Mechatronics
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Lecture - 10
Acceleration and Vibration Measurement Sensors

Good morning everyone. Today, in this lecture on Mechatronics, we are going to discuss the Acceleration and Vibration Measurement Sensors. An accelerometer is a sensor that is designed to measure acceleration or rate of change of a speed due to motion, vibration, and impact events. These accelerometers which measure acceleration are normally mechanically attached or bonded to an object structure for which the acceleration has to be measured.

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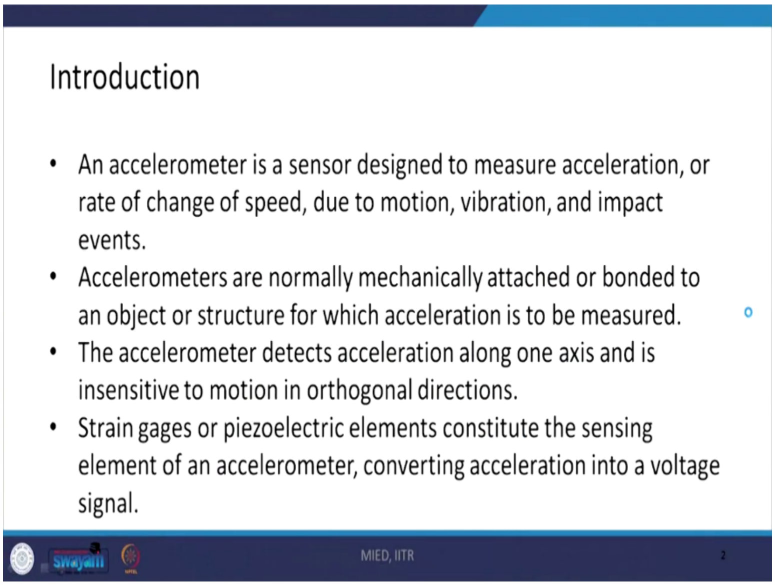

The accelerometer detects acceleration along one axis and they are insensitive to motion in the orthogonal direction. The strain gauge gauges or piezoelectric element constitute the sensing element of an accelerometer which senses the acceleration and how they sense the acceleration that we will be discussing during the next 30 minutes.

These devices that are piezoelectric elements or strain gauges, convert acceleration into the voltage signal and these voltage signals are sensed.

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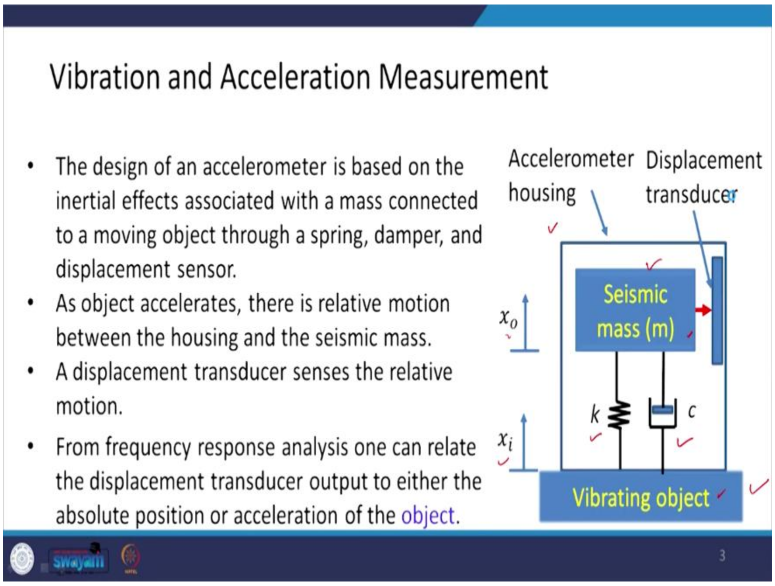

The design of the accelerometer is something like this. So, you have the vibrating object over here, whose acceleration is to be measured and there is a seismic mass and this seismic mass is attached to the vibrating object with the help of a spring and damper as you can see here and there is a displacement transducer a which constitute the measuring device. These all elements are placed inside a housing what we call the accelerometer housing and in this one, the basic principle is that as the object accelerates, there is relative motion between this seismic mass and the vibrating object. So, here I am representing by x_i the vibrating object motion that is I am using subscript i to indicate the input, and x_o, I am indicating say the displacement of the seismic mass which indicates the output. The output is the relative displacement between the seismic mass and the vibrating object. A displacement transducer can be used to sense the relative motion between the seismic mass and the vibrating object and as we will be seeing, the measure of this relative displacement is the measure of the acceleration and this, we could achieve with the help of the frequency response analysis. From there, we can find out that the displacement transducer output to either the position or the acceleration of the object. So, one can relate from frequency response analysis, the displacement transducer output to either the absolute position or the acceleration of the object.

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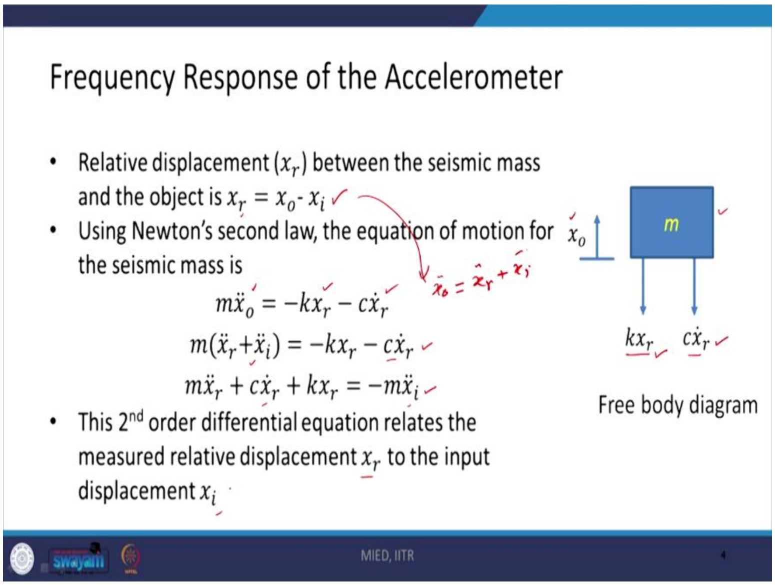

To perform the frequency response analysis of the accelerometer, let us draw the free body diagram of the seismic mass m here. So, I am representing the relative displacement between the seismic mass and the object as x_r that is here. This is x_o and x_i.

So, the relative displacement is represented by,

$$x_r = x_o - x_i$$

This is when I am considering x_o in this direction.

So, if the direction of motion is in this direction, the spring force will be kx_r here and the damper force will be $c\dot{x}_r$.

From this free body diagram, we can write the equation of motion for this system as,

$$m\ddot{x}_o = -kx_r - c\dot{x}_r$$

Now, from here I can substitute for x_o

$$m(\ddot{x}_r + \ddot{x}_i) = -kx_r - c\dot{x}_r$$

So, this is what I get,

$$m\ddot{x}_r + c\dot{x}_r + kx_r = -m\ddot{x}_i$$

So, here you can see that right-hand side, I have got the input and expression related with input, and left-hand side, I have got the expression related to the relative displacement of the seismic mass.

That is the displacement between the seismic mass and the body whose acceleration we are trying to measure. So, this equation is a second-order differential equation that measures as I said the relative displacement x_r or rather which relates the relative displacement x_r to the input displacement x_i.

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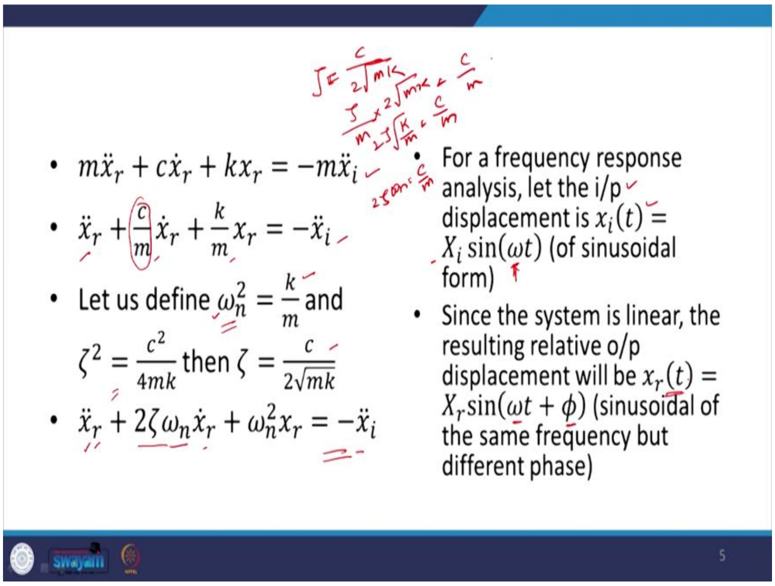

Now, let us divide by m,

$$\ddot{x}_r + \frac{c}{m}\dot{x}_r + \frac{k}{m}x_r = -m\ddot{x}_i$$

Now, let us define these two terms,

$$\omega_n^2 = \frac{k}{m}$$

And

$$\zeta^2 = \frac{c^2}{4mk}$$

So, from here ζ is,

$$\zeta = \frac{c}{2\sqrt{mk}}$$

let us substitute these values in this equation,

$$\ddot{x}_r + 2\zeta\omega_n\dot{x}_r + {\omega_n}^2 x_r = \ddot{x}_i$$

So, this is how I get the equation of motion for the seismic mass which is written in terms of the relative displacement as the output between the relative displacement of the seismic mass with respect to the object and x_i is the input. Now, for a frequency response analysis, we can assume a certain input displacement.

So, let me have,

$$x_i(t) = X_i \sin(\omega t + \emptyset)$$

and this is of sinusoidal form and this has got a frequency ω and the say the amplitude is X_i.

Now, since the system is linear, the resulting output displacement will be given as,

$$x_r(t) = X_r \sin(\omega t + \emptyset)$$

Since the system is linear. So, the output will be also going to be of the same frequency; but different phase different phases.

Now, let us take a look at the equation again. So, this is the equation that we derived in the last sheet.

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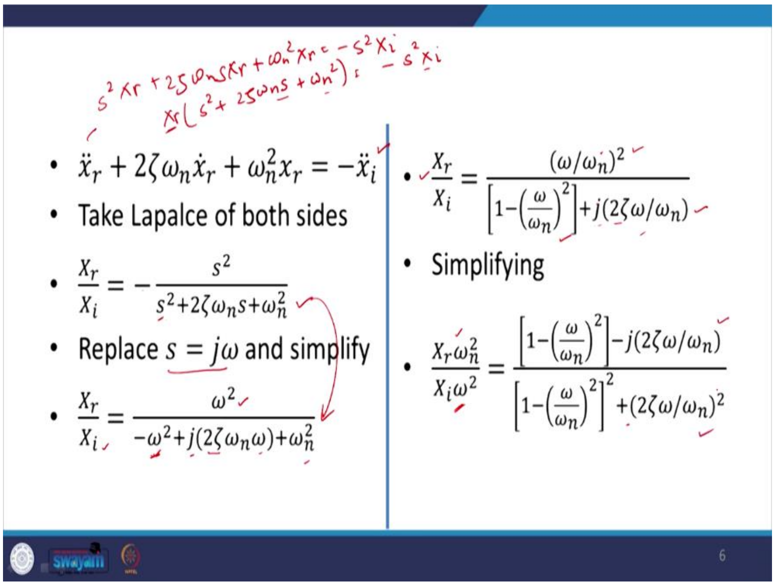

If I take the Laplace of both the sides, then what we get over here is,

$$\frac{X_r}{X_i} = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n{}^2}$$

So, from here, I can take x r out.

Now, what we do is that we replace $s = j\omega$.

$$\frac{X_r}{X_i} = \frac{\omega^2}{-\omega^2 + j(2\zeta\omega_n\omega) + \omega_n{}^2}$$

Now, let us divide the numerator and denominator by $\omega_n{}^2$. So, what we have here is,

$$\frac{X_r}{X_i} = \frac{(\omega/\omega_n)^2}{[1 - (\frac{\omega}{\omega_n})^2] + j(2\zeta\omega_n\omega) + \omega_n{}^2}$$

So, what I do is that I take this ω this side. So, I have,

$$\frac{X_r {\omega_n}^2}{X_i \omega^2} = \frac{[1-(\frac{\omega}{\omega_n})^2]^2 - j(\frac{2\zeta\omega}{\omega_n})}{[1-(\frac{\omega}{\omega_n})^2] + (\frac{2\zeta\omega}{\omega_n})^2}$$

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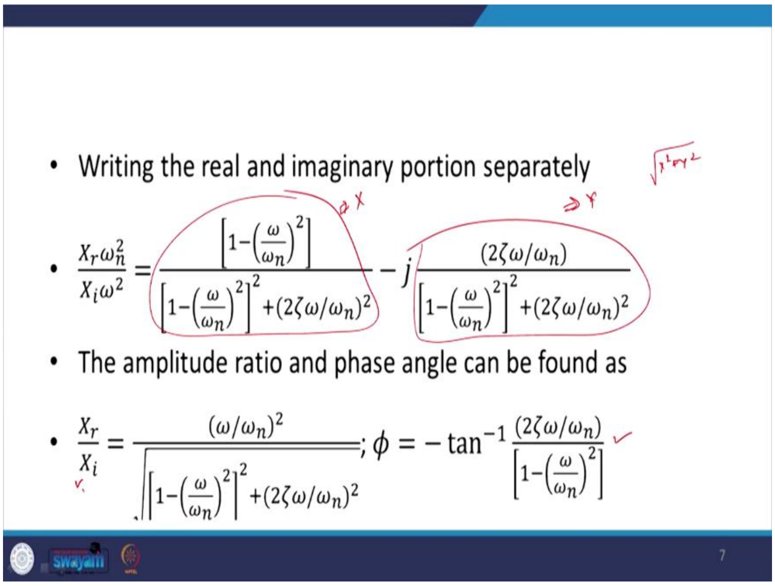

So, this one I can write the real and imaginary portion separately over here.

$$\frac{X_r {\omega_n}^2}{X_i \omega^2} = \frac{[1-(\frac{\omega}{\omega_n})^2]^2}{[1-(\frac{\omega}{\omega_n})^2] + (\frac{2\zeta\omega}{\omega_n})^2} - j\frac{(\frac{2\zeta\omega}{\omega_n})}{[1-(\frac{\omega}{\omega_n})^2] + (\frac{2\zeta\omega}{\omega_n})^2}$$

So, as you can see that this portion is my real part and this portion is my imaginary part. So, from this one, I can find out the amplitude ratio and phase angle. So, amplitude and amplitude ratio, I can find out say this is if this is your x and say this is your y. So, what will be the amplitude ratio?

Amplitude will be x square plus y square. So, that we can find out. So, if we do that, then from that expression, this ratio is going to be,

$$\frac{X_r}{X_i} = \frac{(\omega/\omega_n)^2}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2 + (2\zeta\omega_n\omega)^2}}$$

and the phase will be,

$$\emptyset = -tan^{-1}\frac{(\frac{2\zeta\omega}{\omega_n})}{[1-(\frac{\omega}{\omega_n})^2]}$$

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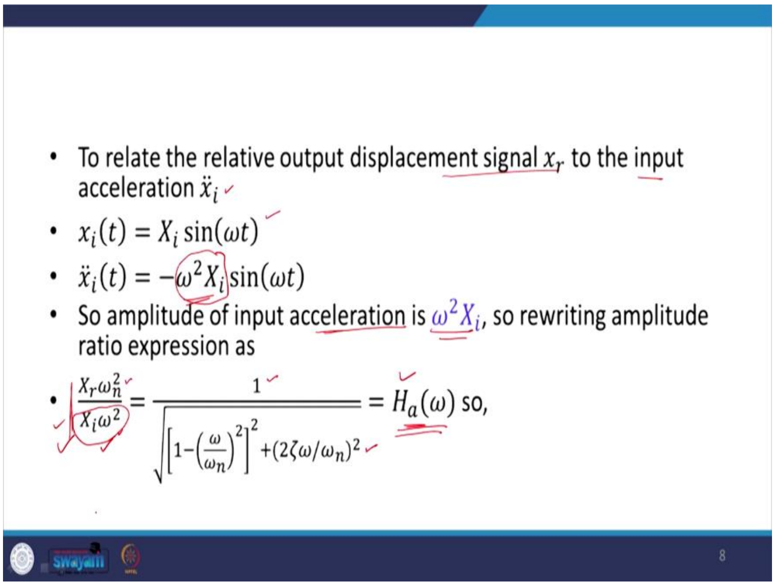

Now, let us look at these expressions. First let us look at this expression here. I give this some name H_a omega.

$$\frac{X_r{\omega_n}^2}{X_i\omega^2} = \frac{1}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2+(2\zeta\omega_n\omega)^2}} = H_a(\omega)$$

Now, to relate the relative output displacement signal x_r to the input signal input acceleration that is $\ddot{x}_i$, this is what I have been talking to you since beginning. So, this is what I assume my input signal to be $X_i\sin(\omega t)$.

So, here, we can see that the amplitude of the input acceleration is

$$\ddot{x}_i = -\omega^2 X_i\sin(\omega t).$$

So, $-\omega^2 X_i$ is the amplitude of the input acceleration as you can see and where is that term? That term is over here. That is $\omega_i^2 x_i$,

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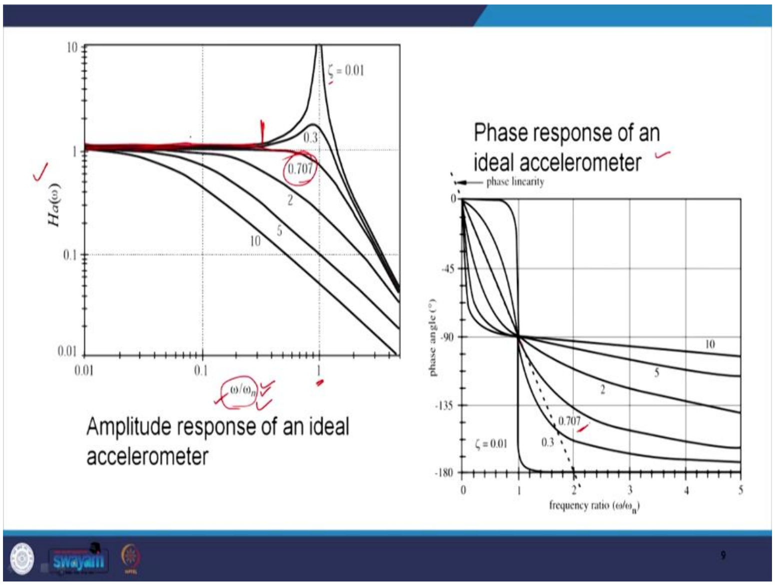

So, next let us see if I plot this Ha this term which I have written over here. If I plot this H a with ω/ω_n, this is what the plot is going to be. Now, here I want to mention one thing that at the damping ratio 0.707, you will see that most of this part is straight, until unless it reaches a ω/ω_n value of 1.

So, one observation is that most of this part is straight and the other observation is that this for this to have a straight line portion to have this omega by omega n value should be small. Rather omega n and this omega by omega n can be small, if your omega n is large. I will be coming back to this point little later and if I plot the phase response of an ideal accelerometer, then you can see that this value that is around 0.707, we get almost a linear type of curve.

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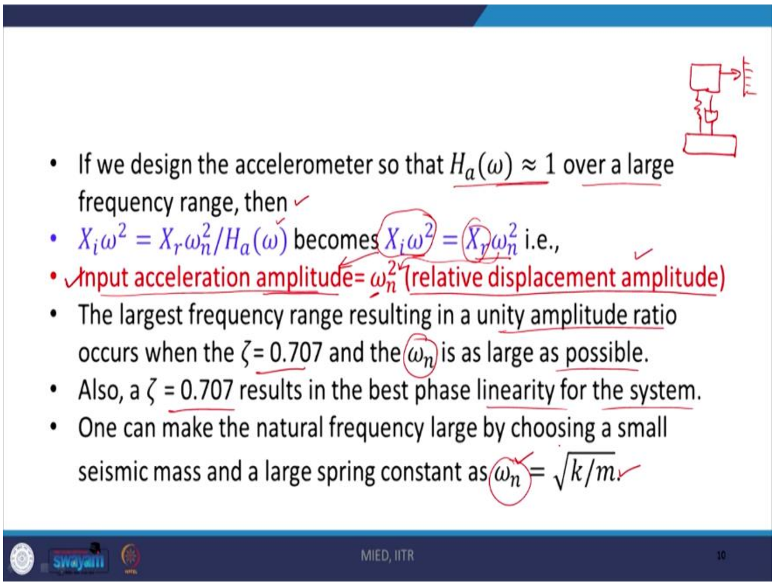

So, if what I was talking to you here about this range , where we have the $H_a(\omega)$ as 1 at ζ is equal to 0.707. So, what I was saying is if we design the accelerometer so that this $H_a(\omega)$ is almost equal to 1 over a large frequency range, then I can write from this equation, I can write from this equation ,

$$X_i\omega^2 = X_r{\omega_n}^2/H_a(\omega)$$

and this H a omega as I said this is 1.

So, what does this becomes? This becomes,

$$X_i\omega^2 = X_r{\omega_n}^2$$

and I have already said that this X_i omega square is the acceleration; input acceleration amplitude. So, this input acceleration amplitude is equal to $X_r{\omega_n}^2$ that is the relative displacement amplitude. So, what does this means? This means that if you are able to measure the relative displacement amplitude, you can measure the input acceleration amplitude, and based on this principle, the accelerometers are made. So, based on this principle, accelerometers are made.

So, here as I was telling you the seismic mass and we had the object here and we try to measure this value this gives us the X_r value. So, from where do we get this relative displacement amplitude? And so, from here, if I multiplied this relative displacement amplitude with $\omega_n{}^2$, I can get the input acceleration amplitude.

And I have already talked that the largest frequency range resulting in unity amplitude ratio occurs when this zeta is 0.707 and ω_n is as large as possible. Why ω_n is as large as possible? Because if your ω_n is large, your ω/ω_n value is going to be is smaller and the smaller the ω/ω_n value here, you can see that the more linear range you will be getting over here.

So, that is there and also, ζ is equal to 0.707 results in the best phase linearity for the system. Now, as I said we want ω_n as large as possible and how can we make ω_n as large as possible, by making this mass as small as possible. So, if your mass is small, your omega n is going to be a large value.

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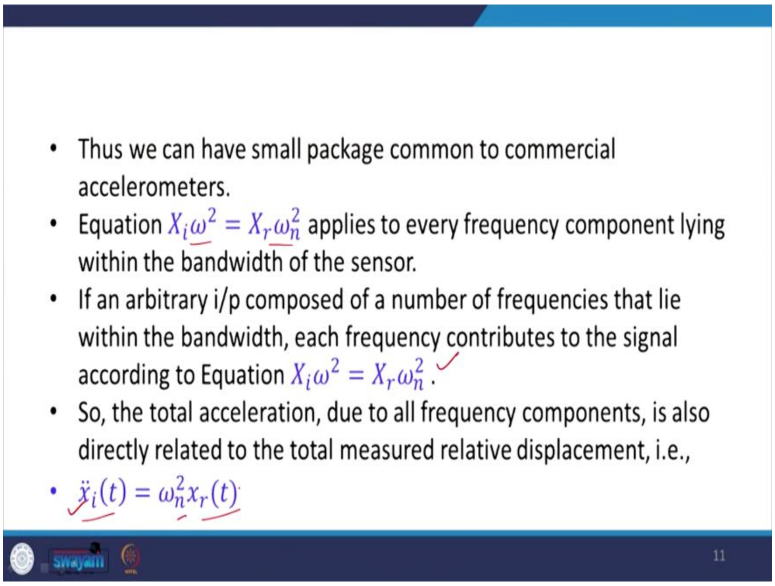

So, because of this accelerometer have got the small seismic mass. So, thus, we can have a small package common to the commercial accelerometers and this equation

$$X_i\omega^2 = X_r\omega_n{}^2$$

applies to every frequency component lying within the bandwidth of the sensor. So, if an arbitrary input that is which is coming from the object which is accelerating.

So, if an arbitrary input composed of a number of frequencies that lie within the bandwidth, each frequency contributes to the signal according to this equation. So, the total acceleration due to the all frequency components is also directly related to the total measured relative displacement. So, that is there. So, from here, we can get into the time domain

$$\ddot{x}_i(t) = {\omega_n}^2 x_r(t)$$

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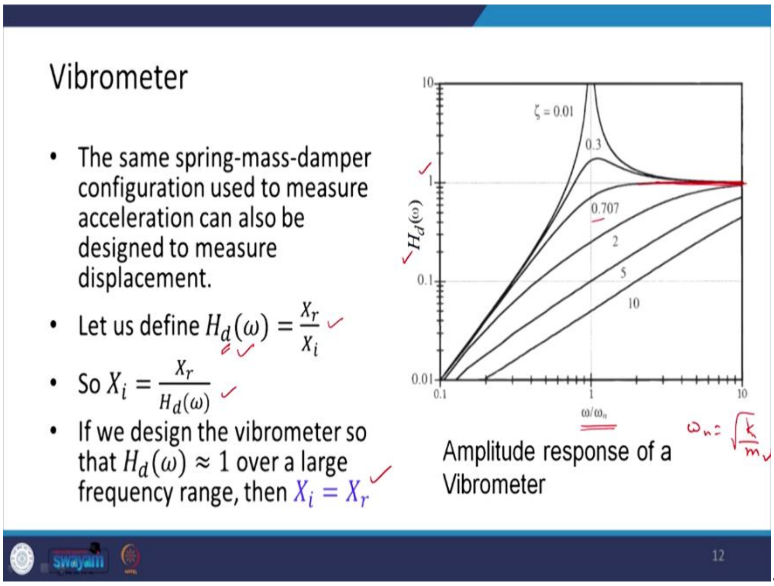

Next, let us look at another device vibrometer. So, in the accelerometer our intention was to measure the acceleration of the object. In case of vibrometer, our interest is to measure the displacement of the object. So, we can use the same spring mass damper configuration to measure the displacement of the object and we can define say,

$$H_d(\omega) = \frac{X_r}{X_i}$$

Here, I am using the subscript d, just to indicate that I am trying to measure the displacement .

So, X_i will be $X_r/\ H_d(\omega)$.

Now, here if I do the plotting for $H_d(\omega)$ for different values of ζ, you may say that how do we get this $H_d(\omega)$? So, this $H_d(\omega)$ will be X_r/X_i which we can get from here.

If I plot that H_d for a different ω/ω_n ratio, then again you can see that here for this damping ratio of 0.707, for this portion this is a straight line that is almost equal to 1 .

So, if we design the vibrometer. So, that H_d is equal to 1 over a large frequency range, then we can see that X_i is equal to X_r. So, that is there and you can see that this portion, we are getting where? When the ω/ω_n value is high and now, in order to get the ω/ω_n high, you need to have the lower value of ω_n and how do we get the lower value of ω_n? Because of the lower value of ω_n we can get, if I use the high value of the mass. That is why these vibrometers have a larger mass.

So, the largest frequency range resulting in H_d is equal to $H_d\omega$ is equal to 1 occurs when ζ is 0.707 as I said over here and natural frequency is as small as possible. So, this is what I discussed in the previous slide and because we want the smaller value of ω_n.

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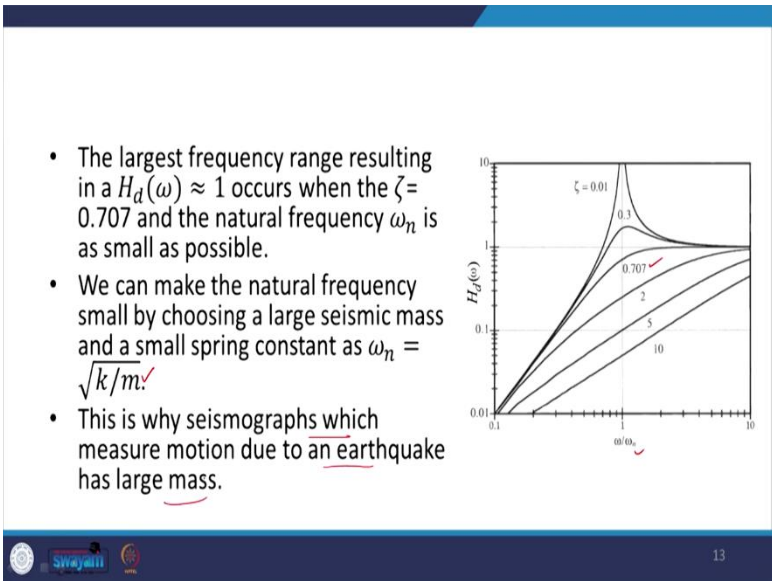

So, that we can get the by making the mass larger and that is why the seismographs which measure motion due to earthquake has a larger mass. So, I hope this must have cleared your concept of the accelerometer and the vibrometer.

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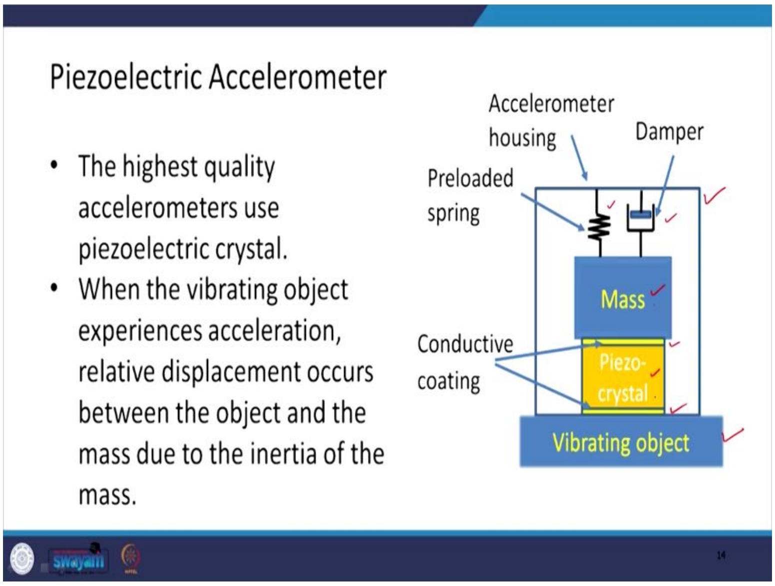

Now, let us look at the piezoelectric accelerometer all right. So, in this piezoelectric accelerometer the system is almost similar. The only thing is the displacement relative delay displacement, we are going to measure with the help of piezo crystals. So, I have got the vibrating object over here.

There is an accelerometer housing and this is my mass systemic mass and what we have is a spring a preloaded spring is there, a damper is there and here is your piezo crystal. So, this piezo crystal is placed between the mass and the vibrating object and here, you can see that there are two conductive coating which is here.

So, the highest quality accelerometer uses piezoelectric crystals. Now, when the vibrating object experiences acceleration, we have seen that the relative displacement occurs between the object and the mass due to the inertia of the mass.

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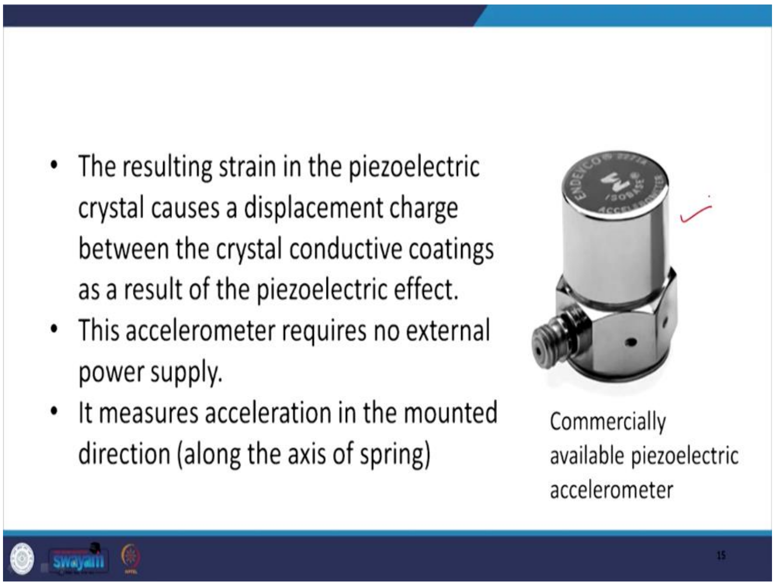

This resulting strain in the piezoelectric crystal causes a displacement charge between the crystal conductive coating and the as a result of the piezoelectric effect and this accelerometer requires no external power supply and it measures acceleration in the mounted direction that is the along the axis of the spring and this is the commercially available piezoelectric accelerometer which is there in the market and one can use it and the place it over the object whose acceleration one wants to measure.

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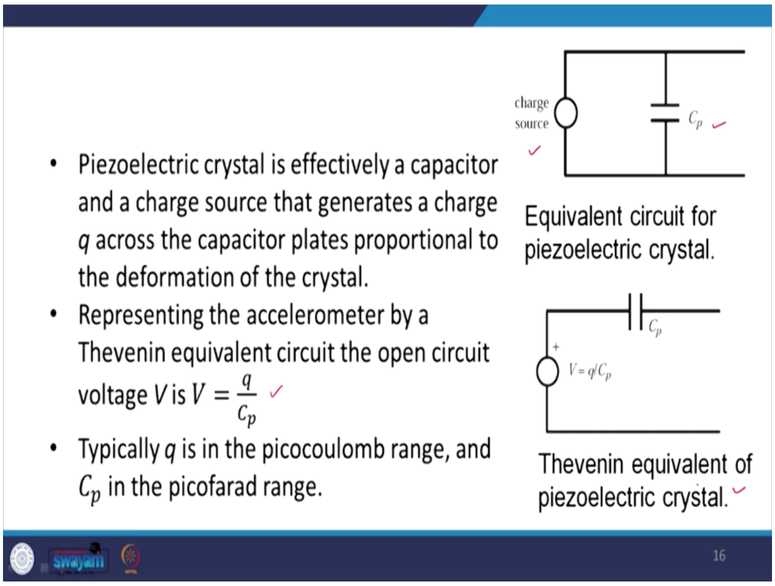

The piezoelectric crystal is effectively a capacitor because a charge source that generates a charge q across the capacitor plate proportional to deformation of the crystal and this deformation occurs because of the relative motion between the mass and the object . So, representing the accelerometer by the Thevenin equivalent circuit, the open circuit voltage

$$V = \frac{q}{C_p}$$

And this is the equivalent circuit for a piezoelectric crystal, where we have the charge source and this is our C_p and typically q is in the Picocoulomb range and C_p in the Picofarad range . So, this is the Thevenin equivalent of the piezoelectric crystal.

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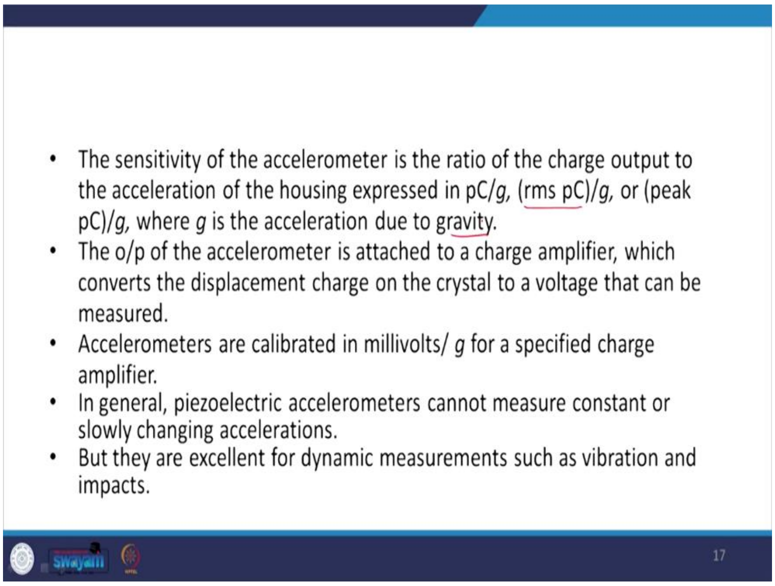

The sensitivity of the accelerometer is defined as the ratio of the charge output to the acceleration of the housing expressed in pC/g or this is also expressed as rms value of the pico Coulomb per g or peak PicoCoulomb per g; where, g is the acceleration due to gravity .

The output of the accelerometer is attached to a charge amplifier which converts the displacement charge on the crystal to a voltage and that voltage can be measured and these accelerometers are calibrated in millivolt per g for a specific charge amplifier. In general, piezoelectric accelerometers cannot measure constant or slowly changing accelerations. But they are very excellent for dynamic measurement such as the vibration and the impact.

Next, let us see the semiconductor sensor and MEM devices which are the Micro Electro Mechanical devices. So, you see there has been lot of development about producing IC's and the technique of producing IC's has developed a new class of semiconductor sensor and actuation called the MEM devices or micro electro mechanical devices.

In 1980, first MEM sensor was developed using this integrated circuit technology to etch silicon and produce a device that responds to acceleration and it consists of tiny silicon cantilever with integrated semiconductor strain gauge. I hope you, all of you must be knowing the cantilever.

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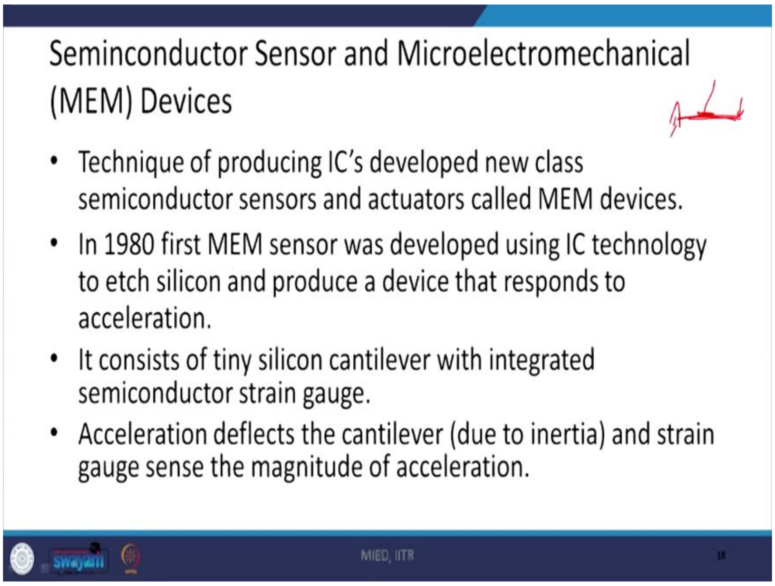

We have a small device it consists of tiny silicon cantilever with integrated semiconductor strain gauges. So, here could be strain gauge and this is your cantilever. Now, what happens? Acceleration deflects this cantilever due to inertia and the strain gauge sense the magnitude of the acceleration.

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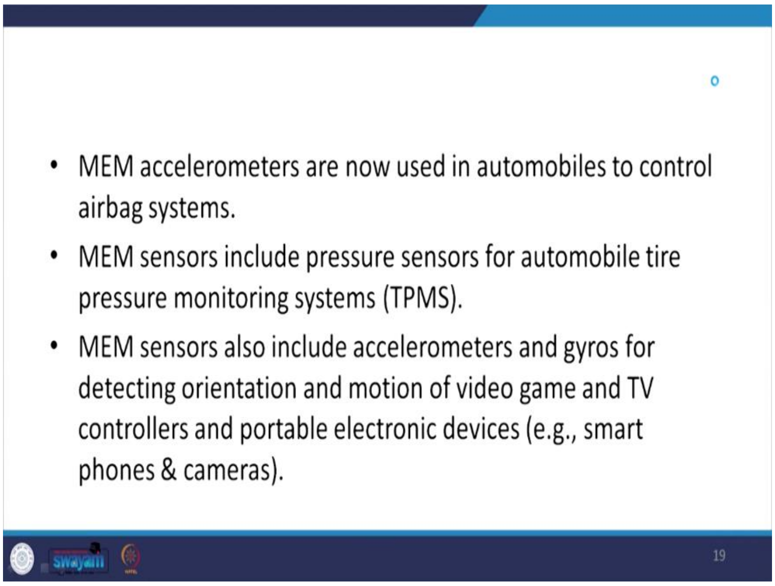

MEM accelerometers are now used at variety of places. So, they are used in automobile to control the airbag system and the also, they are used as a pressure sensor for your auto mobile tire pressure monitoring system that is TPMS devices and MEM sensors also include accelerometers and gyros for detecting orientation and motion of video game and TV controllers and portable other portable electronic devices such as smartphones and cameras.

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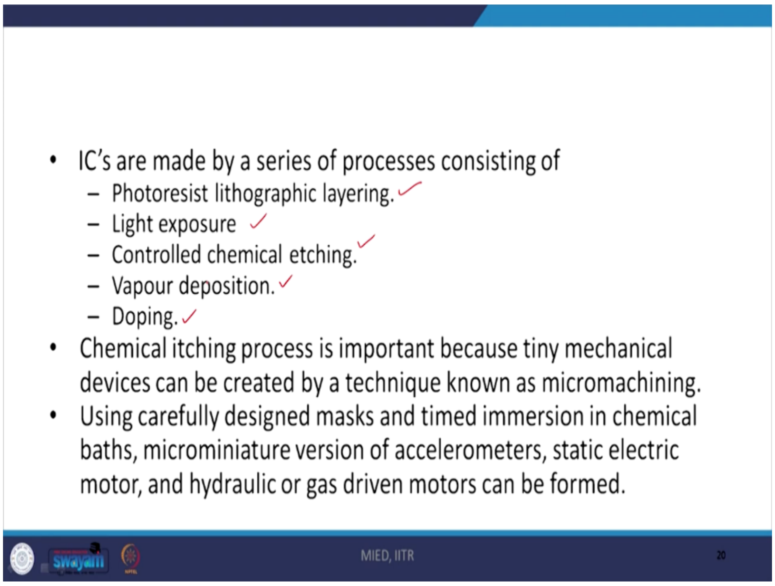

The IC's are made by a series of processes and these processes consist of Photoresist lithographic layering, light exposure, followed by controlled chemical itching, followed by vapor deposition and the doping.

The chemical itching process is important because tiny mechanical devices can be created by a technique known as micro machining and usually, using carefully designed mask and time immersion in chemical bath, micro miniature version of accelerometers, static electric motors and hydraulic or gas driven motors can be formed.

Semiconductor sensors design are based on different electromagnetic properties of doped silicon and gallium arsenide and a variety of ways that they function in different physical environments. The piezoresistive characteristic of doped silicon, the coupling between resistance change and the deformation is the basis for semiconductor strain gauges and pressure sensors.

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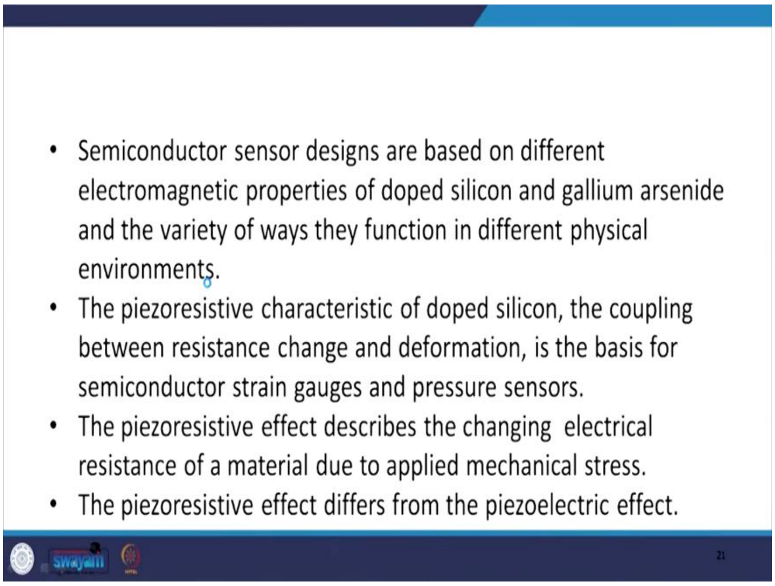

The piezoresistive effect describes the change in the electrical resistance of a material due to applied mechanical stresses. So, you know that the piezo residue effect is differ from the piezoelectric effect. Now, in contrast to the piezoelectric effect, the piezoresistive effect only causes change in resistance and it does not produce an electrical potential which is there are produced in case of the piezoelectric effect.

The magnetic characteristic of doped silicon, principally the Hall effect which I have discussed during my previous lectures are the basis of semiconductor magnetic transistors, where the collector current can be modulated by an external magnetic field.

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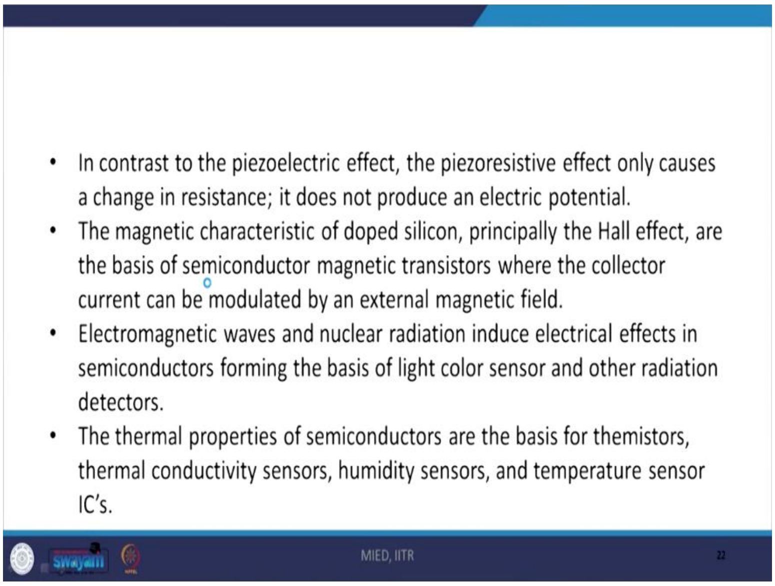

Electromagnetic waves and nuclear radiation induce electrical effects in semiconductor forming the basis of the light colour sensor and other radiation detectors. The thermal properties of semiconductors are the basis of sensors such as thermistors, thermal conductivity sensors, humidity sensors, as well as the temperature sensors IC’s. Now, let us look at the SAW that is the Surface Acoustic Wave. So, these are important class of MEM sensors.

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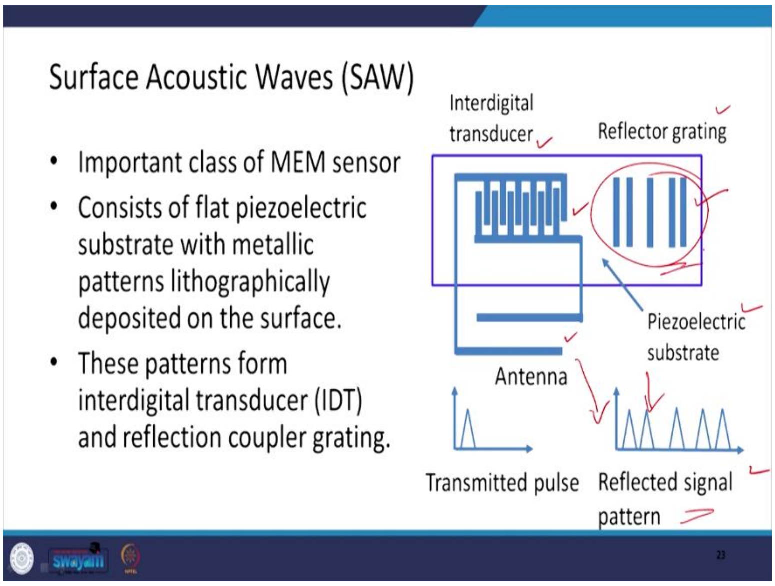

It consists of the flat piezoelectric substrate with a metallic pattern lithographically deposited on the surface. These patterns form the interdigital transducers and reflection coupling grating. So, this is the interdigital transducer and this is the reflector grating and there is an antenna. So, that is there and these antennas receive and the transmit the signals and we will be here seeing the use of the SAW in these type of sensors.

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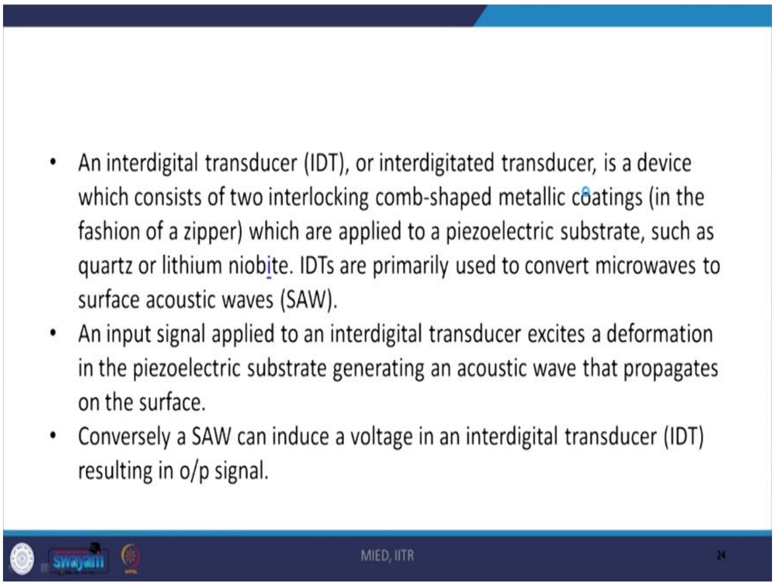

An integrated sorry an interdigital transducer that is IDT or is a device which consists of two interlocking comb shaped metallic coating. You can see here this is the interlocking comb shaped metallic coating, in the fashion of zipper which are applied to a piezoelectric substrate such as the quartz or lithium niobite. So, that is there. IDTs are primarily used to convert the microwave to the SAW that is the surface acoustic wave. An input signal applied to a an inter digital transducer excites a deformation in the piezoelectric substrate grating and an acoustic wave that propagates on the surface and conversely, SAW can induce voltage in the interdigital transducer resulting in an output signal. Now, let us see the application of this. Its application can be found in wireless identification systems, identification systems of say cars.

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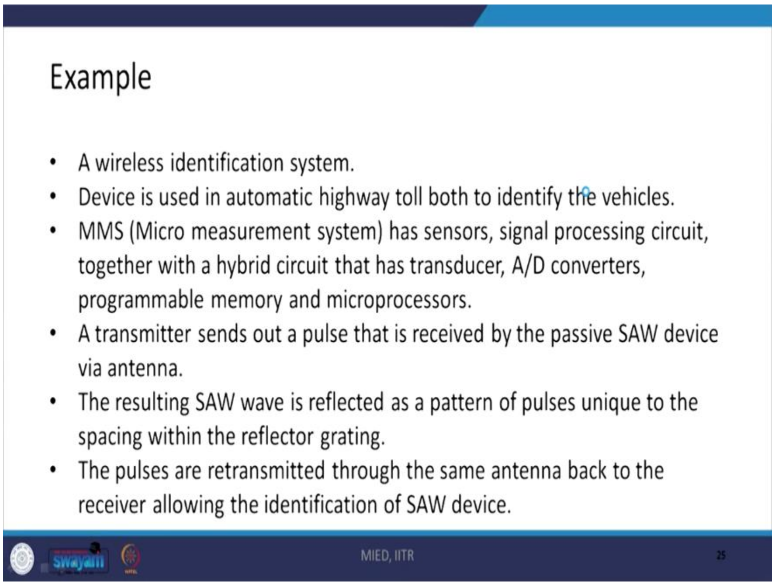

The device is also used in the automatic highway toll plaza to identify the vehicle. So, how this process work? How the system works basically? So, the micro measurement system has sensors; naturally sensors will have along with that the signal processing circuit together with hybrid circuit that has the transducer analog to digital converter, programmable memory and the microprocessor. A transmitter sends out a pulse that is received by the passive SAW device via the antenna. Now, the resulting SAW wave is reflected as a pattern of pulses, unique to the spacing within the reflector grating. Here this is the reflector grating. The reflected signal pattern will be depending on how this reflector grating is formed. The pulses are retransmitted through the same antenna back to the receiver along with the identification of a SAW device. After reflection from this grating, the signal can be transmitted from the antenna and this reflected signal pattern can be there and with the help of this one, based on this grating we can identify the vehicle. So, this is the use of this technology.

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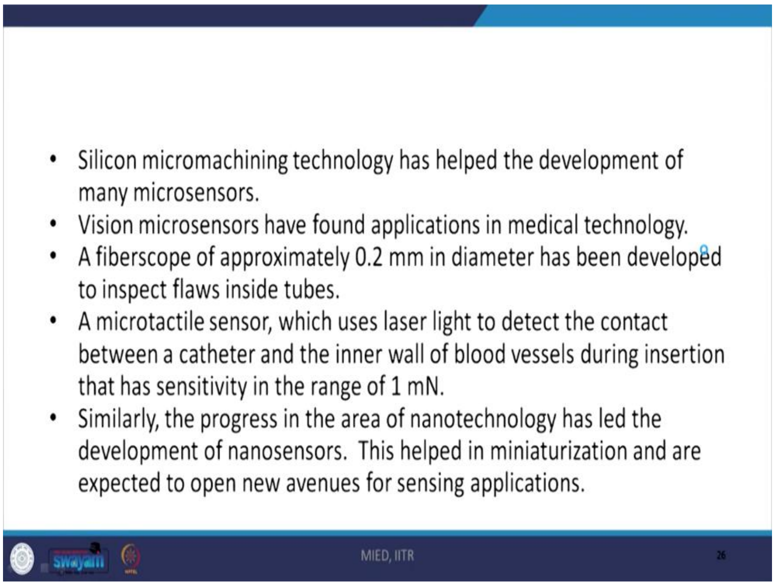

There could be further applications. The silicon micro machining technique help in the development of many micro sensors. Vision micro sensor have found application in the medical technology. A fiberscope of approximately say 0.2 mm in diameter has been developed to inspect flaws inside tubes. See the dimension size 0.2 mm is there. A micro tactile sensor, which uses laser light to detect the contact between the catheter and the inner wall of the blood vessels during insertion has sensitivity in the range of 1 milli Newton. So, if this much 1 milli Newton force is there, then even this much force can be detected.

Similarly, the progress in the area of nanotechnology had led to the development of Nanosensors, and this helps in miniaturization and is expected to open new avenues for sensing applications.

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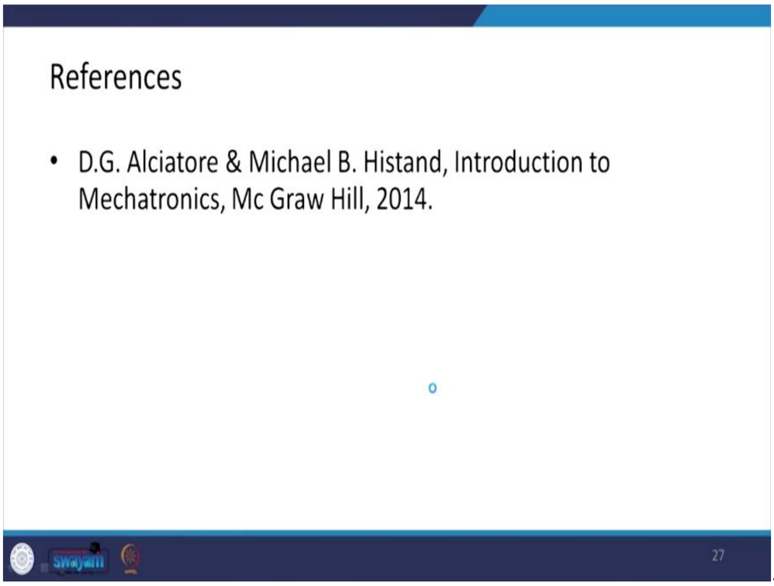

This is the reference Alciatore and Histand, Introduction to Mechatronics by Mc Graw Hill, published in 2014. This book has got very good material whatever I have discussed. In that, so you can refer this book for the further reading and doing the exercise problems.

Thank you.