Mechatronics
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Lecture – 19
Digital to Analogue Converters

I welcome you all to today's NPTEL online certification course on Mechatronics. Today we are going to talk about Digital to Analogue Converter. And, before that, I would like to discuss the representation of any signal as a Fourier series.

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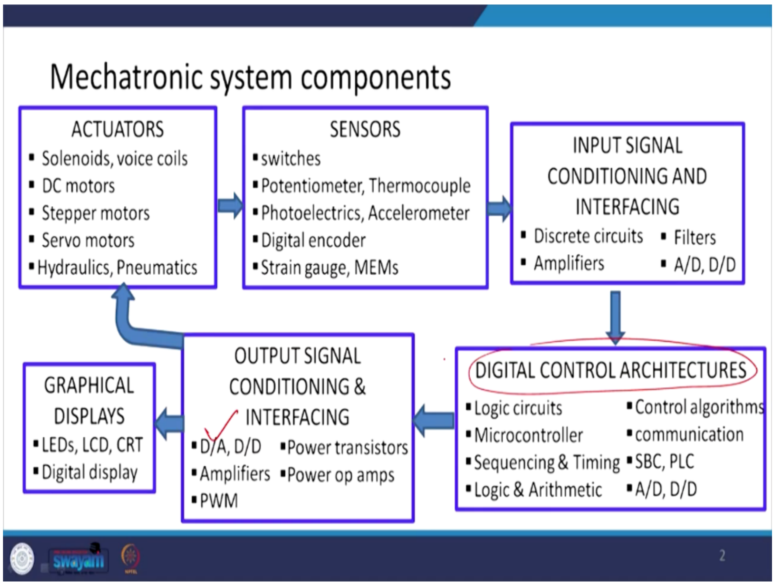

A digital to analog converter is used in the output signal conditioning and interfacing. So, as I have been talking to you that the signals which we get from the microprocessor are digital control architecture. These signals are digital in nature and these signals need to be converted into the analog signal if you want to send or if you want to send those signals to an actuator. Then we require the digital to analog converter.

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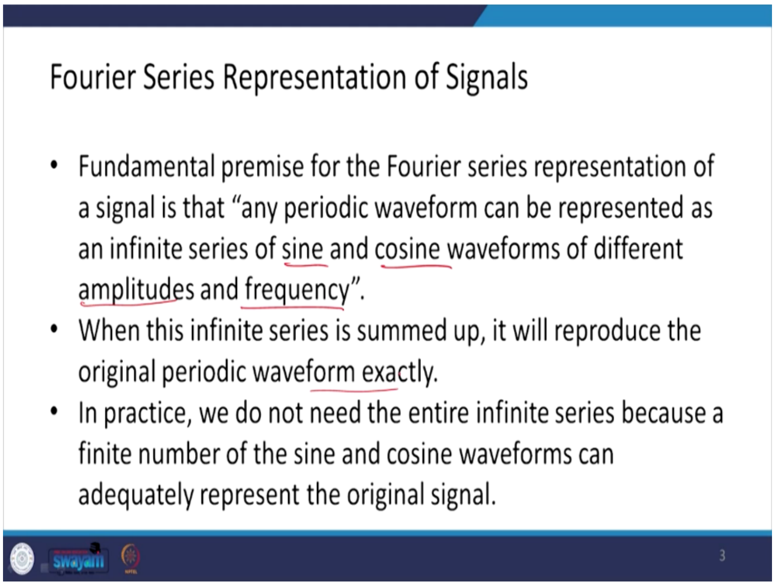

Fourier series representation of the signal, I wanted to have a little discussion on this topic here before I present to you the digital to analog converter. This gives us some visualization about analog signal and its mathematical representation that is the mathematical representation of the analog signal.

The fundamental premise for the Fourier series representation of a signal is that any periodic waveform can be represented as an infinite series of sine and cosine waveforms of different amplitudes and frequencies. When this infinite series is summed up it will reproduce the original periodic waveform exactly. In practice, we do not need the entire infinite series because a finite number of sine and cosine waveforms can adequately represent the original signal.

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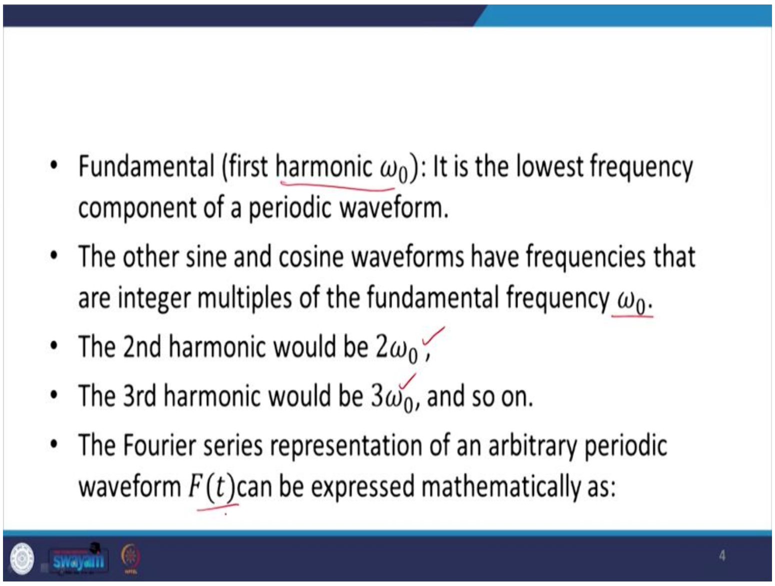

So, the fundamental or the first harmonic frequency ω_0 is defined as the lowest frequency component of a periodic waveform. The other sine and cosine waveforms have frequencies that are integral multiples of the fundamental frequency ω_0. So, the 2nd harmonic would be represented as $2\omega_0$; the 3rd harmonic will be represented at $3\omega_0$ and so on. The Fourier series representation of an arbitrary periodic waveform $F(t)$ can be expressed mathematically as this.

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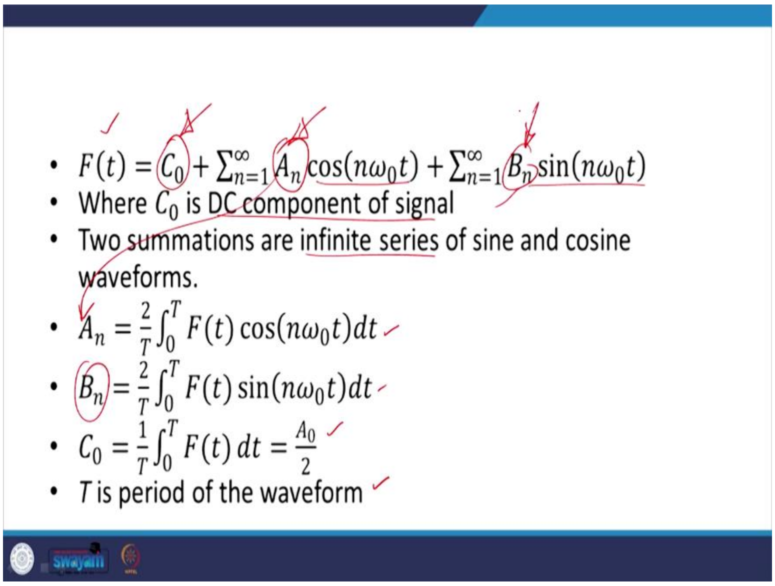

So,

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

Where C_0 is a DC component of the signal. Then, the 2 summations here are the infinite series of sine and cosine waveforms.

Now, here this A_n is defined like,

$$A_n = \frac{2}{T}\int_0^T F(t)\ \cos(n\omega_0 t)dt$$

$$B_n = \frac{2}{T}\int_0^T F(t)\ \sin(n\omega_0 t)dt$$

And,

$$C_0 = \frac{1}{T}\int_0^T F(t)\, dt = \frac{A_0}{2}$$

Here T is the period of the waveform.

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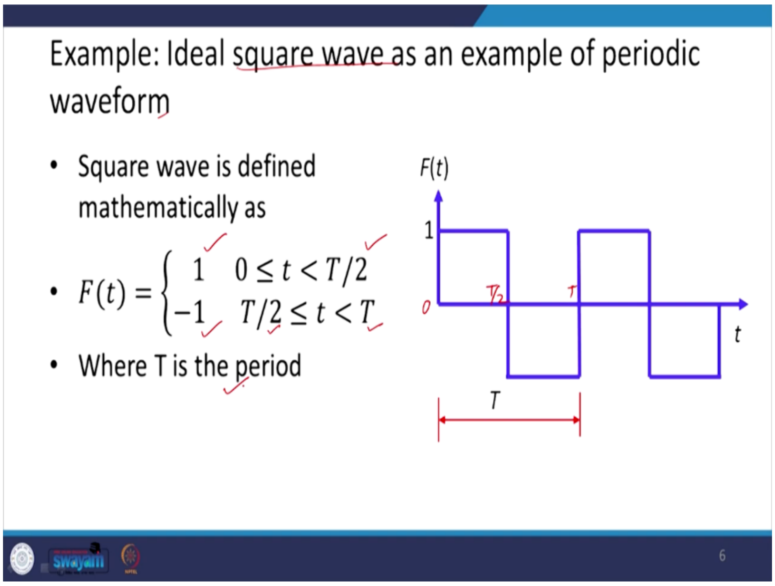

So, suppose we have a ideal square wave as an example of the periodic waveform. So, I have an ideal square wave whose magnitude is 1 between 0 to *T/ 2*, and between *T/ 2* and *T,* it is -1, where *T* is the period. So, the square waveform is defined like this.

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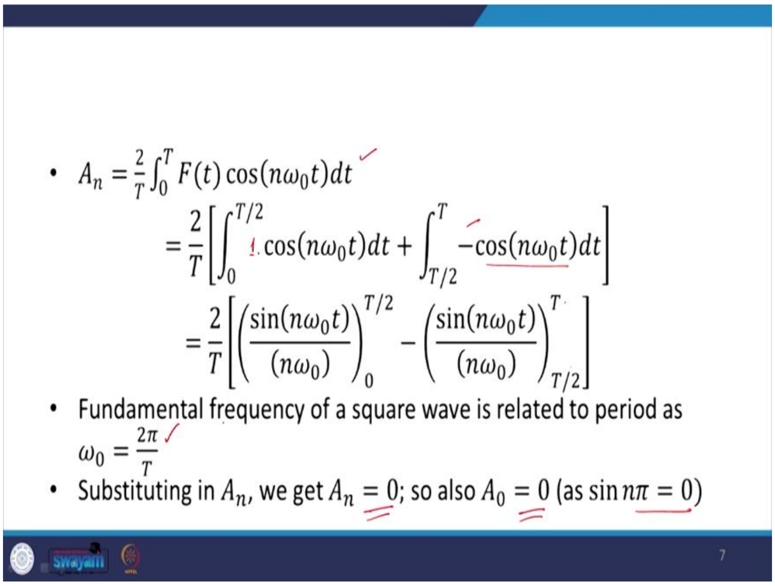

Now, this waveform if I want to represent mathematically that is the using the Fourier series, then we can evaluate the different components here. That is, we can evaluate C_0, we can evaluate A_n and we can evaluate B_n, then we can see that how we can represent this signal. So,

$$A_n = \frac{2}{T}\int_0^T F(t)\ \cos(n\omega_0 t)dt = \frac{2}{T}[\int_0^{\frac{T}{2}} \cos(n\omega_0 t)\ dt + \int_{\frac{T}{2}}^{T} -\cos(n\omega_0 t)\ dt]$$

$$= \frac{2}{T}[\left(\frac{\sin(n\omega_0 t)}{n\omega_0}\right)_0^{\frac{T}{2}} - \left(\frac{\sin(n\omega_0 t)}{n\omega_0}\right)_{T/2}^{T}]$$

And, we know that the fundamental frequency of a square wave is related to period as $\omega_0 = \frac{2\pi}{T}$. If we substitute here,

We get A_n = 0 and so, also A_0 = 0 because here we will be getting $\sin(n\pi)$ type of terms. If I am putting $\omega_0 = \frac{2\pi}{T}$. This is what I am going to get.

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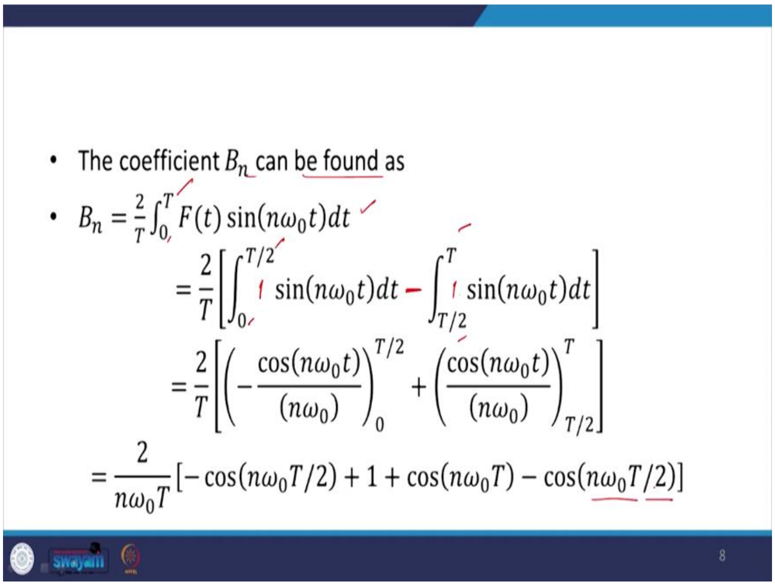

Then we can similarly find out the,

$$B_n = \frac{2}{T}\int_0^T F(t)\ \sin(n\omega_0 t)dt = \frac{2}{T}[\int_0^{\frac{T}{2}} \sin(n\omega_0 t)\,dt + \int_{\frac{T}{2}}^{T} -\sin(n\omega_0 t)\,dt]$$

$$= \frac{2}{T}[\left(-\frac{\cos(n\omega_0 t)}{n\omega_0}\right)_0^{\frac{T}{2}} - \left(\frac{\cos(n\omega_0 t)}{n\omega_0}\right)_{T/2}^{T}]$$

$$= \frac{2}{n\omega_0 T}[-\cos\left(\frac{n\omega_0 T}{2}\right) + 1 + -\cos(n\omega_0 T) - \cos(n\omega_0 T/2)]$$

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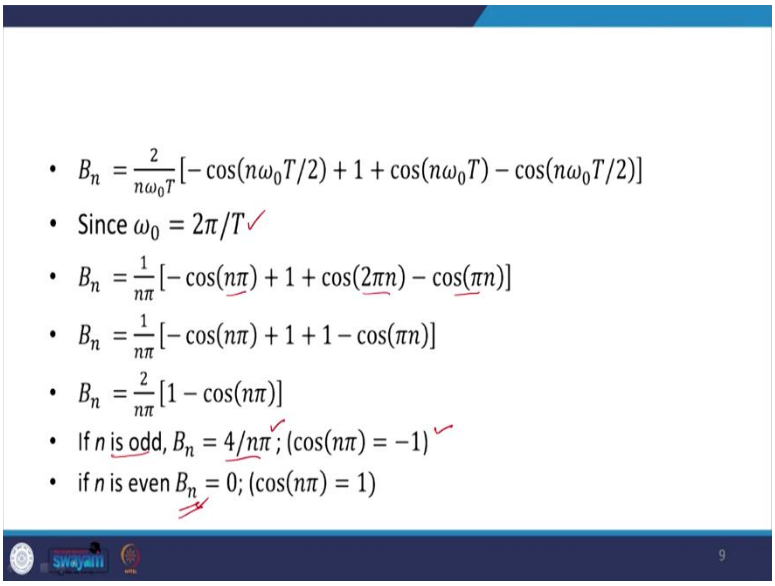

So, this since we have $\omega_0 = \frac{2\pi}{T}$, I can substitute it over here and evaluate these terms. So, this is what I am going to get. And, you see $\cos(n\pi) = -1$, and if n is odd so, I can evaluate

$$B_n = 4/n\pi$$

and if n is even

$$B_n = 0$$

$$\cos(n\pi) = 1$$

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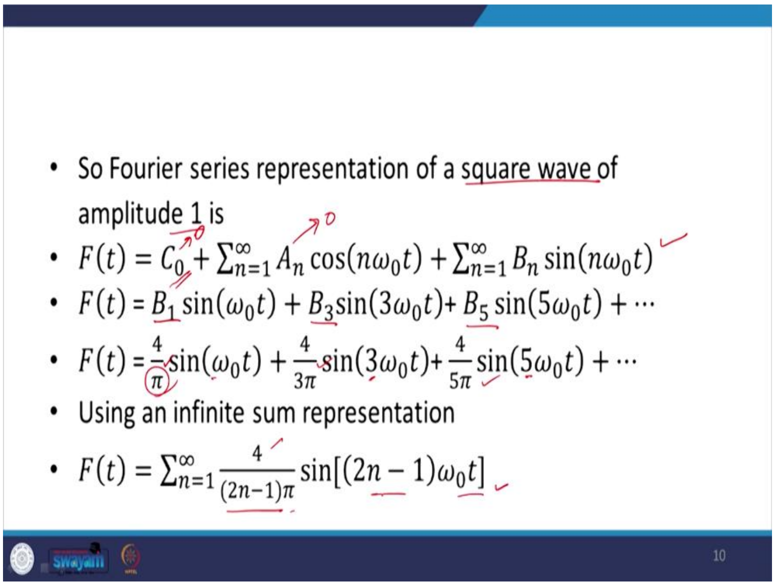

So, the Fourier series representation of a square wave of amplitude 1 is given by this one. So, this is our original equation C_0 we have evaluated as 0, A_n is 0, C_0 we have seen this is 0. So, here we will be getting only odd terms because even terms we have seen that, in case of even terms B_n = 0.

So, this is what I am going to get B_1, $B_{3,}$ and B_5 and then I can substitute for these 4/π here, 4/3π, 4/5π and so on. And, using an infinite sum I can represent this as 4 by here you have the odd terms. So, (2n-1)π and *sin* here again you have the odd term.

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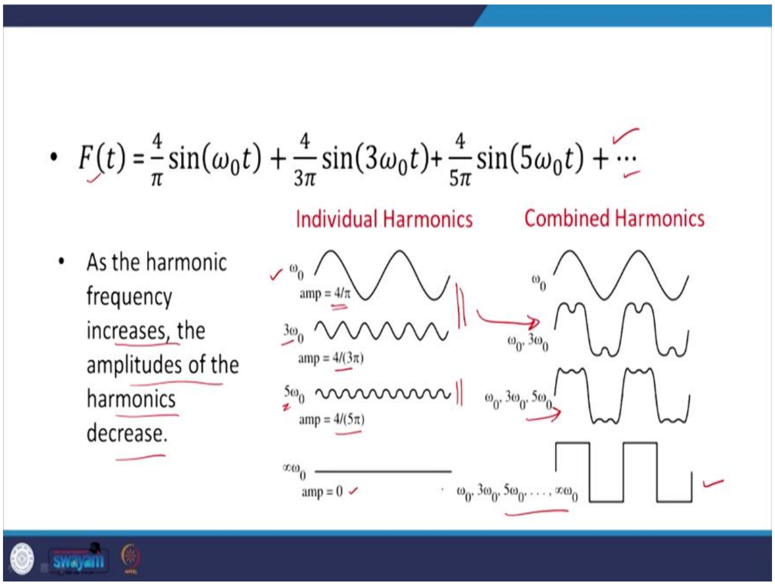

So, this is how *F (t)* is represented over here. So, this is the Fourier series representation of the square wave signal of the unit amplitude; so, now if you look at the individual harmonics and combined harmonics. So, this ω_0 frequency here we have the amplitude as $4/\pi$, this is the waveform, and $3\omega_0$ as amplitude $4/3\pi$.

So, if I add up these 2, this is what I get.

And, this is the fifth harmonic $5\omega_0$amplitude is $4/5\pi$ and if I add up these 2 this is what I am going to get. And, this is the last one infinite ω_0 amplitude is 0 and if I add it up all these terms so, this is what I am going to get. So, what I am wanted intended to show it here is that our square wave signal can be represented in this way. That is by the signal sinusoidal signal of different frequencies and different amplitude. And, as the harmonic frequency increases the amplitude of the harmonic decreases which we have already seen.

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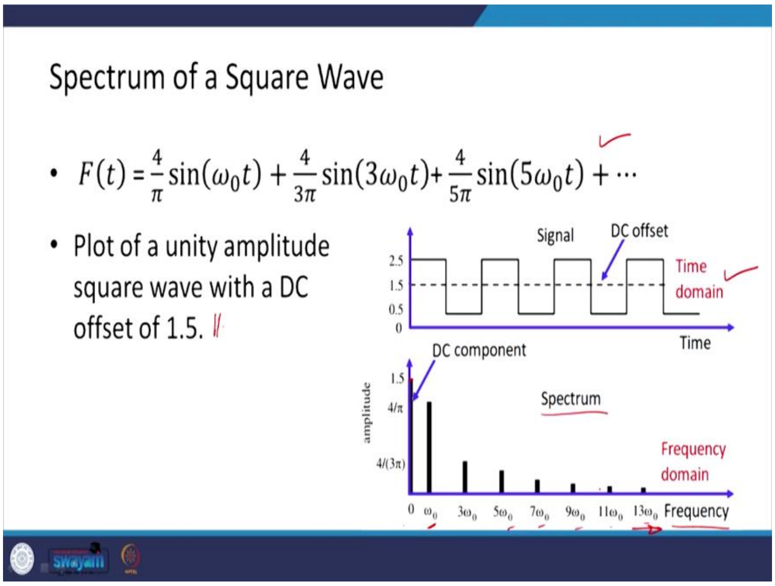

So, the spectrum of a square wave this is your F(t). So, plot of unity amplitude square wave or with a DC offset if this is 1.5 volt. Then in the time domain if I plot it, this is how the signal is going to lo like and its spectrum is something like this frequency.

So, this is the zeroth one you have the ω_0 frequency; this is that 1 by 5 the DC offset which I was talking to you and then you have ω_0 frequency $3\omega_0$, $5\omega_0$, $7\omega_0$ and so on. So, as you can see that as your frequency increases their amplitude decreases and when we sum up all these different terms we get the required signal.

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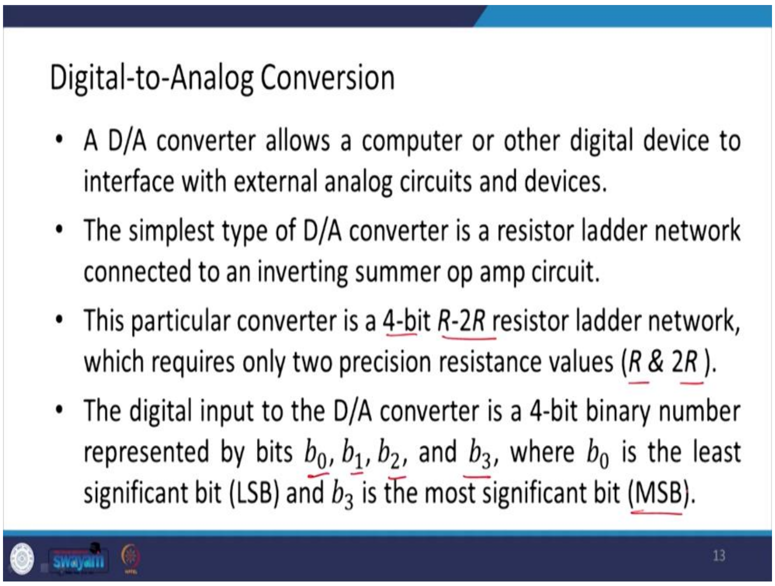

Now, let us talk about digital to analog conversion. The digital to analog converter allows a computer or other digital device to interface with external analog circuit and devices as I talk to you. These devices could be the actuator. The simplest type of digital to analog converter is a resistor ladder network connected to an inverter summing operational amplifier circuit. So, that I am going to discuss with you.

This particular converter is a 4-bit R-2R resistor ladder network that requires only 2 precise resistance values that is R and 2R and the digital input to the D to A converter is a 4-bit binary number represented by bits' b_0, b_1, $b_2$and $b_3$and where b_0 is the LSB or the least significant bit and b_3 is the most significant bit or what we call in short as MSB.

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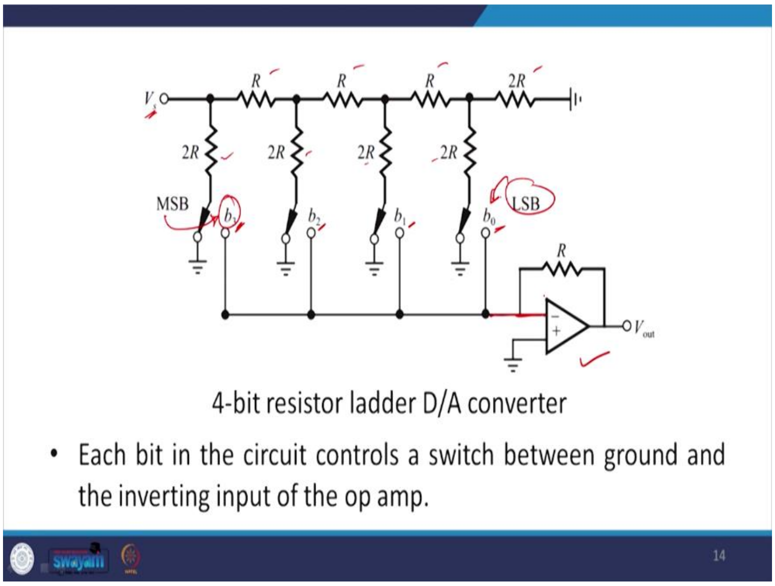

So, this is how this R-2R ladder network comprises of. So, this is your inverting amplifier operational amplifier. And, here you can see that these source volts and you have R, R, R here and you have 2R resistor 2R here and this is 2R and here R the switches to switch between the ground and the input to the inverting amplifier here.

So, these are the b_0, b_1, $b_2$and b_3 which I talked to you with this b_0 is the least significant bit and this b_3 represents the most significant bit. And, each bit in the circuit controls a switch between the ground and the inverting input of the operational amplifier. So, this is the inverting input as you can see. So, this is how each bit makes a connection to that.

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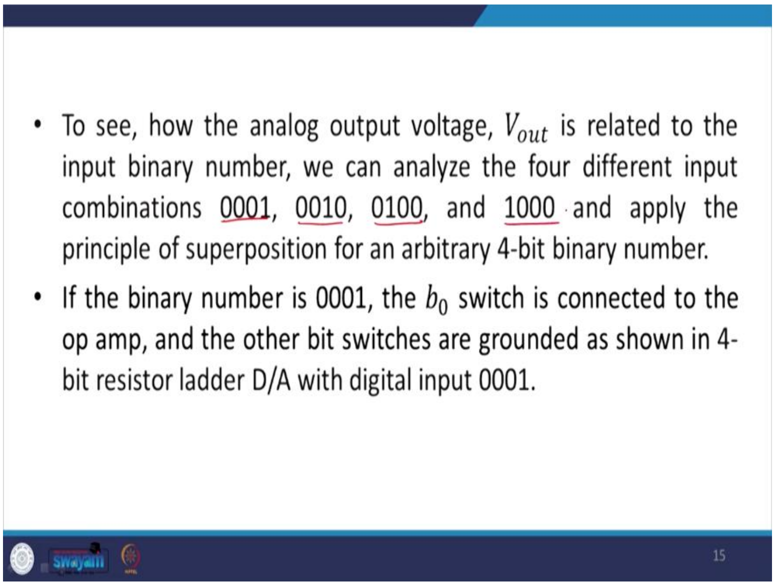

Now, to see how the analog output voltage V_{out} is related to the input binary number. We can analyze the four different input combinations 0001, 0010, 0100 and 1000 and apply the principle of superposition for any arbitrary 4-bit number, and let us start with the first one. If the binary number is 0001 the b_0 switch is connected to the op-amp and the other bit switches are grounded.

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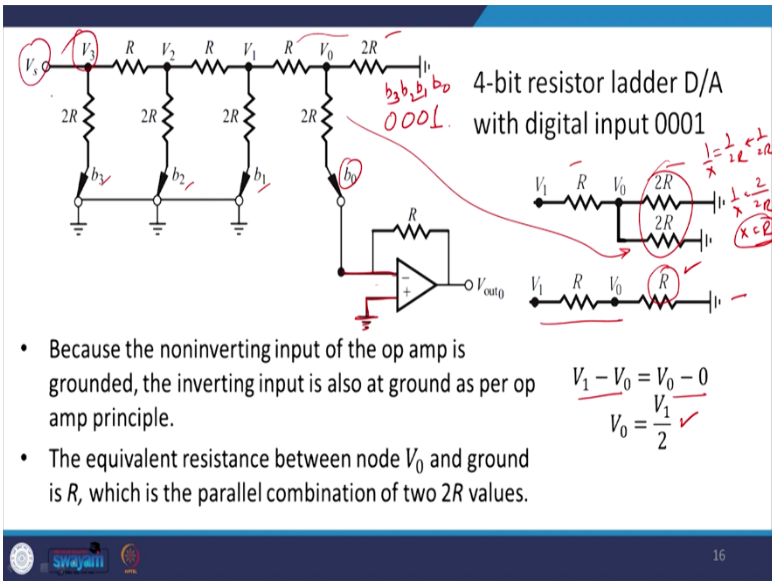

So, as we can see over here. So, this corresponds to 0001. So, here you can see that this one b 0 is connected to the op amp inverting terminal over here and b 1, b 2, b 3 these are grounded. So, this 001 is like this b_0, b_1, $b_2$and b_3. So, this is there. Now, with this what happens? So, with this because you see non-inverting input of the op-amp is grounded. So, the inverting input is also grounded as per the op-amp principle. You can see that this is grounded over here. So, this one is also grounded and this is what we have assumed when we have discussed the operational amplifier. So, with this, if this becomes grounded then this is how our configuration is. So, this R is here this 2R is here and this 2R becomes something like this. Now, you can see that this 2R and this 2R are in parallel. So, I can find out the equivalent of that 1/X is (1/2R +1/ 2R). So, this is 1/X is 2/2R or your X = R. So, the equivalent of this one is a resistor of R ohm.

Now, you see here if I take the drop across this resistor is $V_1 - V_0$ and this is also the same resistor. So, that drop is going to be the same and that is equal to V_0 minus this is grounded. So, this is 0. So, from here I can get,

$$V_0 = \frac{V_1}{2}$$

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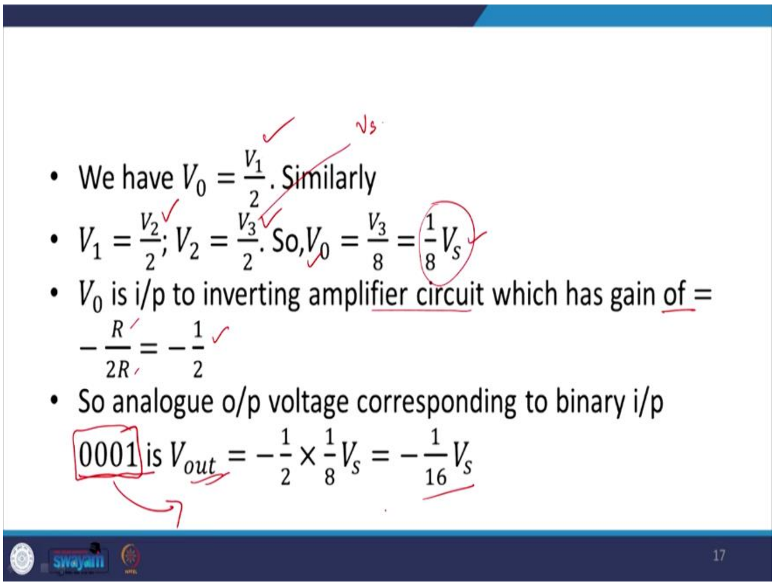

Similarly, I can have,

$$V_1 = \frac{V_2}{2};\ V_2 = \frac{V_3}{2}$$

And,

$$V_0 = \frac{V_3}{8} = \frac{V_s}{8}$$

And, also you see that V_0 is the input to the inverting amplifier circuit and it has a gain of,

$$-\frac{R}{2R} = -\frac{1}{2}$$

So, that is there. So, the analog output voltage corresponding to binary input 001 will be what? It will be amplified by,

$$V_{out} = -\frac{1}{2} \times \frac{1}{8} V_s$$

So, what does this means that if your input is 0001, your output that is if for this digital input your output is going to be,

$$V_{out} = -\frac{1}{16} V_s$$

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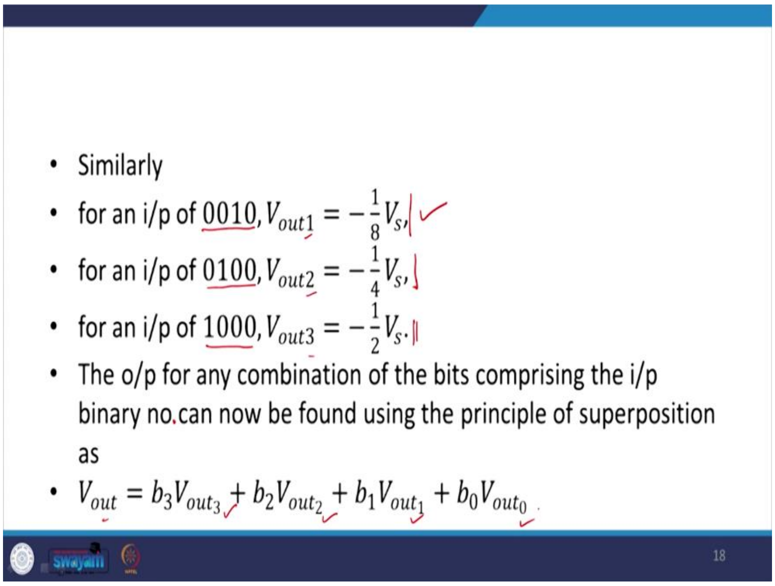

Similarly, if I carry out the similar analysis for an input of 0010,

$$V_{out1} = -\frac{1}{8} V_s$$

if it is 0100 this is,

$$V_{out2} = -\frac{1}{4}V_s$$

and if it is 100 0 this is going to be,

$$V_{out3} = -\frac{1}{2}V_s$$

Now, the output for any combination of the bits comprising the input binaries number can be found using a number can now be found using the principle of superposition as this one.

$$V_{out} = b_3V_{out3} + b_2V_{out2} + b_1V_{out1} + b_0V_{out0}$$

So, this way we can have any combinations of the output voltage using the principle of superposition.

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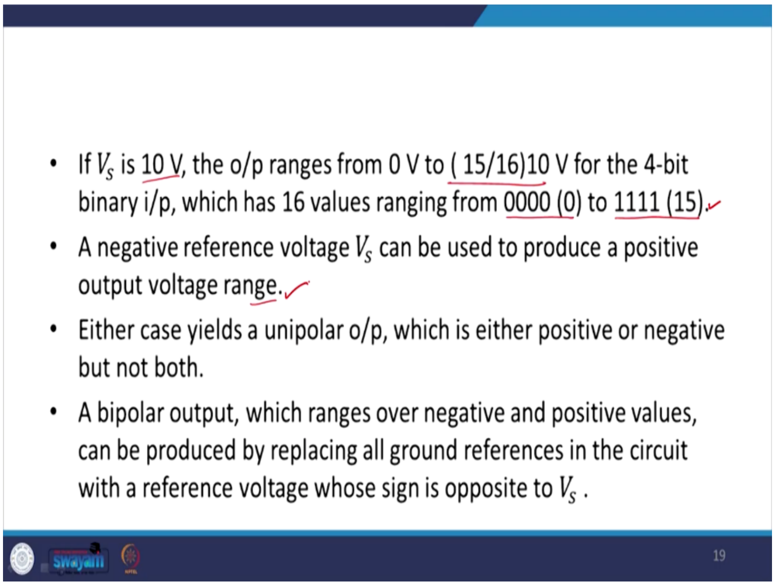

Now, if V_S is 10 volt the output ranges from 0 volt to (15/16)10 volt for the 4 bit binary input which has 16 values ranging from 0000 which is the 0th one to 1111 which is the 15th one. And, a negative reference voltage (V_S) can be used to produce a positive output voltage range.

And, either case yields a unipolar output which is either positive or negative, but not the both and if you need a bipolar output that is which ranges over negative and positive values that can be produced by replacing all the ground references in the circuit with a reference voltage whose sign is opposite to that of the V_s.

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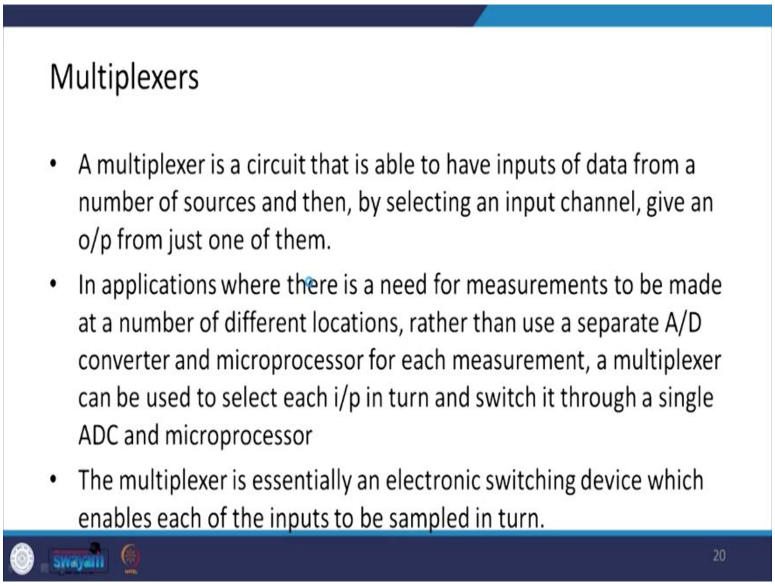

Next another very important component is the multiplexer which is a type of switch electronic switch device which are enable each of the input to be sampled in a term. So, a multiplexer is a circuit that is able to have input of data from a number of sources and then by selecting an input channel gives an output from just one of them. So, it can select an input channel and for corresponding to that one it can give an output. In an application where there is a need for measurement to be made at a number of different locations rather than using a separate analog to digital converter and microprocessor for each measurement, a multiplexer can be used to select each input in turn and switch it through a single analog to digital converter and microprocessor.

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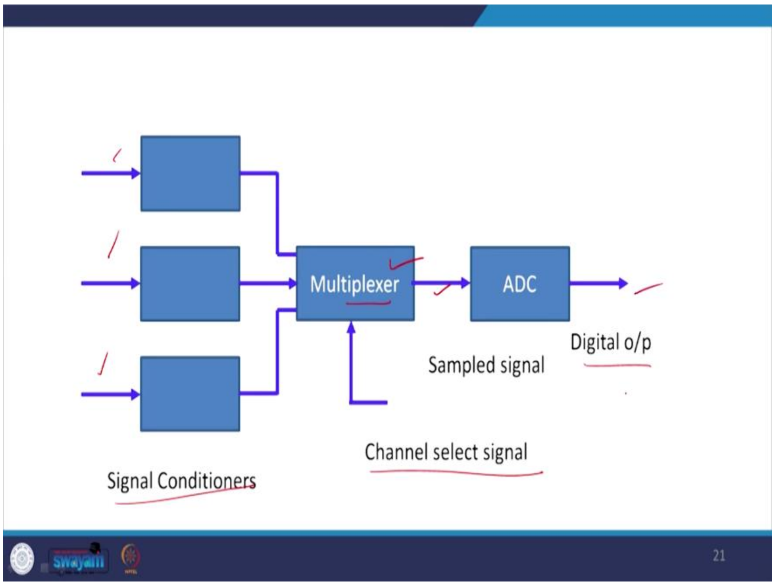

The scheme could be like, you have a signal conditioning conditioner unit. There are multiple signals like this. You pass it through a multiplexer over here. There is a channel to be selected. And, you have a sampled signal here and then it can be sent to analog to digital converter and you get the digital output from over here.

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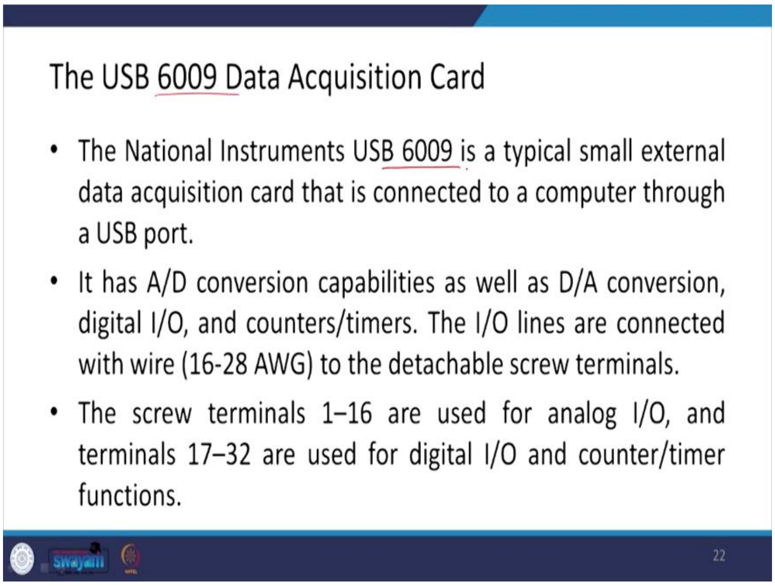

Now, you see that in the last lecture, I have talked about analog to digital converter, in this lecture I have talked about digital to analog converter. Now, these things are typically available in the form of a data acquisition card.

So, I will like to have a small discussion on USB 6009 data acquisition card. So, the national instrument USB 6009 is a typical small external data acquisition card that is connected to a computer through a USB port.

It has analog to digital conversion capabilities as well as digital to analog conversion, digital input-output, and counters as well as timers. The input-output lines are connected with wires 16 to 28 AWG to the detachable screw terminals. The screw terminal 1 to 16 are used for the analog input-output and terminals 17 to 32 are used for the digital input-output and counter timer function.

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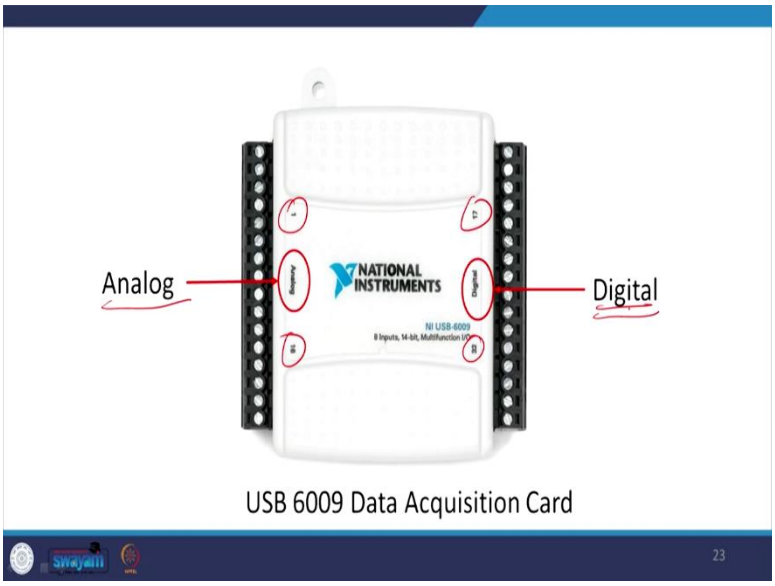

So, here you can see that this 1 to 16 are used for the analog input output whereas, 17 to 32 are used for the digital input output.

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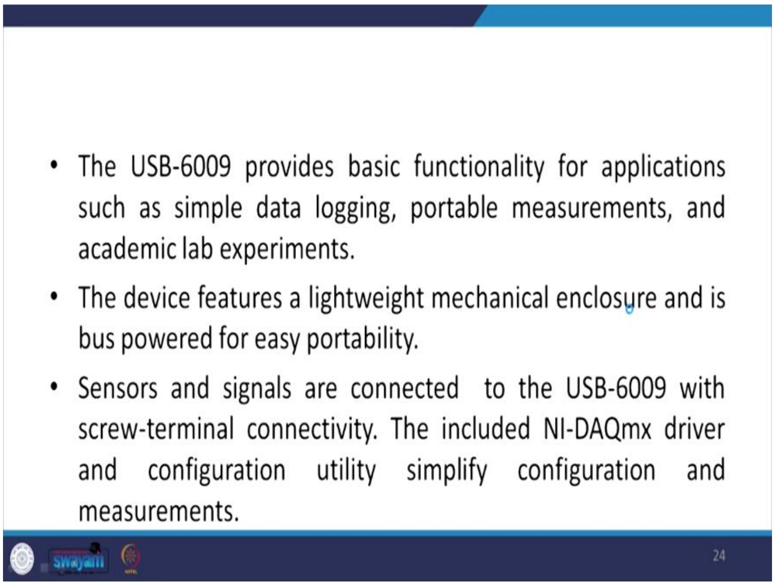

The USB 6009 provides basic functionality for applications such as simple data logging, portable measurements and academic lab experiments. The device features a lightweight mechanical enclosure and is bus powered for easy portability. Sensors and signals are connected to the USB 6009 which is screw terminal connectivity. The included NI-DAQmx driver and configuration utility simplify configuration and the measurement.

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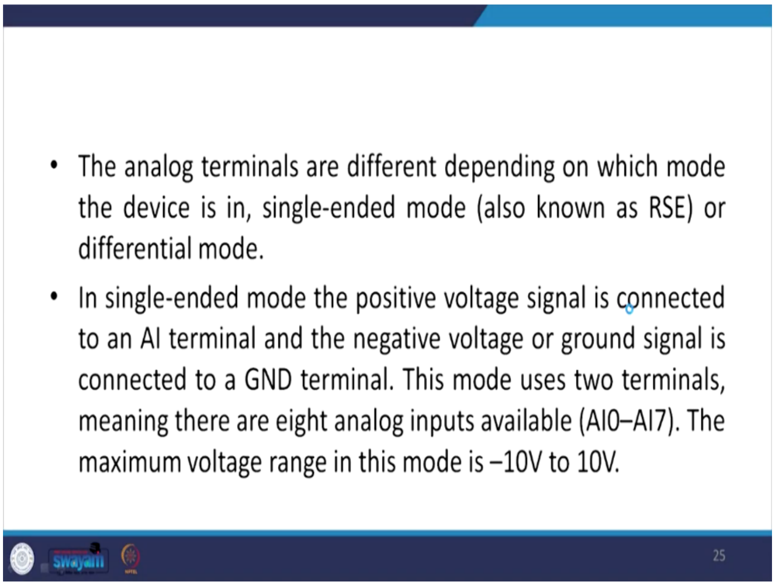

The analog terminals are different depending on which mode the device is in single-ended mode also known as RSE or different mode. In single-ended the positive voltage signal is connected to an AI terminal and in the negative voltage and the negative voltage or ground signal is connected to the ground terminal. This mode uses 2 terminal meaning there are 8 analog inputs available that is AI0 to AI7 and the maximum voltage range is in this mode is -10 to 10 volts.

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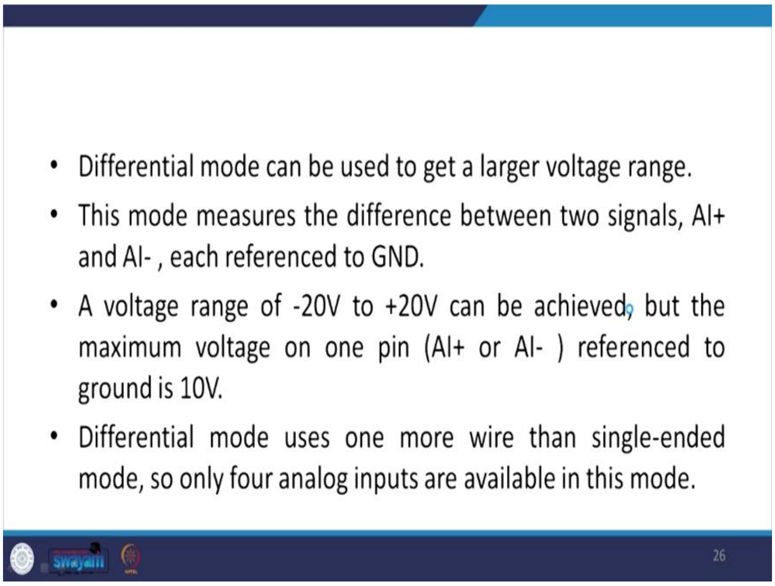

Differential mode can also be used to get larger voltage range and this mode measures the difference between the two signals. So, AI + and AI- each reference to the ground. A voltage ranges of -20 volt to 20 volts can be achieved, but the maximum voltage on one pin that is AI+ or AI- reference to the ground is 10 volts. Differential mode uses one more wire then single ended mode, so, only 4 analog inputs are available in this mode.

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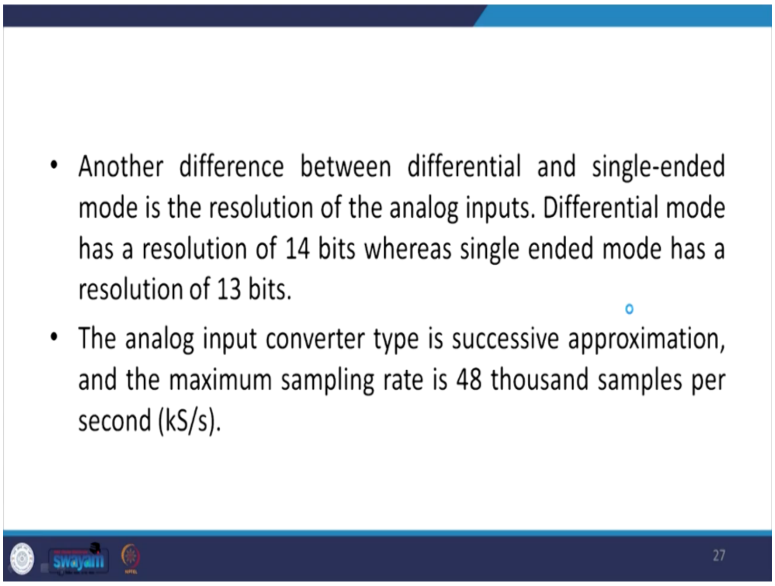

Another difference between differential and single ended mode is the resolution of the analog input differential mode has a resolution of 14 bit whereas, single ended mode has a resolution of 13 bit. The analog input converter type is a successive approximation which I have already discussed with you and the maximum sampling rate is 48 thousand samples per second.

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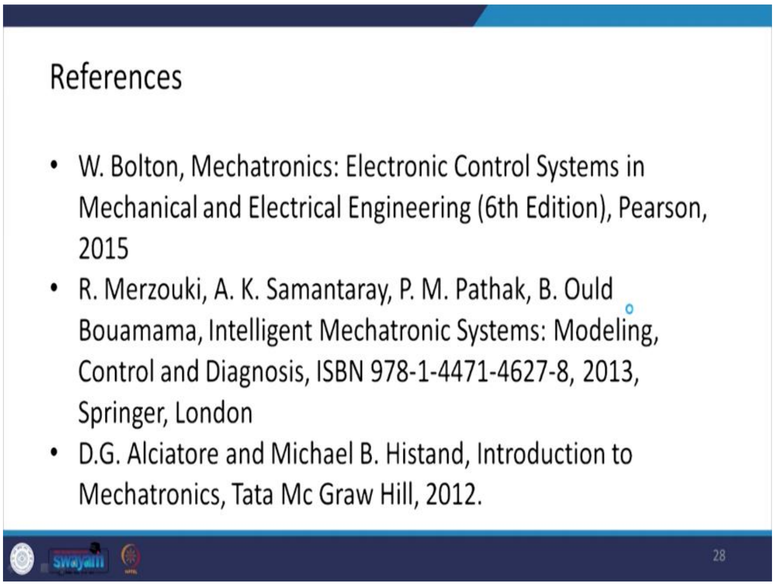

These are the further references. If you wish you can read specifically the Alciatore and Histand.

Thank you.