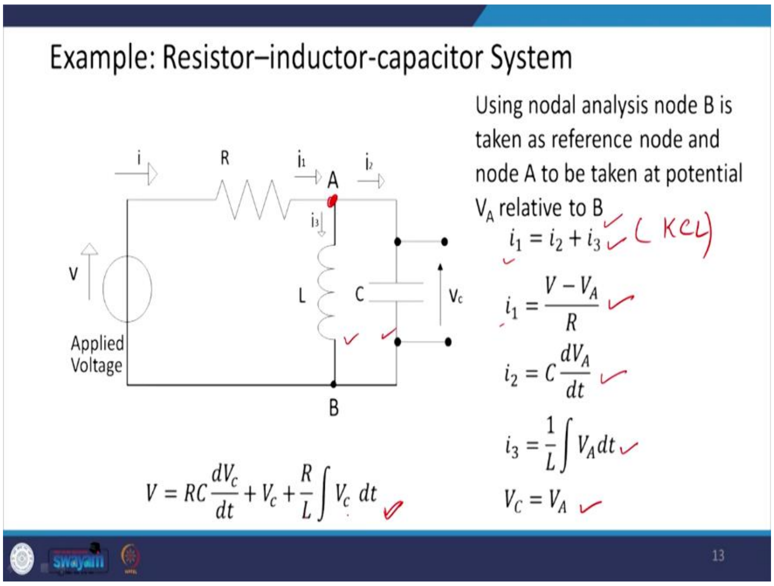

If we have a resistor inductor capacitor system here; in the previous case, all these R, L, and C, were in series and suppose here, they are in a parallel rest inductor and capacitor are in parallel, then, in this case, we can make a nodal analysis using Kirchhoff's current law. So, I will be getting this equation,

$$V = RC\frac{dV_C}{dt} + V_C + \frac{R}{L}V_C dt$$

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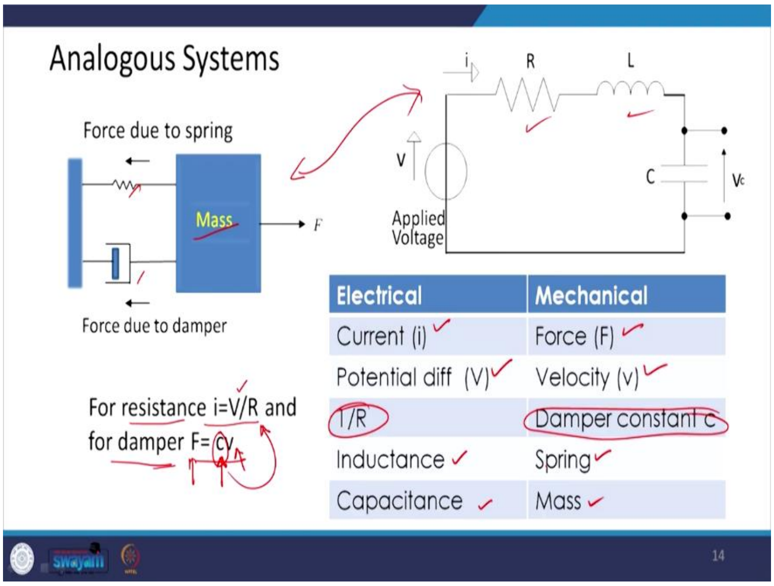

So, this way, using KCL and KVL, we can write the equation of the system. So, you see often there is an analogy which is made between the electrical system and a mechanical system. So, we have seen the mechanical system in the previous lecture, so where you have a mass, you have a spring, and you have a damper over here, and its analogous electrical system is an RLC circuit consisting of a resistor, inductor, and the capacitor.

So, see for resistance from ohm's law, we get $i = V R$ for the electrical system and for a mechanical system, for a damper $F = cv$; where c is the damping coefficient, v is the velocity of the damper and velocity of the piston inside the damper and F is the resistive force which is going to act. So, if I make a comparison of that, you see that this C is analogous to your 1 by R over here. So, damping constant C is analogous to 1/R in the electrical system, and your force in the mechanical is analogous to current in the electrical. So, force in mechanical is analogous to current in electrical, and velocity in mechanical is analogous to the voltage in electrical; velocity in mechanical is analogous to voltage or potential difference in electrical, and inductance is analogous to spring, and capacitance is analogous to mass. So, this way, we can make an analogy between the electrical and mechanical systems.

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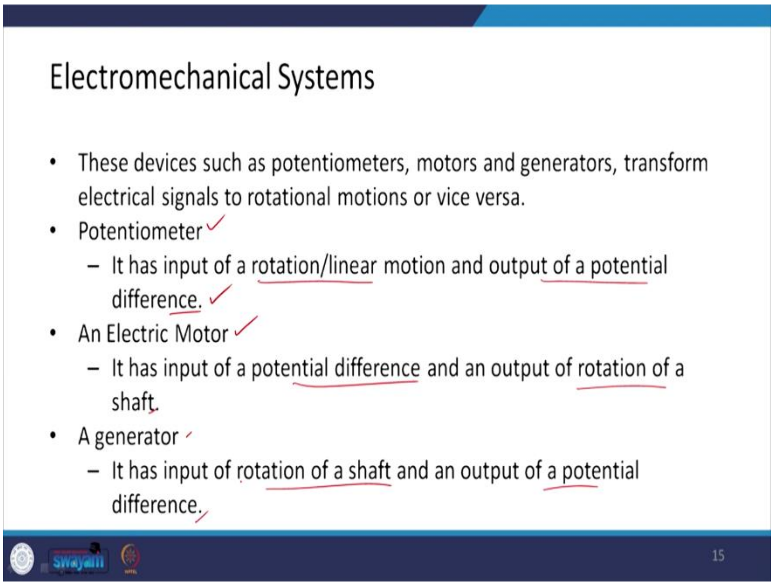

Next, before we take an example of a further example of a motor, let us look at the little basics of the electromechanical system. These devices, such as a potentiometer, motor, and generator, transform the electrical signal to the rotational motion or vice versa. So, what does a potentiometer does? It has the input of a rotation or linear motion and the output of a potential difference. And electrical motor has the input of potential difference and output of rotation of the motion of the shaft. In a generator, it has the input of rotation of the shaft and output of the potential difference.

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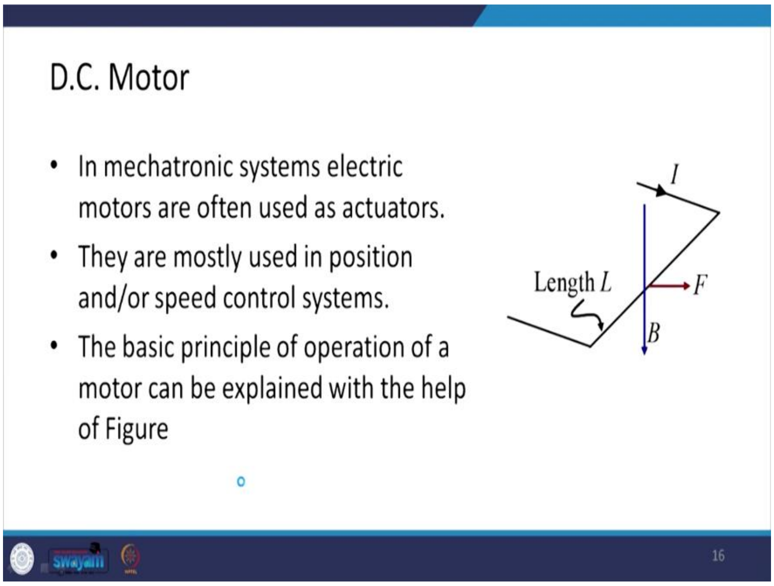

So, these are the three examples of the electromechanical system. So, now let us look at a little concept of the D.C. motor, and then, we will see how we can use the basic building blocks in deriving the system equation for the motor. That is the armature excited as well as the field excited motor. We will see both cases.

Now, in the mechatronic system, electrical motors are often used as actuators. We have seen that, and they are mostly used for in the position or speed control system, and the basic principle operation of a motor can be explained with the help of this figure. So, here is that if a current-carrying conductor is placed in a magnetic field, it experiences a force.

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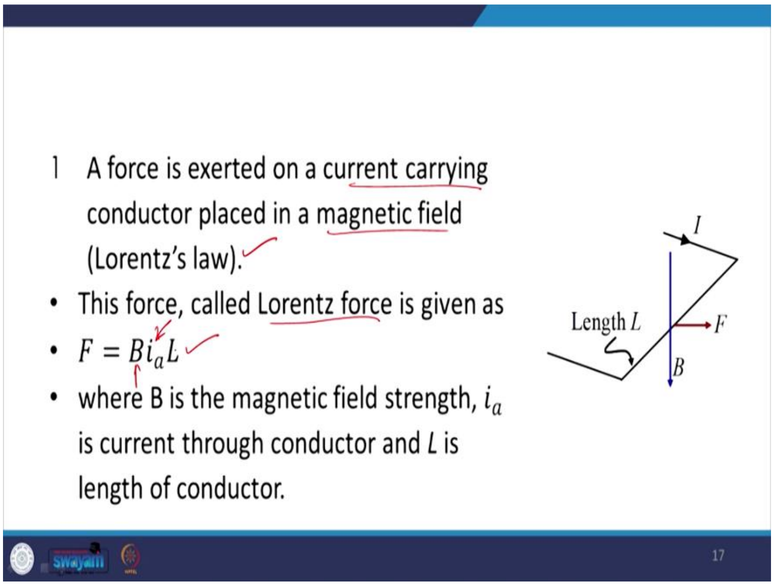

So, that is the basic principle. So, a force is exerted on the current-carrying conductor placed in the magnetic field. This is Lorentz law, and this force called Lorentz force is given by,

$$F = Bi_aL$$

where B is the strength of the magnetic field, i_a is the current through a conductor, and L is the length of the conductor.

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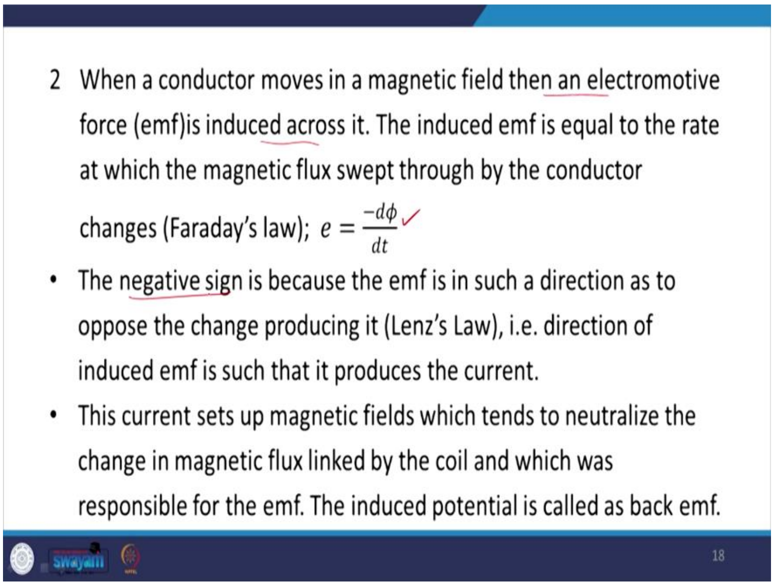

And then, the second law is Faraday's law that is when a conductor moves in a magnetic field, then an electromagnet emf is induced across it, and the induced emf is equal to the rate at which the magnetic field magnetic flux swept through it by the conductor changes or very popularly known as,

$$e = -\frac{d\phi}{dt}$$

So, that is the negative sign because the emf is in such a direction as to oppose the change producing it and which we call Lenz's law. So, that is, the direction of induced emf is such that it produces the current. Now, these current sets of a magnetic field tend to neutralize the change in the magnetic flux linked by the coil and which was responsible for this emf, and that is why this potential is often called the back emf.

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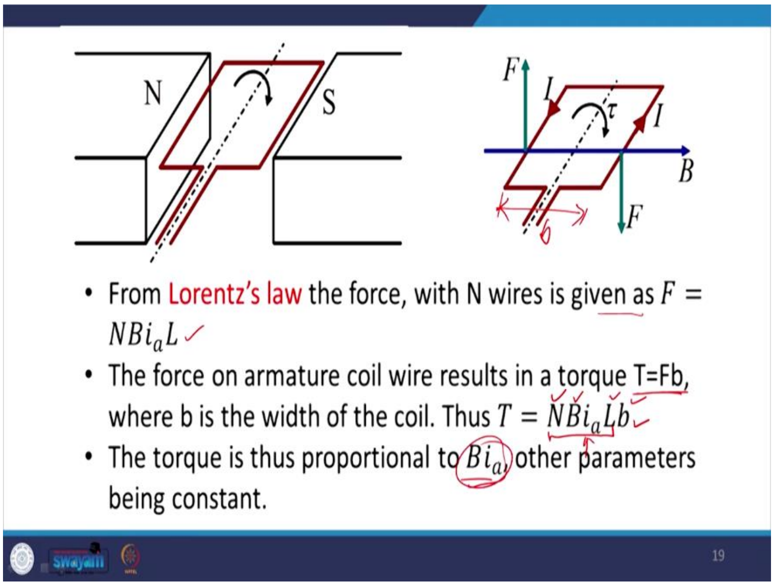

Now, from. So, from Lorentz's law, the force, with N wires can be given as,

$$F = NBi_aL$$

so, the torque if this is having a width b, then you are going to have the,

torque as F .b and,

$$T = NBi_aLb$$

So, here this N is constant field strength is constant, L is constant, b is constant. So, your torque produced is proportional to the current through the armature, other parameters being constant.

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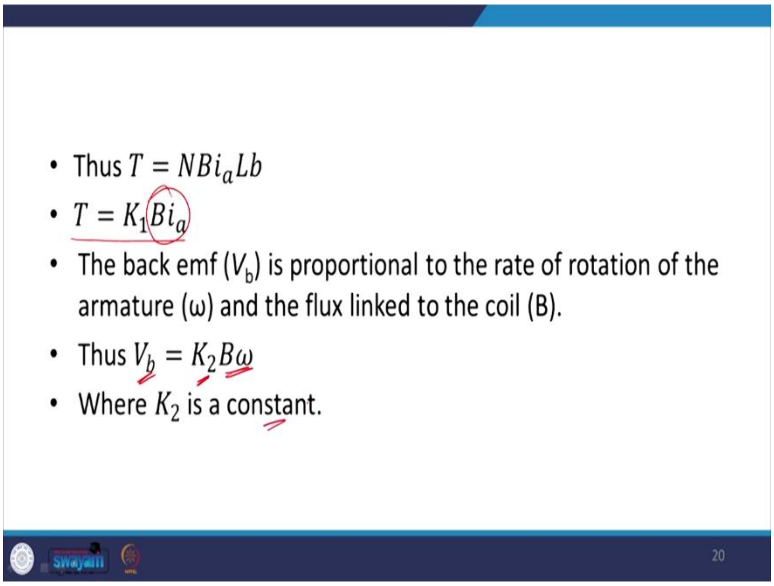

So, here suppose, if I am taking that B is also not constant, then we can take that the torque is proportional to B i_a . So, this is my equation,

$$T = K_1Bi_a$$

and you see that this back emf is proportional to the rate of rotation of the armature and the flux linked to the coil. So, this,

$$V_b = K_2B\omega$$

and this K 2 is constant.

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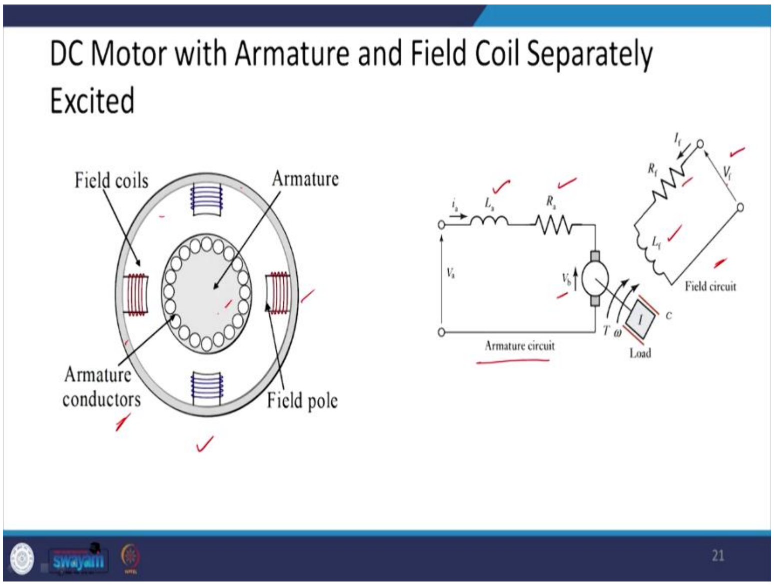

So, we will be using that concept further. So, here let us see the dc motor with armature and field coil separately excited. So, here this indicates the schematic of the D.C. motor. So, you have the armature conductor over here, and you have the field coils over here, and this is your armature. If you model it, we have the armature circuit, which is modeled as an inductor-resistor. This is the back emf, and you have a load over here, and this is the field circuit model as the R_f and L_f and this is the field voltage.

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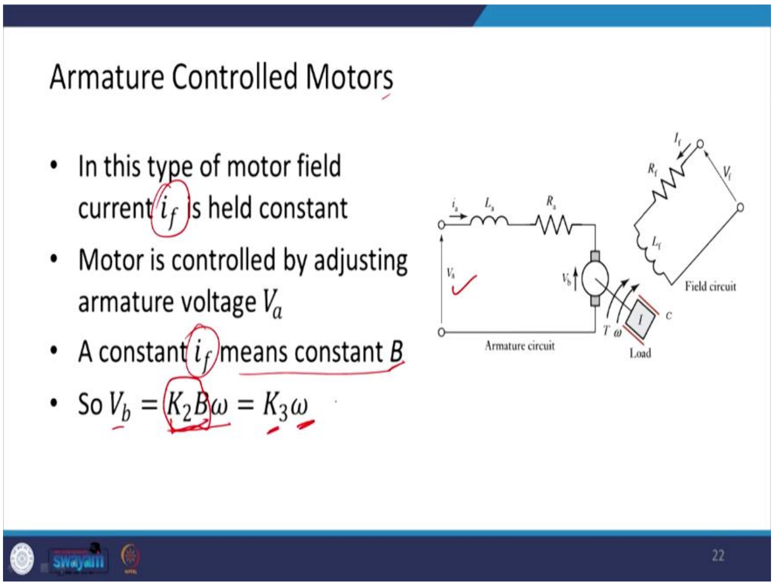

So, first, we will be looking at the armature control motor, and then, I will be talking about the field control motors. So, in the armature control motor, this type of motor field current is held constant because we are controlling it through the armature, and the motor is controlled by adjusting the armature voltage V_a, this one. So, a constant i_f means the constant B. As I said that we are taking i_f constant; so, a constant field current means a constant B. So,

$$V_b = K_2 B\omega = K_3\omega$$

B is constant. So, I can take these two together constant. So, this is K_3 times omega.

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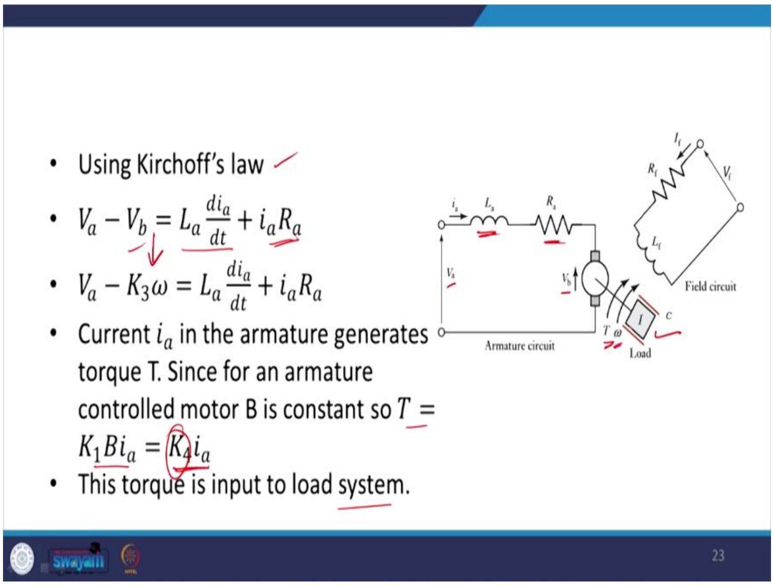

So, then I can apply Kirchhoff's voltage law. So, the net voltage,

$$V_a - V_b = L_a \frac{di_a}{dt} + i_a R_a$$

And then, I substitute for V_b by $K_3\omega$ over here.

$$V_a - K_3\omega = L_a \frac{di_a}{dt} + i_a R_a$$

So, this is the equation that I get. So, the current through armature generates the torque, and since armature is since for an armature control, motor *b* is constant. So,

$$T = K_1 B i_a = K_4 i_a$$

This K_4 I can call it the constant, the torque constant of the motor, and this torque is the input for the load system.

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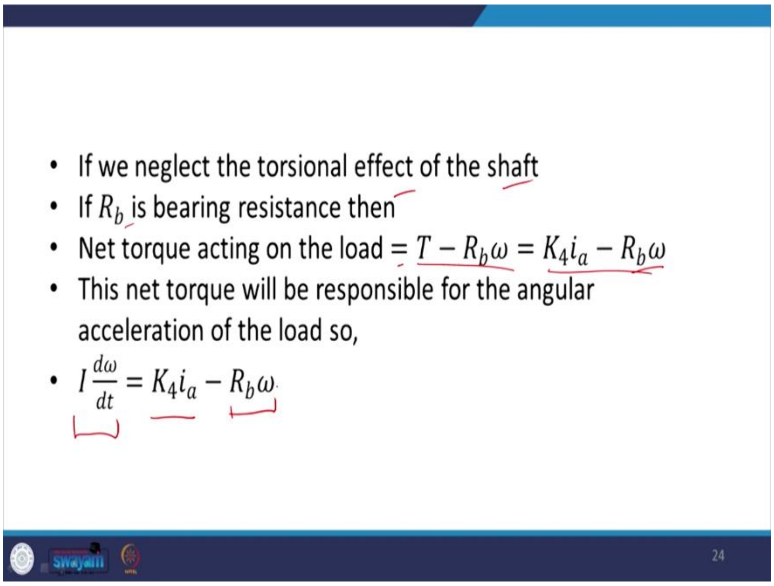

And if we neglect the torsional effect of the shaft and R_b is the bearing resistance, then net torque acting on the load is,

$$\text{The torque acting on the load} = T - R_b\omega = K_4 i_a - R_b\omega$$

and there is a torque T, and the resistance in the bearing will be R_b omega over here. So, we have this, and this net torque is going to be responsible for the angular acceleration of the load. So, this is,

$$I\frac{d\omega}{dt} = K_4 i_a - R_b\omega$$

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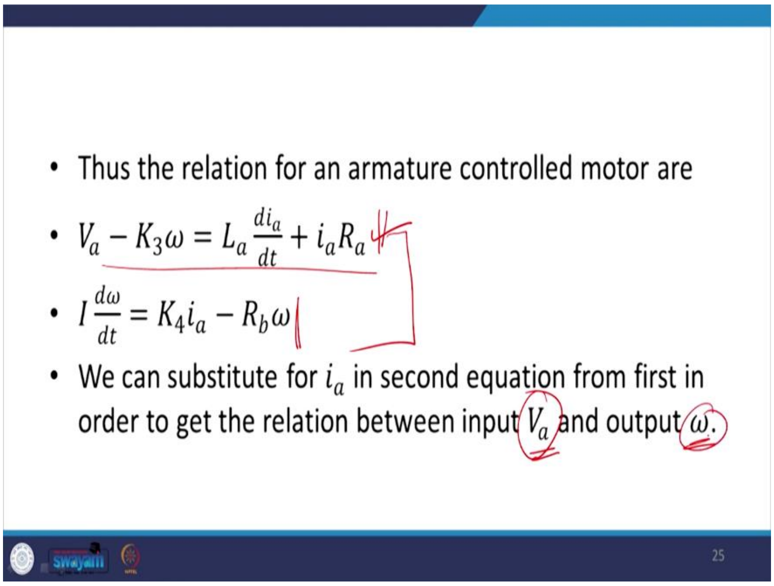

So, in the case of the armature control motor, we have got two relationships. This relationship which we have applied the Kirchhoff's voltage law for the armature and we have got and this relationship, we have applied for the load. So, this we have got.

So, here we can substitute for i_a in the second equation from the first in order to get the relationship between the input voltage V_a and the output ω. So, these two equations we can use to get the relationship between our forcing function, which is the input voltage V_a, and the output ω of the motor.

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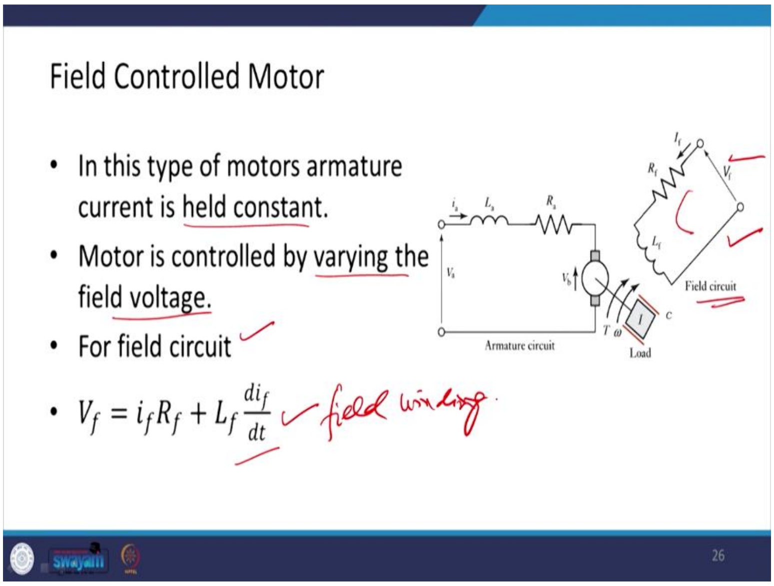

Next, let us take the field control motor. So, in the case of the field control motor, we are going to change the field over here. So, in this type of motor, armature current is held constant. So, the motor is controlled by varying the field voltage. So, for the field circuit here, I can write this equation that is,

$$V_f = i_f R_f + L_f \frac{di_f}{dt}$$

So, this is for the field, field winding rather.

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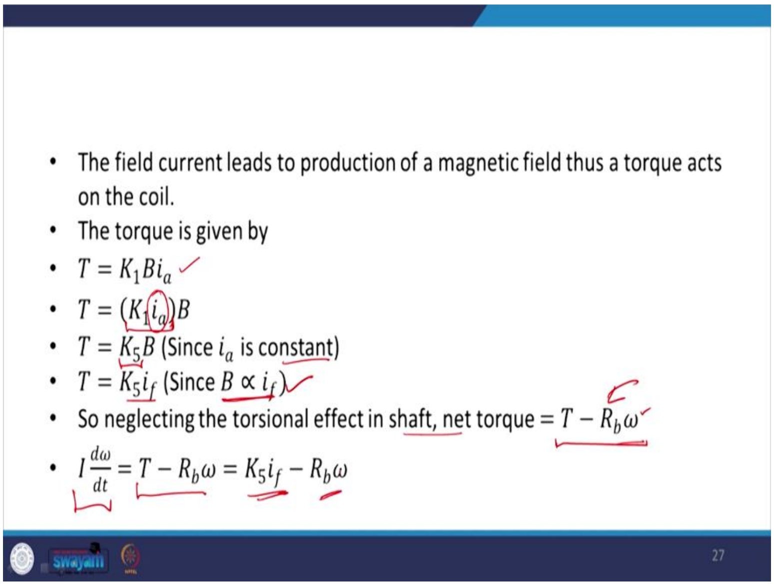

So, the field current leads to the production of a magnetic field. Thus, a torque at on the coil and this torque is given by,

$$T = K_1 B i_a$$

Now, here as I said $K_1 i_a$, i_a I am keeping constant; armature current, I am not changing. So, this is together, and I can take it as a constant. So, this is ,

$$T = K_5 B$$

K_5 is a constant. So

$$T = K_5 i_f$$

So, if I neglect the torsional effect in the shaft, net torque = $T - R_b \omega$

And this is the factor that is responsible for the acceleration of the shaft. So, I equate it to,

$$I\frac{d\omega}{dt} = T - R_b\omega = K_5 i_f - R_b\omega$$

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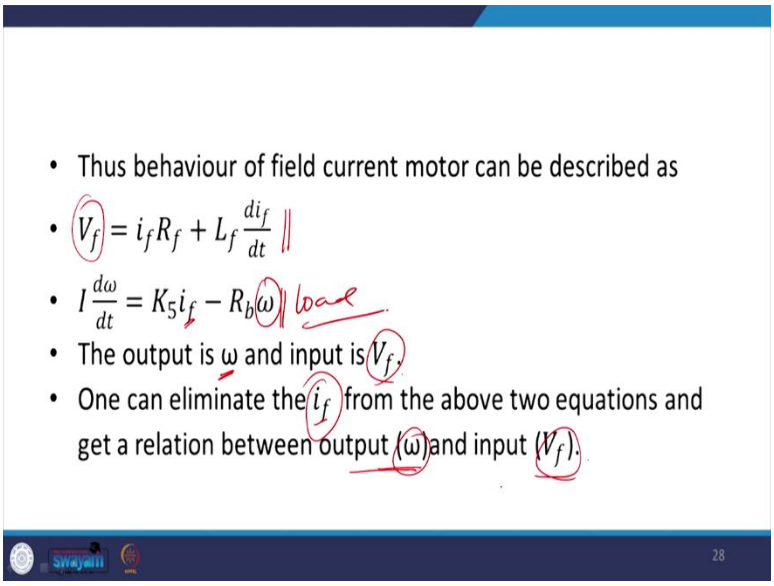

So, this is what I get. This is my load equation. So, $I\frac{d\omega}{dt}$ is $K_5 i_f - R_b\omega$, and this is my relationship which I have got for from the field winding. So, the output is ω here, and our input is V_f. So, this is my output, and input is my V f. So, we can eliminate i_f . So, i_f is common in these two equations, so I can eliminate i_f from the above two equations and get a relationship between the output, which is my ω, and the input, which is my V_f .

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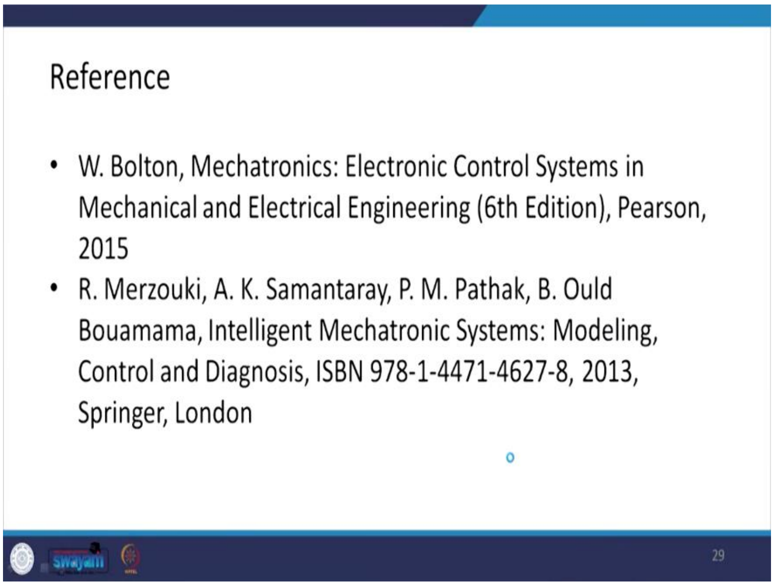

So, you see that here, we have used the basic building blocks to model both the field control D.C. motor as well as the armature control D.C. motor. So, in this lecture, we have seen the basic building blocks for the electrical system, and then, we have used the combination of blocks to analyze a system, and then, we have used the same concept of combination of the block to analyze the armature control and the field control D.C. motor.

So, these are the references. If you want to read it further, please go through them.

Thank you.